Determination of DC-OPF Dispatch & LMP Solutions in the AMES Testbed

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www.econ.iastate.edu/tesfatsi/AMESLMPDetermination.LT.pdf

Topic Presentation

- Preliminary review of demand, supply, competitive market clearing equilibrium, and "Total Net Surplus" (TNS)
- Calculation of TNS in AMES
- Standard DC-OPF Problem Formulation used for the AMES ISO: Max TNS subject to generation & capacity constraints
- DC-OPF as a General Nonlinear Programming Problem (GNPP)
- LMPs as GNPP Lagrange Multiplier Solutions

Basic Demand/Supply Concepts

- In standard economic market analyses:
  - ***Ordinary*** supply and demand schedules give ***quantity Q for each (per unit) price P:***

$$Q = S^o(P); \; Q = D^o(P)$$

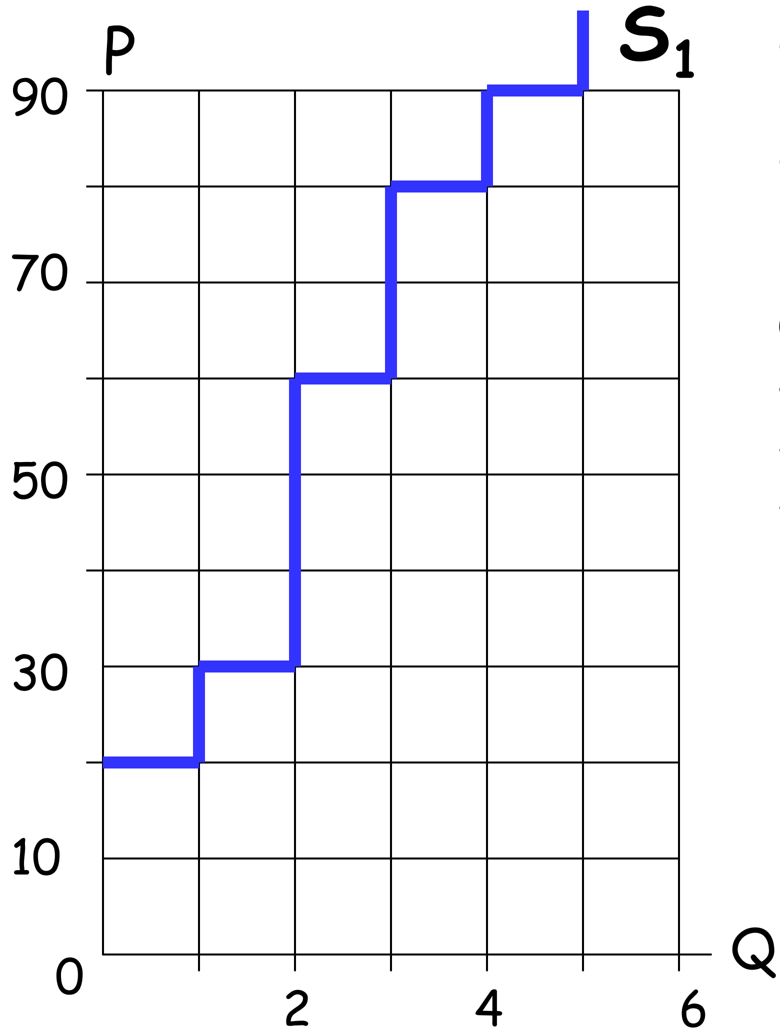
  - ***Inverse*** supply and demand schedules give ***(per unit) price P for each quantity:***

$$P = S(Q); \quad P = D(Q)$$

DEMAND/SUPPLY EXAMPLE

Seller S1 Supply Schedule

Inverse Form $P = S_1(Q)$



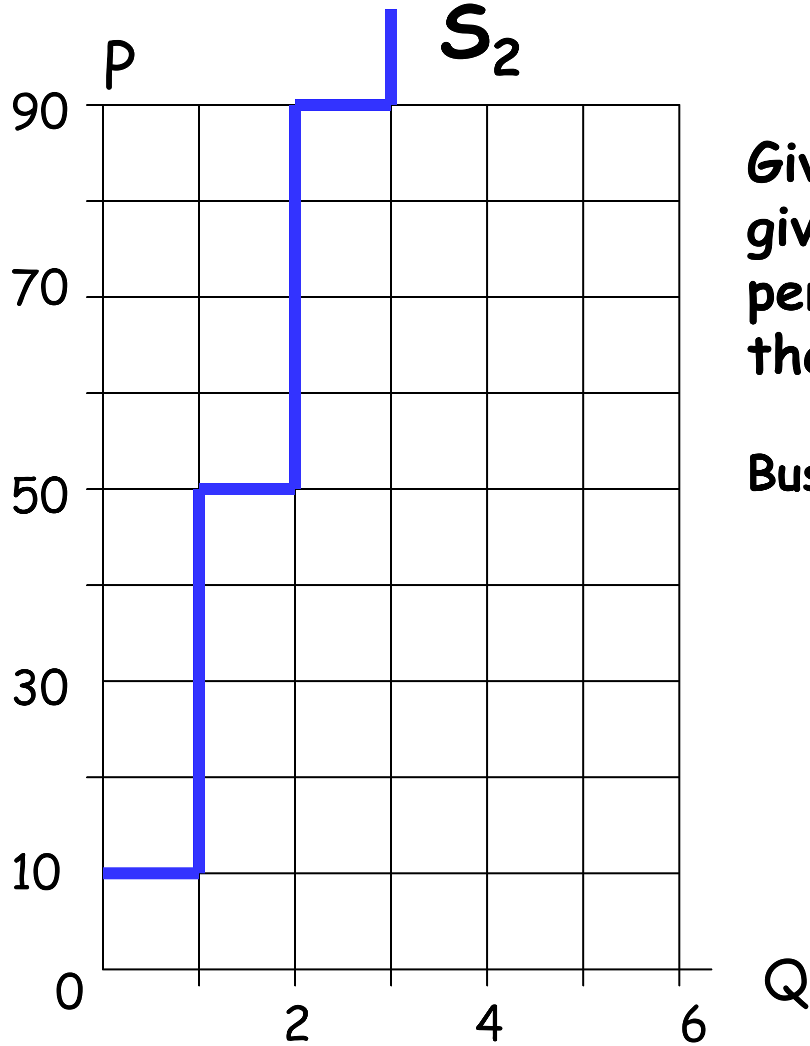
Let Q = Apple Amount (in bushels)

Let P = Per-unit price of apples (i.e., dollars $ per bushel)

Given any Q, the function $P=S_1(Q)$ gives Seller S1's minimum acceptable per-unit sale price ($/bushel) for the "last" unit supplied at this Q.

Bushel Unit	Seller S1 Min Sale Price
1	$20
2	$30
3	$60
4	$80
5	$90 (Max Cap = 5)
6	$∞

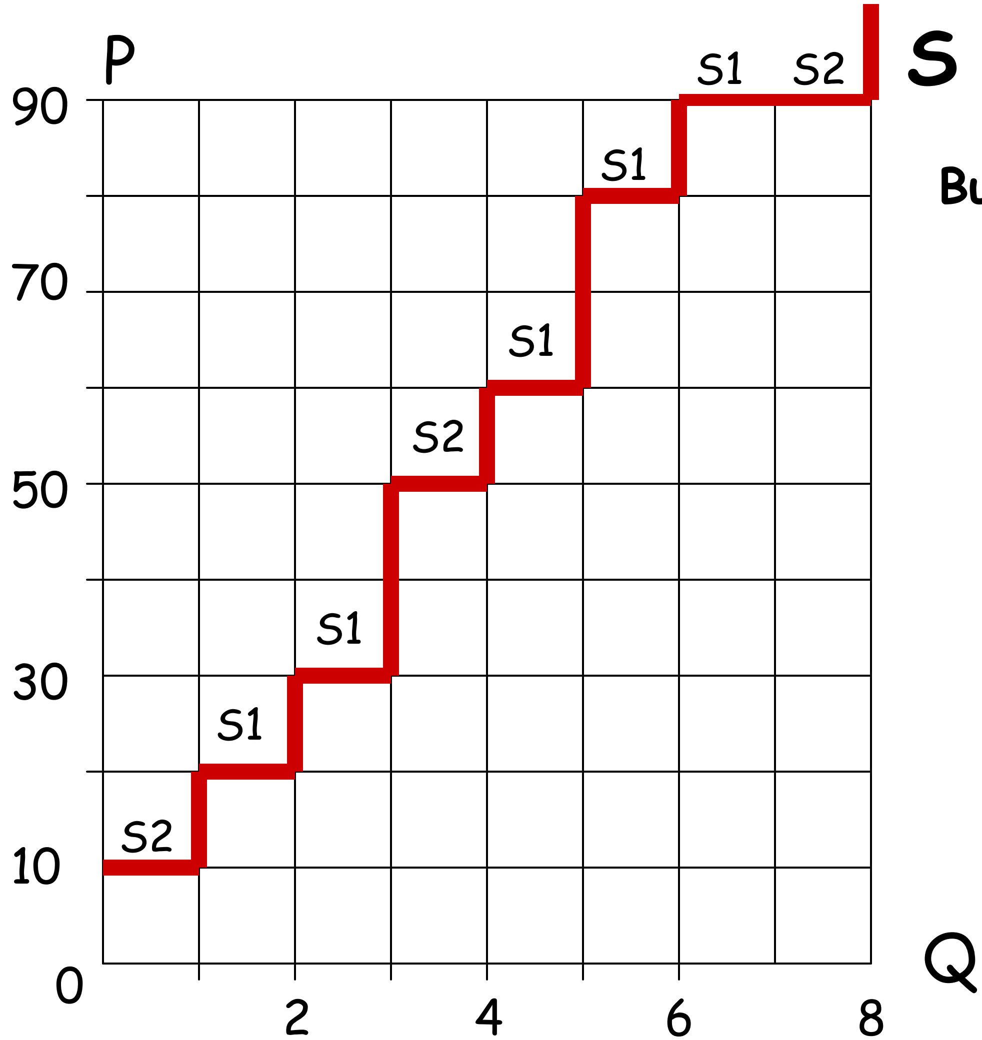
Seller S2 Supply Schedule
Inverse Form P = $S_2(Q)$



Given any Q, the function $P=S_2(Q)$ gives Seller S2's minimum acceptable per-unit sale price ($/bushel) for the "last" unit supplied at this Q.

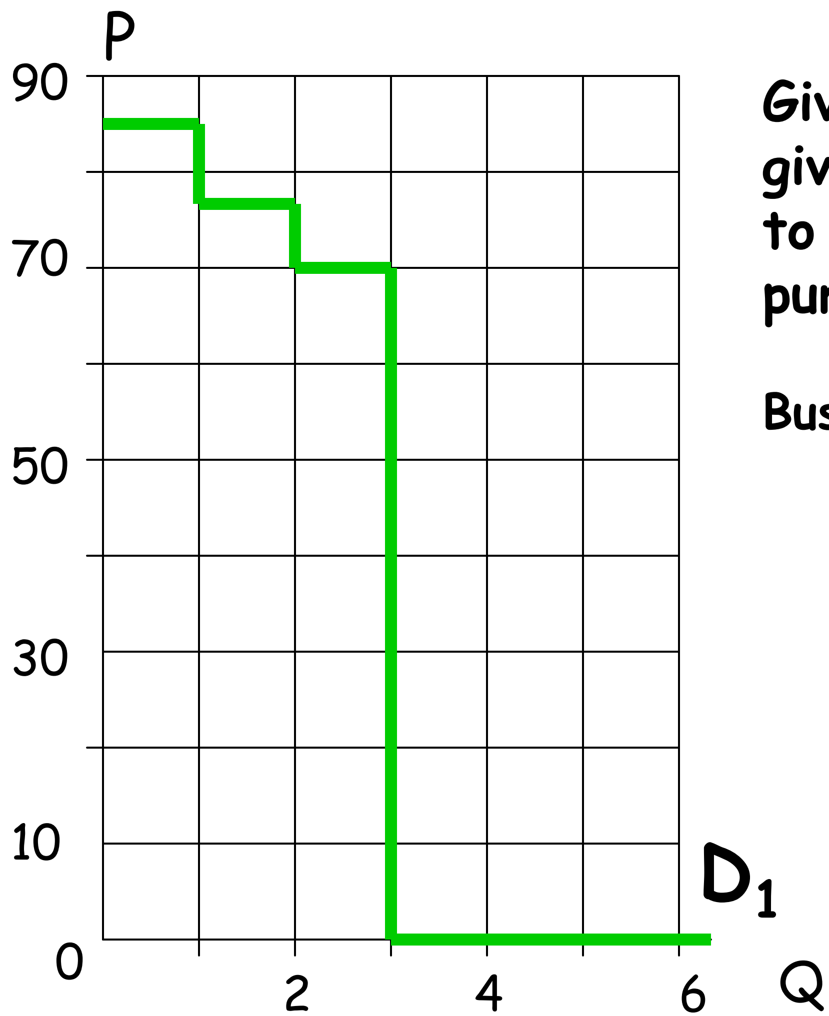
Bushel Unit	**Seller S2 Min Sale Price**
1	$10
2	$50
3	$90 (Max Cap = 3)
4	$ ∞

Total Supply Schedule S (Sellers S1 & S2)
Inverse Form P = S(Q)



Bushel Unit	Min Seller Price	
1	$10 (S2)	
2	$20 (S1)	
3	$30 (S1)	
4	$50 (S2)	
5	$60 (S1)	
6	$80 (S1)	
7	$90 (S1/S2)	
8	$90 (S2/S1)	Max Cap = 8
9	$ ∞	

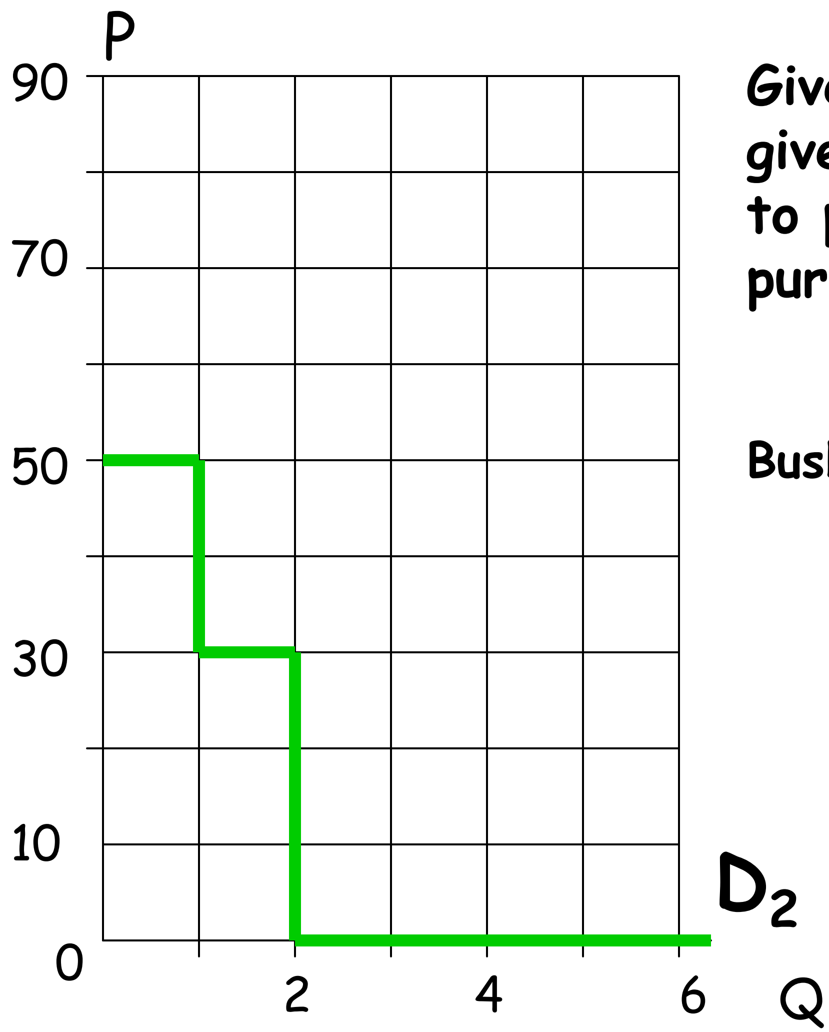
Buyer B1 Demand Schedule
Inverse Form $P = D_1(Q)$



Given any Q, the function $P=D_1(Q)$ gives Buyer B1's maximum willingness to pay ($/bushel) for the "last" unit purchased at this Q.

Bushel Unit	Buyer B1's Max Per-Unit Price
1	$84
2	$76
3	$70 (Max Units = 3)
4	$ 0

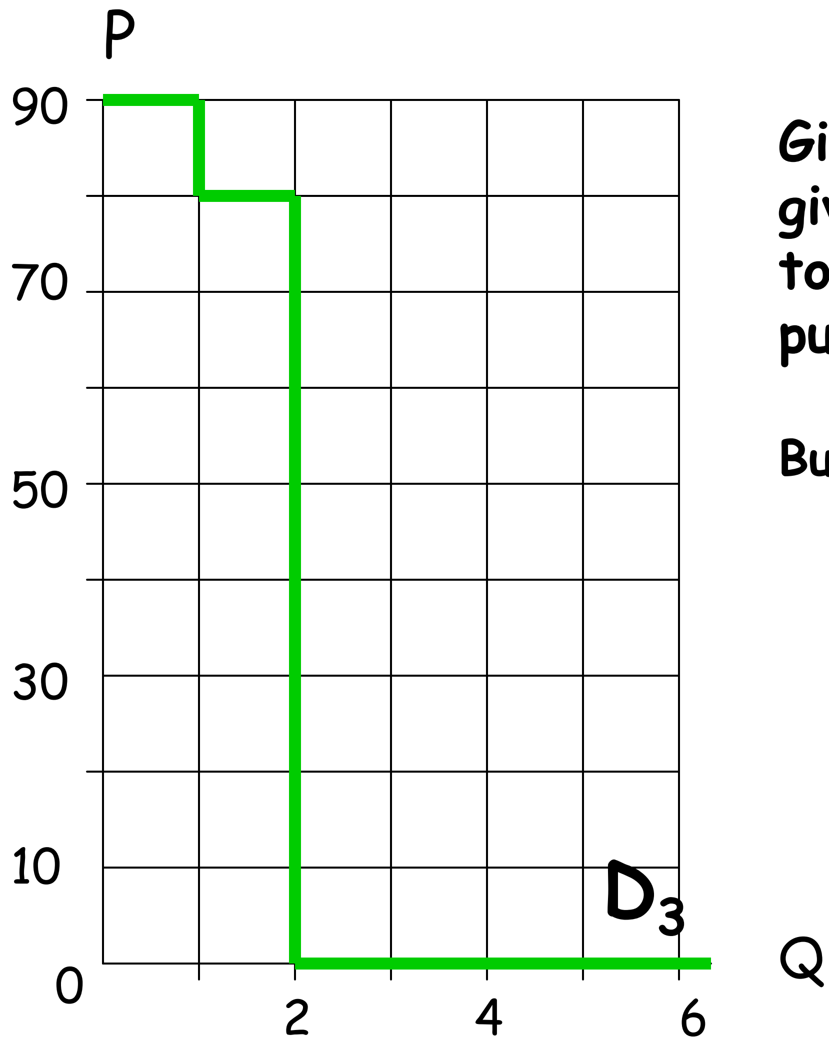
Buyer B2 Demand Schedule
Inverse Form $P = D_2(Q)$



Given any Q, the function $P=D_2(Q)$ gives Buyer B2's maximum willingness to pay ($/bushel) for the "last" unit purchased at this Q.

Bushel Unit	Buyer B2's Max Per-Unit Price
1	$50
2	$30 (Max Units = 2)
3	$ 0

Buyer B3 Demand Schedule
Inverse Form P = $D_3(Q)$

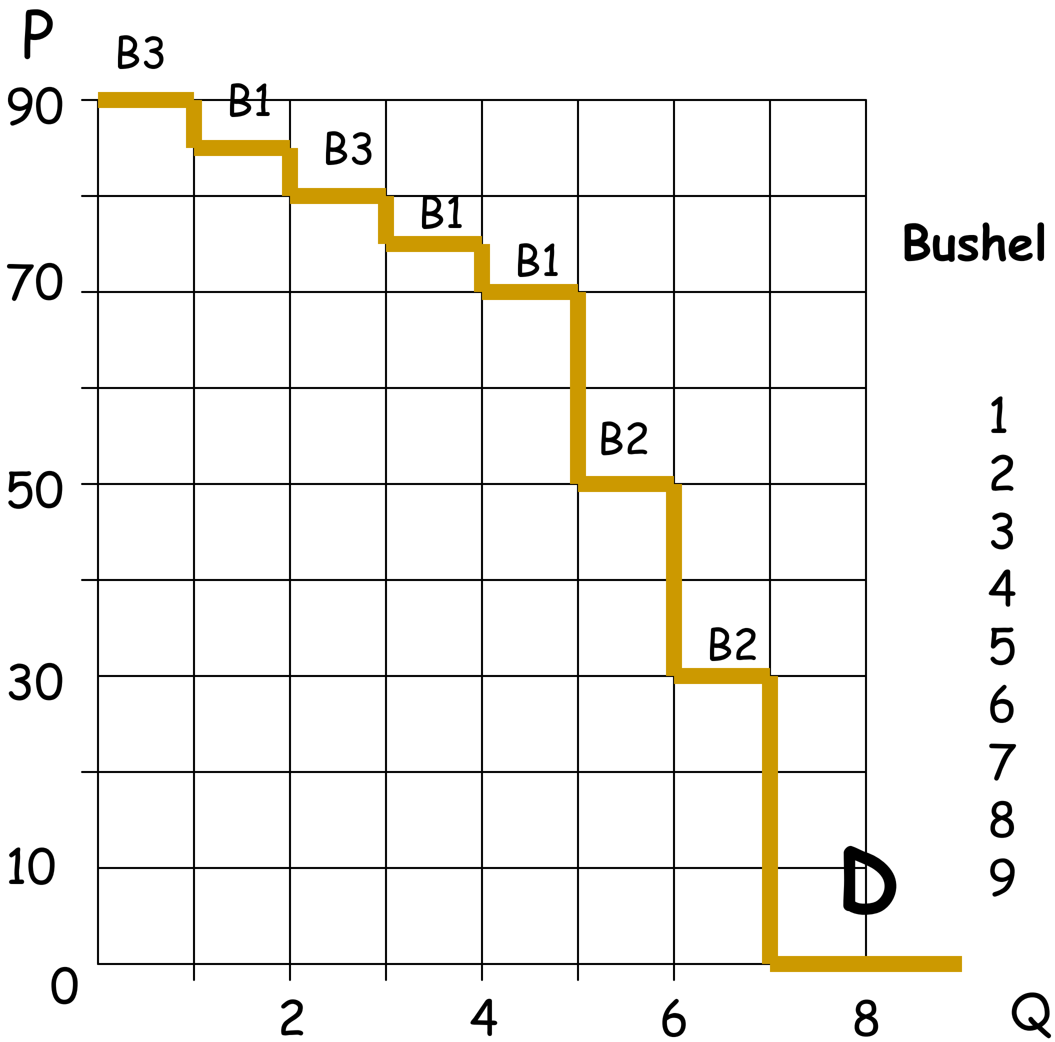


Given any Q, the function P=D_3(Q) gives Buyer B3's maximum willingness to pay ($/bushel) for the "last" unit purchased at this Q.

Bushel Unit	**Buyer B3's Max Per-Unit Price**
1	$90
2	$80 (Max Units = 2)
3	$ 0

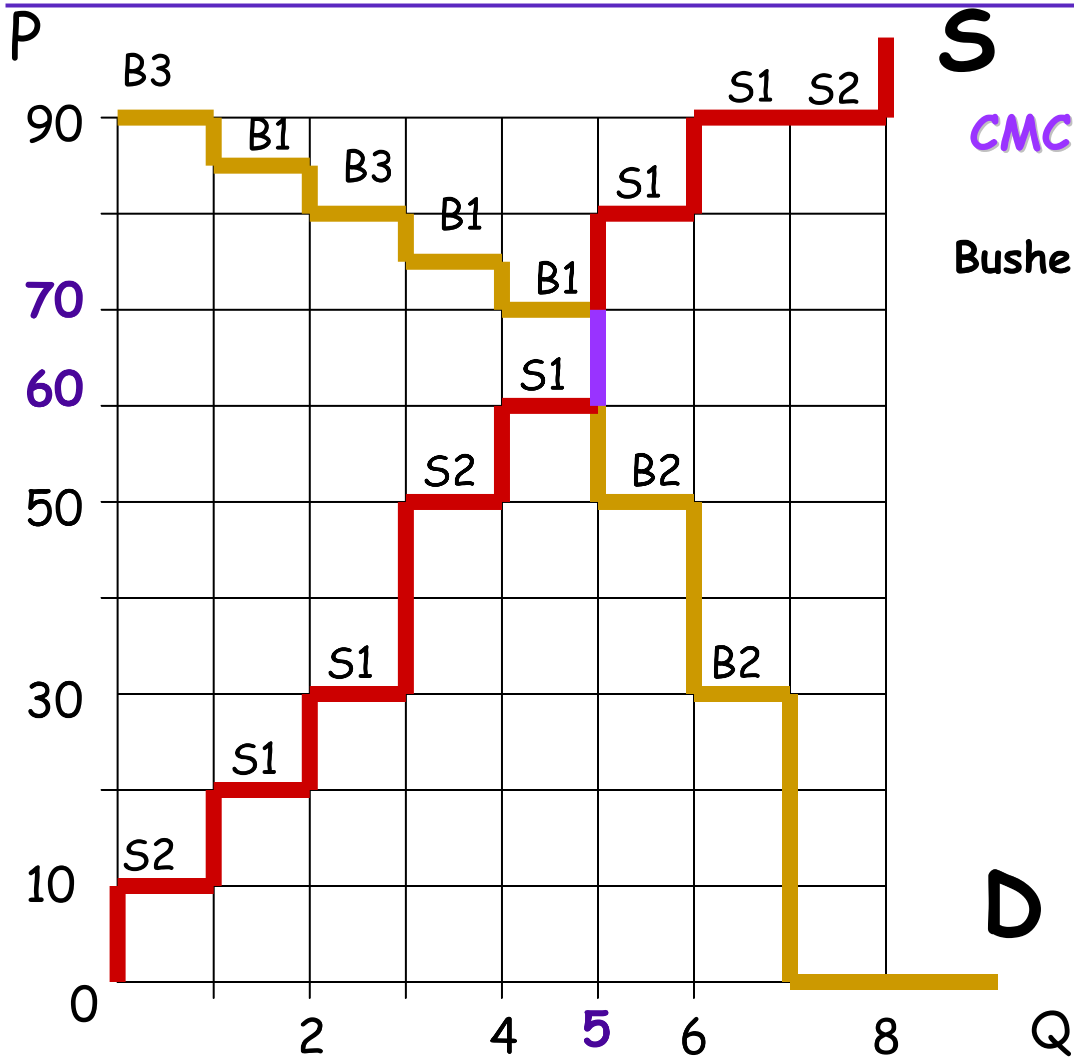
Total Demand Schedule D (Buyers B1, B2, & B3)

Inverse Form P = D(Q)



Bushel Unit	Max Buyer Per-Unit Price	
1	$90	(B3)
2	$84	(B1)
3	$80	(B3)
4	$76	(B1)
5	$70	(B1)
6	$50	(B2)
7	$30	(B2) Max Units=7
8	$0	
9	$0	

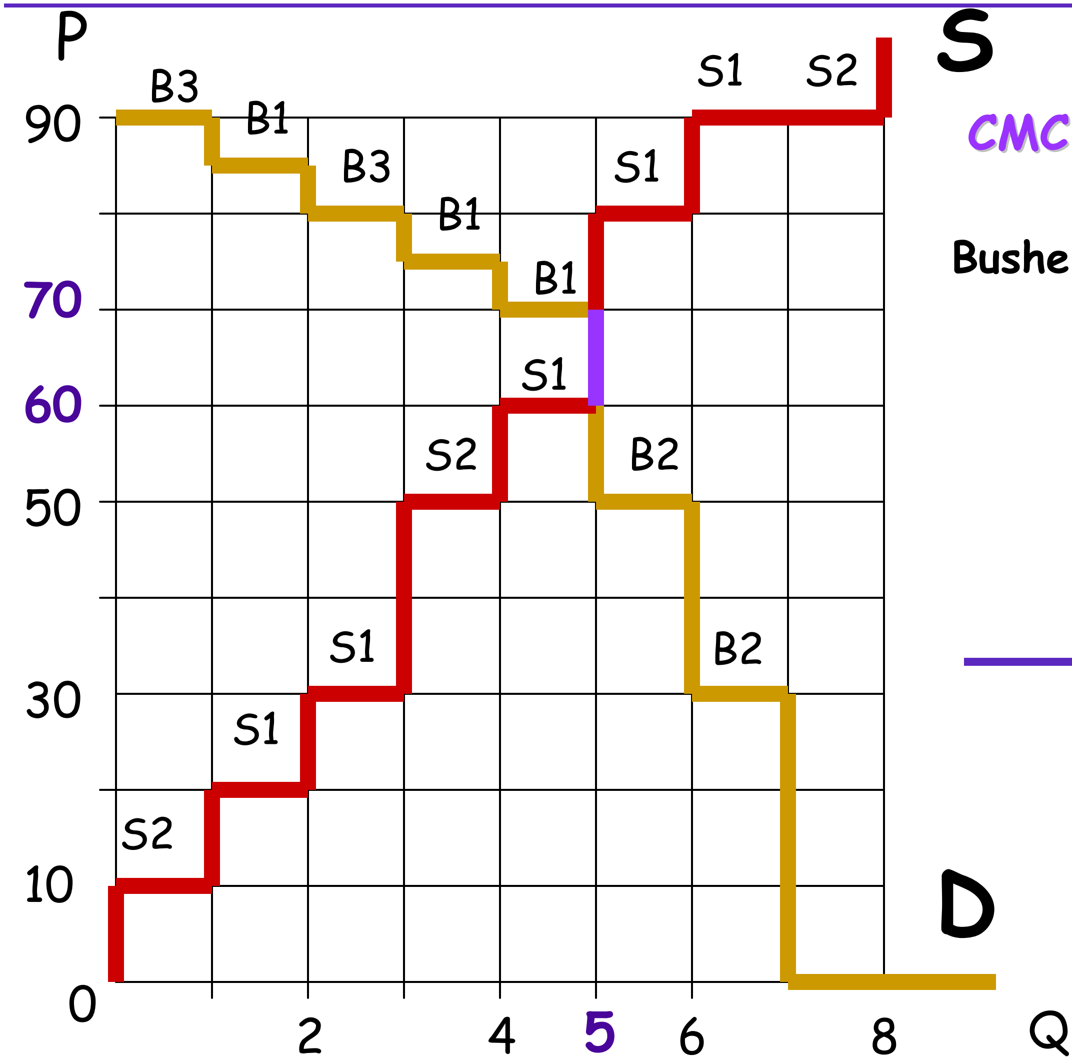
Competitive Market Clearing (CMC) Points: Intersection of S and D Schedules



CMC Pts: Q*=5, $60 ≤ P* ≤ $70

Bushel Unit	MaxBuyPrice	MinSellPrice
1	$90	$10
2	$84	$20
3	$80	$30
4	$76	$50
5	$70	$60
6	$50	$80
7	$30	$90
8	$0	$90
9	$0	$∞

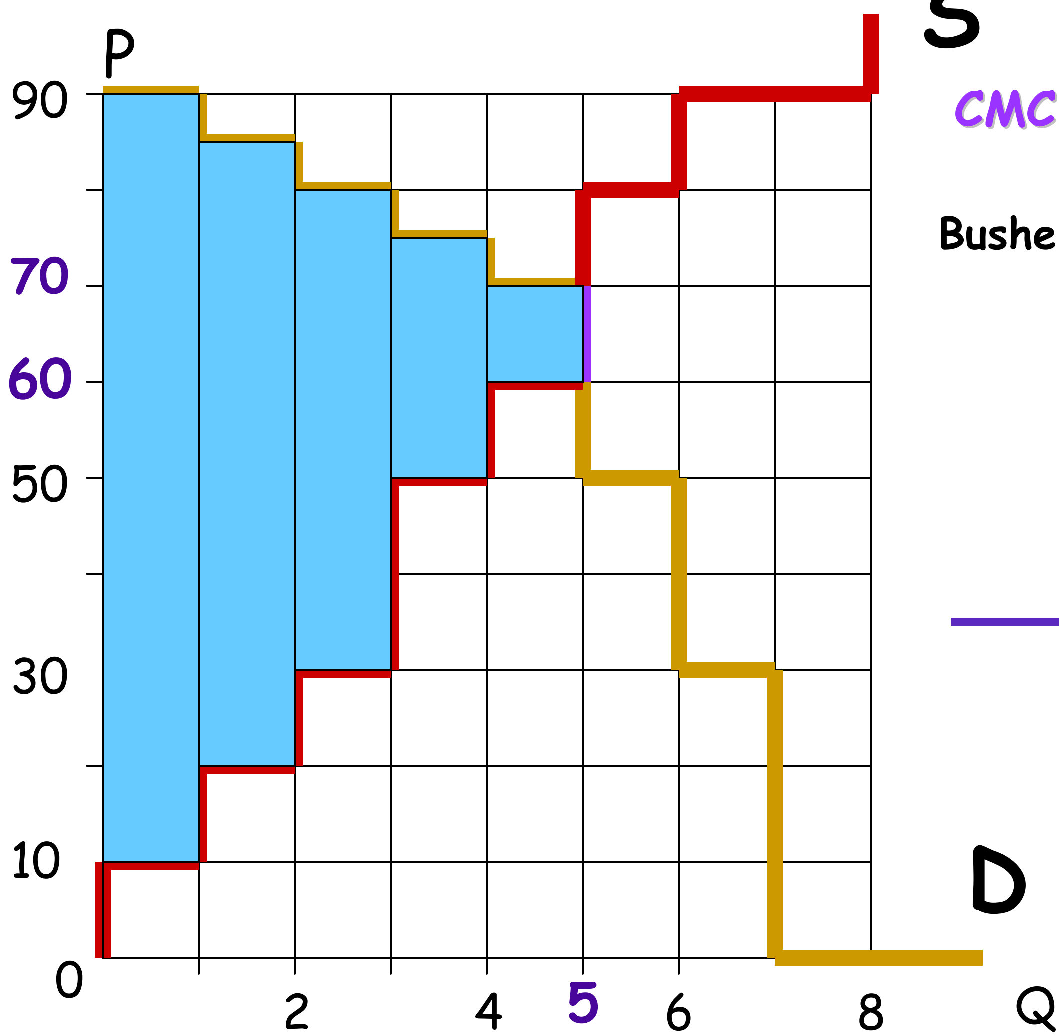
Remark: *Inframarginal* (traded) units versus *extramarginal* (non-traded) units at CMC Pts



CMC Pts: Q*=5, \$60 ≤ P*≤ \$70

Bushel Unit	MaxBuyPrice	MinSellPrice
1	\$90	\$10
2	\$84	\$20
3	\$80	\$30
4	\$76	\$50
5	\$70	\$60
6	\$50	\$80
7	\$30	\$90
8	\$ 0	\$90
9	\$ 0	$\$\infty$

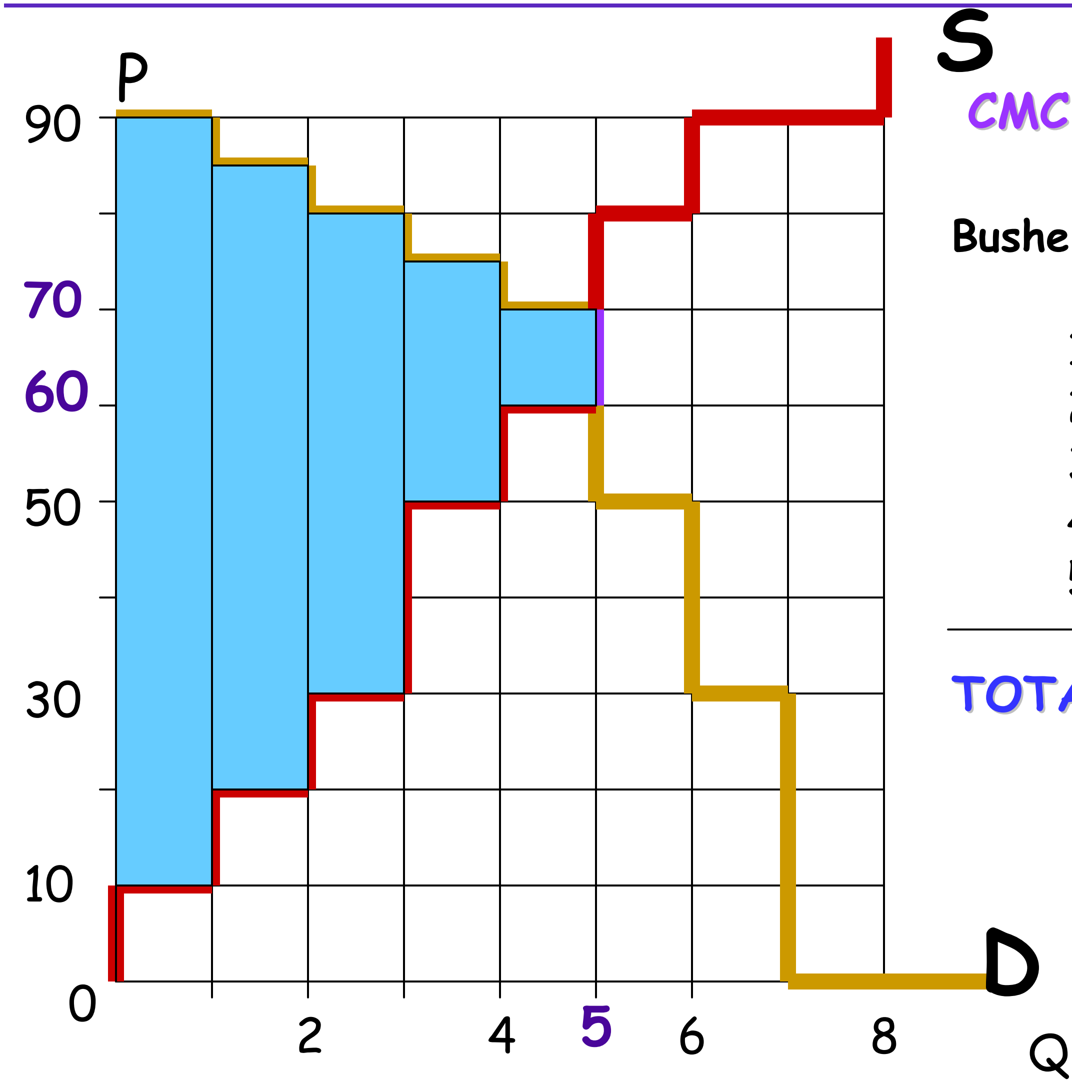
Total Net Surplus at CMC Points
(invariant to particular choice of CMC Point)



CMC Pts: Q*=5, $60 ≤ P*≤ $70

Bushel Unit	MaxBuyPrice	MinSellPrice
1	$90	$10
2	$84	$20
3	$80	$30
4	$76	$50
5	$70	$60
6	$50	$80
7	$30	$90
8	$ 0	$90
9	$ 0	$∞

Total Net Surplus at CMC Points…

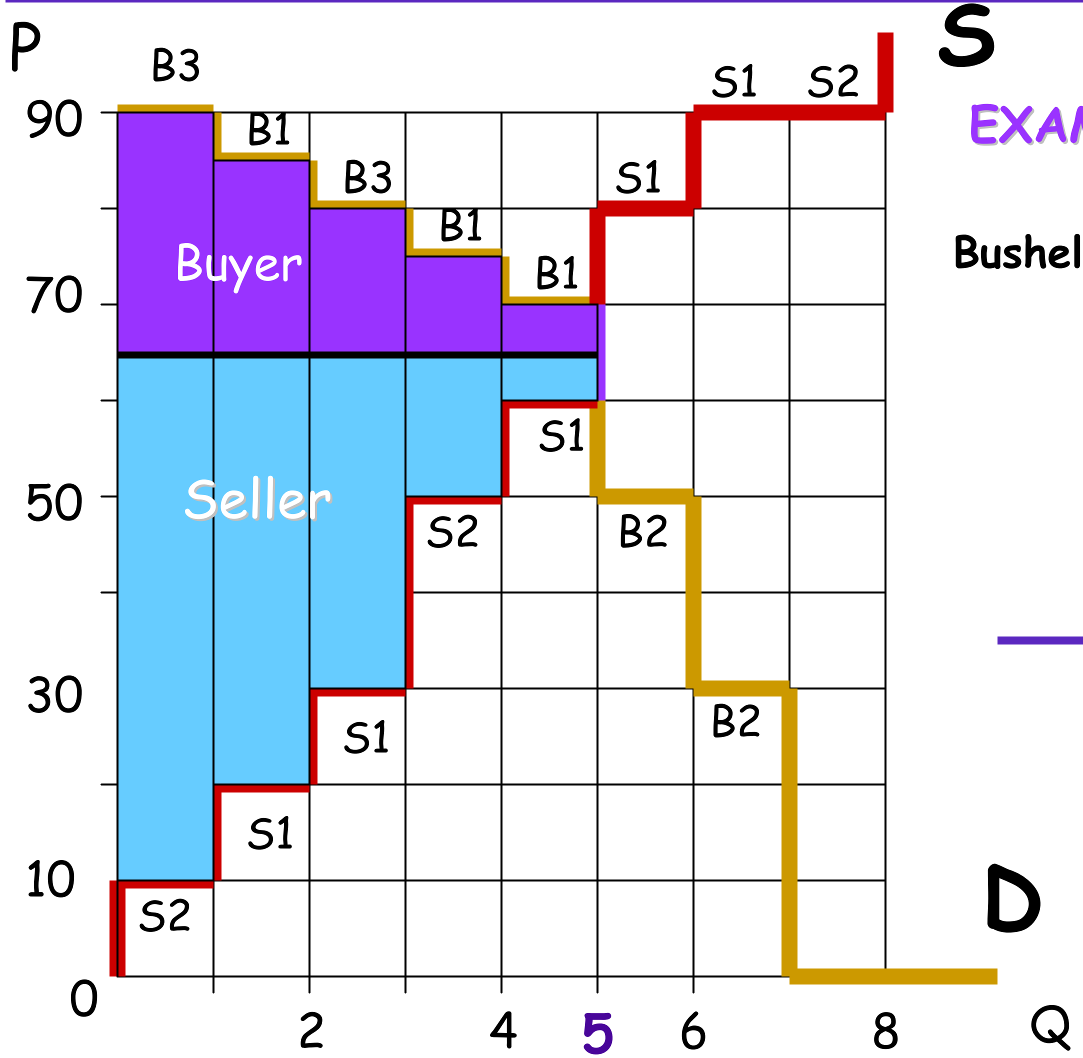


CMC Pts: Q*=5, $60 ≤ P*≤ $70

BushelUnit	MaxBuyP		MinSellP		Net Surplus
1	$90	-	$10	=	$80
2	$84	-	$20	=	$64
3	$80	-	$30	=	$50
4	$76	-	$50	=	$26
5	$70	-	$60	=	$10

TOTAL NET SURPLUS: $230

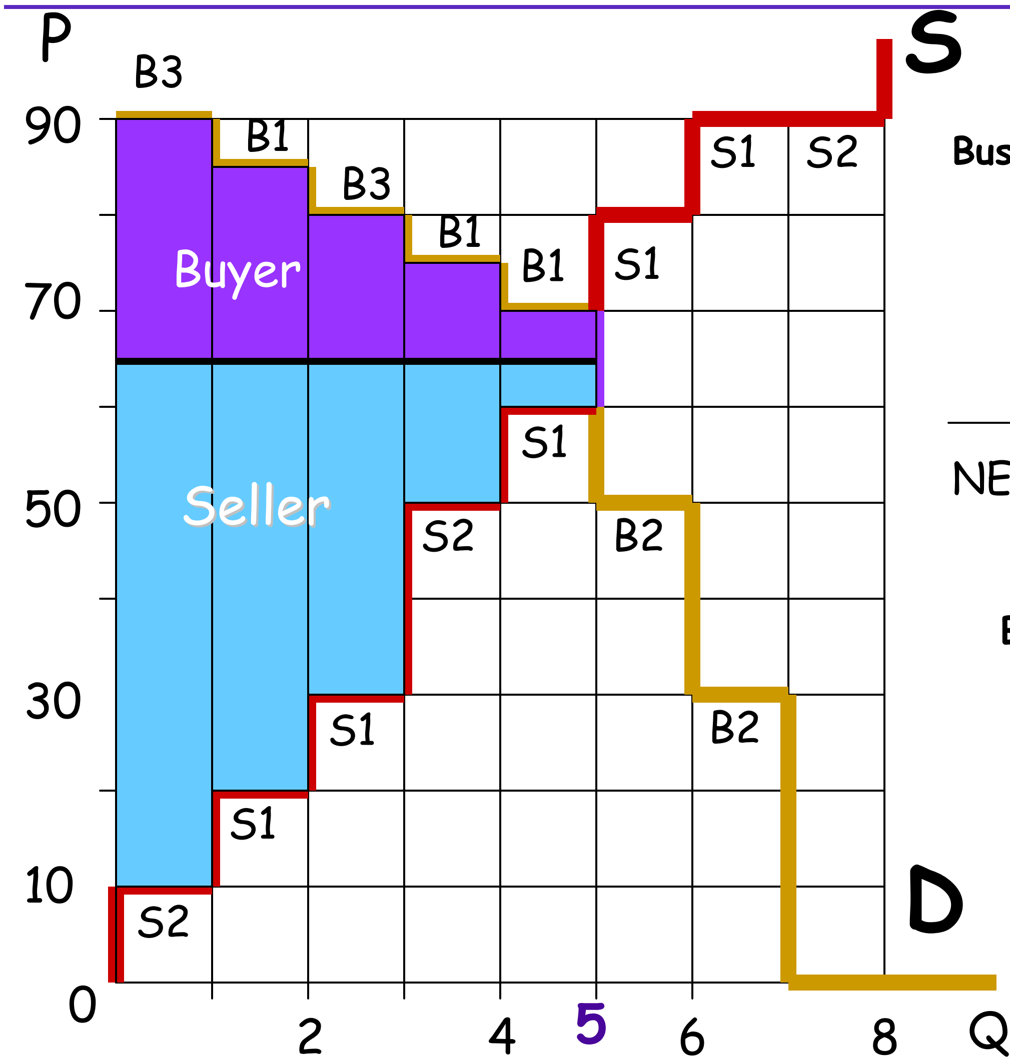
Net Buyer/Seller Surplus at CMC Points (surplus division DOES depend on CMC point)



EXAMPLE: Q*=5, P*= $65

Bushel Unit	MaxBuyPrice	MinSellPrice
1	$90	$10
2	$84	$20
3	$80	$30
4	$76	$50
5	$70	$60
6	$50	$80
7	$30	$90
8	$ 0	$90
9	$ 0	$∞

Net Buyer/Seller Surplus at CMC Points…



EXAMPLE: Q*=5, P*= $65

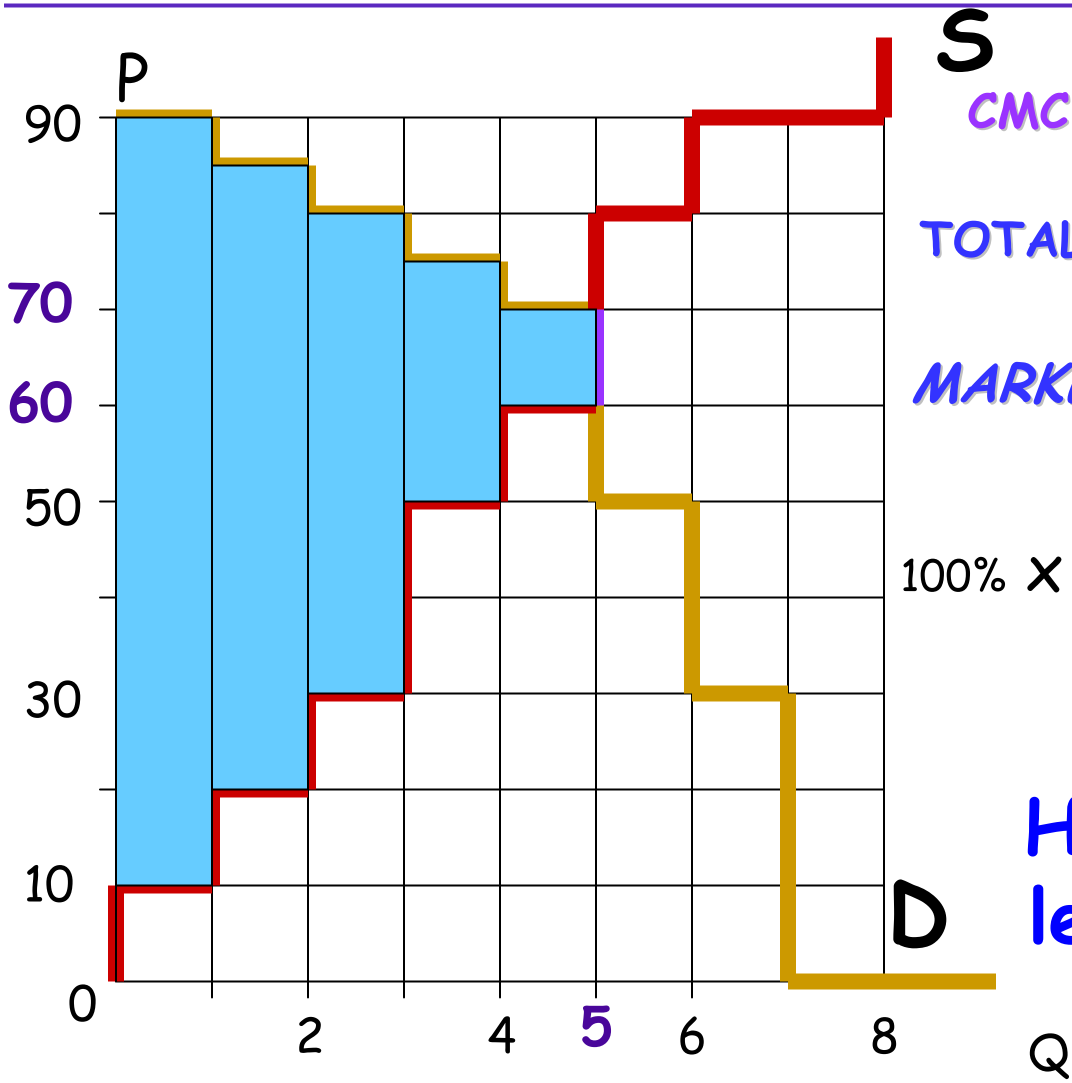
BushelUnit	MaxBPrice		P*=65		BuySurplus	
1	$90	-	$65	=	$25	B3
2	$84	-	$65	=	$19	B1
3	$80	-	$65	=	$15	B3
4	$76	-	$65	=	$11	B1
5	$70	-	$65	=	$ 5	B1

NET BUYER SURPLUS: $75

BushelUnit	P*=65		MinSPrice		SellSurplus	
1	$65	-	$10	=	$55	S2
2	$65	-	$20	=	$45	S1
3	$65	-	$30	=	$35	S1
4	$65	-	$50	=	$15	S2
5	$65	-	$60	=	$ 5	S1

NET SELLER SURPLUS: $155

Market Efficiency (ME)



CMC Pts: Q*=5, $60 ≤ P*≤ $70

TOTAL NET SURPLUS (TNS): $230

MARKET EFFICIENCY (ME) =

100% X $\frac{\text{Actual Extracted Surplus}}{\text{Max Possible TNS}}$

How could ME be less than 100% ?

ME < 100% under What Conditions?

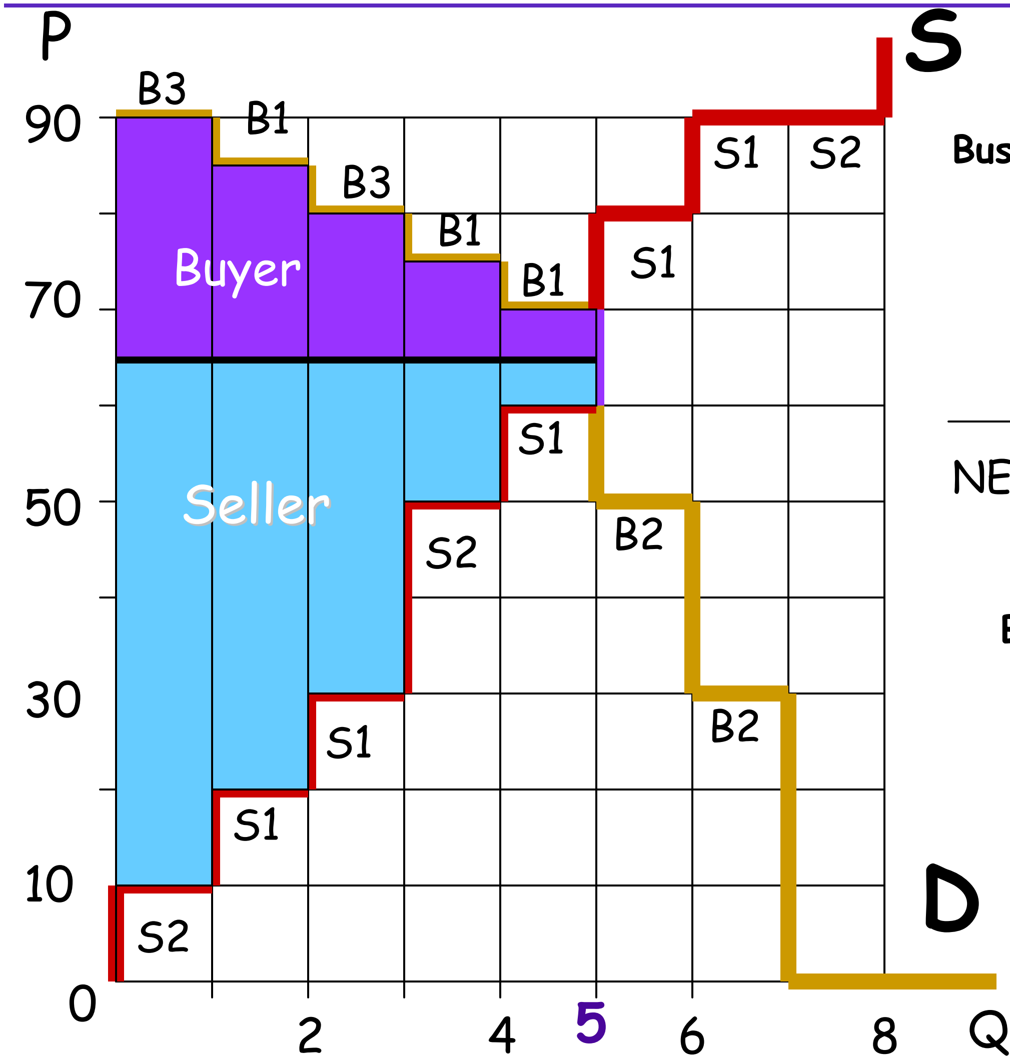
- Some "inframarginal" quantity unit FAILS to trade

- Or some "extramarginal" quantity unit SUCCEEDS in being traded

NOTE: If the price received by the seller of some quantity unit is LESS than the price paid by the buyer of this quantity unit (so some net surplus is extracted by a "third party"), then Buyer Net Surplus + Seller Net Surplus < 100%

→ ISO's in wholesale power markets !

Market Power: Ability of a trader to extract *more* surplus than he would get at CMC Point



EXAMPLE: Q*=5, P*= $65

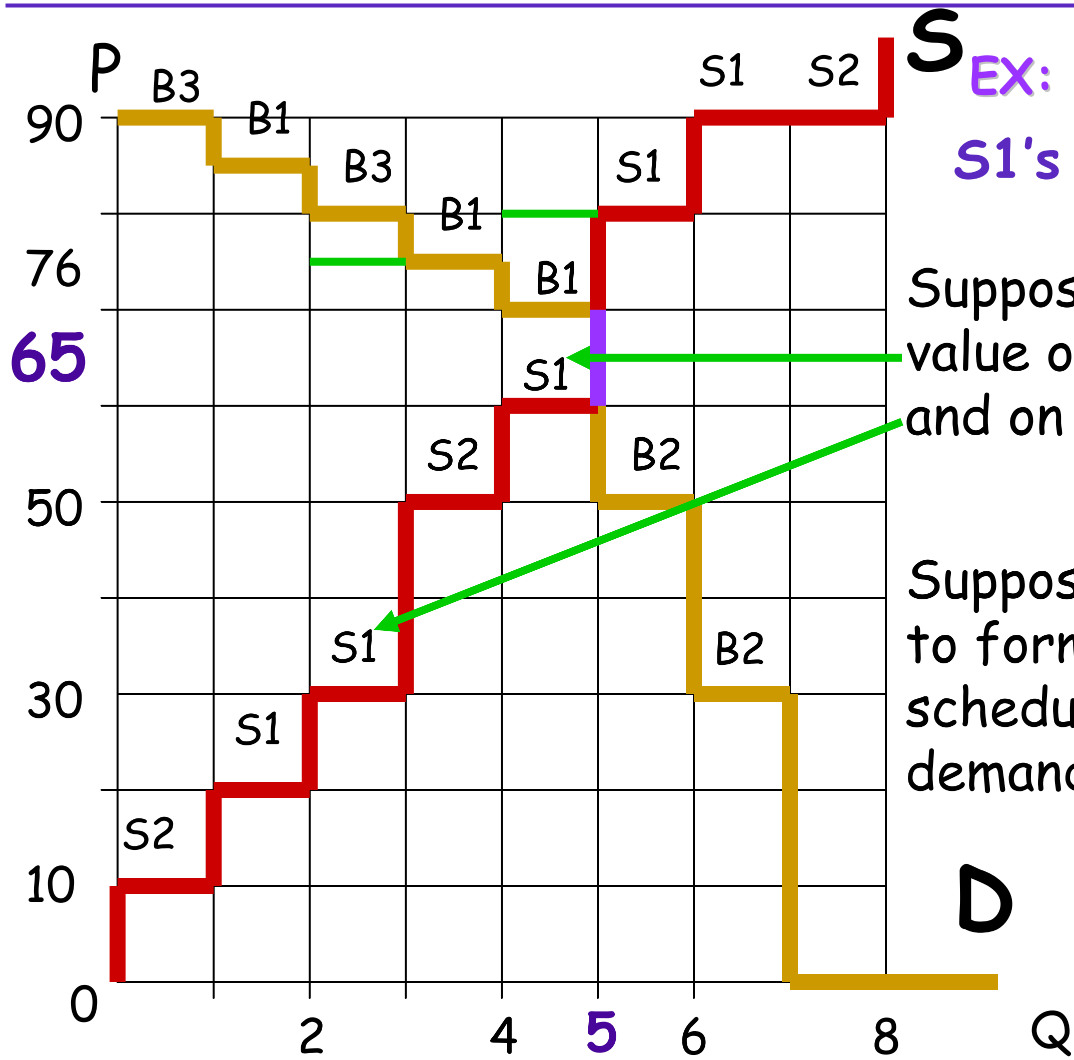
BushelUnit	MaxBPrice		P*=65		BuySurplus	
1	$90	-	$65	=	$25	B3
2	$84	-	$65	=	$19	B1
3	$80	-	$65	=	$15	B3
4	$76	-	$65	=	$11	B1
5	$70	-	$65	=	$ 5	B1

NET BUYER SURPLUS: $75

BushelUnit	P*=65		MinSPrice		SellSurplus	
1	$65	-	$10	=	$55	S2
2	$65	-	$20	=	$45	S1
3	$65	-	$30	=	$35	S1
4	$65	-	$50	=	$15	S2
5	$65	-	$60	=	$ 5	S1

NET SELLER SURPLUS: $155

Example: Does S1 have any market power ?



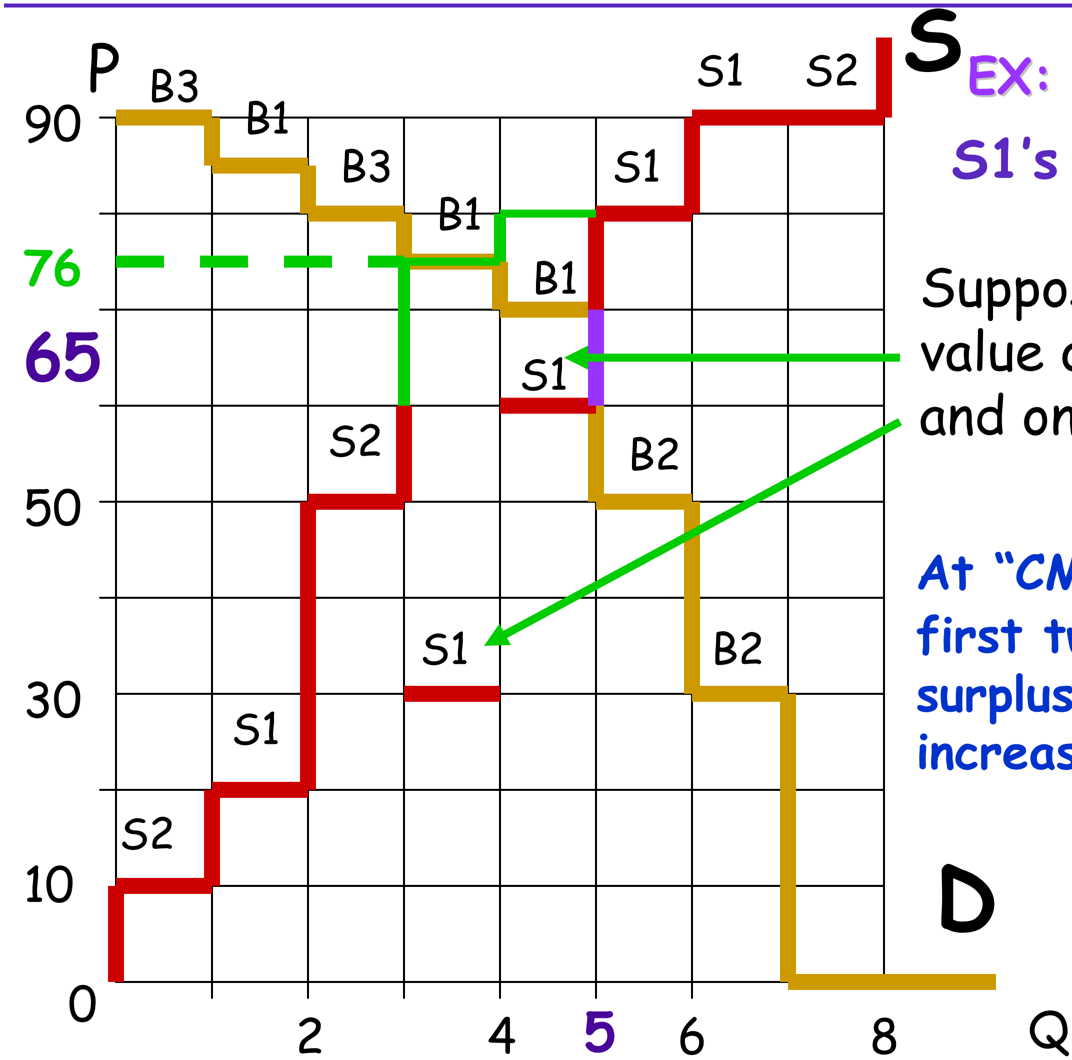
EX: CMC Pt: Q*=5, P*= $65

S1's CMC NetSellerSurplus=$85

Suppose S1 **REPORTS** a reservation value on his 3rd unit equal to $80 and on his 2nd unit equal to $76?

Suppose the market operator has to form total demand and supply schedules based only on REPORTED demands and supplies.

S1 has market power ? … YES!



EX: CMC Pt: Q*=5, P*= $65

S1's CMC NetSellerSurplus=$85

Suppose S1 **REPORTS** a reservation value on his 3rd unit equal to $80 and on his 2nd unit equal to $76?

At "CMC" price $76, S1 only sells his first two units, but his net seller surplus on these two units alone increases to $102 = [$56+$46] !

AMES DC-OPF Formulation

Caution: Notation Switch

- P (in MWs) now denotes *amounts of power*

- LMP_k ($/MWh) = Locational Marginal Price at bus k, roughly defined as the least cost of servicing one additional MW of fixed demand at bus k from any feasible source of generation anywhere on transmission grid

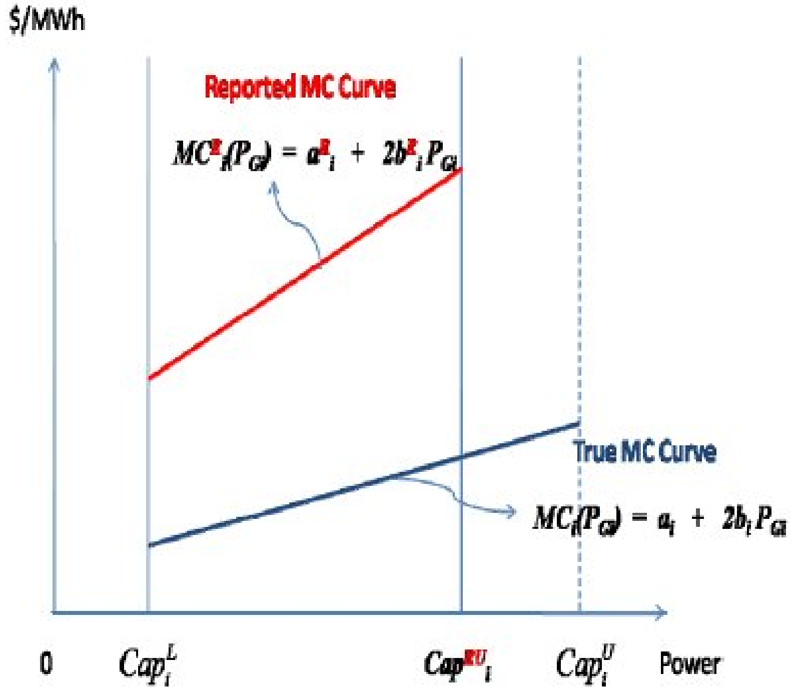
AMES GenCo Supply Offers

Hourly supply offer for each GenCo i = ***Reported*** linear marginal cost function over a ***reported*** operating capacity interval for real power p_{Gi} (in MWs):

$$MC^R_i(p_{Gi}) = a^R_i + 2b^R_i\, p_{Gi}$$

$$Cap_i^L \leq p_{Gi} \leq Cap_i^{RU}$$

GenCos can learn to report ***higher-than-true*** marginal costs and/or to report ***lower-than-true*** maximum capacity.



AMES LSE Demand Bids

Hourly demand bid for each LSE j = ***Fixed demand bid*** +
Price-sensitive demand bid

- Fixed demand bid = p^F_{Lj} (MWs)
- Price-sensitive demand bid

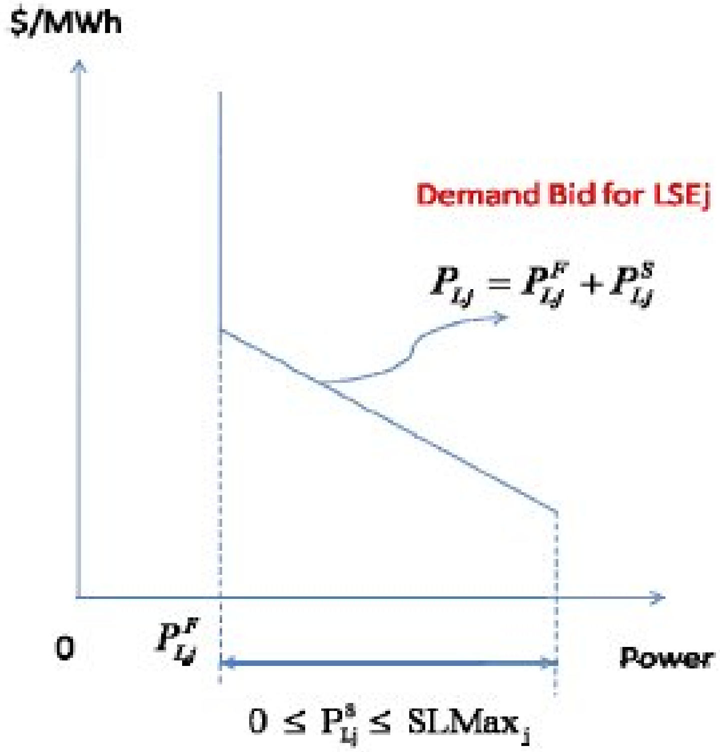
  = Inverse demand function for

  real power p^S_{Lj} (MWs) over

  a purchase capacity interval:

$$F_j(p^S_{Lj}) = c_j - 2d_j\, p^S_{Lj}$$

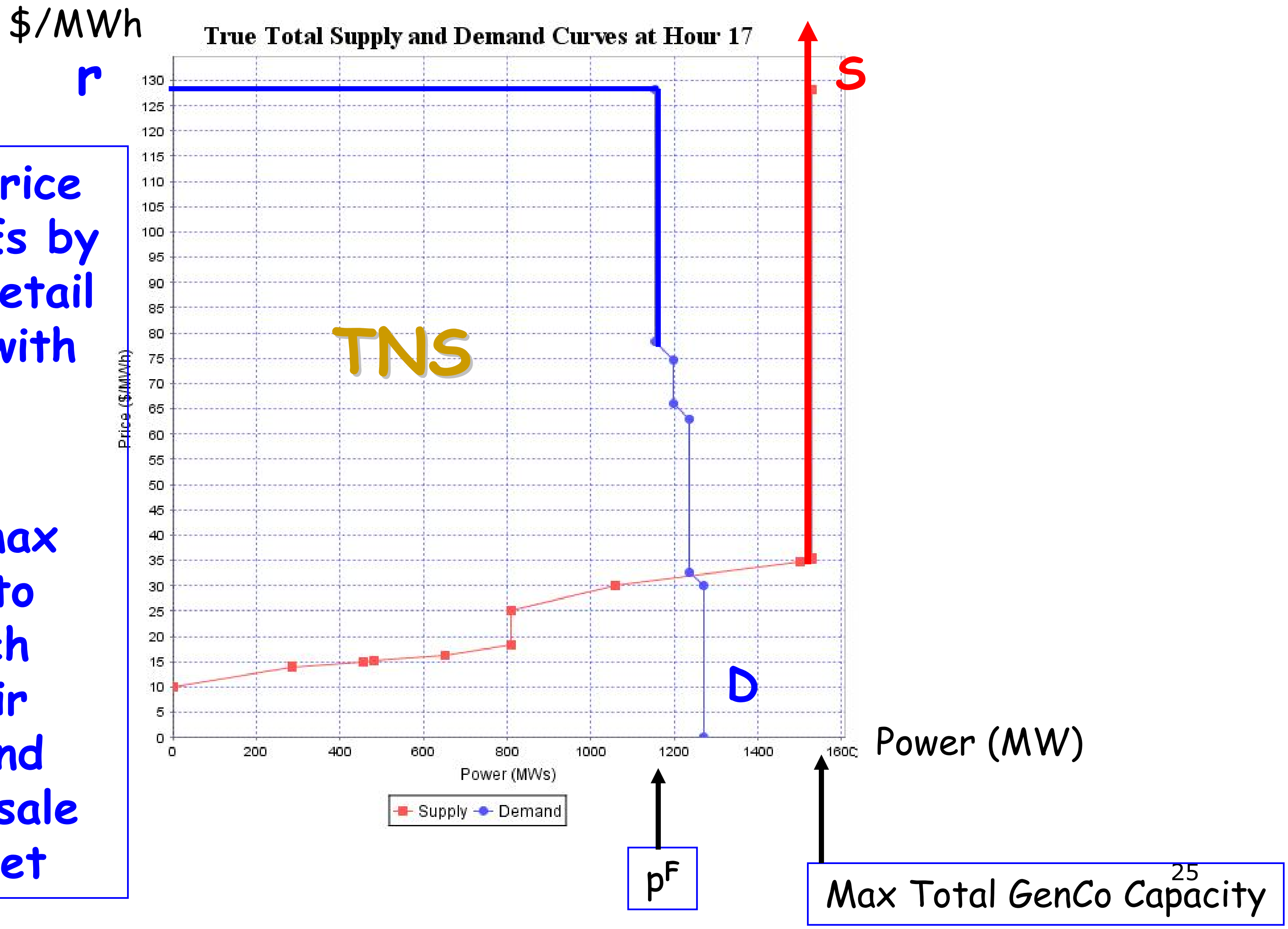
$$0 \leq p^S_{Lj} \leq SLMax_j$$



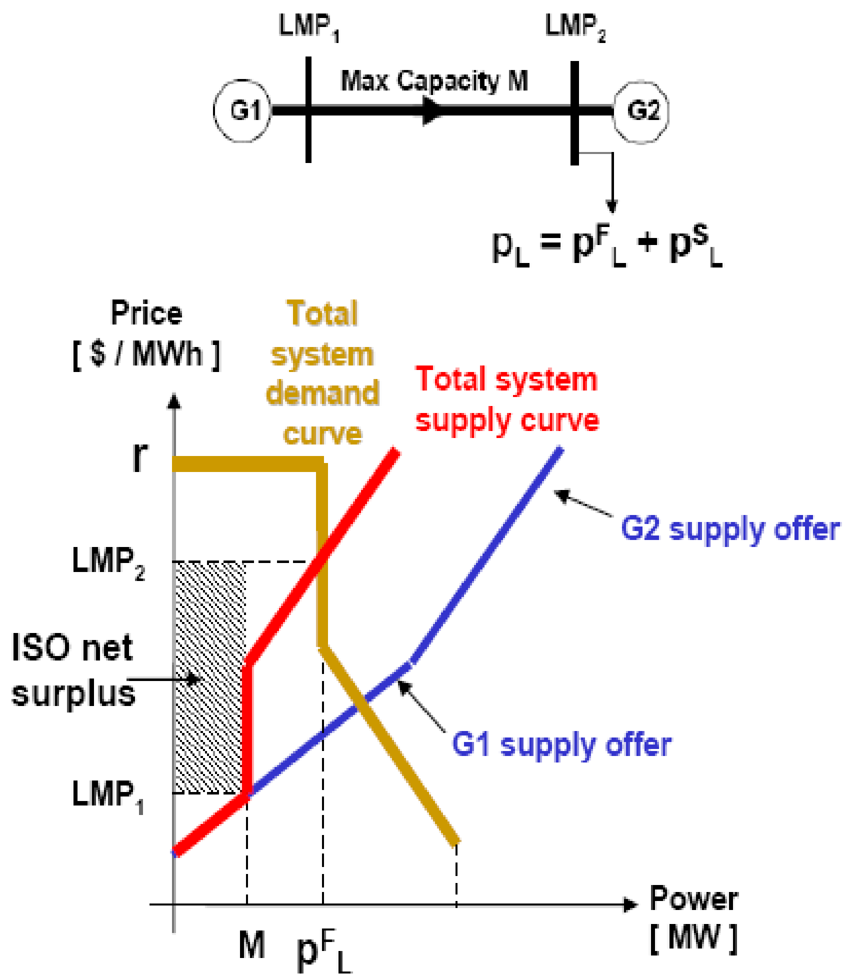
AMES Illustration: Total Net Surplus (TNS) in Hour 17 for 5-Bus Test Case with 5 GenCos and 3 LSEs

r = Fixed price paid to LSEs by the LSEs' retail customers with flat-price contracts

= LSEs' max willingness to pay for each MW of their fixed demand pF in wholesale power market



AMES Calculation of TNS: 2-Bus Example



Cleared load = p^F_L. LSE at bus 2 pays $LMP_2 > LMP_1$ for each unit of p^F_L. M units of p^F_L are supplied by cheaper G1 at bus 1 who receives only LMP_1 per unit.

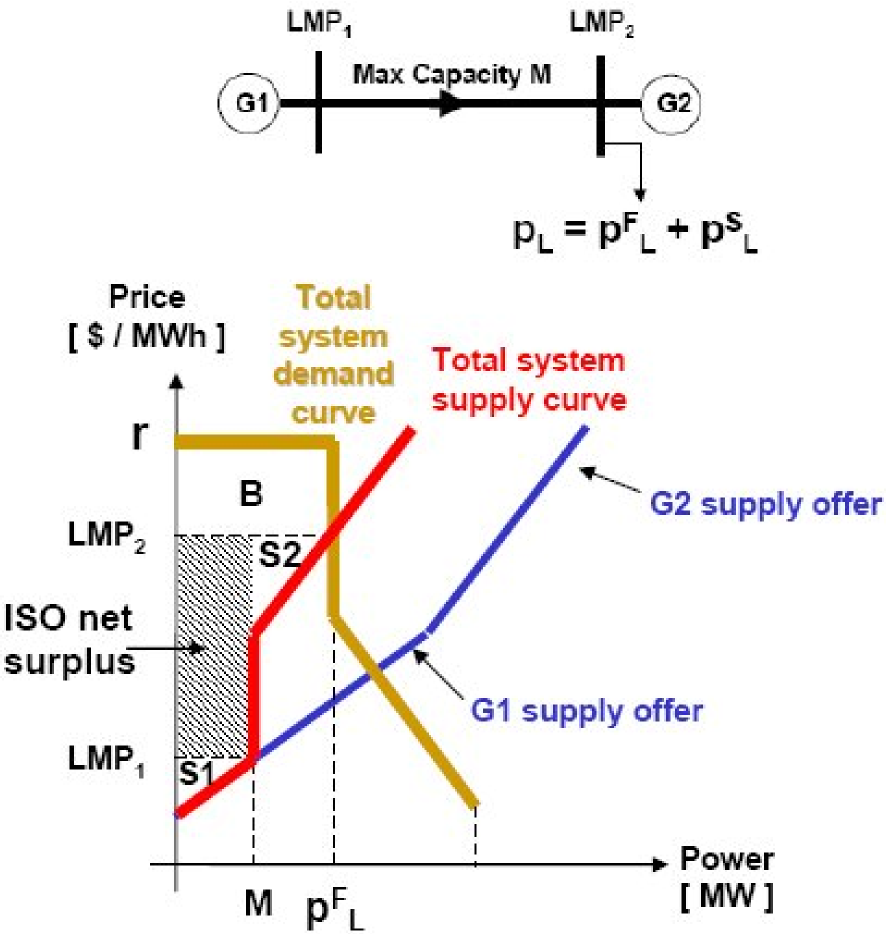
ISO collects difference:

ISO Net Surplus

= [LSE Payments
– GenCo Revenues]

= $M \times [LMP_2 - LMP_1]$

AMES Calculation of TNS: 2-Bus Example … Cont'd



ISO Net Surplus:

INS=M x [LMP_2 – LMP_1]

GenCo Net Surplus:

Area S1 + Area S2

LSE Net Surplus:

Area B

Total Net Surplus:

TNS = [INS+S1+S2+B]

ISO Objective (DC-OPF):

maximize **TNS** subject to trans/gen constraints.

AMES Calculation of TNS: General Form
(Note LMPs cancel out of TNS expression!)

Total Net Surplus for Hour H of Day D+1, based on Day D Supply Offers and Demand Bids:

$$TNS(H,D)$$

$$= \text{LSENetSur}(H,D) + \text{GenNetSur}(H,D) + \text{ISONetSur}(H,D)$$

$$= \sum_{j=1}^{J} GS_j(H,D) - \sum_{i=1}^{I} [\mathrm{C}_i^a(H,D)]$$

where

$$GS_j(H,D) = [r \cdot p_{Lj}^F(H,D) + \int_0^{p_{Lj}^S(H,D)} \mathrm{F}_{jHD}(p)dp]$$

$$\mathrm{C}_i^a(H,D) = \int_0^{p_{Gi}(H,D)} \mathrm{MC}_i(p)dp$$

LSE j's gross surplus from its retail fixed demand sales

LSE j's gross surplus from its retail price-sensitive demand sales

GenCo i's total avoidable costs of production

AMES Basic DC-OPF Formulation:

SI unit representation for AMES ISO's DC-OPF problem for hour H of the day-ahead market on day D+1, solved on day D.

DC-OPF formulation is derived from AC-OPF under three assumptions:

(a) Resistance on each branch km = 0

(b) Voltage magnitude at each bus k= base voltage V_o

(c)Voltage angle difference d_{km} = [$delta_k$ - $delta_m$] across each branch km is small so that $\cos(d_{km}) \cong 1$ and $\sin(d_{km}) \cong d_{km}$

$$\max \quad \text{TNS}^R \tag{15}$$

with respect to LSE real-power price-sensitive demands, GenCo real-power generation levels, and voltage angles

$$p_{Lj}^S,\ j=1,...,J;\ p_{Gi},\ i=1,...,I;\ \delta_k,\ k=1,...,K \tag{16}$$

subject to

(i) a real-power balance constraint for each bus k=1,...,K:

$$\sum_{i\in I_k} p_{Gi} - \sum_{j\in J_k} p_{Lj}^S - \sum_{km} P_{km} = \sum_{j\in J_k} p_{Lj}^F \tag{17}$$

where, letting x_{km} (ohms) denote reactance for branch km, and V_o denote the base voltage (in line-to-line kV),

$$P_{km} = [V_o]^2 \cdot [1/x_{km}] \cdot [\delta_k - \delta_m]$$

(ii) a limit on real-power flow for each branch km:

$$|P_{km}| \leq P_{km}^U \tag{18}$$

(iii) a real-power operating capacity interval for each GenCo i = 1,...,I:

$$\text{Cap}_i^L \leq p_{Gi} \leq \text{Cap}_i^U \tag{19}$$

(iv) a real-power purchase capacity interval for price-sensitive demand for each LSE j = 1,...,J:

$$0 \leq p_{Lj}^S \leq \text{SLMax}_j \tag{20}$$

(v) and a voltage angle setting at angle reference bus 1:

$$\delta_1 = 0 \tag{21}$$

TNS^R = Total Net Surplus based on *reported* GenCo marginal cost functions rather than *true* GenCo marginal cost functions.

Lagrange multiplier (or "shadow price") solution for the bus-k balance constraint (17) gives the LMP_k at bus k

AMES DC-OPF problem is a special type of GNPP, and LMPs are Lagrange Multiplier Solutions for this GNPP

General Nonlinear Programming Problem (GNPP):

- $\mathbf{x}$ = n×1 choice vector;
- $\mathbf{c}$ = m×1 vector & $\mathbf{d}$ = s×1 vector (constraint constants)
- f($\mathbf{x}$) maps $\mathbf{x}$ into R (all real numbers)
- $\mathbf{h}$($\mathbf{x}$) maps $\mathbf{x}$ into R^m (all m-dimensional vectors)
- $\mathbf{z}$($\mathbf{x}$) maps $\mathbf{x}$ into R^s (all s-dimensional vectors)

GNPP: Minimize f(x) with respect to x subject to

$\mathbf{h}(\mathbf{x}) = \mathbf{c}$ (e.g., DC-OPF bus balance constraints)

$\mathbf{z}(\mathbf{x}) \geq \mathbf{d}$ (e.g., DC-OPF branch constraints & GenCo capacity constraints)

AME DC-OPF as a GNPP…Continued

- Define the ***Lagrangean Function*** as

$$L(\mathbf{x},\lambda,\mu,\mathbf{c},\mathbf{d}) = f(\mathbf{x}) + \lambda^T[\mathbf{c} - h(\mathbf{x})] + \mu^T[\mathbf{d} - \mathbf{z}(\mathbf{x})]$$

- Assume ***Kuhn-Tucker Constraint Qualification (KTCQ)*** holds at $\mathbf{x}^*$, roughly stated as follows:

The true set of feasible directions at $\mathbf{x}^*$

= Set of feasible directions at $\mathbf{x}^*$ assuming a linearized set of constraints in place of the original set of constraints.

AMES DC-OPF as a GNPP … Continued

- Given KTCQ, the ***First-Order Necessary Conditions (FONC)*** for $\mathbf{x}^*$ to solve (GNPP) are: There exist vectors λ^* and μ^* of ***Lagrange multipliers (or "shadow prices")*** such that ($\mathbf{x}^*$, λ^*, μ^*) satisfies:

$$0 = \nabla_x L(\mathbf{x}^*,\lambda^*,\mu^*,\mathbf{c},\mathbf{d})$$

$$= [\, \nabla_x f(\mathbf{x}^*) \; - \lambda^{*T}\bullet\nabla_x h(\mathbf{x}^*) - \mu^{*T}\bullet\nabla_x z(\mathbf{x}^*) \,] \, ;$$

$$\mathbf{h}(\mathbf{x}^*) = \mathbf{c} \, ;$$

$$\mathbf{z}(\mathbf{x}^*) \geq \mathbf{d}; \quad \mu^{*T}\bullet[\mathbf{d} - \mathbf{z}(\mathbf{x}^*)] = 0; \quad \mu^* \geq \mathbf{0}$$

✷ **These FONC are often referred to as the *Karush-Kuhn-Tucker (KKT) conditions.***

Solution as a Function of (c,d)

By construction, the components of the solution vector (**x***, λ*****, μ*****) are *functions* of the constraint constant vectors c and d

- **x*** = **x**(c,d)
- λ^* = λ(c,d)
- μ^* = μ(c,d)

GNPP Lagrange Multipliers as Shadow Prices

Given certain additional regularity conditions…

- The solution λ^* for the $m \times 1$ multiplier vector λ is the derivative of the minimized value $f(\mathbf{x}^*)$ of the objective function $f(\mathbf{x})$ with respect to the constraint vector $\mathbf{c}$, all other problem data remaining the same.

$$\partial f(\mathbf{x}^*)/\partial \mathbf{c} = \partial f(\mathbf{x}(\mathbf{c},\mathbf{d}))/\partial \mathbf{c} = \lambda^{*T}$$

GNPP Lagrange Multipliers as Shadow Prices…

Given certain additional regularity conditions…

- The solution μ^* for the $s \times 1$ multiplier vector μ is the derivative of the minimized value $f(\mathbf{x}^*)$ of the objective function $f(\mathbf{x})$ with respect to the constraint vector $\mathbf{d}$, all other problem data remaining the same.

$$0 \le \partial f(\mathbf{x}^*)/\partial \mathbf{d} = \partial f(\mathbf{x}(\mathbf{c},\mathbf{d}))/\partial \mathbf{d} = \mu^{*T}$$

GNPP Lagrange Multipliers as Shadow Prices…

Consequently…

- The solution λ^* for the multiplier vector λ thus essentially gives the ***prices (values)*** associated with unit changes in the components of the constraint vector **c**, all other problem data remaining the same.
- The solution μ^* for the multiplier vector μ thus essentially gives the ***prices (values)*** associated with unit changes in the components of the constraint vector **d**, all other problem data remaining the same.
- Each component of λ^* can take on ***any sign***
- Each component of μ^* must be ***nonnegative***

Counterpart to Constraint Vector c for AMES DC-OPF?

AMES DC-OPF Has K Equality Constraints:

(i) a real-power balance constraint for each bus k=1,...,K:

$$\sum_{i \in I_k} p_{Gi} - \sum_{j \in J_k} p^S_{Lj} - \sum_{km} P_{km} = \sum_{j \in J_k} p^F_{Lj} \qquad (17)$$

Fixed demand of LSE j

Index set for LSEs located at bus k

kth Component of Kx1 Constraint Vector c :

$$\sum_{j \in J_k} p^F_{Lj}$$ = FD_k = Total Fixed Demand at Bus k

LMP as Lagrange Multiplier

- **TNS*(H,D)** = Maximized Value of TNS(H,D) from the ISO's DC-OPF solution on Day D for hour H of the day-ahead market on Day D+1

- **LMP_k (H,D)** = Least cost of servicing one additional MW of fixed demand at bus k during hour H of day-ahead market on day D+1

$$\mathbf{LMP_k(H,D)} = \frac{\partial TNS^*(H,D)}{\partial FD_k}$$

Online Resources

- Notes on DC-OPF Formulation in AMES
  www.econ.iastate.edu/tesfatsi/DCOPFInAMES.LT.pdf
- AMES Wholesale Power Market Testbed
  www.econ.iastate.edu/tesfatsi/AMESMarketHome.htm
- Market Basics for Price-Setting Agents
  www.econ.iastate.edu/classes/econ458/tesfatsion/MBasics.SlidesIncluded.pdf
- Optimization Basics for Electric Power Markets
  www.econ.iastate.edu/classes/econ458/tesfatsion/OptimizationBasics.LT458.pdf
- Power Market Trading with Transmission Constraints
  www.econ.iastate.edu/classes/econ458/tesfatsion/OPFTransConstraintsLMP.KS6.1-6.3.2.9.pdf