An agent-based computational model for bank formation and interbank networks

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Abstract

We introduce a simple framework where banks emerge as a response to a natural need in a society of individuals with heterogeneous liquidity preferences. We examine bank failures and the conditions for an interbank market is to be established.

We start with an economy consisting of a group of individuals arranged in a 2-dimensional cellular automaton and two types of assets available for investment. Because of uncertainty, individuals might change their investing preferences and accordingly seek their surroundings neighbours as trading partners to satisfy their new preferences. We demonstrate that the individual uncertainty regarding preference shocks coupled with the possibility of not finding a suitable trading partners when needed give rise to banks as liquidity providers. Using a simple learning process, individuals decide whether or not to join the banks, and through a feedback mechanism we illustrate how banks get established in the society. We then show how the same uncertainty in individual investing preferences that gave rise to banks also causes bank failures. In the second level of our analysis, in a similar fashion, banks are treated as agents and use their own learning process to avoid failures and create an interbank market.

In addition to providing a bottom up model for the formation of banks and interbank markets, our model allows us to address under what conditions bank oligopolies and frequent banks failures are to be observed, and when an interbank market leads to a more stable system with fewer failures and less concentrated market players.

1 Introduction

In one of the most insightful analysis of the recent financial crisis published to date [10], Alan Kirman examines the role played by standard macroeconomic models and concludes that we are witnessing a crisis for economic theory itself. He argues that the events leading to the subprime crisis tell "a story of contagion, interdependence, interaction, networks, and trust", all of which are absent from economic models where utility maximizing agents act in isolation and only interact through the price system. In addition, instead of being the result of short–term exogenous shocks to a stable system, historical evidence [14] strongly suggests that financial crises originate from long–term endogenous buildups of instability, being therefore the result of the "disruptive internal processes" pointed out by Minsky [12]. Finally, the policy responses deployed in the immediate aftermath of the recent crisis, notably massive injections of liquidity in the banking system, were also at odds with macroeconomic models based on the assumption that the economy, once perturbed by an external shock, adjusts itself to a new equilibrium.

After tracing the (unhappy) intellectual history of the quest to base macroeconomic theory on solid micro foundation, culminating in the Dynamic Stochastic General Equilibrium (DSGE) synthesis, Kirman calls for a new class of models that satisfactorily address both the way agents make forecasts to guide their decisions and the problem of aggregation of individual behaviour. This recognizes both that 'rational expectations' are an inherently inappropriate way to make forecasts under frequent unanticipated changes in the environment and that models based on 'representative agents' merely assume away the solution of the aggregation problem, entirely disregarding the powerful negative results on stability and uniqueness of equilibrium provided by the Sonnenschein [16], Mantel [11] and Debreu [6] theorems. In this paper we begin to answer his call in the context of banking systems through the use of an agent–based computational model.

Agent–based modelling (ABM) is defined in [3] as "the modelling of systems as a collection of autonomous interacting entities (agents) with encapsulated functionality that operate within a computational world." In this context, encapsulation includes both how information is sent and received by agents (data) and the way agents act on this information (methods). As emphasized in [8], the crux in the definition above is autonomy, which means that "agents are endowed with behavioral rules that can tell them what to do in any given situation, independently of each other's rules, even when no one has access to a correct model of the economy."

As surveyed in [3], ABM has been applied to a variety of problems in social sciences, including electricity markets, industrial organization, storage and management of information, and transportation systems, to cite only a few. In many applications the modeller has a descriptive goal: to use ABM to reliably reproduce observed empirical phenomenal from specified initial conditions. A related goal is to gain insight into what conditions might lead to unanticipated behaviour for the system under study. In yet other applications ABM can be used culture–dish experiments to investigate the large–scale effects of structural changes affecting individual agents. In the application to banking systems we have a combination of all of these goals in mind: we want to see (1) how banks arise as a response to a given need in society and how they organize themselves into banking networks, (2) what are the conditions that lead to crises and systemic failures, and (3) how the behaviour of the system alters in response to structural changes, for example through the introduction of new regulations. In this paper we implement a simple model addressing the first of these questions and hope to convince the reader that it is rich enough to warrant further research effort pushing it towards addressing the other two and possibly more.

Our guiding modelling principle is to view a bank as a provider of liquidity, that is, an institution that transforms illiquid products (e.g non-marketable loans) into liquid ones (e.g demand deposits) by 'borrowing short and lending long' in the way described in [7]. Accordingly, we start in Section 2 with a society where banks are absent and individual agents can invest directly into liquid and illiquid assets. We endow each agent with an innate preference between being an early or late consumer, but subject them to frequent shocks that temporarily alter these preferences. The risk of making an investment decision based on innate preferences and then regret it because of the shock leads agents to search for trading partners as a way of insurance. To predict whether or not they will find such a trading partner in case they need one, agents use a learning mechanism based on the type of inductive reasoning proposed Brian Arthur in his seminal work [2] on bounded rationality. As agents understand their environment better with time, they become more aware of the real possibility of finding themselves in the bad scenario where they would like to trade with a partner with opposite preferences but cannot find one. This creates the opportunity for another type of agent to emerge, namely one that can provide liquidity by pooling resources from the society. In other words, we arrive at a propitious environment for the emergence of banks.

We treat the creation of banks in Section 3 using the framework proposed in [9] for the emergence of economic organizations. Specifically, we model a bank as a particular agent receiving deposits from its neighbours and offering in return a demand deposit with payoffs (c_1, c_2) for early and late withdraws respectively. Given that a bank exists in their neighbourhood, other agents need to decide whether it is better to deposit in it or to invest directly in the liquid and illiquid assets with payoffs $(1, 1)$ and (r, R). To establish a bank to begin with, an agent struck by 'the idea of entrepreneurship' first makes an estimate of the proportion of impatient agents in his neighbourhood and decides whether or not it is possible to allocate funds between the liquid and illiquid assets in order to satisfy the liquidity needs of potential clients. As the realize proportion of impatient clients becomes known, the bank can either fail or survive depending on its allocation of funds and the size of the error in the estimated proportion. Banks that survive update their estimate of the proportion of impatient clients based on the realized proportion and the model moves to the next period. In this setting banks operate in isolation and are left to their own devices to cope with liquidity shortages. The numerical simulations show that this leads to frequent bank failures and eventual monopolies and oligopolies of banks formed in the society.

We then consider the creation of an interbank market in Section 4. We endow banks with a learning mechanism similar to what agents themselves use, and let them forecast the adequacy of their estimate of impatient clients. When a bank forecasts an inadequate estimate it tries to prevent failure by establishing a link with another bank. In this way, liquidity shortages are smoothed over society across different banks, and numerical simulations shows a strengthened banking system with fewer failures and less oligopolies.

In Section 5 we consider liquidity shocks that affect an entire region in a strongly correlated way and are thought to be responsible for bank panics both in the original Diamond and Dybvig model and its generalizations surveyed in [5]. By imposing shocks that affect a large number of agents of a region in the same way, we introduce disturbances to the learning mechanism used by banks and therefore provide further incentive for interbank links. The numerical simulations show that when the shocks take the form of preference regions of opposite type, so that there is no overall shortage of liquidity in the system, the interbank market plays its expected stabilizing role. In the extreme case of frequent large shocks in the form of preference regions of the same type, so that the system experiences severe temporary liquidity shortages, the presence of an interbank market does not make it safer, although we find that it does not make it riskier either.

We conclude in Section 6 by suggesting several ways in which the model can be extended to incorporate more realistic features into our agent–based computational model for a banking system.

2 The pre–banking society

2.1 Agents, investment choices, and preference shocks

We follow [7] and model a society consisting of agents facing uncertainty about their intertemporal consumption preferences. Specifically, we initially consider a model with three times $t = 0, 1, 2$, a homogeneous consumption good used as a numeraire, and a productive technology that yields $R > 1$ units of output at time 2 for each unit of input at time 0. However, investment in the productive technology is *illiquid* in the sense that it yields $r \le 1$ if consumed at time 1. By contrast, investment in the numeraire itself is deemed to be liquid, as it yields one unit of consumption either at $t = 1$ or $t = 2$ for each unit owned at 0.

Agents are initially endowed with one unit of the numeraire and can be either impatient (type 1) or patient (type 2), depending on whether they prefer consumption at an earlier or later date. Patient agents prefer to invest in illiquid asset, represented here by the productive technology, whereas impatient agents favour investment in liquid ones, such as the consumption good itself. The first essential ingredient in the Diamond and Dybvig model is that investment decisions must be made at time 0 when agents do not know what their liquidity preferences will be at subsequent times. Inasmuch as these random future preferences are uncorrelated, insurance possibilities arise in the form of mutual contracts between agents with different liquidity needs.

This is modelled in [7] by assuming that agents want to maximize a utility function of the form

$$U(c_1, c_2) = \begin{cases} u(c_1), & \text{with probability } \omega \\ u(c_1 + c_2), & \text{with probability } 1 - \omega, \end{cases} \tag{1}$$

where $u(\cdot)$ is a classical utility function. Denoting the consumption of agents of type i at time k by c^i_k, letting ω be fixed, and and assuming that types are *publicly* known at 1, they show that there exist an optimal sharing of output between patient and impatient agents satisfying

$$c^2_1 = c^1_2 = 0 \tag{2}$$

$$u'(c^1_1) = Ru'(c^2_2) \tag{3}$$

$$\omega c^1_1 + (1 - \omega)\frac{c^2_2}{R} = 1 \tag{4}$$

Using the facts that $R > 1$ and that the utility function u is increasing and strictly convex, it follows that

$$1 < c^1_1 < c^2_2 < R. \tag{5}$$

Equation (2) simply means that those who can, delay consumption, equation (3) is a first order condition relating marginal utility to marginal productivity, and (4) is a resource constraint.

The optimal solution above realizes all the insurance possibilities between agents, much in the same way as regular casualty insurance, where claims are publicly information. The second essential ingredient in the Diamond and Dybvig model, however, consists of postulating that realized liquidity preferences are *private* information, which in principle allows agents to misrepresent their preference, thereby compromising its achievability as an equilibrium. Fortunately, equations (2)–(5) can be used to show that the optimal solution in this case happens to satisfy a self–selection constraint, which in turn implies that there is necessarily a contract structure which implements it as a Nash equilibrium. Their key insight is that such contract can take the form of a demand deposit offered by a bank, which we will discuss in the Section 3.

In the spirit of ABM described in the Section 1, our purpose is to present a model tackling the same problems as the Diamond and Dybvig one, but without assuming identical utility–maximizing agents and the equilibrium results that follows. Instead, we consider N heterogeneous agents and assign liquidity preferences to them at $t = 0$ according to independent uniform random variables ω^i, for $i = 1 \ldots, N$ on $[0, 1]$: if $\omega^i < p$, agent i is said to be of type 1 (impatient), otherwise it is said to be of type 2 (patient). We then introduce the privately observed risk associated with changing type by imposing a preference shock: at $t = 1$ independent shocks b^i, ε^i affect all individuals, modifying their preferences according to

$$W^i = \omega^i + (-1)^{b^i} \frac{\varepsilon^i}{2}, \qquad (6)$$

where $b^i \in \{0, 1\}$ is a Bernoulli distributed random variable with equal probabilities and ε^i is a uniformly distributed random variable on $[0, 1]$. Setting $q = 2p - 1/2$, agent i is then deemed to be impatient if $W^i < q$ and patient otherwise. Therefore, sufficiently large shocks can change the type of an agent at time 1. For the remainder of the paper we use $p = 0.5$ for concreteness, but it is clear that any other overall fraction could be consider.

2.2 Searching for partners

Agents are endowed with one unit of the numeraire at time 0 and can invest either in the numeraire itself (liquid asset) or in the productive technology (illiquid asset). It then follows that the (uncorrelated) discrepancies between their initial preferences and subsequent liquidity needs gives them an incentive to explore the society in search for partners to trade. For example, consider an agent who is initially patient and therefore invests in the illiquid asset at time 0. Suppose that the preference shock for this agent is large enough that he becomes impatient at time 1 (that is, a sudden desire for early consumption), propelling him to interrupt the productive technology and consume $r \leq 1$. On the other hand, consider an agent who is initially impatient and therefore hoards the numeraire at time 0. Suppose also that this agent suffers a sufficiently large preference shock and becomes patient at time 1, making her wish that she had invested in the productive technology instead. This pair of agents will clearly be better off if they are allowed to simply swap their assets: the first agent will receive one unit of the consumption good at time 1, which is at least as good as r, while the second agent will receive one unit of the productive technology and be allowed to consume R at time 2, which is more than her originally held unit of the numeraire.

As mentioned in Section 1, a key ingredient in ABM consists of dropping the assumption of perfect knowledge and instead recognize the limited capacity of agents to gather information about themselves, the environment, and other agents. Accordingly, we use a two-dimensional rectangular cellular automaton framework and assume that each agent i occupies a single cell and interacts with the eight neighbours in its Moore neighbourhood according to the following order:

5	1	6
2	i	3
7	4	8

At $t = 1$, agents with preference shocks large enough to cause a change in liquidity preference declare their willingness to trade, while those who did not change their preferences remain mute. We then pick a cell at random from those that want to trade and match it with the closest of its Von Neumann neighbours for which a trade is possible. We then repeat the matching process till all cells that need partners and could be matched with their neighbours on the Von Neumann neighbourhood have been matched. We next repeat the same for all cells that still need partners by searching in the remaining cells of their Moore neighbourhoods.

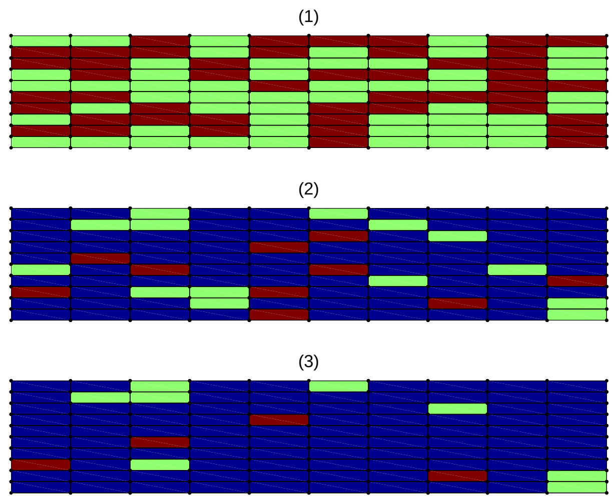


Figure 1: Society, preference shock, and search for partners.

The graphs in Figure 1 illustrate this procedure for a small society of 100 agents arranged in a 10×10 grid. The graph at the top shows the initial preferences in the society, with green and red cells representing patient and impatient agents respectively. The middle graph shows the result of preference shocks at time 1, with blue cells representing agents who did not change their preferences, and green and red cells representing those who changed and became patient and impatient respectively. Finally, the graph at the bottom shows the society after all agents have been matched with possible trading partners in their neighbourhoods. Observe that even after all the matching takes place, there might still be some agents who searched for trading partners and failed to find any, represented by the remaining green and red cells. We argue in Section 3 that this residual liquidity mismatch is what gives rise to banks.

2.3 Learning and predicting

Apart from assuming identical agents with perfect knowledge, a common weakness of most economic models is to restrict the analysis to a small number of time periods, since analytic results are seldom available in more general setups. In our computational approach, however, such restriction is unnecessary and we will now proceed to extend the model to an arbitrary number of periods. In doing so, we will also introduce the learning mechanism that agents use to understand their environment and make predictions.

Accordingly, we consider times $t_0, t_1, t_2, \ldots, t_{2n}$ and let the liquidity preference of agent i at times t_{2k}, for $k = 0, 1, 2, \ldots, n$, be determined by the uniform random variable ω^i as in Section 2.1. The random variables ω^i are independent among agents but drawn only at time t_0. We interpret this as determining the innate preference of agent i, which we assume to be unchanged over the relevant time span. By contrast, at times t_{2k+1}, agents are subject to independent shocks (b^i_k, ε^i_k) that temporarily alter their preferences according to (6). We interpret this as liquidity preferences that can occasionally change at times t_{2k+1} by revert back to their innate state at the end of each period $[t_{2k}, t_{2k+2}]$. Notice that the shocks are drawn afresh in each period, so the same agent might or might not change preferences as time goes by.

Instead of assuming, as it is often the case in the economic literature, that the entire structure of the model is known to all agents, we simply assume agents know their own innate preferences (that is, being patient or impatient as determined by the initial random variable ω^i, but *not* the value of ω^i itself) and whether or not they change preferences at times t_{2k+1} (that is, temporarily adopting a reverse preference according to the shocks (b^i_k, ε^i_k), but *not* that values of the shocks themselves).

In addition, agents receive one unit of the consumption good at t_{2k} and decide either to hoard it or to invest in the same production technology as before, that is, yielding $R > 1$ at t_{2k+2} and $r \leq 1$ at t_{2k+1} for each unit invested at t_{2k}. Because of the preference shocks, they perform a search for trading partners at times t_{2k+1} according to the procedure described in Section 1. At the end of the search, they know whether or not they found a trading partner.

Within this dynamic framework, agents need to make forecasts based on the information available to them. Since the underlying structure of the model is not known by the agents, we find ourselves in a suitably typical situation in which to apply the type of reasoning proposed in [2], where agents with bounded rationality use inductive thinking to deal with ill–defined problems. Agents build a representation of reality, recognize patterns, form hypothesis about the environment and strengthen or weaken the confidence in the hypothesis as more evidence become available.

Specifically, agents in our model need to forecast whether or not they will need a trading partner in the next period, and if they do, whether or not they will be able to find one. We denote the set of forecasts by {N,G,B}, where N (neutral) represents a forecast that the agent will not change preferences in the next period, G (good) represents a forecast that the agent will change preferences and be able to find a partner in the next period, and B (bad) represents a forecast that the agent will change preferences but not be able to find a partner in the next period.

To reach a forecast, we endow agents with a simple mechanism for using past information. Namely, we assume individuals have a memory of 5 days and use the following set of 7 predictors:

1. This period will be the same as the last one.
2. This period will be the same as two periods ago.
3. This period will be the same as three periods ago.
4. This period will be the same as four periods ago.
5. This period will be the same as five periods ago.

6. This period will be the same as the mode for the last three periods.

7. This period will be the same as the mode for the last five periods.

At t_0 all predictors are initialized at the neutral forecast and with zero strength. As time goes by, agents update the strength of their respective predictors by ± 1 depending on whether or not the forecast was correct based on the realized outcome. Each agent then uses the forecast of the predictor with the highest strength as their prediction for the next period (with a probabilistic tie-breaking rule if necessary).

While other memory sizes and sets of predictors could be used, we find that even with this simple structure, agents for the most part can correctly predict the environment they will be facing in the next period. Figure 2 shows the results for a society of 400 agents arranged in a 20×20 grid over the course of 100 periods and 100 simulations. As shown, with our simple learning process, the average percentage of individuals predicting the correct outcome increases in time to reach close to 68%.

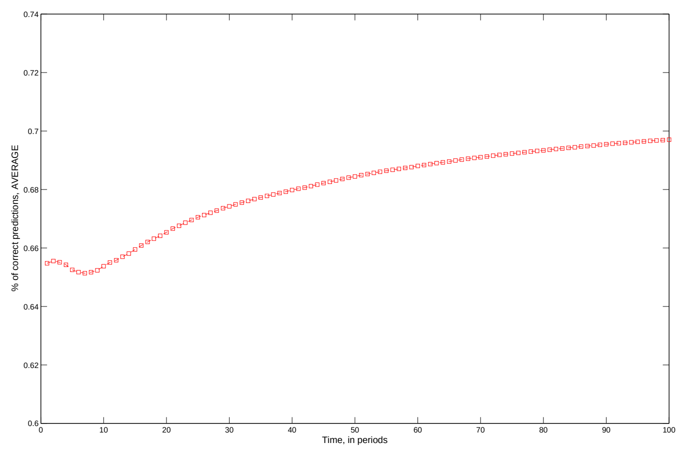


Figure 2: Learning mechanism over time

3 Introducing banks

The Diamond and Dybvig model formalizes the notion of a bank as a liquidity provider. Namely, the function of a bank is to offer a demand deposit contract which, for each unit of the consumption good deposited at time 0, pays $c_1 > 1$ units if the depositor decides to withdraw at time 1 and $1 < c_2 < R$ units if the depositor waits until time 2. The final key ingredient in their model is that depositors are served sequentially based on their position in a waiting line. They then show that the model admits a good equilibrium in which a fraction ω of the depositors receives c_1 at time 1 and the remaining depositors receive c_2 at time 2, thereby achieving the full–information optimal sharing described in Section 2.1. They also show, however, that the nature of the demand deposit contract leads to the existence of a different equilibrium in which depositors expect a higher fraction to withdraw at time 1, making it optimal for all of them to rush to withdraw at time 1. This second equilibrium corresponds to the formalization of the concept of a bank run.

Our purpose in this section is to obtain the phenomena of creation and failure of banks in our ABM context. But before that, let us suppose that a bank already exists and investigate the consequences for the agents in our society.

3.1 The decision to join a bank

In our dynamic model, a bank is an institution that promises to pay $c_1 > 1$ units at time t_{2k+1} and $1 < c_2 < R$ units at time t_{2k+2} for each unit deposited at times t_{2k}, for $k = 0, 1, \ldots, n$. Assume that a bank already exits and that agents have the learning and predicting capabilities described in Section 2. We now proposed a mechanism for each agent to decide whether or not to join bank if given the opportunity. The idea is to take into account the agents innate preferences and compare the payoffs obtained by investing directly in either the consumption good or the productive technology with the payoff promised by the bank, according to the current state of their forecasts and weighted by the strength of each of their predictors.

For illustration, consider a patient agent whose predictors currently have the forecasts and strengths shown in the first two columns of the following table:

	forecast	strength	payoff (join)	payoff (not join)
1	N	-2	c_2	R
2	G	0	c_1	1
3	N	+1	c_2	R
4	B	-1	c_1	r
5	G	+1	c_1	1
6	N	0	c_2	R
7	B	+2	c_1	r

According to the first predictor, this agent is better off not joining the bank, since $c_2 < R$. Conversely, according to the last predictor it is better to join the bank, since $c_1 > r$. The agent then weighs each payoff by the strength of the corresponding predictor and reaches a decision based on whether joining or not joining has the largest weighted sum of payoffs. For example, using $c_1 = 1.1, c_2 = 1.5, r = 1, R = 2$ leads to the decision of joining the bank (a weighted sum equal to 0.7 versus a weighted sum of zero for not joining).

3.2 The birth of a bank

We now adapt to the procedure used [9] for the emergence of economic organizations in agent–based computational models to our framework. In particular, our version of Howitt and Clower's "idea of entrepreneurship" becomes the idea to establish a bank offering the demand deposit contract (c_1, c_2) described above, which we take to be exogenously given.

Specifically, at each time t_{2k}, an agent i is selected randomly from the society and picks a random number W_k^i from the set $\{0, 1/9, 2/9, \ldots, 1\}$. This is the analogue of the "animal spirits" in [9] and is interpreted as an initial estimate of the proportion of impatient agents in the agent's immediate Moore neighbourhood. If any of the agent's 8 neighbours has already joined a bank, then the agent gives up on the idea of establishing one. Otherwise, the agent uses the estimate W_k^i to compute

$$y_k^i = c_1 W_k^i \tag{7}$$

$$x_k^i = \frac{c_2(1 - W_k^i)}{R} \tag{8}$$

If $x_k^i + y_k^i \leq 1$, the agent concludes that it is possible to become a bank and satisfy the expected liquidity needs of its potential clients by allocating a fraction x_i of deposits in the productive technology while leaving the remaining fraction of $(1-x_k^i) \geq y_k^i$ invested in the consumption good. The newly created bank then offers the demand deposit contract to agents in its immediate neighborhood, who then decide whether to join it or not according to the procedure described in the previous section.

Before moving to the next period, we let banks that were established at t_{2k-2} offer their services to new clients in the neighborhood of their existing clients.

3.3 Surviving as a bank

Suppose that a bank located at i has N_k^i clients at time t_{2k}, each depositing one unit of the numeraire. Having computed x_k^i according to (8), the bank invests an amount $x_k^i N_k^i$ in the productive technology and the remaining amount $(1 - x_k^i)N_k^i$ in the consumption good. At t_{2k+1}, each client receives a preference shock according to (6) and their realized preferences after the shock determines the actual proportion $\overline{W}_k^i$ of impatient agents for this period.

If $\overline{W}_i^k \leq W_k^i$, the bank has overestimated the proportion of impatient clients, and we expect it to face a surplus at t_{2k+1} and a shortfall at t_{2k+2}. Namely, the bank can use the $(1 - x_k^i)N_i$ units invested in the numeraire to pay $c_1\overline{W}_k^i N_i$ to clients withdrawing at time t_{2k+1} and still carry forward a surplus

$$(1 - x_k^i - c_1\overline{W}_k^i)N_k^i \geq c_1(W_k^i - \overline{W}_k^i)N_k^i \geq 0$$

to t_{2k+2}, at which point it faces a shortfall

$$[c_2(1 - \overline{W}_k^i) - c_2(1 - W_k^i)]N_k^i = -c_2(W_k^i - \overline{W}_k^i)N_k^i \leq 0.$$

Therefore, if the inequality

$$(1 - x_k^i - c_1\overline{W}_k^i) \geq c_2(W_k^i - \overline{W}_k^i) \tag{9}$$

holds, we find that

$$\begin{aligned} Rx_k^i + (1 - x_k^i - c_1\overline{W}_k^i) &= c_2(1 - W_k^i) + (1 - x_k^i - c_1\overline{W}_k^i) \\ &\geq c_2(1 - \overline{W}_k^i) \end{aligned}$$

and the bank has enough assets to pay c_2 per unit deposited to each patient client. In this case, the difference

$$\Delta R_k^i = N_k^i \left[(1 - x_k^i - c_1\overline{W}_k^i) - c_2(W_k^i - \overline{W}_k^i)\right] \geq 0 \tag{10}$$

is reinvested in the consumption good and deemed to be added to the bank's reserves. On the other hand, if (9) does not hold, then the bank will have to withdraw from previously accumulated reserves (if any) in order to pay the promised amount to each client. When even that is not enough, the bank divides its assets equally and pays an amount smaller than promised to each client. However, since $W^i_k \geq \overline{W}^i_k$, this amount is never smaller than c_1 per unit deposited, for

$$\begin{aligned} Rx^i_k + (1 - x^i_k - c_1\overline{W}^i_k) &= c_2(1 - W^i_k) + (1 - x^i_k - c_1\overline{W}^i_k) \\ &\geq c_2(1 - W^i_k) + c_1 W^i_k - c_1\overline{W}^i_k \\ &= c_1(1 - \overline{W}^i_k) + (c_2 - c_1)(1 - W^i_k 1) \\ &\geq c_1(1 - \overline{W}^i_k). \end{aligned}$$

In other words, each client will receive at least as much at t_{2k+2} as the largest amount that they would have received if they misrepresented their preferences at t_{2k+1}. In view of that, we assume that such clients stay with the bank, which is deemed to have survived for the next period.

Conversely, if $\overline{W}^k_i > W^i_k$, the bank has underestimated the proportion of impatient clients, and we expect it to face a shortfall at t_{2k+1} and a surplus at t_{2k+2}. Observe, however, that we could still have $c_1\overline{W}^i_k N^i_k \leq (1 - x_i)N^i_k$, since $x^i_k + y^i_k \leq 1$ when the bank decides its allocation according to (7)–(8), in which case it is possible for the bank to honour all the withdraws made at t_{2k+1} directly from the investment made in the consumption good at t_{2k}. In this favourable but admittedly unlikely situation, the bank experiences a surplus both at t_{2k+1} and t_{2k+2}, resulting in the amount

$$\Delta R^i_k = N^i_k \left[(1 - x^i_k - c_1\overline{W}^i_k) + c_2(\overline{W}^i_k - W^i_k)\right] \geq 0$$

being added to its reserves. More likely, underestimating the proportion of impatient clients leads to a shortfall of

$$(1 - x_i)N^i_k - c_1\overline{W}^i_k N^i_k < 0 \tag{11}$$

at t_{2k+1}. The bank will then first attempt to cover this shortfall using any accumulated reserves from the previous periods. Failing that, the bank will use the $c_2(\overline{W}^k_i - W^i_k)$ units that can be liquidated from the illiquid asset without decreasing the amount c_2 to be paid to clients at t_{2k+2}. For sufficiently large shortfalls (11), this will not be enough and the bank will be forced to liquidate more than the surplus units of the illiquid asset, thereby causing the amount to be paid to each client at t_{2k+2} to be smaller than c_2 per unit deposited. Consistently with the last paragraph and similarly to [1], we assume that such early liquidation can go on provided the amount paid to each patient client at t_{2k+2} remains larger than c_1 per unit deposited. The rationale is that when this amount becomes smaller than c_1, such clients could have done better by withdrawing at t_{2k+1} instead, which will prompt them to collectively do so in the next period, thereby forcing the bank to fail. Accordingly, the maximum that can be raised by early liquidation is

$$\frac{r}{R}\left[(1 - W^i_k)c_2 - (1 - \overline{W}^i_k)c_1\right] N^i_k.$$

When this amount, plus accumulated reserves, is not enough to cover the shortfall in (11), we declare the bank to have failed in this period and replace it by an ordinary agent located at i.

To sum up, overestimating the proportion of impatient clients might lead to either accumulation or depletion of reserves, but never to failure as a bank, whereas underestimating it might lead to accumulation or depletion of reserves, but also possibly failure. If a bank does survive, however, it needs to make a new estimate of the proportion of impatient clients for the next period. Following [9], we assume that banks update their estimate according to

$$W_{k+1}^i = W_k^i + \alpha(\overline{W}_k^i - W_k^i) \tag{12}$$

where $0 \leq \alpha \leq 1$ represents the speed of adaptation or learning, which is equivalent to assuming that banks use an Exponential Moving Average (EMA) model with a constant smoothing factor α to update their estimates.

3.4 Numerical experiments

The model presented so far corresponds to banks operating in isolation, before the introduction of either interbank links or government guarantees. Accordingly, we find that is models a state of nature where the life of most banks is 'solitary, poor, nasty, brutish, and short'.

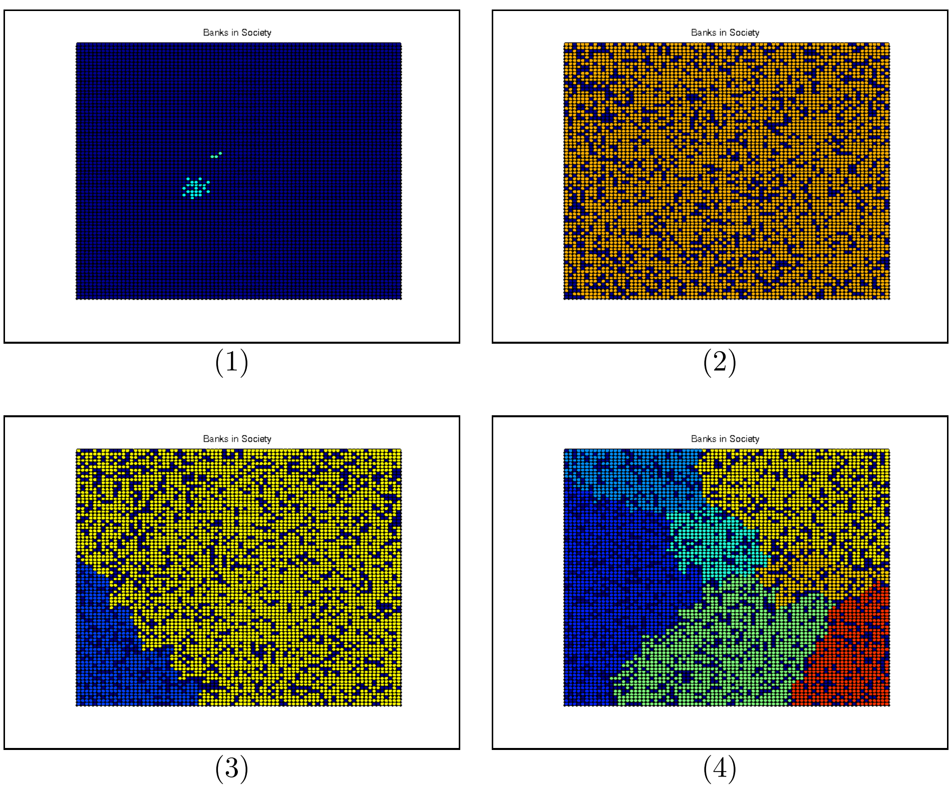


Figure 3: Examples of banks established in a society of 80×80 individuals over the course of 80 periods, with parameters $c_1 = 1.1$, $c_2 = 1.5$, $r = 0.5$, $R = 2$, and $\alpha = 0.7$

Figure 3 shows the outcomes of four different simulations for a society of 80×80 agents over the course of 80 periods. The outcome shown in (1) corresponds to a simulation with frequent bank failures, where a total number of 59 banks tried to establish themselves over the history of the society, with only two infant banks appearing at the end of our time span. The outcome in (2) corresponds to a simulation where a monopolistic bank expanded to the whole society, after 26 banks tried to establish themselves. Similarly, outcomes (3) and (4) show 2 and 7 successful banks out of a total of 24 and 23 banks that tried to establish themselves, respectively.

For more conclusive results, we conducted 50 independent simulations of the model and found that in 7 of them a monopolistic bank emerge, while in 66% of cases an oligopoly was formed (18 simulations having only two banks and 15 with only three banks established). In all cases, the longer a bank lived the higher its chances of survival, as it accumulates reserves and eventually achieves adequate estimates of the liquidity preferences of its clients.

4 Interbank market

In the same way that in the Diamond and Dybvig model clients make deposits in banks as an insurance against uncertain liquidity preferences, it is argued in [1] that banks make deposits in other banks as insurance against liquidity shocks involving entire regions of society. In the Allen and Gale model, liquidity fluctuates in a perfectly anticorrelated way across regions, so that a bank in a region experiencing a liquidity shortage (that is, a higher–than–average number of impatient clients) can satisfy this demand by withdrawing deposits from a bank experiencing a liquidity surplus in the anticorrelated region. In a stylized model with four banks, they were able to show how a decentralized allocation in the form of interbank deposits can achieve the optimal liquidity transformation that a central planner would implement after observing the anticorrelated fluctuations in each region. Typically for an equilibrium model, it assumes that the banks have perfect knowledge about the way liquidity preferences fluctuate. The purpose of this section is to develop an interbank market with the same motivation as in the Allen and Gale model, but in our ABM context.

In the previous section we used the process (12) suggested in [9] for banks to update their estimate of the proportion of impatient clients based on the observed proportion in each period. The goal for each bank was to gain knowledge about the immediate liquidity needs of its clients and allocate deposits in a way that allowed it to survive and accumulate reserves through time. As we have seen, this version of the model leads to fast and frequent bank failures and the establishment of a few oligopolies in the long run. In particular, there was no mechanism or even an incentive for banks to establish links between themselves. We now extend the learning process used by banks with a view to the eventual establishment of an interbank market.

4.1 Learning as a bank

Similar to individual agents in our society, banks try to achieve targets by observing their environment and learning from past experience. We assume a bank's main target is to avoid failure, which in turn translates into obtaining better estimates of the proportion of impatient clients and avoiding low levels of reserves.

As before, we assume that a bank located at i updates its estimate of the proportion of impatient clients according to the EMA process (12). In addition, it determines the adequacy of the estimate based on two criteria: its statistical accuracy and the current levels of bank reserves.

For the accuracy, we use the Wilson score confidence interval $[z_k^i - \sigma_k^i, z_k^i + \sigma_k^i]$ with centre

$$z_k^i = \frac{\overline{W}_k^i + \frac{1}{2N_k^i} Z_{1-\frac{\alpha}{2}}^2}{1 + \frac{1}{N_k^i} Z_{1-\frac{\alpha}{2}}^2} \tag{13}$$

and half-width

$$\sigma_k^i = \frac{Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\overline{W}_k^i (1-\overline{W}_k^i)}{N_k^i} + \frac{Z_{1-\frac{\alpha}{2}}^2}{4(N_k^i)^2}}}{1 + \frac{1}{N_k^i} Z_{1-\frac{\alpha}{2}}^2}, \tag{14}$$

where $\overline{W}_k^i$ is the realized proportion of impatient clients at time t_{2k+1} and $Z_{1-\frac{\alpha}{2}}$ is the $(1-\alpha/2)$–percentile of a standard normal random variable (see [4] for details). A bank judges its estimate W_k^i to have been adequate for this period if it falls within the bounds of the confidence interval and the accumulated reserves per deposit at the end of the period are above a predetermined threshold R_{min}. Conversely, the bank judges it to be inadequate if it falls outside the confidence interval or leads to alarmingly low levels of reserves.

Just as individuals make predictions based on the observed history, we assume now that banks use the last 5 periods to predict the adequacy of their estimate for the next period. For simplicity, we assume that all banks use the same 7 predictors described Section 2, each forecasting one of the possible states {N,G,B}, where G (good) represents a forecast that the estimate of the proportion of impatient client in the next period will be adequate, B (bad) represents a forecast that the estimate will be inadequate, and N (neutral) is the initial state for each predictor, when there is not enough information to forecast either an adequate or inadequate estimate.

Similar to the mechanism described in Section 2, banks update the strength of their respective predictors by ± 1 depending on whether or not the forecast was correct based on the realized outcome. Each bank then uses the forecast of the predictor with the highest strength as their prediction for the next period (with a probabilistic tie-breaking rule if necessary).

We argue that banks predicting an adequate forecast for the next period have no incentive to seek links with other banks, as they believe that their allocations alone will be enough to satisfy the liquidity needs of their clients and maintain a minimum level of reserves. On the other hand, banks predicting an inadequate estimate have an incentive to seek links with other banks. The precise mechanism for links to be establish depends on whether the bank believes to be overestimating or underestimating the true proportion of impatient clients and will be described in the next section. We notice for now that the very possibility of forming such links depends on the existence of at least two banks with insufficient confidence in their estimates. We call a period when such a pair exists a window of opportunity for the creation of an interbank link.

Figure 4 shows how the adequacy of the estimates evolve in time. Each row correspond to one of the 17 different banks that tried to get established in society of 80×80 individuals over the course of 80 periods, with a yellow, light blue, and dark blue cell representing respectively a prediction of a neutral, bad (inadequate), and good (adequate) estimate for the next period. We observe windows of opportunity for an interbank link (i.e two or more light blue cells at the same time) in less than half of the periods. As time goes by, their frequency decrease as surviving banks become increasingly confident, with 4 out of the 5 banks present at the end of the simulation predicting an adequate estimate and therefore no need to establish a link with other banks.

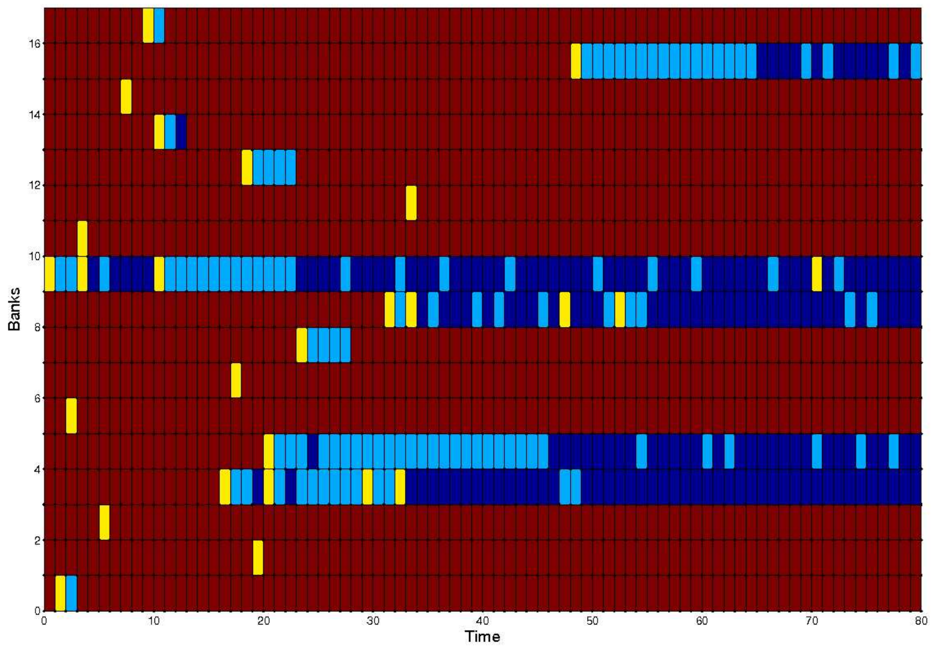


Figure 4: Adequacy of the forecast for banks established in a society of 80x80 individuals over 80 periods (yellow = neutral, light blue = inadequate, dark blue = adequate).

4.2 The need for interbank deposits

Suppose that, according to the learning mechanism described in Section 4.1, a bank at site i forecasts that its estimate W^i_k of impatient clients for the next period k is inadequate, thereby having an incentive to try to prevent a drop in reserves by entering a deposit contract with another bank. For simplicity, we assume that such interbank deposit works in much the same way as a regular demand deposit for bank clients: for each unit deposited at t_{2k}, the bank receiving the deposit promises to pay back either c_1 units on demand at t_{2k+1} or c_2 units at t_{2k+2}.

It should be clear that the decision to either make or accept an interbank deposit depends on whether the bank believes W^i_k to be an underestimate or an overestimate of the true proportion $\overline{W}^i_k$. Specifically, a bank at site i forecasting the possibility of an underestimate $W^i_k \ll \overline{W}^i_k$ has an incentive to raise funds by accepting deposits from other banks at at t_{2k} to pay for its higher–than–estimated share of impatient clients at t_{2k+1}.

Conversely, if the bank forecasts the possibility of an overestimate $W^i_k \gg \overline{W}^i_k$, then it can make a deposit with another bank j and wait until t_{2k+2} to withdraw from it, thereby helping it pay for its higher–than–estimated share of patient clients. Notice that this is better for bank i then investing a higher amount directly in the illiquid asset, because in case the estimate W_k^i turns out to be accurate (that is, the actual proportion $\overline{W}_k^i$ ends up not being so small after all), it can still withdraw at a rate c_1 from bank j instead of liquidate the illiquid asset at a rate r.

The question that follows is how can a bank determine whether it is overestimating or underestimating the true proportion of impatient clients, given that it forecasts it to be inadequate for the next period. Simply comparing the estimate W_k^i with the realized proportion $\overline{W}_{k-1}^i$ in the previous period does not immediately answer this question, since the updating equation (12) already incorporates this information. We therefore make the additional assumption that banks can observe the realized proportion of impatient agents among the entire population of clients for all existing banks at the period $k-1$, which we denote simply by $\overline{W}_{k-1}$, without any superscript since it is a global variable.

When a bank forecasts that its estimate W_k^i will be inadequate, it compares it with $\overline{W}_{k-1}$. If $W_k^i > \overline{W}_{k-1}$, the bank concludes that W_k^i is likely to be an overestimate and will seek to deposit an amount equal to

$$O_k^i := N_k^i(W_k^i - W_{k-1}) \tag{15}$$

with other banks. This amount will be allocated using its accumulated reserves first and if not enough part of the originally planned investment $(1 - x_k^i)N_k^i$ in the liquid asset, where x_k^i is given by (8).

Conversely, if $W_k^i \leq \overline{W}_{k-1}$ the bank concludes that W_k^i is likely to be an underestimate and will accept deposits from other banks up to the amount

$$I_k^i := N_k^i(W_{k-1} - W_k^i) \tag{16}$$

and keep it invested in the liquid asset.

4.3 Building interbank links

Let $m := m(k)$ be the total number banks seeking to establish interbank links at the beginning of period k and L_k denote the $m \times m$ matrix of interbank exposures. The purpose of this section is to described an algorithm to determined the entries l_k^{ij}, corresponding to the amount deposited by bank i into bank j, based on the desired amounts defined in (15) and (16). In other words, we want to populate the matrix L_k in such a way that its i–th row adds up to O_k^i and its j–th column adds up to I_k^j, as represented below:

$$L_k = \left[\begin{array}{cccc|c} l_k^{11} & l_k^{12} & \cdots & l_k^{1m} & O_k^1 \\ l_k^{21} & l_k^{22} & \cdots & l_k^{2m} & O_k^2 \\ \vdots & & \ddots & \vdots & \\ l_k^{m1} & l_k^{m2} & \cdots & l_k^{mm} & O_k^m \\ \hline I_k^1 & I_k^2 & \cdots & I_k^m & \end{array}\right] \tag{17}$$

Our algorithm proceeds as follows: first we order banks from 1 to m according to their establishment date, from the earliest to the most recently established. We then assign values to each of the links according to

$$l_k^{ii} = 0, \tag{18}$$

and

$$l_k^{ij} = \min\left(O_k^i - \sum_{n=2}^{j-1} l_k^{in}, I_k^j - \sum_{n=2}^{i-1} l_k^{nj}\right). \tag{19}$$

In this way we satisfy the deposit requirement of older banks first, essentially following a preferential attachment rule [13] for the establishment of the interbank market.

4.4 Dissolving interbank links

Having made deposits according to the matrix of exposures L_k, banks observe their actual percentage of early customers $\overline{W}_k^i$ after the preference shock that takes place at t_{2k+1}.

If $\overline{W}_k^i > W_k^i$, bank i experiences a shortfall at t_{2k+1}. After using its planned investment $(1 - x_k^i)N_k^i$ in the liquid asset, the bank will withdraw from its deposits in other banks, following the order specified by the indices 1 to m, while observing a netting procedure in case any of the other banks also faces excess liquidity demands at the same time, but with priority over clients in any of the banks where it has deposited. Only after withdrawing all of its deposits the bank will attempt to use any available reserves R_k^i or liquidate its holdings in the illiquid asset to pay impatient clients as described in Section 3.3.

Conversely, if $\overline{W}_k^i \leq W_k^i$ the bank has a surplus at time t_{2k+1}. Differently from Section 3.3, before carrying forward this surplus to time t_{2k+2}, the bank needs to use it together with accumulated reserves to pay for the withdraws from any other banks that have deposited in it and face excess demand for liquidity at t_{2k+1}. When that is not enough, the bank will have to liquidate part of its holdings in the illiquid assets to pay for the withdraws from other banks.

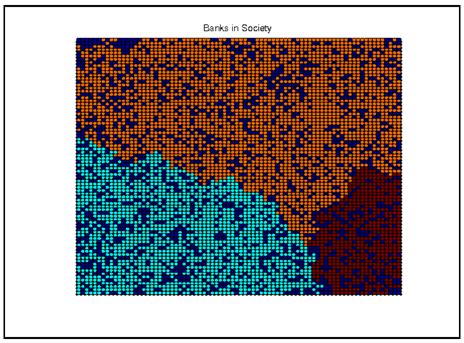
At time t_{2k+2} all remaining interbank links are dissolved. Banks use the payoff from their own investment in the illiquid asset, plus reserves and any surplus carried over from t_{2k+1} to pay their own patient clients and any outstanding deposits from other banks, all treated equally. If the bank has enough funds to payc_2 per unit deposited, any amount left over is added to its accumulated reserves. If not, clients and other banks alike will receive less than the promised amount c_2 per unit deposited. According to the rationale discussed in Section 3.3, if this amount is less than c_1 we declare that the bank fails in this period.

Observe that the presence of interbank links leads a much richer set of possible outcomes than in Section 3.3. For example, a bank that underestimated the proportion of impatient clients but made enough deposits with banks in the opposite situation will need to liquidate a smaller fraction of its illiquid assets and therefore be less likely to fail at the end of the period. On the other hand, a bank that overestimated the proportion of impatient clients but accepted enough deposits from banks in the opposite situation might be forced to liquidate part of its illiquid holdings because of such interbank links, which could ultimately lead it to fail at the end of the period. Many more combinations of similar benign and malign effects of interbank links are clearly possible. It is inherent to ABM that the range and likelihoods of possible outcomes are not determined in advance, but rather emerge from the interactive dynamics of the agents. In the next section we discuss some of these possible outcomes.

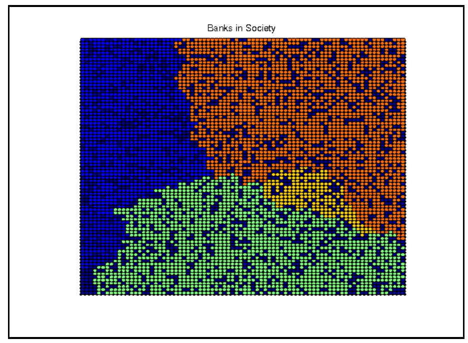
4.5 Numerical experiments

We saw that in the version of our model with no interbank deposits some banks were able to eventually establish themselves, but the banking system as a whole was characterized by frequent bank failures, few market players (oligopoly), and a complete monopoly in some cases.

We now observe that allowing banks to interact with each other significantly changes the characteristic of our banking system. As before, we implement the model with interbank deposits for a society of 80×80 individual agents over the course of 80 periods. Figure 5 shows four different simulations with more established banks than in the case without interbank links. The number of established banks out of the total that tried to get established but failed are 3 out of 31 for the first outcome, 4 out of 26 for the second, 5 out of 19 for the third, and 6 out of 28 for the fourth.



(1)



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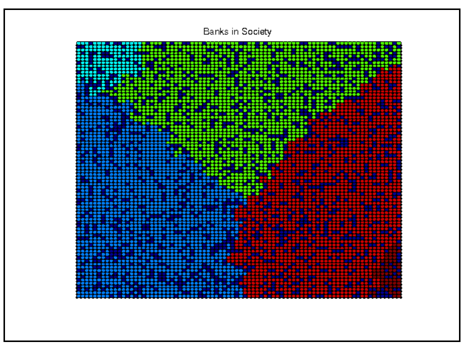

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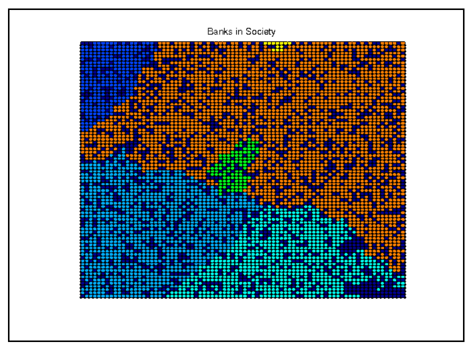


(4)

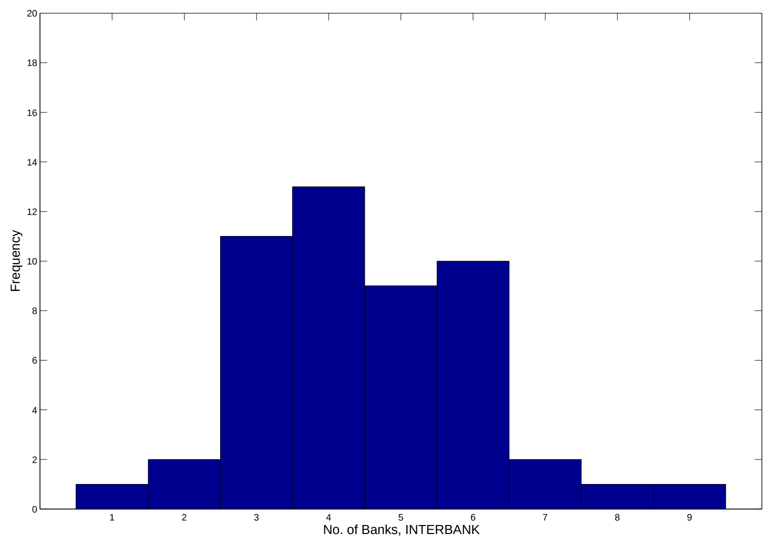
Figure 5: Banks in an 80x80 society over the course of 80 periods, with parameters: $c_1 = 1.1$, $c_2 = 1.5$, $r = 0.5$, $R = 2$, $\alpha = 0.7$, and $R_{\min} = 0.2$.

Again for more conclusive results, we implemented 50 independent simulations of the model and compare the cases with and without interbank links according to three criteria: (i) the fraction of surviving banks to total number of banks that tried to be established, (ii) the size of the largest bank in the society, and (iii) the percentage of agents who joined a bank by the end of the simulation.

For the first criterion, we find that more banks are able to survive in the case of the interbank market. The average number of banks present at the end of the simulation was 4.52 when banks are allowed to interact and 2.74 when there is no interbank market, corresponding to the histograms shown in Figure 6. More significantly, the average percentage of surviving banks out all banks created during the simulation was 18.87% with interbank links and 10.07% with no interbank market, and their difference was found to be statistically significant.

Regarding the second criterion, we find that the largest bank in the case of no interbank system has an average of 70.28% of total size of the banking system (measured by number of clients), while in the case where banks are allowed to interact this average was significantly lower and equal to 52.1%. In only one instance a monopoly was observed when interbank links are allowed, compared to 7 instances with no interbank market.

Interestingly, for the third criterion we found that the bank coverage by the end of the simulation was not significantly different in the two cases: an average of 69.7% of agents joined a bank when no interbank links are allowed, while 66.2% of agents joined when an interbank market was allowed.



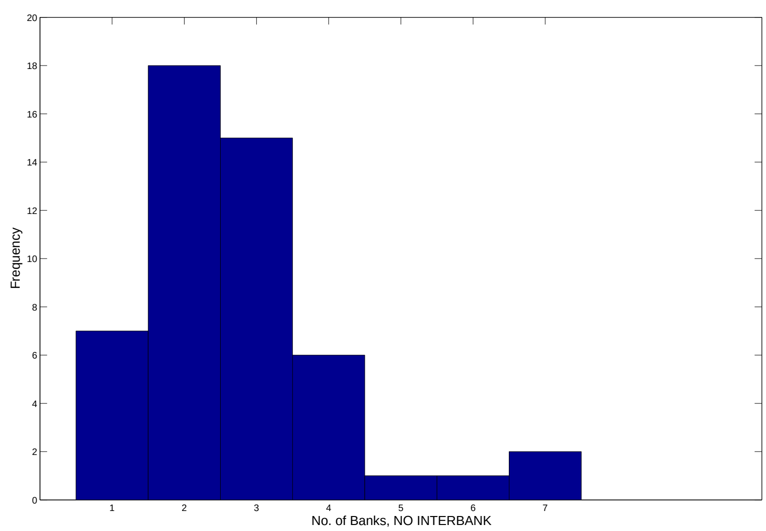


Figure 6: Histogram of number of banks established by the end of 50 simulations for the cases with (upper graph) and without (lower graph) an interbank market.

The results above suggest that the interbank market strengthens the banking system, leading to few monopolies, less bank runs and more market players. On the other hand it has almost no effect on bank coverage, which should not be surprising given that banks offer the same fixed contract (c_1, c_2) and agents deciding to join the bank do not consider the possibility of bank failures.

Notice, however, that these conclusions hold for societies where liquidity preferences and shocks are randomly distributed as described in Section 2.1. As such, the model so far excludes 'blocks' or 'communities' with highly correlated preferences, for example arising when agents are exposed to a common source of news or information, or when agents influence each other preferences, for example through rumours or fads. In the next section we add a simple modification allowing for more interesting patterns of behaviour in the society.

5 Communities of correlated preferences

As we mentioned before, one of the main contributions in [7] is that their model predicts the existence of a bad equilibrium associated to a bank run in the form of a self–fulfilling prophecy: when agents believe that a high number of agents will withdraw from their deposits, it becomes optimal for each of them to withdraw as well. Instead of proposing a specific mechanism for such beliefs to develop, it is simply suggested in [7] that they can be caused by "a random earnings report, a commonly observed run at some other bank, a negative government forecast, or even sunspots".

More specific causes of bank panics, defined as sudden withdraws by the clients of many banks forcing them to either fail or take drastic action to prevent a failure (e.g suspension of convertibility, clearing–house loan guarantees, etc), were further in the literature following the original Diamond and Dybvig model. This line of research is well–summarized in [5], where the origins of bank panics are broadly divided into 'random withdraws' and 'asymmetric information' models. Under the first group of models, withdraws are primarily motivated by real consumption needs and panics originate from location specific shocks such as seasonal demand for cash to satisfy agricultural payment procedures. On the other hand, according to the second group of models, withdraws are motivated by clients rationally changing their views about the riskiness of a bank or group of banks, for example by receiving new information about their portfolio of assets. More generally, collective withdraws might be the result of rumours and fads propagating through society in the manner described in [15] in the context of stock price dynamics.

Whatever their reason, the collective withdraws leading to bank panics require a strongly correlated change in liquidity preferences in a way that cannot be implemented by the independent shocks defined in (6). The purpose of this section is to propose a simple modification of our agent–based model allowing for these correlated shocks. Our proposed implementation of correlated shocks also allows for collective changes towards late-withdraw preferences, as is the case for example in the Allen and Gale model [1] discussed before, where some regions experience a higher–than–average proportion of impatient clients, whereas others experience the opposite type of shock.

5.1 Preference regions

Our society so far has been characterized by agents with independently distributed initial preferences who are periodically subject to independent shocks. We now define a preference region to be a spatial concentration of individuals with identical liquidity preferences.

To construct a preference region, we supplement the shocks introduced in (6) with the following procedure. We first randomly select an individual from the society to serve as the base for region. Next we set a positive integer M to be the largest reach of a correlated preference shock and randomly select a fraction (say 3/4) of the agents whose Chebyshev distance to the base is at most M. We then declare these agents to be all either patient (an 'early–preference' region) or impatient (a 'late–preference' region). In accordance with our previous distinction between temporary and innate preferences, we assume that the preference regions disappear at the end of each period and the agents revert back to their initial preferences.

For example, to model a rumour leading to early withdraws we select an agent at random and construct an early–preference region around her. More generally, to construct communities with opposite preference as in the Allen and Gale model, one or more individuals are selected and early or late preference regions are formed around them, as shown for example in Figure 7.

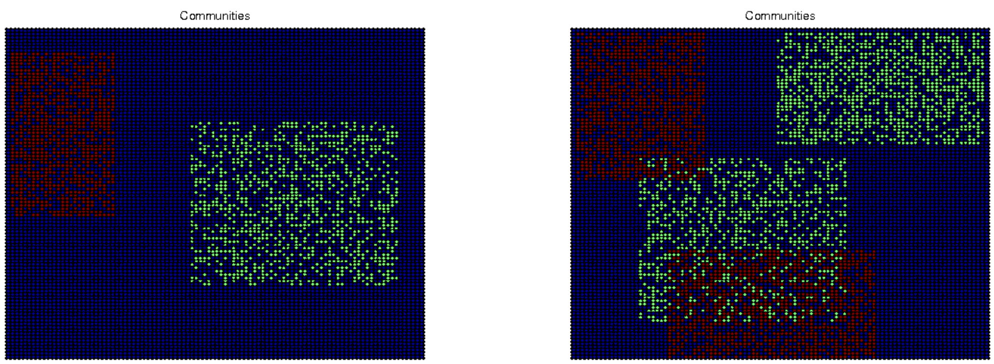


Figure 7: Communities with opposite liquidity preferences in a 100×100 society with $M = 25$.

5.2 Numerical experiments

Many different structures of preference regions are of interest and could be investigated within our framework. We restrict ourselves to a few test cases intended to highlight the differences between banking systems with and without interbank links.

In the first test case we shock the system with two opposite preference regions (i.e one early and one late–preference region) formed at random every 15 periods of our 80 periods-history (i.e different new regions are formed every time the shock is applied). In doing so we occasionally perturbed the learning processes for the banks and put them in a state of confusion leading to more windows of opportunity for the formation of interbank links.

As before, we conducted 50 independent simulations for a society of 80×80 agents with and without interbank links. We find that the presence preference regions with correlated shocks led to slightly more bank failures and a smaller number of established banks than in the version of the model used in Section 4.5.

We still find that more banks are able to survive in the case of interbank market, with an average of 4.04 against an average of 2.57 when there is no interbank market, corresponding to the histograms shown in Figure 8. The average percentage of surviving banks out of all banks created during the simulation was 9.9% with no interbank market and 15.56% with interbank market, and the difference was again found to be statistically significant. The average size of the largest bank established when there is no interbank system was was 74.13 of the total size of the banking system versus a significantly lower size of 55.13 when interbank links are allowed. Finally, the banking coverage by the end of the simulation was 71.48% when no interbank links are allowed, versus 75.8% when an interbank market was allowed, and the difference was found to be not significant.

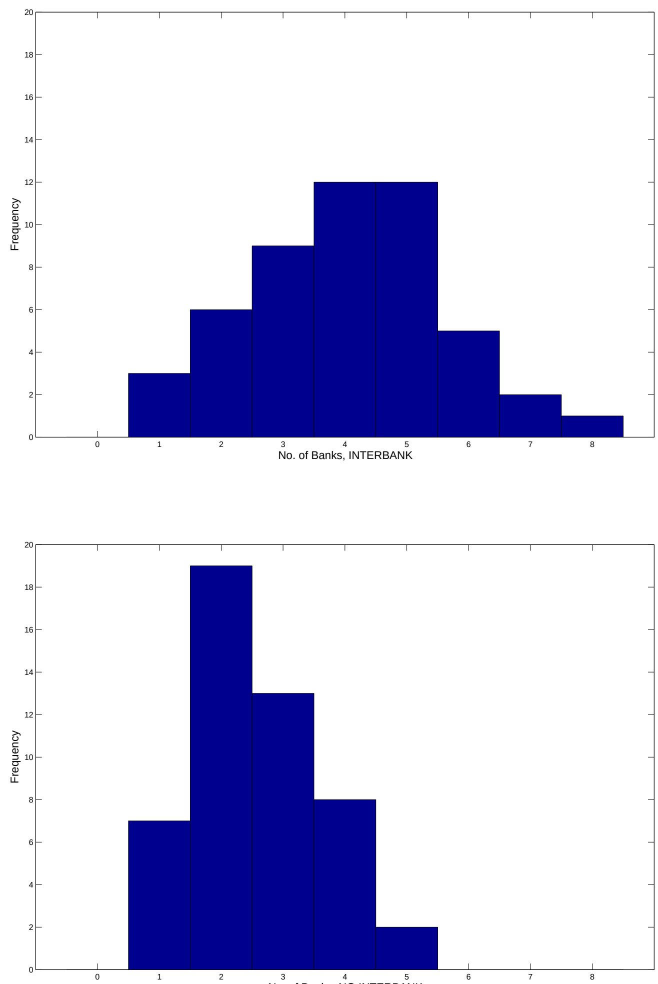


Figure 8: Histogram of number of banks established by the end of 50 simulations for the cases with (upper graph) and without (lower graph) an interbank market, both subject to shocks in the form of opposite preference regions with $M = 20$ every 15 periods.

Next we consider a more severe case and impose shocks in the form of opposite preference regions constructed every other period, leading to much fewer and more concentrated banks. The average number of banks established by the end of the simulations was 2.92 when interbank links were allowed and 2.00 in the absence of an interbank market, corresponding to the histograms shown in Figure 9. In the case with an interbank market we observe an average of 9.25% successfully established banks out of all banks created during the simulation, compared to an of 5.84% when interbank links were not allowed, and the difference was found to be statistically significant. Similarly, the average size of the largest bank as a percentage to total banks size was 81.89% for no interbank case, which is significantly higher than the average of 67.81% for the case with interbank links. Finally there is no significant difference between the percentage of people joining the banks, with an average of 59.95% for the no interbank case and 64.19% for the interbank case.

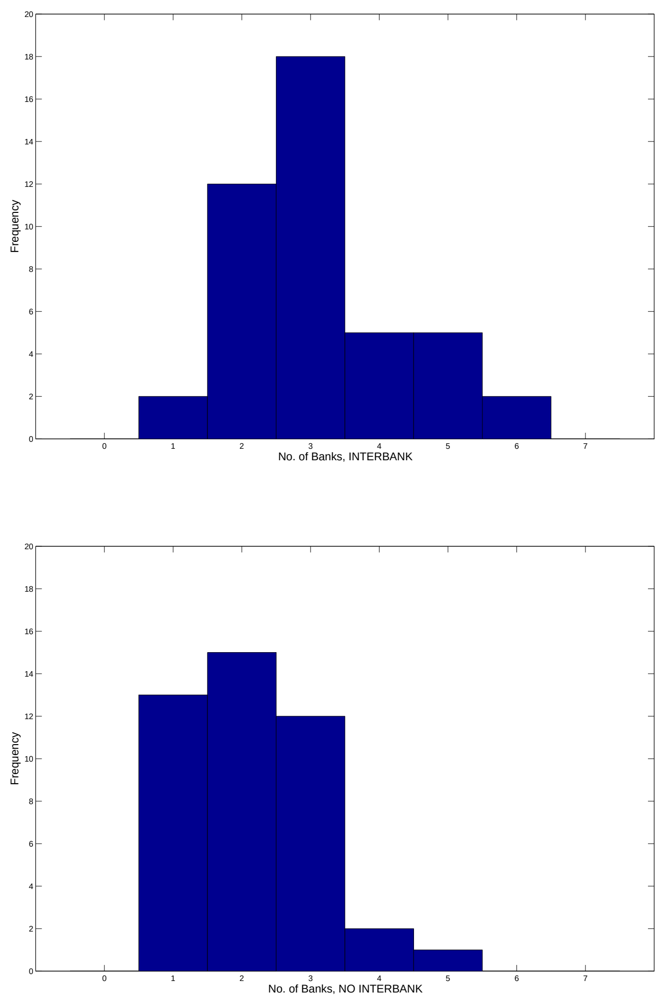


Figure 9: Histogram of number of banks established by the end of 50 simulations for the cases with (upper graph) and without (lower graph) an interbank market, both subject to shocks in the form of opposite preference regions with $M = 20$ every other period.

We notice that as extreme as creating preference regions every other period may sound, we still held the total amount of demand for liquidity roughly constant throughout the system, since the regions were always created with opposite preferences, following the view that underlies, for example, the Allen and Gale model. Consequently, there is no overall shortage of liquidity and interbank links play their expected role of improving its allocation across different regions of society. It is therefore not surprising that systems with interbank links outperforms those without them.

For our last set of simulations we examine the problem genuine liquidity shortages in the society as a whole by imposing the creation of two random early–preference regions every other period. In this admittedly more extreme situation the banking system falls apart much more easily: a large number of our 50 simulations failed to have banks covering more than 10% of the population (23 simulations when no interbank links were allowed and 18 when there was an interbank market). In addition, the cases with and without an interbank market showed no significant advantages in any of our three comparison criteria. Interestingly, however, while interbank links did not make the system safer against generalized liquidity shortages, they did not make it riskier either.

6 Conclusions and further directions

We have shown how to construct an agent–based computational model implementing the basic insights of both the Diamond and Dybvig model for bank creation and the Allen and Gale model for interbank links. In contrast to traditional economic models, our construction does not rely on agents and banks all agreeing on the underlying nature of the model, but let them use inductive reasoning to learn about their environment, make predictions, and act on the basis of their information and forecasts. The construction does not rely on any free–floating notion of equilibrium either, with the possible outcomes being the result of the interactive dynamics for agents and banks instead, as is the case with most complex adaptive systems.

Even at the simple level presented here, we were able to obtain several important features: agents decide to join banks because they provide valuable solutions for their real need for liquidity; banks survive or fail depending on the adequacy of their estimate of the preferences of surrounding agents; frequent failures and eventual oligopolies are common when banks are forced to act in isolation; the presence of interbank links leads to fewer failures and less concentrated banking systems; and correlated shocks across regions are better absorbed when an interbank market is formed.

Possible extensions abound. Still within the basic framework presented here, we could modify the demand deposit contract to allow for early withdraws at any time in a finer partition of each period, as well as let the payoff structure be determined endogenously as part of the learning and predicting mechanism used by banks. This should be accompanied by the introduction of more sophisticated predictors to cope with the extra complexity, for example by the use of a genetic algorithm to replace unsuccessful predictors with newly created ones. The creation of preference regions could also be endogenized by the introduction of an explicit dynamics for the propagation of information between agents. More significant changes include a more detailed modelling of the balance sheet for banks, taking into account investment into different classes of external assets, leverage ratios, capital requirements, shareholders interests and the like. The nature of the interbank exposure could also be extended from simple deposits to the more complicated derivative contracts commonly traded between banks.

In all cases, extensive computer simulations are required to address all the goals mentioned at the beginning of this paper. We are confident, however, that agent–based computational models such as the one introduced here constitute an important new weapon in the arsenal of statistical, mathematical, and economic methods deployed to understand and mitigate systemic risk in modern banking systems.

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References

[1] Franklin Allen and Douglas Gale. Financial contagion. *Journal of Political Economy*, 108(1):pp. 1–33, 2000.

[2] W. Brian Arthur. Inductive reasoning and bounded rationality. *The American Economic Review*, 84(2):pp. 406–411, 1994.

[3] Paul L. Borrill and Leigh Tesfatsion. Agent-based modeling: The right mathematics for the social sciences? Staff General Research Papers 31674, Iowa State University, Department of Economics, July 2010.

[4] Lawrence D. Brown, T. Tony Cai, and Anirban DasGupta. Interval estimation for a binomial proportion. *Statist. Sci.*, 16(2):101–133, 2001. With comments and a rejoinder by the authors.

[5] Charles W. Calomiris and Gary Gorton. The origins of banking panics: Models, facts, and bank regulation. In *Financial Markets and Financial Crises*, NBER Chapters, pages 109–174. National Bureau of Economic Research, Inc, November 1991.

[6] Gerard Debreu. Excess demand functions. *Journal of Mathematical Economics*, 1(1):15 – 21, 1974.

[7] Douglas W. Diamond and Philip H. Dybvig. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91(3):pp. 401–419, 1983.

[8] Peter Howitt. Macroeconomics with intelligent autonomous agents. In Roger Farmer, editor, *Macroeconomics in the Small and the Large: Essays on Microfoundations, Macroeconomic Applications and Economic History in Honor of Axel Leijonhufvud*, Cheltenham, 2008. Edward Elgar.

[9] Peter Howitt and Robert Clower. The emergence of economic organization. *Journal of Economic Behavior and Organization*, page 84, 2000.

[10] Alan Kirman. The economic crisis is a crisis for economic theory. *CESifo Economic Studies*, 56(4):498–535, December 2010.

[11] Rolf R. Mantel. On the characterization of aggregate excess demand. *Journal of Economic Theory*, 7(3):348 – 353, 1974.

[12] H. P. Misnky. *Can 'it' happen again ?* M E Sharpe, 1982.

[13] M.E.J. Newman. *Networks: an introduction.* Oxford University Press, 2010.

[14] Carmen M. Reinhart and Kenneth Rogoff. *This Time is Different: Eight Centuries of Financial Folly.* Princeton University Press, 1st edition edition, September 2009.

[15] Robert J. Shiller. Stock prices and social dynamics. Cowles Foundation Discussion Papers 719R, Cowles Foundation for Research in Economics, Yale University, October 1984.

[16] Hugo Sonnenschein. Market excess demand functions. *Econometrica*, 40(3):pp. 549–563, 1972.