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UNSAFE SEX, AIDS, AND DEVELOPMENT

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Abstract

Much of Africa has been ravaged by the AIDS epidemic. There, heterosexual contact is the primary mode of transmission for the HIV virus. Even when access to condoms is good and their price low, a large fraction of young Africans continue to engage in unprotected sex. In this paper, we propose a simple two period rational model of sexual behavior that has the potential to explain why a large proportion of sexual activity in poor countries maybe unprotected. In the model economy, even when agents are perfectly cognizant of the risk involved in unsafe sexual activity, and fully internalize the effects of their own sexual behavior on their chance of catching the virus, they may rationally choose to engage in such risky behavior. Our results indicate that safe sexual practice is essentially a “normal good” and that development may be key to reducing HIV infectivity.
1 Introduction

In much of Africa and in many other parts of the world, the presence of HIV/AIDS has created an intimate tangle of sex and death. The numbers are staggering and mind-numbing at the same time. Upwards of 25 million people worldwide have died from AIDS; another 40 million live with the disease today. Of all regions in the world, sub-Saharan Africa is the hardest hit; while only 10% of the world’s population lives there, the region contains around 70% of people living with the disease. With a HIV prevalence rate of 12-25%, and an average duration from HIV infection to death of about 9 years, the lifetime chances of dying from AIDS is around 40 percent in Eastern and Southern Africa as a whole and 70 percent in Zimbabwe and Botswana. What remains mind-boggling is why, faced with these statistics, so many people “insist on playing the Russian-roulette game of unprotected sex”. (Joseph, 2005)

Perhaps the most immediate barrier to safer sex is inaccurate risk perception. In a recent survey (cited in Joseph, 2005) conducted in South Africa by Nelson Mandela, 63 percent of HIV-positive South Africans (who didn’t know their status) said it was close to impossible that they were infected.¹ This suggests that South Africans assess themselves as being at substantially lower risk than dispassionate evidence would suggest. A similar pattern of self-delusion exists everywhere.² On the face of it, this would suggest that a large part of the solution to the AIDS crisis would involve mass education, information, and awareness programs. Not surprisingly, that is exactly where the AIDS-fighting community has spent most of its scarce resources. These programs, however, have not resulted in sustained behavior change, and have largely failed to prevent the general spread of the disease in developing countries.³ As Caldwell (2000) puts it bluntly, “the epidemic cannot

¹Caldwell (2000) reports that “It almost takes an act of faith to relate the appearance of AIDS symptoms to sexual encounters that occurred almost a decade ago, given that at least 95 percent of seropositive Africans do not know of their infection until the symptomatic AIDS stage.”.

²In practically all countries surveyed in a 2002 UN report “HIV/AIDS: Awareness and Behavior”, including those where HIV prevalence is high, at least two thirds of female respondents and 8 of 10 male respondents said that they are either at no risk at all or at small risk of getting AIDS.

³In the UN report “HIV/AIDS: Awareness and Behavior” (2002), only a small percentage of respondents began using condoms to prevent HIV transmission. Fewer than 8 per cent of women in all countries surveyed report that they have changed their behaviour by using condoms.
be defeated by more education.”

This paper argues for consideration of a wider context of sexual behavior. Following Caldwell (2000), it formalizes the notion that even when individuals are acutely aware of the risk of contracting HIV, a sense of disillusionment with their own future can easily help rationalize high-risk and unsafe sexual practices. Specifically, if their future is uncertain to even exist (presumably because of general high mortality) or appears unattractive because of current stagnation and poverty, individuals may take on a “distinctly careless attitude to life” and cease to fear death as much.4

The formalization of this argument is conducted in a model environment whose details are as follows. Consider an economy in which individuals live potentially for two full periods. When young, they work for a fixed wage, engage in safe and unsafe sex, and save for the future in a fixed return asset. We postulate that both safe and unsafe sex provide utility to the young although unprotected sex raises the probability of contracting the HIV virus and hence hastens death. We also assume that the total volume of sexual activity across countries is fixed at unity. The principal novelty here is that agents are fully aware of the link between their own sexual activity and their probability of catching the HIV virus.

The baseline model is deliberately kept simple so as to generate analytical tractability. We are able to show that there exists a unique fraction of coital activity that is protected. This fraction is greater than one-half, because agents internalize the effect of their decision to have unsafe sex on their probability of reaching the future. We also prove that this fraction increases with development. The implication is clear. As countries become richer, their citizenry will increase the fraction of all sexual activity that is protected. In the model economy, this translates into a lower lifetime probability of contracting the virus. When utility conditional on being infected goes up (presumably because of better treatments and so on), then we find that unsafe sexual activity increases. The basic intuition underlying these results is simple. When the future looks attractive (and in the model, it does so

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4 As Caldwell (2000) reports, “some men practicing high-risk sex say that, if the latency period until onset of AIDS is nearly a decade, they are not worried because they are likely to die of something else in such a long time.”
when a country develops), people try to make sure that they will be around to enjoy it; they achieve that by reducing the fraction of all sexual activity that is unsafe. The predictions of the baseline model are consistent with simple correlations observed in the data. For example, data from a long list of countries suggest that condom use is positively associated with development; indeed condom use among young people who engage in high risk sex is also positively correlated with development. HIV infections rates, similarly, are negatively correlated with the level of development of a country.5

We go onto the extend the baseline model in two ways. First, we incorporate the effect of general public health and public health systems into the probability of contracting AIDS. Now, an agent’s probability of catching AIDS depends on his own sexual activity as well as on the general public health facilities he is surrounded by. Here we find that interestingly, the relationship between protected sexual activity and development is no longer monotonic as before. Curiously enough, unsafe sexual activity may in fact increase slightly after a critical level of development has been reached. This appears to be consistent with recent evidence on the resurgence of unsafe sexual practices in the Western world. In the model economy, this happens because agents substitute out of protected sexual activity once they realize that their governments are spending more on public health and this reduces their risk of contracting the disease.

The paper goes on to investigate the possibility of multiple equilibria. It shows that under an extension of the baseline model, there can be two equilibria, one in which most sex is protected, and the other in which it is not. Countries may find themselves in one of these; indeed, countries at very similar levels of development may differ vastly in the popularity of protected sexual activity (and hence HIV infectivity). This is quite consistent with cross section data. After all, HIV prevalence rates and condom use in many poor countries, even poor countries within Africa, are not close to being similar.

The upshot is that even when agents are perfectly cognizant of the risk involved in unsafe sexual activity, and fully internalize the effects of their own sexual behavior on their chance of catching the dreaded disease, they may rationally choose to engage in such risky

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5Bloom et. al. (2001) find evidence of a positive correlation between countries’ HIV prevalence and poverty, as measured by per capita income, income inequality, or absolute poverty.
behavior. The novelty here is that agents, by their choice of sexual activity, influence the discount factor between the current and the future. Our results indicate that safe sexual practice is essentially a “normal good” and that development with a promise of a better future may be key to reducing HIV infectivity. What is also interesting is that once public health improves substantially and quality of life under AIDS ceases to be dismal, unsafe sex may become popular again.

The paper closest in spirit to the current endeavor is Oster (2007). Like us, she models the sexual behavior response of rational individuals who face the prospect of contracting HIV. Our work differs from hers in two important respects. First, we allow agents to internalize the effects of their own behavioral response (via a choice of safe versus risky sex) on their risk of getting infected; in her setup, agents choose the number of partners to have sex with and treat other things as being parametric. Perhaps more importantly, she does not focus on the decision to engage in protected versus unprotected sex. Notwithstanding these differences, she finds that the sexual behavioral response is sharper for the rich and for those with high non-HIV life expectancy, a result analogous to our finding that those for whom the future is worth waiting for will reduce their demand for unsafe sex more than those with dim futures. Overall, she find strong empirical support with African data for her aforementioned hypotheses.

Our paper is also in line with a new and evolving literature that incorporates mortality concerns in growth models. The seminal papers in this literature are Blackburn and Cipriani (2002) and Chakraborty (2004). Blackburn and Cipriani (2002) endogenize old-age mortality concerns in a growth model. They assume that in a three-period overlapping generations model, the probability with which an agent survives to the third period depends on the stock of human capital or more generally, on the level of development of the economy itself. Chakraborty (2004) introduces endogenous mortality in an otherwise standard overlapping generations model with production; in particular, the probability with which a young agent survives on to old age depends on public health expenditures which are in turn funded by income taxes on labor income. In more recent work, Chakraborty

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6 Almost all the work in the current paper was done before the authors became aware of Oster (2007).
and Das (2005) and Chakraborty et. al (2007) allow for the possibility that private investments may be made to alter one’s probability of death. Our focus is quite different from all these papers but we do share a thematic continuity with them insofar as they investigate endogenous mortality concerns.

The plan for the rest of the paper is as follows. In Section 2, we present the baseline model and prove existence and uniqueness of the equilibrium while in Section 3 we derive our main analytical results connecting the demand for unsafe sex and its relationship with development. In Section 4, we explore two alternative reformulations of the baseline model and study the possibility of multiple equilibria while Section 5 concludes.

2 The baseline model

2.1 Preliminaries

We study a simple economy in which a unit-mass continuum of agents live for a maximum of two periods. In the first period of life, when agents are young, they work in a competitive labor market and earn a wage \( w \); they are retired in the second period of life, i.e., when they are old. When young, agents engage in protected \((p)\) and unprotected \((u)\) sex, consume, and save for old age at the gross world interest rate of \(R\). When old, they consume the return from their previous savings, and die towards the end of the period.

We impose the condition

\[ p + u = 1. \]

The reason for this condition is that almost nothing is known about the total volume of sexual activity across countries. There is some information on the relative frequency of safe and unsafe sex. To prevent our results from getting tainted by the largely unknown total volume of sexual activity, we fixed that number to 1 across all countries.\(^7\)

\(^7\)The 2007 Durex Global Sexual Wellbeing Survey questioned more than 26,000 people in 26 countries about all aspects of their sex lives and found that the global average is 103 times a year: South Africans (120), Americans (114), Greeks (164 times), Brazilians (145); Poles and Russians (143) have the most sex and the Japanese the least (48 times). 75% of all countries surveyed have sex around a 100 times a year.
Any agent may survive onto old age disease-free with probability $1 - \theta$. We think of $\theta$ as the \textit{lifetime cumulative} probability of contracting the HIV virus or, more simply, getting AIDS.\footnote{Sexual transmission is responsible for an estimated 80 percent of the world’s H.I.V. infections. The estimated risk of H.I.V. transmission during a single act of vaginal sex is on the order of 1 in 1,000 if the woman is the infected party, and somewhat higher for an infected man. Anal sex has a higher associated risk, and oral sex a lower one.} In the baseline model, we assume
\begin{equation}
\theta(u) = u^\lambda; \quad \lambda > 1
\end{equation}
where $u$ is the fraction of all sexual encounters that is unprotected (i.e., sex without a condom) an agent has had when young and $\lambda$ is a parameter. It is easily verified that $\theta$ is convex in $u$ implying that the higher the amount of unprotected sex an agent has, the higher the probability with which he contracts AIDS and the latter increases at an increasing rate.\footnote{The risk of HIV transmission via heterosexual intercourse is highest early in the course of HIV infection, before most infected people know their HIV status, according to the latest research. This line of research, following a cohort of over 15,000 adults living in rural villages in Rakai, Uganda, suggests that the rate of heterosexual HIV transmission per coital act varies over the course of HIV infection. Among partners with newly acquired HIV infection, more than 40 percent transmitted to their partners within approximately 5 months. (see Wawer, et. al. 2005).}

The preferences of agents are described by the utility function $U(c^o, p)$ where
\begin{equation}
U(c^o, p) = \frac{p^{1-\gamma}}{1-\gamma} + \frac{u^{1-\gamma}}{1-\gamma} + \theta(u) \left[ \frac{\delta(c^o)^{1-\gamma}}{1-\gamma} \right] + [1 - \theta(u)] \frac{(c^o)^{1-\gamma}}{1-\gamma}; \quad 0 < \gamma < 2
\end{equation}
Here $c^o$ denotes old age consumption, $p$ denotes the quantity of protected sex, $u$, the quantity of unsafe or unprotected sex (or sex without the use of a condom as a barrier against infection), and $\gamma$ is a parameter.\footnote{The connection between $\theta$ and $u$ is strong. The mode of transmission, for example, for nearly 90% of AIDS cases between 1996-99 in Africa was unprotected heterosexual contact.} Also, $\delta$ is a parameter capturing the fraction of old-age utility one gets after having contracted the virus during one’s youth. It may also be interpreted as the fraction of old-age one gets to enjoy after getting infected. As an indirect measure of $\delta$, it has been noted that life expectancy in Zimbabwe and Botswana would be 25 years higher than what it would be had AIDS not affected these countries!

There are several additional things to note here. First, notice that $\theta$ depends on $u$ but not $p$; this is in line with epidemiological evidence [see Davis and Weller (1999)] that...
finds “condom’s efficacy at reducing heterosexual transmission may be comparable to or slightly lower than its effectiveness at preventing pregnancy”, roughly 90%. Second, in the way we have defined these preferences, people are assumed to have an independent demand for both types of sex. This seems natural. After all, there may be many reasons (pregnancy prevention, disease prevention, etc.) that may create a demand for protected sex; similarly, as has been documented by epidemiologists, reduced sexual pleasure or hindrance to spontaneity and so on, are well known reasons why many people who often have protected sex may occasionally have unprotected sex. The percentage of people (both men and women) aged 15–24 reporting the use of a condom during sexual intercourse with a non-regular partner in Sub-Saharan Africa between 2001–2005 is the range of 25-45%.  

Since agents care only about old-age consumption, and hence save everything they earn, the budget constraints facing an agent in this baseline model are simply 

\[ w = S \] (4)

and 

\[ c^o = RS \] (5)

where \( S \) is saving.  

When (1) holds, the utility from sexual activity rises with protected sex until \( p = 0.5 \) and then falls.

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\( ^{12} \)Gender power inequities are believed to play a key role in the HIV epidemic through their effects on women’s power in sexual relationships. For example, among sexually experienced, 15- to 24-year-old women in South Africa, women with “low relationship control” were 2.10 times more likely to use condoms inconsistently and inconsistent condom use was, in turn, significantly associated with HIV infection. See Pettifor et al (2004) for details.

\( ^{13} \)We have not added a cost to protected sex to keep things simple here; there is evidence to suggest that cost of condoms to adolescents may be quite high especially in developing countries. We revisit this issue in Section 4.
Below, we will show that the rational agent will choose $p > 0.5$ because he will internalize the effect of his decision to have unsafe sex on his likelihood of catching the disease.

### 2.2 Existence and uniqueness of equilibrium

The problem of a young agent is to figure out the optimal fraction of coital acts that should be protected. His problem may be rewritten as

$$
\max_p \quad \frac{p^{1-\gamma}}{1-\gamma} + \frac{(1-p)^{1-\gamma}}{1-\gamma} + [1 - (1 - \delta)\theta (1 - p)] \frac{(Rw)^{1-\gamma}}{1-\gamma} \quad (6)
$$

Note that we can use (2) to write

$$
[1 - (1 - \delta)\theta (1 - p)] = 1 - (1 - \delta) (1 - p)^\lambda. \quad (2)
$$

Then, the first order conditions to the agent’s problem (interior solution) is given by

$$
p^{-\gamma} - (1-p)^{-\gamma} + \left[\lambda(1-\delta)(1-p)^{\lambda-1}\right] \frac{(Rw)^{1-\gamma}}{1-\gamma} = 0. \quad (7)
$$

Let

$$
\nu \equiv [\lambda(1-\delta)] \frac{(Rw)^{1-\gamma}}{1-\gamma}. \quad (4)
$$

Then, simplifying (7) further, we get

$$
(1-p)^\gamma - p^\gamma = -\nu (1-p)^{\lambda+\gamma-1} p^\gamma. \quad (8)
$$
Define
\[ G(p) \equiv (1 - p)\gamma - p\gamma, \]
and
\[ K(p; w) \equiv -\nu (1 - p)^{\lambda + \gamma - 1} p\gamma. \]

Then eq. (8) may be compactly expressed as
\[ G(p) = K(p; w) \tag{9} \]

For future reference, note that at a valid maximal solution \( p^* \), the following second order condition must hold:
\[ G'(p^*) - K'(p^*; w) < 0. \tag{10} \]

It will be useful to accumulate some pertinent information about the functions \( G \) and \( K \). First, it is easy to show that \( G'(p) < 0 \). Next, it straightforwardly follows that
\[
G(p) = \begin{cases} 
1 & \text{if } p = 0 \\
> 0 & \text{if } p \in (0, 0.5) \\
0 & \text{if } p = 0.5 \\
< 0 & \text{if } p \in (0.5, 1) \\
-1 & \text{if } p = 1 
\end{cases}
\]
holds. Therefore, if there exists a solution \( p^* \) to (9), it must be the case that \( p^* > 0.5 \) since \( K(p) < 0 \) for all \( p \in [0, 1] \).

It is also easy to verify that \( K(0) = 0 \) and \( K(1) = -\infty \). Also,
\[ K'(p) = (p\lambda - \gamma - p + 2p\gamma) \left( p\gamma^{-1} \right) \left( (1 - p)^{\lambda + \gamma - 2} \right) \nu. \]

Notice that \( K'(p) = 0 \) when \( p = \bar{p} \equiv \frac{\gamma}{\lambda + 2\gamma - 1} \). If the turning point for \( K(p) \), i.e., \( \bar{p} \), occurs to the left of \( 1/2 \), then it follows that to the right of \( p = 1/2 \), \( G \) is strictly falling and \( K \) is strictly increasing, implying a single intersection. It is easy to check that \( \bar{p} < 1/2 \) holds for \( \lambda > 1 \). We are then ready to state our primary existence and uniqueness result.
Proposition 1 Eq. (9) has a unique solution, $p^*$. 

For future reference, an equilibrium is defined as a $p^*$ which satisfies (10). Presumably $p^*$ is heavily influenced by $w$, our proxy for the level of development. We will investigate this link below. In passing, also note that since agents do not care about young-age consumption, their savings are not influenced by their choice of $p$.

3 Protected sex

3.1 Protected sex and development

An important motivation of this paper has been to investigate the impact of development on the private allocation of sex to its safe and unsafe forms. In other words, how does $w$ affect the choice of protected sex? Below, we show that the effect of $w$ on $p$ is monotonic.

Proposition 2 $p^*$ increases with $w$, i.e., the fraction of all coital activity that is protected increases with development.

Proof. From (9), it follows that

$$G(p^*) \equiv K(p^*, w).$$

Then straightforward differentiation yields,

$$\frac{\partial p^*}{\partial w} = \frac{\partial K}{\partial w} \frac{G'(p^*) - K'(p^*)}{G'(p^*)}$$

Since (10) holds at $p^*$, the denominator is negative. Then

$$\text{sign} \left( \frac{\partial p^*}{\partial w} \right) = - \text{sign} \left( \frac{\partial K}{\partial w} \right)$$

It is easy to check that $(\frac{\partial K}{\partial w}) < 0$ and hence $\frac{\partial p^*}{\partial w} > 0$ holds.

The implication of Proposition 2 is that as poorer nations experience development, citizens in those countries will conduct an increasing fraction of their sexual activity in safe forms.
The intuition is simple. With development, wages rise and agents desiring to smooth consumption wish to spread some of that increase in wages to the future. The future then becomes more attractive to wait for. In the model, this attractiveness of the future gets translated into a substitution of sexual activity from unsafe to safe forms.

Is there any evidence we can bring to bear to support the empirical predictions of our simple model? First, our model hypothesizes that richer countries should have lower HIV infection rates, since $\theta$ falls with $w$. The following figure seems to suggest that at a broad level of aggregation, the model’s prediction is roughly in line with reality. The poorer nations, on average, seem to have larger fractions of their population living with HIV.

**HIV/AIDS and Africa**

Figure 2: HIV incidence and development

Second, our model predicts that condom use (or rather fraction of all sexual activity that is protected) should rise with income.
This picture illustrates condom use across a cross section of about 150 countries. A quadratic trend line has been added in for visual convenience. The data is for couples who use condoms for contraceptive purposes. Since using condoms is the only way to have protected sex, we believe it is fair to use this “condom use” data as a rough proxy for $p$ even though the twin goals of contraception and protection from STDs are presumably not shared by all couples. As is clear from the picture, the association between the increase in condom use with development is fairly pronounced in poorer countries.

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14 The condom use data is the percentage of couples where the woman is of reproductive age (15–49 years) using condoms for contraceptive purposes. Data are for the most recent year available. Source: World Contraceptive Use 2004, database maintained by the United Nations, Department of Economic and Social Affairs, Population Division. Data may be accessed from: http://www.un.org/esa/population/publications/POP_HIVAIDS2005/POP_HIVAIDS.htm. GDP per capita numbers are for 2006.

15 A case in point is Uganda. There “between 1988 and 1995, the percent of married Ugandan women who were currently using condoms rose from 0 percent to 0.8 percent and from 0 percent to 15.4 percent for sexually active unmarried women. This trend continued between 1995 and 2000: the percent of married Ugandan women who were using condoms rose from 0.8 percent to 1.9 percent and for sexually active unmarried women it rose from 15.4 percent to 29 percent. Between 1995 and 2000 condom use among married men rose slightly from three percent to five percent. However, among unmarried men aged 15–24, reported condom use at last sex increased sharply —from 39 percent to 57 percent. Thus marital use of condoms increased only slightly while non-marital increases were dramatic. However, a large proportion of
There is scattered data (mainly from African nations) in the World Health Statistics (2006) on condom use at higher risk sex in young people aged 15-24 years both for males and females. When plotted against GDP per capita, as Figure 3b suggests, the correlation is positive. A quadratic trend line has been added in for visual convenience.

Figure 3b: Condom use at higher risk sex in young people aged 15-24 years (males) against GDP per capita

3.2 Protected sex and loss of utility from contracting AIDS

It is well known that the health-related quality of life in patients with HIV disease may be particularly important because of the disease’s chronic debilitating course and the uncertain effects of current treatments on morbidity: new antiretroviral treatments improve survival, but the durability of the therapeutic effect is uncertain and severe side effects are common. There is strong evidence to suggest that physical functioning and emotional well-being among patients with symptomatic HIV disease, or who had AIDS, were worse than for sexually active unmarried youth, particularly young women, do not use condoms at all”. HIV prevalence has reduced from a median of 19.8 per cent in 1995 to 5.5 per cent in 2001 among women attending antenatal care at sentinel sites but the rate of decline is lower in the 1996-2001 period (1.25 per cent per year) compared to 1992-95 (2.43 per cent per year). See Murphy et.al (2006) for details.
patients with several other chronic diseases. In addition, lingering fatigue, depression (often arising from loss of employment), and so on plague patients for many years. In sum, AIDS contributes to a sharply reduced quality of life.

In our setup, we can investigate the effects of this reduced quality of life by adjusting the parameter $\delta$. Lower $\delta$ implies a reduced quality of life conditional on contracting the disease.

**Proposition 3** *Ceteris paribus, a rise in $\delta$ reduces $p^*$, i.e., when quality of life subsequent to being infected rises, the equilibrium response is to engage in more unsafe sex.*

**Proof.** From (9), it follows that

$$G(p^*) \equiv K(p^*, \delta).$$

Then straightforward differentiation yields,

$$\frac{\partial p^*}{\partial \delta} = \frac{\partial K}{\partial \delta} \frac{G'(p^*)}{G'(p^*) - K'(p^*)}.$$

Since (10) holds at $p^*$, the denominator is negative. It is easy to check that $\frac{\partial K}{\partial \delta} > 0$ and hence $\frac{\partial p^*}{\partial \delta} < 0$. $\blacksquare$

The implication is interesting. The model predicts that as quality of life while infected improves (presumably with improved access to better antiretroviral treatments with fewer side-effects), the fraction of sexual encounters that are protected, goes down!

This appears to be consistent with epidemiological evidence suggesting that initiation of HAART (highly active antiretroviral treatment) regimens may be related to continuation of or increases in sexual risk behaviors. Using a sample of 724 American women who initiated HAART between January 1996 and January 2001, Wilson et. al (2004) find that “the risk for unprotected sex was higher after HAART initiation than before HAART initiation among all sexually active women”. More importantly, Proposition 3 is in line with Lakdawala, Sood, and Goldman (2006) who find that in the US “advances in treatment have improved the health of the infected, reinvigorated their sexual activity, and thus rekindled the spread of HIV” and that “increases in HIV infection rates might themselves have resulted from improvements in HIV treatment and the accompanying declines in the deadliness of HIV.”
4 Alternative formulations

The baseline model was kept deliberately simple to facilitate derivation of clean analytical results. In this section, we attempt, often via numerical examples, to showcase several alternative model formulations. Importantly, the possibility of multiple equilibria may arise.

4.1 Role of public health

While unsafe sex is a primary factor in the transmission of HIV, it must be noted that general level of public health in the economy is also important. To capture this effect, we assume

\[ \theta(u, h) = \phi \frac{u^\lambda}{h}; \quad \lambda > 1, \phi \in [0, 1] \]

where \( h \) is a measure of the general health of the population, and \( \lambda \) and \( \phi \) are parameters.

In postulating the connection between \( \theta \) and \( h \), we have the following in mind. First, we believe that if there are very few infected individuals in a society, the probability of catching the HIV virus from an unprotected sexual encounter has to be low. Even if the number of infected individuals is high but everyone is made aware of their infection status via costly public health programs, then again the risk of contracting the virus from an act of unsafe sex is low. Second, it is clear that in poorer countries, the level of public health and public health services is low and the general population is generally susceptible to many other competing diseases whose presence may increase HIV infectivity. \(^{16}\) For example, Corbett, et.al (2002) find that “the high prevalence of untreated STD infections has been a major factor facilitating the spread of HIV-1 in Africa; with the synergistic interaction between HIV-1 transmission and genital herpes being of especial concern for control of both diseases.”

\(^{16}\)In this sense, \( h \) captures the idea that non-HIV life expectancy (used in Oster, 2007) is high in countries with superior public health systems.
Below, we are going to assume, following Chakraborty (2004) and others, that

\[ h = \tau w \]

where \( \tau \) is the tax rate on wage income. The idea, of course, is that countries that spend a lot on maintaining a high level of public health tend to have generally healthy populations.\(^{17}\)

It is easily verified that

\[ \theta(u_t, h_t) \in (0, 1): \theta_1(u_t, h_t) > 0; \theta_2(u_t, h_t) < 0 \]  

(12)

holds. The implication is that ceteris paribus, the higher the amount of unprotected sex an agent has, the higher the probability with which he contracts AIDS; similarly, the higher the general health of the population, given a level of unsafe sex, the lower is the probability of catching the disease.

\[ \text{Figure 4: } \theta \text{ against } u \ (\phi = 0.2, h = 1; \lambda = 1.2 \text{ in black, and } \lambda = 1.6 \text{ in red}) \]

\(^{17}\)The WHO (2000) report presents the same idea as follows: “If Sweden enjoys better health than Uganda — life expectancy is almost exactly twice as long — it is in large part because it spends exactly 35 times as much per capita in its health systems”.

17
The dependence of $\theta$ on $\lambda$ deserves mention. As is clear from Figure 4, an increase in $\lambda$ has an important effect on the probability of catching AIDS. Notice that the marginal impact on the AIDS probability of an additional unit of unprotected sex is much higher at low levels of unprotected sex when $\lambda$ is low. There is evidence to suggest that about half of all new infections happen in people in the 15-24 age group. The WHO report “Protecting Youth from HIV” argues that the risk of contracting the virus is greater if “someone starts sex at a young age and has multiple partners. Many young people have sex before marriage with more than one partner. The first sexual experience for many girls is forced or coerced. Many boys have their first sexual experience with a sex worker. Girls who have sex with older men are vulnerable: immature bodies are more susceptible to HIV.”

In the model economy, the budget constraint (4) is replaced by $w(1 - \tau) = S$. Then, routine algebra verifies that (7) is now replaced by

$$p^{-\gamma} - (1 - p)^{-\gamma} + \left[ \frac{\phi \lambda h (1 - \delta) (1 - p)^\lambda - 1}{(h + (1 - p)^\lambda)^2} \right] \frac{(Rw(1 - \tau))^{1 - \gamma}}{1 - \gamma} = 0. \quad (13)$$

Define

$$\hat{K}(p) \equiv -\hat{\nu} \left[ \frac{(1 - p)^\lambda - 1}{(h + (1 - p)^\lambda)^2} \right]$$

where

$$\hat{\nu} \equiv \frac{\lambda h (1 - \delta) (Rw(1 - \tau))^{1 - \gamma}}{1 - \gamma}.$$

**Proposition 4** Eq. (13) has a unique solution, $p^*$.

**Proof.** Define

$$\hat{G}(p) \equiv p^{-\gamma} - (1 - p)^{-\gamma}.$$

---

18 Morison (2001) find that “the chance that a person becomes infected with HIV during one particular sexual contact varies greatly and depends on many factors. Male to female penile-vaginal transmission appears to be 2—3 times more efficient than female to male transmission and there is some evidence that first sexual intercourse for females may be associated with particularly high transmission probabilities.”
It is easy to show that $\hat{G}'(p) < 0$. Next, it follows that

$$
\hat{G}(p) = \begin{cases} 
\infty & \text{if } p = 0 \\
> 0 & \text{if } p \in (0, 0.5) \\
0 & \text{if } p = 0.5 \\
< 0 & \text{if } p \in (0.5, 1) \\
-\infty & \text{if } p = 1
\end{cases}
$$

holds. It is also easy to verify that $\hat{K}(0) = -\frac{\varphi}{(h+1)^2} < 0$ and $\hat{K}(1) = 0$ since $\theta_{11} > 0$ (because $\lambda > 1$ holds; see below). Also,

$$
\hat{K}'(p) = \nu(1-p)^\lambda - 2\frac{(\lambda - 1)h - (\lambda + 1)(1-p)^\lambda}{(h + (1-p)^\lambda)^3} > 0
$$

because

$$
\theta_{11} = \phi\lambda h \frac{(\lambda - 1)h - (\lambda + 1)(1-p)^\lambda}{(h + (1-p)^\lambda)^3} > 0.
$$

Since $\hat{G}(p)$ is monotonically decreasing and $\hat{K}(p)$ is monotonically increasing, $\hat{K}(0) < \hat{G}(0)$ and $\hat{K}(1) > \hat{G}(1)$, there must exist one and only one solution, $p^*$. ■

We are now in a position to show that unsafe sex may respond non-monotonically to development even when public health directly affects the probability of catching AIDS.

**Proposition 5** Suppose $p^*$ exists for all $w$. Then, at sufficiently low (high) levels of $w$, $p^*$ rises (falls) with $w$.

**Proof.** From (13), it follows that

$$
G(p^*) = \hat{K}(p^*, w).
$$

Then straightforward differentiation yields,

$$
\frac{\partial p^*}{\partial w} = \frac{\partial \hat{K}}{\partial w} \frac{\partial \hat{K}}{\partial p^*} G'(p^*) - \hat{K}'(p^*)
$$

Since (10) holds at $p^*$, the denominator is negative. Noting that $\hat{K}$ may be rewritten as

$$
\hat{K} = -\lambda\tau(1-\delta)(R(1-\tau))^{1-\gamma}(1-p^*)^{\lambda-1} \left[ \frac{w^{2-\gamma}}{(\tau w + (1-p^*)^\lambda)^2} \right],
$$

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we have
\[
\frac{\partial \hat{K}}{\partial w} = -\frac{\lambda \tau (1 - \delta) (R(1 - \tau))^{1-\gamma} (1 - p^{*})^{\lambda-1}}{1 - \gamma} \frac{\partial}{\partial w} \left( \frac{w^{2-\gamma}}{(\tau w + (1 - p^{*})^{\lambda})^2} \right)
\]
\[
= \frac{\lambda \tau (1 - \delta) (R(1 - \tau))^{1-\gamma} (1 - p^{*})^{\lambda-1}}{1 - \gamma} \frac{w^{1-\gamma}}{(\tau w + (1 - p^{*})^{\lambda})^3} \left( w \tau \gamma - (2 - \gamma) (1 - p^{*})^{\lambda} \right)
\]

Notice that
\[
\text{sign} \left( \frac{\partial p^{*}}{\partial w} \right) = - \text{sign} \left( \frac{\partial \hat{K}}{\partial w} \right) = \text{sign} \left( \frac{(2 - \gamma) (1 - p^{*})^{\lambda} - w \tau \gamma}{(\tau w + (1 - p^{*})^{\lambda})^3} w^{1-\gamma} (1 - p^{*})^{\lambda-1} \right).
\]

Note that \((1 - p^{*}) \in (0, 1/2]\) for all \(w \geq 0\).

First, we can show that as \(w \to 0\), \(p^{*} \to 1/2\) and \(\frac{\partial p^{*}}{\partial w} \to 0\). However, as \(w\) increases by a sufficiently small amount, \(\frac{\partial p^{*}}{\partial w} > 0\). Next, since \(p^{*} \leq 1/2\), it is easy to verify that \(\frac{\partial p^{*}}{\partial w} < 0\) holds when \(w > \frac{2 - \gamma}{\tau \gamma} 2^{-\lambda}\). And it remains so as \(w\) rises. Finally, \(\frac{\partial p^{*}}{\partial w} \to 0\) as \(w \to \infty\). □

The implication of Proposition 2 is somewhat surprising. It says that as poorer nations experience development, citizens in those countries will conduct an increasing fraction of their sexual activity in safe forms; but after a critical level of development is reached, the practice of unsafe sex may start to go up again.\(^{19}\)

The intuition is as follows. When wages/incomes are low, saving is low and public health is low keeping the risk of contracting AIDS fairly high. As incomes rise, saving increases as does expenditures on public health reducing the risk of contracting AIDS. Rational agents face the following tension: as more of the sex people have is protected, future expected utility rises but the current utility from lack of unsafe sex falls. When agents are sufficiently poor, the aforementioned loss in current utility is outweighed by the gain in future utility because the risk of catching AIDS is high; when agents are rich enough, the former effect (loss in utility from lack of unsafe sex) starts to dominate because the risk of contracting HIV is quite low.

We illustrate our result using a numerical example.

\(^{19}\)The 2005 Durex Global Sex Survey finds that roughly 70% of respondents from rich countries such as Norway, Sweden, Greece confess to having had unprotected sex with a partner of unknown sexual history; that number was below 30% for China, Spain, Hong Kong, and India. Exceptions are South Africa (64%) and Germany (30%).
Example 1  Let \( R = 1.2, \gamma = 0.95, \tau = 0.05, \delta = 0.7, \phi = 0.3, \) and \( \lambda = 0.3. \) Then,

\[ \frac{w - 0.536}{0.542} \]

Figure 5a: \( p \) against \( w \) (Example 1)

\[ \frac{w - 0.125}{0.275} \]

Figure 5b: \( \theta \) against \( w \) (Example 1)

An important thing to note from the figure is that \( p \) rises quite sharply with \( w \) but the fall in \( p \) beyond a certain level of development is far more gradual. In the same way, the model predicts that development has a much stronger effect on HIV infectivity \( (\theta) \) in the early stages of development when compared to the later stages.

4.2 A non-trivial savings decision

Thus far, we have worked with models that presented a trivial savings decision. Since young agents were assumed not to care about their current consumption, they simply saved everything. In this section, we allow for a non-trivial savings decision when young. A brief sketch of the changes follow.

Redefine preferences in (3) as

\[
U (c^y, c^o, p) = \left( \frac{c^o}{1 - \gamma} \right)^{1-\gamma} + \left( \frac{p}{1 - \gamma} \right)^{1-\gamma} + \left( \frac{u}{1 - \gamma} \right)^{1-\gamma} + \theta(u, h) \left[ \frac{\delta (c^o)^{1-\gamma}}{1 - \gamma} \right] + [1 - \theta(u, h)] \left( \frac{c^o}{1 - \gamma} \right)^{1-\gamma}
\]

where \( \theta(u, h) \) is defined in (11). The only other change is that (4) is replaced by

\[
w(1 - \tau) = c^y + S.
\]

Reworking the problem yields

\[
S(p; h) = \frac{w(1 - \tau)}{1 + [1 - (1 - \delta)\theta(1 - p, h)]^{1 - \gamma} R^{-\gamma}}
\]
from where it is clear that the saving decision and the decision to have unsafe sex are intertwined. In fact, it is possible to show that

$$\frac{dS}{dp} > 0$$

implying that when agents raise their saving in response to an increase in safe sex. Routine algebra verifies that (13) is now replaced by

$$p^{-\gamma} - (1 - p)^{-\gamma} + \left[ \frac{\phi \lambda h (1 - \delta) (1 - p)^{\lambda - 1}}{(h + (1-p)^{\lambda})^2} \right] \frac{(RS(p; h))^{1-\gamma}}{1-\gamma} = 0. \quad (17)$$

**Example 2** Retain the same parametric specification of Example 1.

As example 2 illustrates, if $p^*$ is unique, then the qualitative nature of the response of $p^*$ to $w$ is largely unchanged by the introduction of a non-trivial saving decision.

### 4.3 Multiple equilibria: Including sex as a composite and adding a cost of protected sex

Thus far, we have focussed on settings in which agents have separate and independent tastes for each kind of sexual activity, protected and unprotected. This approach yielded sufficient analytical tractability for sure. Yet, one could argue that human beings have preferences for sexual activity as a composite with differential weights on the protected and unprotected varieties. To that end, let us allow for the possibility that agents care
about the composite “good”, \((\alpha p_t + (1 - \alpha) u_t)\) where \(\alpha\) is the weight attached to safe sex.

Another aspect we’ve ignored thus far is the cost of safe sex. While such a cost may be interpreted quite literally as the cost of procuring condoms, one may broaden the scope a bit to include elements such as the cost of staying in one partner sexual relationships and avoiding promiscuity, embarrassment of purchasing condoms, and so on. Even when they are freely distributed via public health clinics, provider bias or disapproval of adolescent sexual activity, and overall lack of confidentiality may hinder their continued use (especially among adolescents) as barriers against infection. In developed countries, the cost of having safe sex is miniscule compared to people’s incomes but that is not the case in many poor nations.

When both these new elements along with young-age consumption \((c^y)\) are introduced, the model, while analytically intractable, provides some new insights. We quickly sketch the revised model below. Preferences are now described by

\[
U(c^y, c^o, p) = \frac{(c^y)^{1-\gamma}}{1-\gamma} + \frac{\alpha p + (1 - \alpha) u}{1-\gamma} \left[ \frac{\delta (c^o)^{1-\gamma}}{1-\gamma} + [1 - \theta(u, h)] \frac{(c^o)^{1-\gamma}}{1-\gamma} \right] + \theta(u, h)
\]

and the budget constraints by

\[
w(1 - \tau) = c^y + S + ap
\]

---

20 Cross country evidence on \(\alpha\) is, of course, hard to find. Scattered anecdotal evidence suggests that African men feel a strong distaste for protected sex with phrases like “One does not take showers with a raincoat on” often used to express such dislike. In a landmark survey study, Agha et. al (2002) try to determine why sexually experienced males and females from multiple countries in sub-Saharan Africa do not use condoms. They find: “Males and females most frequently reported trusting their partner as the main reason for not using a condom in last sex with a marital or a regular (non-marital) partner. This suggests that low personal risk perception is the most important reason for not using a condom with a marital or a regular partner. A dislike of condoms is the most frequently cited reason for not using a condom with a casual partner.”

21 Clive Simpkins writes in a letter dated March 17, 1995 in the Mail & Guardian (Johannesburg): “What’s not immediately obvious or important to the average Caucasian, is the resulting astonishing cost of safer sex. In a random suss-out, all condoms cost more than R2 per unit. Add a pretty essential accompaniment, to avoid friction which might create invisible and lethal lacerations in the latex, namely Johnson & Johnson’s KY lubricating jelly at about R20 – yes, you read it right – for the large tube. Let’s assume parsimonious use of the KY, and you’ll get a dozen uses out of it. That’s R2,50 per stint, bringing the total cost of the indulgence to R4,50. Given the statistics, which indicate a three times per night frequency as not unusual in certain communities – particularly the economically disadvantaged and condom-averse ones – and you’re up to R7,50 per night to practise safer sex.” [1 US $ ≈ R 10 in 2003].

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where \( a \) is the cost of a unit of safe sex, and \( \alpha \) is a fractional parameter. The remaining aspects of the model are the same as that discussed in Section 4.2 above. It follows that

\[
\alpha p + (1 - \alpha) u = (2\alpha - 1) p + (1 - \alpha)
\]

and so, to generate increasing utility from the composite sexual activity good, we restrict \( \alpha > 1/2 \). The agent’s problem reduces to

\[
\max_{S,p} \left( \frac{w (1 - \tau) - S - ap}{1 - \gamma} + \frac{((2\alpha - 1) p + (1 - \alpha))^{1-\gamma}}{1 - \gamma} + \left[ 1 - \left( \frac{1 - \delta}{h + (1 - p)^\lambda} \right) \right] \frac{(RS)^{1-\gamma}}{1 - \gamma} \right).
\]

It can be checked that the first order conditions (for an interior solution) are given by

\[
a (w (1 - \tau) - S - ap)^{-\gamma} = ((2\alpha - 1) p + (1 - \alpha))^{-\gamma} + \frac{(1 - \delta) \lambda h (1 - p)^{\lambda-1} (RS)^{1-\gamma}}{(h + (1 - p)^\lambda)^2} \frac{1 - \gamma}{1 - \gamma}. \tag{18}
\]

and saving is given by

\[
S = \frac{(1 - \tau) w - ap}{1 + R^{1-\frac{1}{\gamma}} [1 - (1 - \delta) \theta ((1 - p), h)]^{-\frac{1}{\gamma}}}. \tag{19}
\]

One can incorporate (19) into (18) to get a single equation in \( p \).

**Example 3** Let \( \alpha = 0.7898104, R = 1.281895, \gamma = 0.6393279, a = 5.831592, \tau = 0.2756531, \delta = 0.1867340, \) and \( \lambda = 0.6681025 \). When \( w = 17.34 \), for example, there are two fixed points to (18), \( p^*_h = 0.98 \) and \( p^*_l = 0.10 \).
In this case, there are two equilibria possible, one where most sexual activity is safe, and another where it is not. Thus countries with similar levels of development may find
themselves at either one of these two equilibria; such countries may then exhibit very different levels of HIV infectivity. It is also interesting to note from Example 3 that countries at the high-\( p \) equilibrium may see more unsafe sexual activity with further development and those at the low-\( p \) equilibrium the reverse.

5 Concluding remarks

The received wisdom seems to be that limited risk perception about the real risks of getting infected with the HIV virus is all that matters when it comes to explaining why Africans continue to engage heavily in unsafe sex. In this paper, we show that even when agents are fully aware of the risk involved in unsafe sexual activity, and fully internalize the effects of their own sexual behavior on their chance of catching HIV, they may rationally choose to engage in such risky behavior. Our results indicate that safe sexual practice is essentially a “normal good” and that development with a promise of a better future may be key to reducing HIV infectivity. We also find that once public health improves and quality of life under AIDS rises, unsafe sex may become popular again.
References


