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Abstract

This paper is an overview of "Agent-based Computational Economics" (ACE), an emerging approach to the study of decentralized market economies, in methodological perspective. It summarizes similarities and differences with respect to conventional economic models, outlines the unique methodological characteristics of this approach, and discusses its implications for economic methodology as a whole. While ACE rejoins the reflection on the unintended social consequences of purposeful individual action which is constitutive of economics as a discipline, the paper shows that it complements state-of the-art research in experimental and behavioral economics. In particular, the methods and techniques of ACE have reinforced the laboratory finding that fundamental economic results rely less on rational choice theory than is usually assumed, and have provided insight into the importance of market structures and rules in addition to individual choice. In addition, ACE has enlarged the range of inter-individual interactions that are of interest for economists. In this perspective, ACE provides the economist's toolbox with valuable supplements to existing economic techniques rather than proposing a radical alternative. Despite some open methodological questions, it has potential for better integration into economics in the future.

Keywords: Agent-based Computational Economics, Economic Methodology, Experimental Economics.

Résumé

Cet article est une lecture méthodologique de l'économie computationelle orientée agents (Agent-based Computational Economics, ACE), une approche nouvelle à l'étude d'économies de marché décentralisées. L'article relève ressemblances et différences par rapport aux modèles conventionnels, appréhende les spécificités méthodologiques de l'ACE, et en discute les implications pour la méthodologie économique générale. Ayant souligné que l'ACE se rapproche de la réflexion sur les conséquences non intentionnelles de l'action individuelle, constitutive de l'économie comme discipline, le papier montre qu'il s'agit d'un complément à la recherche de pointe en économie expérimentale et comportementale. En particulier, l'ACE a renforcé le résultat expérimental que certaines propositions centrales de l'analyse économique dépendent moins de la théorie du choix rationnel que l'on ne le croit habituellement, et a jeté de la lumière sur l'importance des structures et règles de marché. L'ACE a aussi élargi le champ des interactions inter-individuelles ayant un intérêt économique. En ce sens, l'ACE enrichit la boîte à outils de l'économiste avec des compléments aux techniques existantes, plus que proposer une alternative radicale. Malgré des questions méthodologiques ouvertes, l'ACE a la potentialité de mieux s'intégrer à la discipline économique à l'avenir.

Mots-clés : économie computationelle orientée agents, méthodologie économique, économie expérimentale.

Agent-based Computational Economics: a Methodological Appraisal

This paper is an overview of "Agent-based Computational Economics" (ACE), an emerging approach to the study of decentralized market economies, in methodological perspective. It summarizes similarities and differences with respect to conventional economic models, outlines the unique methodological characteristics of this approach, and discusses its implications for economic methodology as a whole. By so doing, the paper endeavors to contribute to the wider discussion on the promises and shortcomings of the new approach, which is already lively within the ACE community and is now raising interest among economic methodologists and economists more generally.

ACE uses computer-intensive tools and techniques in the study of economic problems, so that its models take the form of computer programs that can be run to simulate the behavior of the phenomenon under study. Similarly to other areas of research in computational economics, its rise has greatly benefited from the dramatic surge in calculation capacity of the last two decades. Yet it differs from the traditional core of computational economics, whose focus on the search for numerical solutions of equation-based models and the exploration of new optimization techniques is reflected in the first volume of the *Handbook of Computational Economics* (Amman, Kendrick and Rust (1996)). Instead ACE, which is central to the second volume of the *Handbook* (Tesfatsion and Judd (2006)), places emphasis on agents and the dynamics of their interactions and can be defined as "the computational study of economic processes modeled as dynamic systems of interacting agents" (Tesfatsion (2002)).

While the earliest contributions to agent-based economic modeling date back to the late 1980s (see e.g. Anderson, Arrow and Pines (1988)), the field has attracted growing attention in recent years as its areas of application have widened, a number of theoretical and empirical results have been obtained and tested, and a great deal of experience in this area has been accumulated. In the wake of the recent crisis, even the question of whether agent-based computer models could help to better understand economic and financial systems and prevent another meltdown has surfaced (Colander et al. (2009), Buchanan (2009)).

The potential and possibilities for ACE to contribute to economic knowledge are discussed here through the lenses of three inter-related methodological questions:

- What does ACE change, and what does it preserve relative to traditional approaches to the study of decentralized market economies?

- What are the distinctive methodological characteristics of ACE and where is it likely to head in the near future?
- What do recent developments in this field mean for the methodology of economics more generally?

While it would be impossible to have the final word on all these issues in their multiple dimensions and their still evolving character, it is important at this stage of development of the field to assess the state of the art in the hope to provide elements for further reflection.

It is a central argument of this paper that ACE rejoins the long-lasting reflection on the unintended social consequences of purposeful human action and their potential benefits, which has been at the heart of economics throughout its history, from Adam Smith's invisible hand metaphor to modern research on the functioning of markets. Admittedly, this interpretation draws on the similarities between ACE and mainstream economics, but it does not overlook the differences, stressed by many practitioners of the field: rather, the paper suggests that ACE accompanies the process of transformation and renewal of the economics discipline that is already being pursued in other areas of research, notably experimental and behavioral economics. In particular, the paper insists on ACE's role as a complement to laboratory findings on individual economic rationality, the market mechanism, and the linkages between the micro and macro levels of analysis.

The place of the new approach within economics is still somehow controversial, as some ACE scholars openly challenge received wisdom and claim that theirs is a "better way" to understand the behavior of economic systems and to guide policy-making (see e.g. Farmer and Foley (2009)), while many in the profession remain much more cautious. Nonetheless, the paper argues that ongoing transformations in economic methodology are likely to consolidate the position of ACE in the years to come and to better integrate it into economics. As the discipline opens to a wider variety of methods and techniques ranging from statistical tools to software, experimental protocols, and neuro-imaging, ACE can be seen as a set of new, additional items for the economist's toolbox, which potentially complements existing ones without necessarily substituting for them.

Finally, the paper discusses the multi-disciplinary origins of ACE and the strong ties that subsist between the ACE community and like-minded scholars in neighboring disciplines. It emphasizes the benefits of inter-disciplinarity which can enrich economics with new research questions, variables of interest, and methods of analysis; at the same time, it discusses its drawbacks, as closeness with other disciplines may hinder dialogue with different parts of economics rather than enhancing it. In this sense, the paper emphasizes the need for ACE to strike a balance between the two opposing needs to maintain contact with other disciplines while still engaging in a constructive discussion with economics itself.

The remainder of the paper is organized as follows. The first section presents the main features of ACE and discusses similarities and differences with respect to previously existing work in economics in general, and in computational economics in particular. Based on that, the second section outlines the methodological features of this approach, its most recent developments, and the challenges it currently faces; special emphasis is on its experimental character, its approach to decision-making processes, validation and verification issues, and heterogeneity of model types and objects of study. Examples are mainly drawn from research on markets, even though ACE can be, and actually is, applied to various topics ranging from economic growth to technological change, use of natural resources, and the environment. Finally, the paper assesses the impact of ACE on the methodology of economics as a whole. The last section briefly concludes.

1. ACE as a subfield within Economics and Computational Economics

This section first outlines the main characteristics of ACE. Focus is on similarities and differences with respect to received economic theories and, more specifically, computational economics. Then, the section briefly presents the major influences and the milestones that have marked the development of ACE, at the interface between economics and neighboring disciplines, and its subsequent, gradual acceptance as a legitimate field within economics.

1.1 Similarities and differences

In general terms, it can be said that ACE aims to explain aggregate regularities as so-called “emergent” properties of an economic system, arising over time from repeated interactions between autonomous, heterogeneous agents. Depending on the problem under study, the latter can be individuals or entities such as households, firms, and organizations. The notion of emergence refers to cases in which the whole cannot be taken as the sum of its parts: put differently, individual behavior alone is insufficient to fully predict large-scale outcomes, and some understanding of how agents interact with one another is necessary to bridge the micro and macro levels of analysis. A typical example is the well-known urban segregation model developed by Thomas Schelling (1978), in which a weak individual preference for having a minimum of less than half their neighbors of the same type (originally interpreted as ethnic group) ends up splitting agents into completely separated neighborhoods within which agents are all of the same type.

In turn, it is acknowledged that aggregate-level outcomes may induce changes in individual behavior, in a two-way feedback between the agent and the system. The perspective is clearly dynamic in the sense that the modeler first formulates hypotheses about the behavior of agents and their interactions with others, and then uses computer simulation to generate "histories" that bring to light the implications of these hypotheses. Agents' behaviors and interactions usually depend on their past experience, and in many models, agents update their behavior based on that experience. This generates path-dependence, a property that can be related to heterogeneity of agents. Indeed even if the attributes of two agents are identical at the beginning, they may make different choices and evolve along distinct trajectories, so as to distinguish themselves progressively from each other (Rouchier 2008).

With its focus on decentralized decision-making by a multitude of autonomous agents and the rise of non-fully-predictable macro-level regularities from micro-level actions, ACE echoes a conception of the market system that has always been at the heart of economics. It appears as a form of comeback to the long-lasting reflection on the unintended social consequences of purposeful human action, with contributions ranging from Adam Smith's "invisible hand" to Hayek's conception of the economic order and Walrasian general equilibrium. While ACE prolongs this reflection, its ambition is to provide a new way to look at the "mystery", one that leaves aside the equilibrium conditions that older economic approaches used to impose from the outside, somewhat artificially: from the Walrasian auctioneer to common knowledge assumptions, representative agents, and rational expectations. Today's methodological advances, it is believed, should enable to truly decentralize decision-making.

Despite this commonality, ACE's insistence on path-dependence and heterogeneity of agents reveals how profoundly different it actually is from mainstream economics. It is to some extent closer to evolutionary theory, which emphasizes adaptation of the individual to its environment and the development of inter-individual and inter-group variation. More to the point, standard utility maximization and perfect information assumptions are not normally present in ACE models, which rather tend to explore alternative notions of cognition, rationality, and learning. Typically, bounded rationality is assumed together with agents' capacity to revise their decisions as interaction plays out; central issues are the acquisition, accumulation and use of information at individual level, and the circulation of information at system level. Induction prevails over deduction in that agents move from their own specific experiences to broader generalizations through learning processes, rather than deriving conclusions from given assumptions. This is also true for the modeler: a simulation run gives rise to one possible realization only and strictly speaking, results do not have the status of proofs; some form of generalization is only possible inductively if results are reproduced many times and under different conditions. Simulation enables to identify sufficient, but generally not necessary, conditions for a phenomenon to emerge; in this sense, ACE is more similar to engineering and the experimental sciences rather than to deductive logic or mathematics. Accordingly, the core questions of ACE are closer to those of experimental and behavioral economics as well as (to some extent) game theory, than to those of neoclassical economics. It can thus be said that this approach participates to some extent in the ongoing renewal of the economic theory of individual behavior.

With respect to these other approaches, the specificity of ACE is its focus on interpersonal, social interactions in a dynamic perspective. As a matter of fact, the motivations for economists to explore agent-based modeling include dissatisfaction not only with utility maximization but also with mainstream economics' difficulty in deriving aggregate properties from individual behavior, the "representative agent" being a particularly unsatisfactory solution (Kirman (1992)). Likewise, ACE's emphasis on the time factor can be seen as a response to the standard equilibrium approach and its inadequacy to deal with out-of-equilibrium situations. Awareness of these potential strengths is widespread among practitioners of the field, and is one of the reasons why some of them loudly promote ACE as a better alternative.

Focus on agent interactions in a dynamic framework distinguishes ACE also from other types of computer-intensive work in economics; in a certain, definite sense, the fact that all use computers is only a loose analogy (Axtell (2000)). One may go as far as to say that computational techniques play only a small part in the definition of ACE: indeed Schelling's segregation model, widely considered as an important precursor of the field, was originally developed by simply moving dimes and pennies on a chessboard by hand. The relatively greater importance of agents and their interactions with respect to the use of computer-based tools is also reflected in the name that Leigh Tesfatsion, a major contributor to the field, initially proposed for it: it was simply "ABE", agent-based economics. She later changed it to ACE for greater clarity, given that after all, there are some types of agents also in standard economic models[1]. Be that as it may, the similar denomination of ABM (agent-based

[1] See http://www.econ.iastate.edu/tesfatsi/news0297.htm.

modeling), also with no reference to computational tools, is still commonly used to denote this approach methodologically[2].

1.2 The multi-disciplinary origins of ACE-ABM and its integration into economics

This terminological duplicity reflects another specific trait of the field. While the ACE label refers explicitly to economics, ABM does not and in fact, discloses the multi-disciplinary origins of this approach. It is true that Schelling, an economist, was a pioneer in showing how the complex pattern of interactions among agents in society may give rise to unexpected collective outcomes. Other recognized influences, however, include John Holland, whose book on adaptation ((1975), 2nd ed. (1992)), drew heavily on both evolutionary biology and computer science; and Robert Axelrod, a political scientist, with his work on the emergence of cooperation in an Iterated Prisoner's Dilemma game (1984). The influence of complex systems physics has also been heavy, with the major role of the Santa Fe Institute (SFI) in bringing together economists and physicists (Anderson, Arrow and Pines (1988)) and the development of the closely related "econophysics" research program (Rosser (2008)). At least in the early days of the SFI, the approach even had the ambition to provide a unifying perspective to the study of nature, human life, and society. Its development has also benefited from contributions of representatives of the wider social science, especially in Europe (Gilbert and Doran (1994); Gilbert and Conte (1995)). Today, agent-based models are applied in a variety of disciplines, including not only economics but also management, political science, sociology, anthropology, geography, biology, ecology, and even archaeology and linguistics. Agent-based modelers form a well integrated multi-disciplinary community, and share common outlets for their publications such as *Complexity*, *Advances in Complex Systems*, and (especially in Europe) the web-based *Journal of Artificial Societies and Social Simulation*[3].

While honoring its multi-disciplinary origins, agent-based modeling has aroused particular interest within economics since the early days, when the first SFI conference on "The Economy as a Complex System" was held in 1987, followed by the foundation of a new SFI economics program, and the publication of first research results in top journals (Arthur 1991; Holland and Miller 1991). Support from renowned economists including Kenneth Arrow, Alan Kirman, and Axel Leijonhufvud contributed to progressively reinforcing the

2 The very term of "agent" in its modern sense seems to have first appeared in an economics journal, namely the *American Economic Review* (Holland and Miller (1991)).
[3] http://jasss.soc.surrey.ac.uk/JASSS.html.

field. As early as 1996, Tesfatsion created a dedicated website[4], which she has maintained since then and is now a reference for agent-based modelers, especially in economics. ACE has gradually secured a place in the Society for Computational Economics and in specialized journals (*Computational Economics*, *Journal of Economic Behavior and Organization*, *Games and Economic Behavior*, *Journal of Economic Dynamics and Control*), while remaining open to collaborations with like-minded researchers in both the social and the natural sciences. Generalist journals ranging from *American Economic Review* to *Journal of Political Economy*, *Quarterly Journal of Economics*, and *The Economic Journal* have occasionally published agent-based models. Though ACE has not quite experienced the extraordinary success of other emerging fields such as behavioral economics, it has stabilized as a subfield, enjoys increasingly solid reputation, and is expanding its presence in both teaching and research programs worldwide.

2. The methodological characteristics of ACE

To identify the strengths and weaknesses of ACE, this section first provides a few introductory details on the underlying logic and founding principles of agent-based models. On this basis, it aims to derive questions for in-depth methodological discussion. Specifically, attention is drawn to four main issues, namely the interpretation of ACE as an experimental methodology; the choice between different models of decision-making, and their methodological implications; validation and verification issues; and finally, diversity of modeling approaches, assumptions, interpretations, and objects of study within ACE.

2.1 The structure of an ACE model

At a very basic level, most models share a similar structure, even though with some variation. Given a population of agents situated in a pre-defined environment and/or social context, at the heart of a model is their decision-making process. As a rule, agents' decisions are about possible exchanges or interactions with others and may include both the choice of an action (e.g. the quantity of a good to buy or sell) and the choice of a partner (e.g. a seller for a buyer, or a buyer for a seller); in other cases, instead, partners (buyers and sellers in this example) are matched randomly by the computer. Depending on the problem under study, the model may represent exchanges of goods or services but also communication, information-

[4] http://www.econ.iastate.edu/tesfatsi/ace.htm

sharing, advice-seeking, and other cognitive or social processes. As mentioned above, the individual decision-making process and the ensuing interactions between agents are iterated several times; at each step, past decisions and actions shape new choices and in some models, agents change their behavior progressively, based on the results of past choices and the ensuing changes in the environment.

The social context may take various forms: for example, it may allow agents to interact with any other agent or with a selected subgroup, which in turn may be defined spatially as a neighborhood or in terms of a network of ties, and may change in size and composition as a result of agents' actions over time. The choice set may be either given or gradually discovered as the agent acquires and accumulates information, while choice-making rules may range from forms of strategic, rational behavior to some "satisficing" criterion *à la* Simon and even to random choice, sometimes allowing for differences within the same population depending on the individual attributes of agents. Technically, many possibilities are open: for instance time may be continuous or discrete, choices may be simultaneous or sequential, and interactions may be bilateral or multilateral. It is the modeler who defines these and all other framing aspects, also including agents' attributes and the state of the system at time 0 (initialization). The modeler also needs to specify in advance some indicators of the properties (of agents and/or of the system as a whole) that are of interest for the investigation, for example patterns of transaction prices and quantities in a market model.

Then, the modeler lets agents interact repeatedly over time and refrains from any further intervention, typically until the system reaches a steady state. At the end, the modeler observes the values of indicators and derives from them answers to the questions under study. The simulation can be replicated for various values of the parameters and of the initial conditions so as to learn how to fine-tune the model to yield different results. This approach, which limits the participation of the modeler to the definition of the starting point and the rules of action, is often referred to as "bottom-up" in that the final result depends only on agent behavior with no imposition of equilibrium conditions (market-clearing, rational expectations, etc.) from the outside ("top-down").

An important remark is that many aspects of the model, including the frame of action, parameters, and detailed behavioral rules, need to be specified; yet the range of possible options is often very wide and, once standard rationality assumptions and externally-imposed equilibrium conditions are removed, little guidance comes from existing economic knowledge. This is one of the main practical challenges for modelers and suggests that in a non-Walrasian framework of analysis and/or with non-fully-rational agents, microeconomic behaviors are still poorly understood and require further research.

2.2 ACE as an experimental methodology

Drawing an analogy from biology, Tesfatsion compares the bottom-up approach of ACE to a “culture-dish laboratory experiment” (2002). More generally, she and other practitioners of the field propose an interpretation of it as an experimental methodology, exploiting controlled conditions as a means of isolating the micro-level sources of macroeconomic phenomena. Running a computer simulation is to some extent comparable to performing an experiment and in fact, ACE is similar to experimental economics under many respects: both generate their own data, and may or may not compare them to external sources (survey or other); both rely primarily on inductive rather than deductive reasoning; and finally, both make extensive use of computers to record data and enable replication. Along these lines, agent-based simulation has been compared to the use of “computational laboratories”, both in the social sciences in general (Dibble (2001)) and in economics in particular (Tesfatsion (2002); Duffy (2006)).

ACE is not fully interchangeable with experimental research with human subjects, though: it cannot, say, test whether actual decision-makers violate rational choice theory, but it can provide insight into the possible consequences of an alternative behavioral assumption in a given social context. Indeed an experiment allows observing, and controlling, the actual object of study (for example, human decision-making criteria), while computer simulation should be conceived as an experiment on a model rather than on the phenomenon itself. Another difference is the focus of ACE on the macro effects of micro behavior, which is not always a concern, and is not always practically feasible, in laboratory experiments.

While the analogy between ACE and experimental economics should not be pushed too far, the differences between them allow for potentially useful complementarities. Indeed a particularly promising direction of research consists in coupling ACE models with human subject experiments (Tesfatsion (2002); Duffy (2006); Contini, Leombruni and Richiardi, (2006); Rouchier (2007)). Laboratory findings can provide rich information on agents’ attributes, cognitive skills, and actual behavior; they thus contribute to designing and fine-tuning the simulation. In turn, the latter can help to better understand results from experiments with human subjects, for instance by allowing a very high number of repetitions of an experiment, or by providing insight into the possible large-scale effects of some observed behavior, which would be difficult to do in the laboratory. Agent-based models can also provide a benchmark: for example in market experiments, researchers can program software buyers and sellers to act according to some pre-specified rule, and compare simulated prices and quantities to those that prevail with human subjects in the lab. By so doing, they can assess similarities and differences between observed human behaviors and those implied by the rule under study, so as to gain further insight into decision-making processes.

The similarities between agent-based modeling and experimental economics, and even more, the increasingly diffused practice of performing parallel experiments with human subjects and artificial agents, are among the reasons that account for the rise of ACE in recent years. To some extent, the sub-field participates in the success of experimental research and its perceived potential to enrich economic knowledge.

2.3 Decision-making in ACE models

To examine more closely the methodological specificity of ACE, with its strengths and weaknesses, it is now important to discuss the individual decision-making process. As mentioned above, researchers in this field overwhelmingly reject neoclassical utility maximization, under a variety of influences ranging from Herbert Simon's "bounded rationality" approach to behavioral economists' claim that real decision-makers do not follow rational choice theory. The search for alternative models of rationality and cognition oscillates between the opposed principles of *simplicity* and *complexity* (Jager (2000), p. 102). They frame the extremes on a continuum and many researchers adopt intermediate positions; nevertheless, debates among supporters of the two visions have often been lively, and for the sake of argument, it is useful to keep them separate. Simplicity has been widely popularized through Axelrod's KISS principle, which allegedly stands for the army slogan "keep it simple, stupid", and is in fact a reformulation of Occam's razor. The idea is that as any other human being, the researcher has limited cognitive ability, so that it is crucial to understand everything that goes into the model in case a surprising result occurs. Simplicity at the agent level allows focusing on the dynamics of interactions among agents and on how they can, alone, lead to complex (and often, unexpected) outcomes at the aggregate level. In Axelrod's own words,

> "The point is that while the topic being investigated may be complicated, the assumptions underlying the agent-based model should be simple. The complexity of agent-based modeling should be in the simulated results, not in the assumptions of the model" (2007).

Simplicity is especially useful in models that need to separate out the effects of the structure and rules of interaction from those of individual behavior. In particular the so-called

“Zero-Intelligence” agents research program on the functioning of markets, inaugurated by a seminal article by Gode and Sunder (1993), assumes random decision-making so as to obtain a high degree of simplicity at agent level, and exploits it to develop tractable models of the bearing of the institutional and regulatory structure on price formation. Random choice should not be taken as a truthful representation of human behavior, but rather as an attempt to isolate the effects of the market structure in which transactions take place. With this approach, Sunder (2006a), (2006b), advocated a new direction of research, focusing more on institutions and structures than on the detailed study of micro behavior that in his view, characterizes much of today’s economics. Along similar lines, researchers at the SFI are currently investigating the role of financial institutions in shaping the price formation process, independently of the behavior of individual traders.

Clearly, a drawback of simplicity is its lack of realism. This is not necessarily a concern: the “as-if” arguments used in economics at least since Milton Friedman (1953) suggest that even if its assumptions are unrealistic, a theory does not need to be rejected provided its predictions are not contradicted by observation. Yet too naïve behavioral hypotheses may hinder the study of relevant issues, for instance in this case, the question of whether and how traders consent to abide by market rules and may even contribute to reshaping and improving them over time (Tubaro (2009)).

A more realistic representation of individual decision-making requires some degree of complexity, endowing agents with relatively sophisticated behavioral and cognitive skills (see e.g. Conte and Paolucci (2001); Edmonds and Moss (2001); Sun and Naveh (2004); Sun (2006)). For instance, there might be a fitness or utility function that enables the agent to evaluate the consequences of its past actions and take them as a basis to improve its choice criteria (“learn”), in a dynamic process. In more complex models, agents learn not only from their own experience but also from others, so that learning becomes a collective rather than an individual process (Vriend (2000)). Correspondingly, learning may take several forms, ranging from relatively simple stimulus-response learning (Arthur (1991), (1993); Roth and Erev (1995); Erev and Roth (1998)) to more sophisticated belief-based learning, which requires an agent to form, and regularly update, beliefs on other agents’ actions (Cheung and Friedman (1997)). Even more complex forms of learning that are found in the literature are genetic algorithms and classifier systems, borrowed from the principles of population biology (Holland (1992)).

On the whole, research along these lines places less emphasis on cognitive limits, and rather stresses the inadequacy of rational choice theory in the case of complex or ill-defined problems or in the presence of widespread, systemic uncertainty –the typical example is chess, where deduction alone does not lead to a solution and other cognitive capacities such as induction or calculation must intervene (Batten (2000)). Generally speaking, reliance on complexity is relatively more widespread in the growing literature that addresses cognitive and information-related issues, often (though not always) relying on insight from psychology or cognitive science. A reason for concern, though, is that simulations tend to be less transparent under these conditions, and may make it difficult to clearly identify the dynamics that relate macro-level outcomes to micro-level behavior.

2.4 Validation and verification

Whether a modeler adopts simplicity or complexity, the question remains of what is an explanation in an agent-based model. How can computer simulation provide insight into a social phenomenon? In the early days of agent-based modeling, it was already a big step forward to generate a social phenomenon artificially:

> "What constitutes an explanation of an observed social phenomenon? Perhaps one day people will interpret the question, 'Can you explain it?' by asking 'Can you *grow* it?' Artificial society modeling allows us to "grow" social structures *in silico* demonstrating that certain sets of microspecifications are *sufficient to generate* the macrophenomena of interest" (Epstein and Axtell (1996), p. 20, italics in original).

While this approach allowed significant progress initially, many questions remained open. How to ensure that the model captures salient dimensions of the problem under study, and reproduces relevant social processes? How to compare it to previously existing theories, and how to test its findings against empirical data? How to check for robustness of results to changes in parameter settings, initial conditions, and software implementation? These questions are of course very general and arise with virtually every modeling approach; however, they are all the more challenging in the particular case of ACE, in that path-dependence and co-evolution of agent behavior and the environment often entail non-linearities or multiple equilibria, so that an analytical solution is hard or even impossible to find. As mentioned above, computer simulation does not provide proofs *stricto sensu* but only allows for inductive reasoning; simulation may even appear as a "black box" in which the modeler defines the inputs (agents' attributes, initial conditions, rules of interaction) and observes the output (indicators), but may have limited understanding of the inner working of the system.

Such issues are behind the reservations of many non-ACE economists, who feel on more solid ground with conventional mathematical and statistical models. In response, the ACE community has become progressively more aware of methodological problems and has devoted increasing attention to validation and verification, with an exponential growth of contributions in recent years (see e.g. Fagiolo, Moneta and Windrum (2007); Fagiolo, Birchenham and Windrum (2007); Galán et al. (2009)). A full account of the whole range of discussions pertaining to validation would be outside the scope of this paper, which will simply outline some of the main lines of reflection; the interested reader is invited to refer to the specialized literature.

Broadly speaking, validation can be theoretical or empirical. The former is particularly appropriate in cases in which agent-based models are used for qualitative insight and theory generation, and basically consists in weighing model results against predictions derived from economic theory: for instance, the extent to which neoclassical supply-and-demand schemes are good predictors of decentralized market outcomes even if, say, exchanges do not follow tâtonnement rules, or agents do not maximize utility, can be assessed by comparing simulated prices and quantities to those that would prevail at the theoretical equilibrium point. Models with Zero-Intelligence agents in a general equilibrium framework are a case in point (Crockett, Spear and Sunder (2008)).

Empirical validation can take several forms, but a common approach starts from identifying a set of "stylized facts" in the real world, that is, following the definition of Nicholas Kaldor, broad tendencies that summarize the empirical data, ignoring that individual detail may be subject to snags and qualifications; then, the modeler builds an agent-based environment and endeavors to reproduce the stylized facts *jointly* in the simulation. Thus, choice of parameter values will be such that the simulated result is closest to the observed facts; put differently, it is the output of the model that is subject to validation, rather than its inputs. This approach is particularly appropriate for models that aim to explain some persistently observed empirical regularities, for instance models of financial markets (LeBaron (2002); Tesfatsion (2008)). One problem that frequently arises with output validation, however, is that different combinations of parameters and initial conditions may be consistent with the set of stylized facts of interest, so that some kind of validation is also desirable for the micro structure of the model.

In fact validation may also concern inputs, especially for models that aim to design some new institutional mechanism or rule, in a normative perspective; in such cases, parameter values should reproduce the observed characteristics of individuals, their interactions, and their local environment as closely as possible, so that the simulation can be trusted to yield plausible results. Examples of this type of research include design of matching mechanisms, of auctions (Sun and Tesfatsion (2006)), and of welfare benefit schemes (Pingle and Tesfatsion (2003)). However with input validation, it may be the case that parameters and/or initial conditions cannot be easily estimated, due to lack of sufficiently rich microdata: in particular, statistical databases very rarely include any details on decision-making and learning rules (Fagiolo, Moneta and Windrum (2007)).

One solution consists, as mentioned above, in performing parallel experiments with artificial and human subjects. They can provide much richer information on agent attributes, cognitive skills, and behavior, than other data sources; they thus contribute to defining the appropriate specification of agent behavior and the parameters of agent-based models. Still another approach to validation is based on involvement of stakeholders (Barreteau (2003)). To address social dilemmas, typically local-level conflicts related to the collective use of natural resources such as land or water, the very people concerned are asked to participate to model building with their own knowledge and understanding of their social context. They provide information that contributes to shaping the model behavioral rules, attributes, and parameters. Validation comes from actors' acceptance of the model as an adequate description of their problem and as a useful tool to address it. This approach is better than experiments in providing insight into agents' representations in the real-world, but allows only limited control and replicability; its findings are often context-dependent, and accumulation of empirically-based knowledge is relatively difficult (Rouchier (2007)). To date, stakeholder involvement models are less common in economics than in other social sciences, more familiar with fieldwork observations.

2.5 Diversity of modeling approaches

Constrained maximization at individual and equilibrium at collective levels form the core principles of neoclassical economics and give it some degree of unity and consistency. In contrast, ACE models share only minimal commonalities and are in fact very diverse. One reason is that the standard utility maximization and perfect information hypotheses admit not one but many alternatives: hence, their removal opens the way to a variety of choice criteria ranging from Zero-Intelligence to sophisticated learning processes, and to differing assumptions about how agents collect, process and accumulate information. Although use of experimental or other data can restrict the range of possible options, no decision-making model has emerged as a universally valid alternative so far; rather, many researchers believe that the context of choice may determine which conception is most appropriate in each case.

Modeling frameworks are also heterogeneous. For instance, ACE representations of the market often follow the experimental economics tradition of assigning reservation values to each buyer and seller, and then deriving aggregate supply and demand schedules by sorting seller values from lowest to highest and buyer values from highest to lowest. While allowing for straightforward linkages between agent simulations and human subject experiments, this approach does not facilitate comparison with standard microeconomics arguments, typically framed in terms of preferences, utility functions, and endowments. The main reason for this gap is that the experimental/ACE method draws heavily on Marshallian analysis, which was very much in the minds of the pioneers of market experiments such as Edward Chamberlin and Vernon Smith, but was at the same time losing ground elsewhere and has now virtually disappeared from microeconomics textbooks (Tubaro (2009)).

The community of agent-based modelers has recognized that heterogeneity of assumptions and frameworks of analysis may hinder comparability and transferability of knowledge between models (Fagiolo, Moneta and Windrum (2007)). Because computer simulation does not allow proofs as in deductive logic or mathematics, but only inductive reasoning, then reliability of a result is higher if it is reproduced by different modelers and/or on different software and hardware platforms. Hence, many researchers insist on the need to replicate, re-write and compare models more systematically (Hales, Rouchier and Edmonds (2003)), and some have gone as far as to propose a common protocol (Leombruni et al. (2006)).

An issue that has been less widely discussed in the literature is ACE's distinctiveness in the choice of its objects of study. To be sure, some of them overlap with those of previous economic theories, for instance the general principles regulating price formation and the coordinating capacities of decentralized market economies (e.g. Gode and Sunder (1993); Weisbuch, Kirman and Herreiner (2000); Axtell (2005); Gintis (2007)). Yet they also include a broad range of less conventional issues. Specifically, ACE goes beyond the traditional neoclassical view in which all inter-individual interactions are mediated by market prices, and explores a larger variety of coordinating processes and mechanisms. For instance, ACE draws on insight from game theory and experimental economics (Ostrom and Walker (2005)) to study how trust and reciprocity may appear in situations of imperfect information or uncertainty, in which agents can choose between cooperative or defective behaviors. Under these conditions, an agent may accept to be vulnerable to the actions of another based on the expectation that the other will cooperate; this is all the more likely as interactions are repeated over time and agents have the possibility to reciprocate. Agent-based models of trust and reciprocity can be used to explain, among other things, the formation of long-term business relationships between firms and their suppliers and customers (e.g. Kim (2009)).

Other questions that ACE models deal with include loyalty, which may arise in cases in which heterogeneous agents interact repeatedly and at each step, choose which agent to interact with and which actions to perform. Loyalty has been especially explored in the study of perishable good markets (fruits, vegetables, and fish), where buyer/seller interactions are repeated on a daily basis due to limited possibilities to form stocks, and participants are very dependent on the regularity, and predictability, of their interactions. It has been shown that loyalty towards the same buyers or sellers increases predictability and thus market efficiency, while opportunism (i.e. shopping around for the cheapest prices) makes transactions less predictable, lowers efficiency, and increases waste (Rouchier (2004), Kirman and Vriend (2000)). Reputation also has a place in ACE models of the market. It matters when buyers need to choose from among a group of sellers but have imperfect information on the quality of the goods or services on offer; in such cases, they can take the reputation of a seller, i.e. other buyers' judgment, as an indicator of quality. This principle may give rise to complex dynamic interactions in which sellers endeavor to improve the image they convey to buyers and the latter exchange with each other information about sellers. In the absence of other sources of information, use of reputation as an indicator can lead to strong potential gains in terms of buyer and seller satisfaction (Rouchier (2008)). To give another example, agent-based models also consider endogeneity of consumer preferences due to social influence. The idea is that tastes and preferences may evolve following the marketing and advertising strategies of firms (Janssen and Jager (2003)) or peer influence, with social processes such as imitation, social comparison and status-seeking (Dosi et al. (1999); Kemp (1999)), which give rise in some cases to forms of conspicuous consumption as those originally described by Thorstein Veblen (Friedman and Ostrov (2008)).

Clearly, inclusion of non-traditional topics is not only due to linkages with experimental economics research; nor does it entirely derive from the specific assumptions of ACE and its emphasis on imperfect information, non-deductive reasoning, non-Walrasian framework of analysis. It is also related to ACE's closeness with neighboring disciplines; in particular, sociology already has a long tradition of investigating relationships between market participants that are not mediated by prices, and may involve coexistence of forms of competition and cooperation (Smelser and Swedberg (2005)).

Combinations of new hypotheses, new tools and new topics allow ACE to explore a wider range of “possible worlds”, and by so doing, to enrich economists’ understanding of the market mechanism. This perspective is all the more promising as an increasing amount of research is done in parallel with experimental and game-theoretic work, not to mention psychological and sociological research. Nevertheless, widening of topics together with inputs from other disciplines may hamper comparison with other parts of economics and make interpretation of ACE models less straightforward. This may discourage other economists from developing an interest in the field.

3. The impact of ACE on the methodology of economics

Based on the discussion conducted so far, it is now possible to frame a broad reflection on the relationships between ACE and economic methodology. A first issue is, of course, that computer simulation is commonly perceived as less rigorous than the logical deduction and mathematics that imposed themselves as primary tools for economic reasoning in the aftermath of World War II, with the seminal contributions of Paul Samuelson and the Arrow-Debreu general equilibrium model. In this sense, computer simulation is bound to be suspicious for many mathematically-trained economists.

Yet the ongoing tendency is to broaden the range of admissible methods of research in economics, and to lessen the centrality of the abstract, deductive mathematical method that was dominant until recently. Increasingly sophisticated statistical and econometric techniques (e.g. instrumental variables, matching, difference-in-difference, etc.) have raised the prestige of empirical research; laboratory experiments are much more widely performed, and field experiments are gaining ground, for instance in the fight against poverty; even neuro-imaging is attracting growing attention by economists. The result is a remarkable diversification of the permissible methodologies. Though not identical to pure mathematics, these methods are all based on some kind of quantitative or scientific reasoning and are increasingly demanding in terms of the technical and methodological skills that are necessary to apply them –a reason that partly explains their perceived solidity and their newly-earned status in economics. To the extent that computer simulation can be seen as part of this trend, it may benefit from it and contribute, in turn, to transform economics.

Further, the growing amount of methodological reflection on validation and verification issues that has been recently developed within the ACE community may have wider repercussions as it raises the very general questions of cross-model comparability, generation of cumulative knowledge, relationship between theory and data, interpretation of empirical findings. It may also cross-fertilize with what is currently being done in other fields, for instance concerning replication, data quality, and reporting of results, which are also widely discussed in relation to (among other things) applied econometrics.

The preceding sections also show that ACE contributes to shaping a novel theory of individual economic behavior, taking into account human cognitive limits on the one hand, and the existence of decision-making situations for which pure deduction is inappropriate on the other hand. In this perspective, agent-based models appear as valuable complements to game theory and experimental/behavioral economics; more precisely by studying interactions and two-way feedback effects, they provide insight into the possible implications of different behavioral assumptions in a given social context. In particular, a major result of ACE research is that neoclassical utility maximization and perfect information hypotheses are much less general than what is usually thought, and apply in fact only to a sub-set of choice problems. Agent-based market models provide especially strong evidence that a number of standard economic results rely less on rational choice theory than is typically assumed. Indeed in single-market models, the same supply-and-demand equilibrium obtains both with utility maximizing and with “Zero-Intelligence” agents (Gode and Sunder (1993); Gode and Sunder (1997)): this result confirms that the familiar supply and demand model, of Marshallian origin, is a good predictor of market outcomes and is robust to changes in individual behavioral assumptions. In light of this result, computer simulation can be said to provide a new way to test the consistency of existing theories and to assess the precise role of each of their underlying assumptions.

Another feature of ACE which raises questions for the methodology of economics is its embeddedness in an interdisciplinary context. Major contributions to the methodological and substantive development of ACE came from fields of research outside economics, and cross-disciplinary dialogue and exchange are widespread within the agent-based modeling community. This process has allowed ACE economists to absorb contents from other disciplines over time, and to transform their work accordingly –in an attitude opposed to the infamous “imperialist” tradition of the discipline. This tendency has both advantages and disadvantages. Interdisciplinary relationships enrich economic reflection with new research questions, variables, and assumptions. The drawback is that insight from other disciplines may hinder comparison with intra-disciplinary theories and results; as a result, it may be difficult to assess ACE findings and their contribution to a better understanding of economic phenomena. In the next few years, the challenge for ACE will be to strike a balance between the two opposite needs of maintaining interdisciplinary contacts and enhancing dialogue with other parts of economics. The extent to which it will succeed in doing so will determine its future place in a renewed, but methodologically highly demanding, economics discipline.

4. Conclusion

This paper has endeavored to provide an overview of ACE from a methodological viewpoint. It has reviewed similarities and differences with respect to existing economics, methodological issues, and wider implications for the discipline as a whole. Emphasis has been placed on how ACE shares the long-lasting concern of the economics discipline for the unintended social consequences of purposeful individual action. More to the point, evidence has been provided of complementarities between ACE and state-of-the-art research in experimental and behavioral economics as well as game theory, in a common effort to revise and renew economic theory. In particular, ACE has reinforced the experimental finding that key economic results rely less on rational choice theory than is usually assumed, and has provided insight into the importance of market structures and rules in yielding market-level outcomes. It has also enlarged the range of behaviors that are of interest for economists to understand the functioning of markets, by taking into account interactions that are not mediated by prices. In this perspective, ACE appears as a useful supplement to existing economic tools rather than a radically different alternative, and has potential for better integration into economics in the years to come. Although methodological issues such as those that concern validation and verification are still to be addressed, a great deal of work has already been done and may cross-fertilize with reflection in other areas of economics. Finally, the historically strong linkages with other disciplines may yield further advantages in the future if they are accompanied by improved intra-disciplinary exchanges.

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