Auction Market System in Electronic Security Trading Platform

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Abstract

Under the background of the electronic security trading platform Xetra operated by Frankfurt Stock Exchange, we consider the Xetra auction market system (XAMS) where the interactions among heterogenous traders and Xetra auction market mechanism generate the non market-clearing price dynamics. First we develop an integrative framework which serves as general guidance to analyze the economic system from 'bottom-up' and to construct the corresponding agent-based model. Then we apply this integrative framework to construct the agent-based model of XAMS. By conducting market experiments with the computer implementation of the agent-based model of XAMS, we investigate the role of the price setter who assumes its trading behavior can manipulate the market price. The main finding of the market experiment is that the introduction of the price setter in the setting of XAMS improves market efficiency while does not significantly influence price volatility of the market.

Keywords: agent-based modelling, computational market experiment, electronic security trading platform, Xetra, non-equilibrium price dynamics, automatic trading.

JEL Classification: B4, C6, C9, D4, D5, D6, G1.

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1 Introduction

Electronic security trading platforms operated by stock exchanges are prevailing in global financial markets, e.g. Xetra (Frankfurt Stock Exchange), SETS (London Stock Exchange), and Universal Trading Platform (NYSE Euronext). This new generation of trading platforms enables market participants (brokers, dealers, institutional and retail investors) to trade on electronic order books via remote computer access. Electronic trading platforms explicitly stipulate market mechanisms for determining the trading price and the trading volume even when the market is not in equilibrium, e.g. see Gruppe Deutsche Börse (2003) and NYSE Euronext (2010). On the other hand, market participants, especially institutional investors, are competing against each other to utilize the knowledge on market mechanisms and the real-time market data to generate trading strategies in order to exploit the market for a profit. The interactions among market participants and market mechanisms in electronic trading platforms generate market dynamics with non market-clearing trading prices.

It is a challenging task to establish a financial market model for the electronic trading platform as financial market models in economic literature generally follow the assumption of the market equilibrium, e.g. see Sharpe (1964) and Merton (1992). The construction of the financial market model for the electronic trading platform requires three aspects. First, it requires a formalization of the market mechanism in the electronic trading platform which allows non-equilibrium trading prices. Market mechanisms vary in electronic trading platforms, although they implement the same fundamental principle of maximum trading volume to determine the trading price. In this work we focus on one specific electronic trading platform – Xetra system operated by Frankfurt Stock Exchange.

According to Gruppe Deutsche Börse (2003), Xetra system is an order driven system which traders participate in by submitting their order specifications such as limit orders or market orders. A limit order in Xetra is an order specification to buy or sell indicated shares of the security at a specific price called limit price or better. A market order is an unlimited order specification to buy or sell the indicated shares at the next trading price. There are two trading forms in Xetra: continuous trading and Xetra auction, which associate with different market mechanisms. Our target is on Xetra auction which belongs to the type of multi-unit double auction. It mainly consists of the call phase and the price determination phase. During the call phase, traders submit order specifications and Xetra auction market mechanism (XAMM) collects these orders into a central order book without giving rise to transactions. The price determination phase follows when the call phase stops randomly after a fixed time span. Given the central order book, XAMM determines the trading price and the trading volume in the price determination phase by applying Xetra auction trading rules specified in Gruppe Deutsche Börse (2003). A first attempt to formalize Xetra auction trading rules has been proposed in Li & Wenzelburger (2005). We follow this line to consider an explicit formulation of XAMM to determine the trading price and the trading volume in the Xetra auction market model.

The second aspect is on the role that the knowledge on XAMM and the real-time market data play in trader's investment decision process. XAMM determines the trading price according to the central order book which is an aggregation of orders submitted by traders. This implies that the trader could potentially influence the current trading price by submitting its order. With the knowledge on XAMM and the real-time market data of the central order book, the trader can discover the function of the current trading price with the control variable of the trading quantity quoted in its order, see Li (2010). By integrating this function into the trader's investment decision to determine the quoted trading quantity in its order, the trader realizes in its investment decision the potential of manipulating the trading price by submitting its order. The trader is then called the price setter as it can manipulate the trading price through its trading behavior in the market. In order to investigate the interaction and feedback among traders and XAMM, Xetra auction market model should include the formulation of the price setter with its investment decision process in which the knowledge on the market mechanism and the real-time market data from XAMM play an important role.

The last aspect is on how to model complex interactions among traders and XAMM in the dynamics of Xetra auction market. The dynamic process of complex interactions among economic entities can be modelled by applying the methodology of Agent-based Computational Economics (ACE) that is a computational study on the dynamical economic system from 'bottom-up', see Tesfatsion (2006). Economists in this strand construct the ACE model for the economic system by modelling the interaction of agents that represent economic entities in the system. An **agent** in the ACE model is a bundle of computational processes to represent the functionality of the economic entity. An **ACE model** is a collection of agents interacting with each other to generate the macroscopic behavior of the economic system.

ACE researchers have successfully handled complex financial market systems and have constructed agent-based financial market models with explicit forms of market mechanisms to determine market prices and trading volumes. For instance, Das (2003) introduced an agent-based financial market model which adopted a simple version of single-unit double auction as the explicit market mechanism to generate the market dynamics. Although the market mechanism considered in Das (2003) does not share the same type of auctions as XAMM that belongs to the type of multi-unit double auction, the success of introducing explicit market mechanism in the agent-based financial market model suggests the possibility of applying the methodology of ACE modelling to construct the agent-based model with an explicit formulation of XAMM. The investment decision process of the price setter is essentially a computational process. This implies the feasibility of applying the concept of agents to model the price setter in Xetra auction market. Thus, it seems appropriate to employ the ACE modelling to construct the agent-based model for Xetra auction market with the explicit formulation of XAMM and of the price setter to investigate Xetra auction market dynamics.

The current difficulty of constructing agent-based models for financial market systems is the lack of general principles that economists can apply to construct agent-based models, see LeBaron (2006). In order to overcome this difficulty, we work in section 2 to develop an integrative framework for ACE modelling that serves as general principles to investigate economic systems from 'bottom-up' and to construct the corresponding ACE models. Then we apply this integrative framework in section 3 to construct the ACE model of Xetra auction market system (XAMS). With the implementation of the ACE model by employing the computer programming language Groovy/Java, section 4 conducts the computational market experiment to simulate XAMS and preforms statistical analysis on simulation results to investigate the impact of the price setter on Xetra auction market. 5 concludes with a brief discussion on implications and possible extensions of our work.

2 Agent-Based Modelling of Economic System

In general, we follow the ACE modelling procedure that is depicted in Figure 1. First, by applying the Agent-Based Modelling (ABM) method which considers modelling the system as a collection of interacting agents, the ACE modeler analyzes the economic system and constructs the corresponding ACE model. Then the ACE modeler employs computer programming languages to implement the ACE model as the computer software system. After that, the ACE modeler initiates and executes the software system to observe in the computer environment the evolution of the software system which represents the dynamics of the economic system.

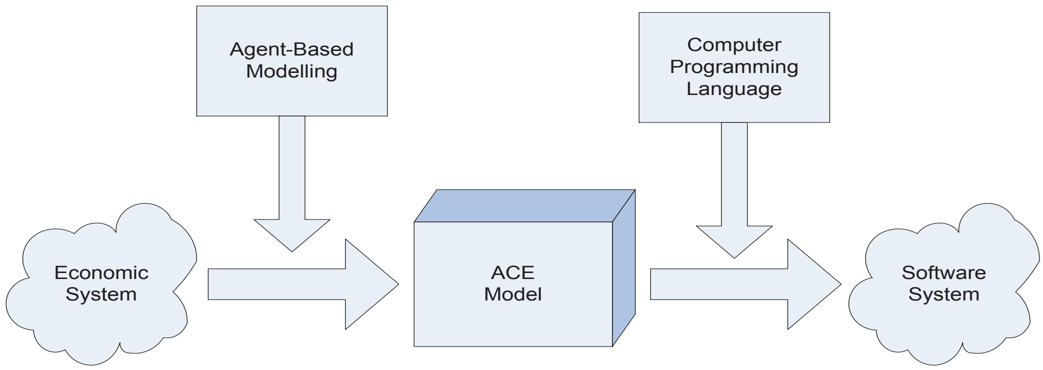

Figure 1: The ACE modelling procedure.

An ACE model is an abstracted representation of the economic system. It is independent of the computer programming language and is not a computer software system. Here we start from the perspective of systems theory and the methodology of ABM to investigate an economic system from ‘bottom-up’ and to derive constructive aspects of the economic system that serve as the foundation of the integrative framework for ACE modelling.

2.1 Constructive Aspects of Economic System

Constructive aspects of economic system have the epistemic root in the systematic principle of economic phenomena and processes, i.e. economic phenomena and processes can be regarded as states and evolutionary processes of the corresponding economic systems.[1]

From systems theory and the methodology of ABM, an economic system can be regarded as a dynamical open system which interacts with its environment in the society. An **economic system** is a collection of economic entities (consumers, firms, commodities, markets, etc.) interacting with each other such that the interactions of economic entities perform macroscopic behavior of the system given the influence from the environment. To explicitly analyze and model an economic

[1]The systematic principle of economic phenomena and processes is the specification of the systems theory in the field of economics, see Bertalanffy (1993).

system from 'bottom-up' is equivalent to stipulating the following **constructive aspects of the economic system**:

I. The scope of the economic system and its environment;

II. The interrelation between the economic system and its environment;

III. Elements of the economic system, which are economic entities considered in the economic system;

IV. The structure of the economic system, which is the interrelation among elements of the economic system.

To model an economic system from 'bottom-up', constructive aspects of the economic system suggest the integrative framework to formulate aspect I to IV of the economic system and their updating rules (also called state transition rules) which represent the dynamics of the economic system.

The economist normally specifies the scope of the economic system and its environment when initiating the economic research. Consider the composite of the economic system and its environment as the economic world and regard the interrelation between the economic system and its environment as information flows.[2] The integrative framework formulates the ACE world which integrates the ACE model of the economic system and its environment with the information flows.

As observed in contemporary economic literature, economists tend to classify economic entities into different types in order to investigate the characteristics of each type. For example, Pindyck & Rubinfeld (2001) clusters microeconomic entities into consumers, producers (firms), commodities, markets, etc. The integrative framework follows this classification of economic entities to stipulate elements in the economic system and the corresponding agents in the ACE model.

The integrative framework represents the structure of the economic system as the information flows among agents in the ACE model. To explicitly depict in the ACE model the structure of the economic system, the integrative framework proposes the **diagram of the relationship** for the ACE model which nodes represent agents in the ACE model and which arcs represent information flows

[2]The concept of information can be termed as diversified meanings. Here the information is considered as quantitative representations of the economic world. Knowledge, methods, and actions are regarded as information in this sense once they can be quantified.

between agents.[3] The diagram of the relationship represents the interrelation between the economic system and its environment as the arcs which connect the corresponding ACE model with its environment.

Now the crucial point for the integrative framework is to model economic entities with the concept of agents. Most economic entities investigated in economic research concern with the functionality and behavior of individual or a group of people in economic world. We denote all these economic entities interpreting the functionality of human subject as **active economic entities** in the sense that they behave actively to fulfill their needs and objectives. Economic entities which are not directly involved with the functionality of human subject, e.g. commodities traded in the markets, are classified as **passive economic entities**. Correspondingly, we denote the agent representing the active economic entity as the **active economic agent** and the agent representing the passive economic entity as the **passive economic agent**.

An economic system can be treated as an economic entity that is used to construct another economic system, whereas an element in the economic system can be treated as an economic system. This property of system-element duality guarantees the hierarchy of economic systems, see Potts (2000). More importantly, this property infers that one can consider the economic entity as a collection of components whose interaction among each other provides the functionality of the economic entity and thus one can model the economic entity by formulating its constructive aspects.

2.2 Constructive Aspects of Active Economic Entity

In economic literature, active economic entities represent the functionality of decision makers in the economic world. The concept of the decision maker has been investigated in various fields in science which include psychology, sociology, computer science, etc. As we are looking for the general framework to construct the ACE model that is to implement as computer software system, we employ the related concepts in computer science and combine with decision theory in economics to model the active economic entity.

The skeleton of the decision maker is stated in the concept of the agent in artificial

[3] The diagram of the relationship for the ACE model serves the same purpose as the class diagram which describes the static structure of objects in a system and their relationships in Unified Modelling Language (UML), see Blaha & Rumbaugh (2004). Comparing with the class diagram in UML, the diagram of the relationship for the ACE model emphasizes on the connections of information flows among agents.

intelligence (AI), a subfield of computer science. An agent in this field is *"anything that can be viewed as perceiving its environment through sensors and acting upon that environment through actuators"* (Russell & Norvig 2003, p. 32). Although it lacks of a unified model of the agent in AI, we take a pragmatic viewpoint and propose a general pattern by integrating behavioral rules of active economic entities with the current concept of the learning agent in AI, see (p. 53, ibid.). This general pattern with the name of the **module of active economic agent (MAEA)** is regarded as constructive aspects of the active economic entity and is composed of the submodule of information acquirement, the submodule of storage, the submodule of learning, the submodule of objectives, the submodule of forecasting, and the submodule of action transmission. One applies MAEA to construct the corresponding active economic agent from 'bottom-up' by specifying its submodules and the interrelation among submodules. We sketch the functionality of each submodule with the structure of MAEA illustrated in Figure 2.

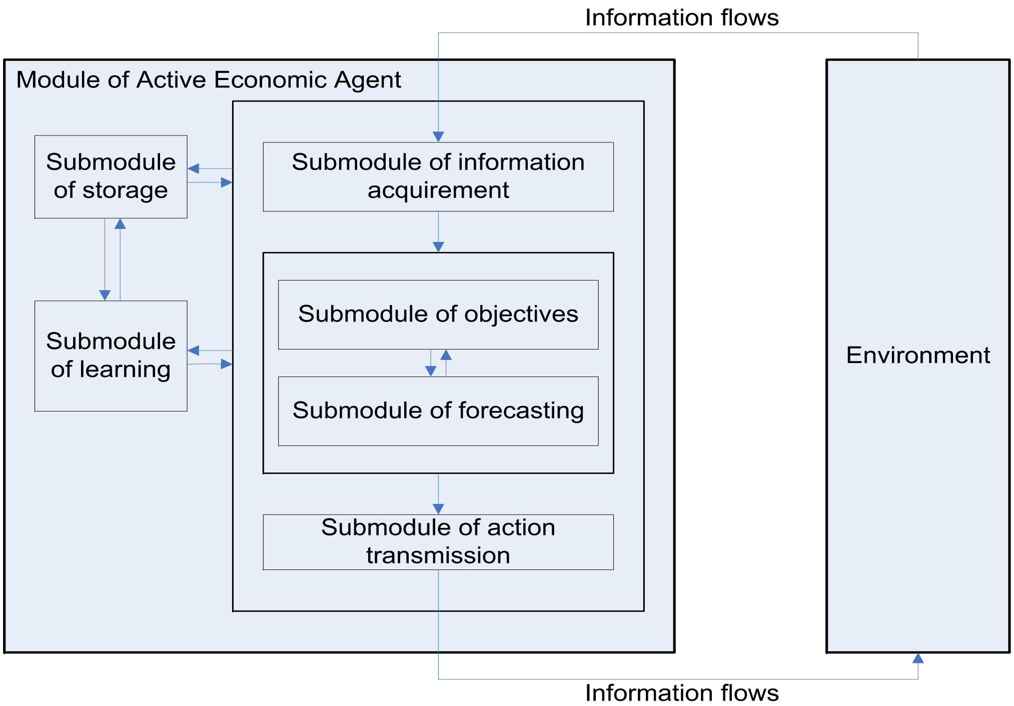

Figure 2: The structure of MAEA.

The environment in Figure 2 represents those parts that are out of the scope of the active economic agent. The information flows between MAEA and the environment represent the interrelation of the agent with others in the economic system.

The submodule of information acquirement establishes the connections with its environment and collects information through the interrelations. The submodule of storage stores the information about the state of the agent. It collects the information transmitted from other submodules and sends out the information on request. The submodule of forecasting generates the forecast on uncertain factors that the agent considers. The submodule of objectives depicts the objectives that the agent intends to achieve, selects the action plan based on its designated objectives, and sends out the action plan to the submodule of action transmission to realize the action. The submodule of action transmission receives action plans from the submodule of objectives and realizes the action through its interrelations with the environment. The submodule of learning considers the learning process of the agent, which might not be constructed as it is not a compulsory piece when modelling the agent in economic literature.

The state of the active economic agent evolves when the agent acts to fulfill its objectives. The updating rules of the agent are thus the decision making process that is presented by interactions among submodules.

The decision making process generally starts when the agent initiates its state. The agent observes information via the submodule of information acquirement and keeps the information in its memory via the submodule of storage. Then it applies the submodule of learning to update itself, e.g. to update the forecasting methods currently applied in the submodule of forecasting in order to provide more accurate forecast on uncertain factors that the agent considers. After that, the agent generates its subjective forecast via the submodule of forecasting, selects the action plan to fulfill its objectives via the submodule of objectives, and transmits the action to the economic system via the submodule of action transmission. Finally, the agent receives from the environment the feedback of its action. This general decision making process is illustrated in Figure 3 and is worked as a benchmark for depicting the updating rules of active economic agents.

2.3 Constructive Aspects of Passive Economic Entity

Passive economic entities do not behave actively to fulfill their objectives. They mainly act as information providers that disseminate information to active economic entities on request. We propose a general pattern named the **module**

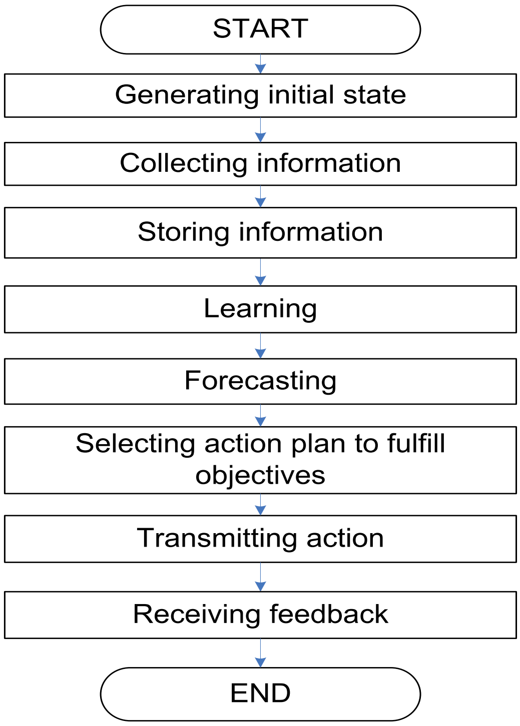

Figure 3: The general decision making process of active economic agent.

of passive economic agent (MPEA) to construct passive economic agents. MPEA consists of a set of **economic properties** that represent the information considered in the agent and associated **operations** such as the operation of updating information that is regarded as the updating rules of the agent.

2.4 Integrative Framework for ACE Modelling

The integrative framework for ACE modelling is a modelling process that applies the constructive aspects of the economic system and of the economic entities to translate the economic system into the corresponding ACE model. Given the economic system in study, the integrative framework starts with specifying constructive aspects of the economic system. Then it applies MAEA and MPEA as templates to formulate the corresponding economic agents in the economic system. It models the updating rules of active economic agents with the decision making process and the updating rules of passive economic agents with the operation of updating information. The updating rules of economic agents constitute the updating rules of the ACE model. Given information flows between the ACE model and its environment, the interactions among agents generate the dynamics of the model. The integrative framework proposes in the form of the flowchart the **diagram of the interaction** for the ACE model that describes the sequence of workflows and activities among agents. This diagram explicitly depicts the interactions among agents in the dynamic process of the ACE model.[4] In summary, the integrative framework contains the modelling procedure as follows:

1. Specify constructive aspects of the economic system;
2. Construct corresponding agents in the ACE model by applying MAEA and MPEA respectively, model the decision making process of active economic agents and the operation of updating information in passive economic agents;
3. Present the diagram of the interaction to describe the sequence of workflows and activities among agents for the dynamics of the ACE model.

3 ACE Model of Xetra Auction Market System

We apply the integrative framework to construct the ACE model of XAMS. Consider an economic world with one risky asset market and one risk-free asset market in trading period $t \in \{1, \dots, T\}$. The economic world applies Euro (€) as the trading currency. The risky asset considered in the market has no dividend and is traded in integer shares, i.e. traders can trade 19 shares of the risky asset but not 19.81 shares. The risk-free asset is divisible in any trading quantity with the trading price normalized to 1. It has a constant interest factor R which can be interpreted as $1 + r$ with r denoting the nominal interest rate. N traders participate in the economic world to trade in the risky asset market and the risk-free asset market. The economic world has no transaction cost and no short sale constraint for traders.

The risky asset market is a Xetra auction market that holds one Xetra auction for each trading period to determine the market price and the trading volume by XAMM. Xetra auction consists of the call phase and the price determination phase. During the call phase, XAMM disseminates the real-time trading data of the central order book in the market. Upon observing the real-time trading

[4]The diagram of the interaction can be regarded as the flowchart version of the activity diagram in Unified Modelling Language (UML) which is to represent the sequence of activities among components in the system, see Blaha & Rumbaugh (2004).

data from the market, traders perform their investment decisions and submit orders. XAMM then collects the submitted orders to the central order book and simultaneously updates the real-time trading data. The call phase stops randomly after a fixed time span and the price determination phase follows to determine Xetra auction price and the final transaction volumes. After that, XAMM cancels the unexecuted part of the orders and conducts the settlement to complete the payment for each transaction. After trading in Xetra auction market, traders interact with the risk-free asset market and trade for the risk-free asset.

We focus on Xetra auction market and regard the risk-free asset market as the environment of Xetra auction market. Given the information flows from the risk-free asset market, the interactions among traders and XAMM provide the functionality of Xetra auction market, i.e. determining the trading price and reallocating the risky asset among traders. We apply the integrative framework to construct the ACE model of XAMS. The first step is to consider constructive aspects of XAMS.

Example 1 (Constructive aspects of XAMS).

I. XAMS considers economic entities which operate in the market, i.e. N traders, XAMM, the numeraire employed in the market, and the risky asset traded in the market. The environment of XAMS is the risk-free asset market.

II. XAMS connects to the risk-free asset market to request the information of the interest factor R as well as to transmit trader's trading request on the risk-free asset. The risk-free asset market connects to XAMS to inform traders the interest factor R as well as their realized trading quantities of the risk-free asset. These information flows represent the interrelation between XAMS and its environment.

III. XAMS is regarded as a dynamical system which implicitly contains the concept of time. We propose the concept of the system clock to provide the information of the time considered in the ACE model. Thus, elements of XAMS as well as the corresponding ACE model are: N traders, XAMM, the numeraire, the risky asset, and the system clock.

IV. Consider a decentralized market. Traders connect with XAMM in order to perform the trading behavior, whereas there is no direct connection among traders. Traders and XAMM connect to the numeraire, the risky asset, and the system clock to have access to the associated information. Denote Xetra auction market center in the ACE model as a composite of XAMM, the numeraire, the risky asset, and the system clock. Then the structure of XAMS follows the type of the star network, see Figure 4. Xetra auction market center as the central node in the star network connects with all other nodes of traders in the ACE model. □

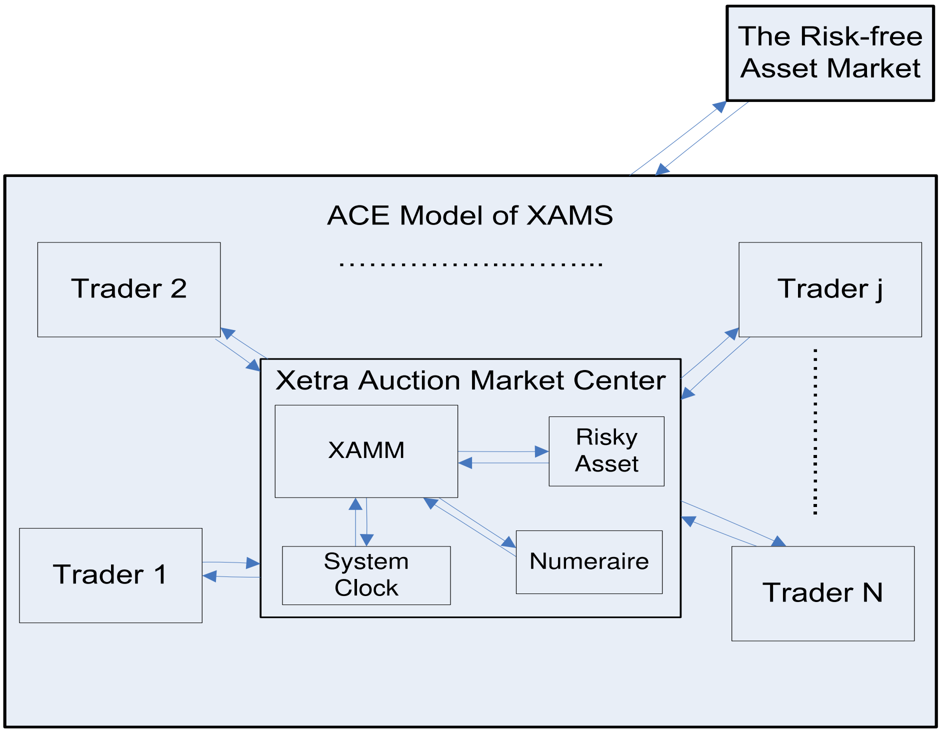

Figure 4: The diagram of the relationship for the ACE model of XAMS.

Classify agents in XAMS as active economic agents of N traders and XAMM with passive economic agents of the numeraire, the risky asset, and the system clock. The second step of the integrative framework is to apply MAEA and MPEA to construct these agents respectively.

We consider three types of traders in XAMS. The first type is the price setter who assumes that, with the knowledge on XAMM and the real-time trading data, it could manipulate the current trading price as well as its transaction volume by its trading behavior. The second type is the price taker who believes that its trading behavior has no impact on the market. The last type is the noise trader who is assumed to act randomly in the market. Assume trader 1 in the ACE model as the price setter, trader $j \in \{2, \dots, N-1\}$ as price takers, and trader N as the noise trader. For simplicity, further assume that each trader submits at most one order in each trading period with price setter and noise trader submitting the market order and price takers submitting the limit order. We apply MAEA to construct trader 1 in Example 2, trader $j \in \{2, \ldots, N-1\}$ in Example 3, and trader N in Example 4 respectively.

Example 2 (Trader 1). As stated in Figure 4, trader 1 connects with Xetra auction market center and the risk-free asset market. At the beginning of the trading period t, trader 1 is with the initial endowment $(y_0^{(1)}[t], Z_0^{(1)}[t])$ where $y_0^{(1)}[t]$ is the initial holding of the risk-free asset and $Z_0^{(1)}[t]$ is the initial holding of the risky asset. Trader 1 obtains through its submodule of information acquirement the interest factor R from the risk-free asset market and the real-time data set $\mathcal{I}_0[t]$ of the central order book from XAMM.

By applying the submodule of forecasting, trader 1 computes the expected mean value $q^{e(1)}[t]$ of the risky asset price for the next trading period $t+1$ and its associated variance $V^{e(1)}[t]$. As trader 1 can manipulate the current trading price as well as its transaction volume by submitting its market order, it performs its subjective forecast $P_X^{e(1)}[t](Q_m)$ on the current Xetra auction price and $Z_X^{e(1)}[t](Q_m)$ on the final transaction volume for period t which are functions with the control variable Q_m of the quoted trading quantity in its market order. The values of $P_X^{e(1)}[t](Q_m)$ and $Z_X^{e(1)}[t](Q_m)$ are the trading price and the final transaction volume that XAMM would calculate by perceiving the trader's market order with quoted trading quantity Q_m would be added to the current central order book in the market.

With its forecast of $\{P_X^{e(1)}[t](Q_m), Z_X^{e(1)}[t](Q_m), q^{e(1)}[t], V^{e(1)}[t]\}$, the trader has the budget constraint:

$$P_X^{e(1)}[t](Q_m) \cdot Z_X^{e(1)}[t](Q_m) + y^{e(1)}[t] = 0, \tag{1}$$

where $y^{e(1)}[t]$ is the trader's expected trading quantity of the risk-free asset in period t. The trader expects the portfolio holding after trading in period t as $(y_0^{(1)}[t] + y^{e(1)}[t], Z_0^{(1)}[t] + Z_X^{e(1)}[t](Q_m))$. Complying with the budget constraint (1), the trader considers the mean value $mean^{(1)}[t]$ of its future wealth at the end of the trading period t as:

$$\begin{aligned} mean^{(1)}[t] \;&=\; \{q^{e(1)}[t] - R \cdot P_X^{e(1)}[t](Q_m)\} \cdot Z_X^{e(1)}[t](Q_m) \\ &\quad + q^{e(1)}[t] \cdot Z_0^{(1)}[t] + R y_0^{(1)}[t]. \end{aligned} \tag{2}$$

The associated variance $var^{(1)}[t]$ is as:

$$var^{(1)}[t] = \{Z_0^{(1)}[t] + Z_X^{e(1)}[t](Q_m)\}^2 \cdot V^{e(1)}[t]. \tag{3}$$

We assume that trader 1 takes the linear mean-variance preference. The trader presents its objective in the submodule of objectives as the portfolio selection problem:

$$
\begin{aligned}
&\max_{Q_m\in\mathbb{Z}} \quad mean^{(1)}[t] - \frac{\alpha_1}{2}\, var^{(1)}[t] \qquad (4)\\
\Leftrightarrow\ &\max_{Q_m\in\mathbb{Z}} \quad \{q^{e(1)}[t] - R\cdot P_X^{e(1)}[t](Q_m)\}\cdot Z_X^{e(1)}[t](Q_m)\\
&\qquad +q^{e(1)}[t]\cdot Z_0^{(1)}[t] + Ry_0^{(1)}[t] - \frac{\alpha_1}{2}\{Z_0^{(1)}[t] + Z_X^{e(1)}[t](Q_m)\}^2\cdot V^{e(1)}[t],
\end{aligned}
$$

where α_1 is a constant measure of absolute risk aversion. Trader 1 solves this portfolio selection problem (4) and obtains the integer maximizer $Q_m^{(1)}[t]$ with $Q_m^{(1)}[t] > 0$ denoting the trading quantity on the demand side and $Q_m^{(1)}[t] < 0$ denoting the trading quantity on the supply side. Then the trader submits the market order with the quoted trading quantity $Q_m^{(1)}[t]$ to XAMM via the submodule of action transmission.

Trader 1 realizes the trading price $P_X[t]$ and its transaction volume $Z_X^{(1)}[t]$ after XAMM determines the trading price and the trading volume in the price determination phase. Then the trader completes with XAMM the payment for its transaction.

After trading the risky asset, trader 1 attains from the risk-free asset market the share $y^{(1)}[t] = -P_X[t]\cdot Z_X^{(1)}[t]$ of the risk-free asset. The portfolio holding that the trader acquires after trading in period t is $(y_0^{(1)}[t] + y^{(1)}[t], Z_0^{(1)}[t] + Z_X^{(1)}[t])$ and the trader's initial endowment of the next trading period $t+1$ is

$$
\begin{cases}
y_0^{(1)}[t+1] &= R(y_0^{(1)}[t] + y^{(1)}[t]),\\
Z_0^{(1)}[t+1] &= Z_0^{(1)}[t] + Z_X^{(1)}[t].
\end{cases} \qquad (5)
$$

The decision making process of trader 1 is illustrated in Figure 5. □

Example 3 (Trader $\mathbf{j = 2, \ldots, N-1}$). Analogous to trader 1, trader j connects with Xetra auction market center and the risk-free asset market. At the beginning of the trading period t, trader j is with the initial endowment $(y_0^{(j)}[t], Z_0^{(j)}[t])$. The trader obtains through its submodule of information acquirement the interest factor R and the real-time order book data set $\mathcal{I}_0[t]$.

By applying the submodule of forecasting, trader j computes the expected mean value $q^{e(j)}[t]$ of the risky asset price for the next trading period $t+1$ and its associated variance $V^{e(j)}[t]$. As the trader has to decide a limit price to quote in its limit order, the trader conducts its subjective forecast $P_X^{e(j)}[t]$ on the current

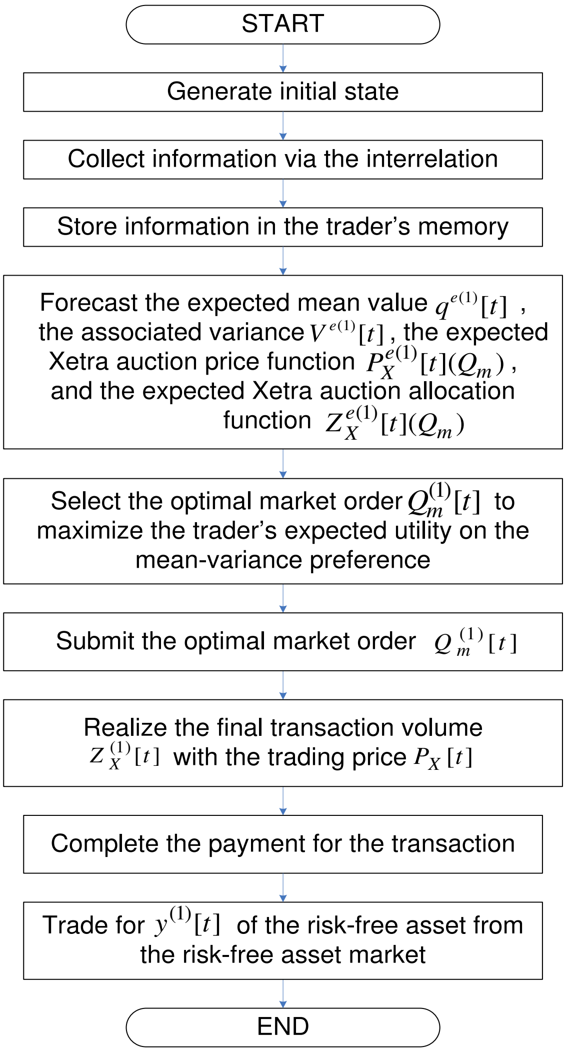

Figure 5: The decision making process of trader 1 in Example 2.

Xetra auction price and regards its forecast as the limit price. The trader expects it will realize from the market the quoted trading quantity Q_l in its limit order. With its forecast of $\{q^{e(j)}[t], V^{e(j)}[t], P_X^{e(j)}[t]\}$, the trader has the budget constraint:

$$P_X^{e(j)}[t] \cdot Q_l + y^{e(j)}[t] = 0, \tag{6}$$

where $y^{e(j)}[t]$ is the trader's expected trading quantity of the risk-free asset in period t. The trader expects the portfolio holding after trading in period t as $(y_0^{(j)}[t] + y^{e(j)}[t], Z_0^{(j)}[t] + Q_l)$. Complying with the budget constraint (6), the trader considers the mean value $mean^{(j)}[t]$ of its future wealth at the end of the trading period t as:

$$mean^{(j)}[t] = \{q^{e(j)}[t] - R \cdot P_X^{e(j)}[t]\} \cdot Q_l + q^{e(j)}[t] \cdot Z_0^{(j)}[t] + Ry_0^{(j)}[t]. \tag{7}$$

The associated variance $var^{(j)}[t]$ is as:

$$var^{(j)}[t] = \{Z_0^{(j)}[t] + Q_l\}^2 \cdot V^{e(j)}[t]. \tag{8}$$

Assume that trader j takes the linear mean-variance preference. The trader presents its objective in the submodule of objectives as the portfolio selection problem:

$$\begin{aligned} &\max_{Q_l \in \mathbb{Z}} \quad mean^{(j)}[t] - \frac{\alpha_j}{2}\, var^{(j)}[t] \\ \Leftrightarrow\ &\max_{Q_l \in \mathbb{Z}} \quad \{q^{e(j)}[t] - R \cdot P_X^{e(j)}[t]\} \cdot Q_l \\ &\qquad\quad + q^{e(j)}[t] \cdot Z_0^{(j)}[t] + Ry_0^{(j)}[t] - \frac{\alpha_j}{2}\{Z_0^{(j)}[t] + Q_l\}^2 \cdot V^{e(j)}[t], \end{aligned} \tag{9}$$

where α_j is a constant measure of absolute risk aversion. Trader j solves this portfolio selection problem (9) and obtains the integer maximizer $Q_l^{(j)}[t]$. Then the trader submits its limit order with the price-quantity pair $(P_X^{e(j)}[t], Q_l^{(j)}[t])$ to XAMM via the submodule of action transmission.

Trader j realizes the trading price $P_X[t]$ and its transaction volume $Z_X^{(j)}[t]$ after XAMM determines the trading price and the trading volume in the price determination phase. Then the trader completes with XAMM the payment for its transaction.

After trading the risky asset, trader j attains from the risk-free asset market the share of risk-free asset $y^{(j)}[t] = -P_X[t] \cdot Z_X^{(j)}[t]$. The portfolio holding that the trader acquires after trading in period t is $(y_0^{(j)}[t] + y^{(j)}[t], Z_0^{(j)}[t] + Z_X^{(j)}[t])$ and the trader's initial endowment of the next trading period $t+1$ is

$$\begin{cases} y_0^{(j)}[t+1] & = R(y_0^{(j)}[t] + y^{(j)}[t]), \\ Z_0^{(j)}[t+1] & = Z_0^{(j)}[t] + Z_X^{(j)}[t]. \end{cases} \tag{10}$$

The decision making process of trader j is illustrated in Figure 6. □

Example 4 (**Trader N**)**.** Consider trader N is with the initial endowment of $(y_0^{(N)}[t], Z_0^{(N)}[t])$ at the beginning of the trading period t. Trader N obtains the interest factor R from the risk-free asset market.

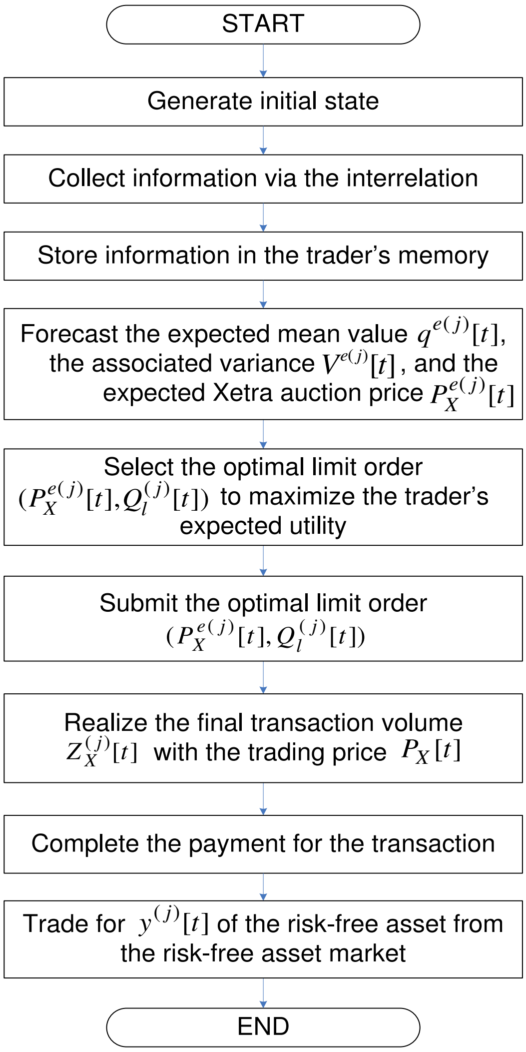

Figure 6: The decision making process of trader j in Example 3.

The trader randomly selects $Q_m^{(N)}[t]$ from the set Q_{range} of all possible trading quantities considered by the noise trader. Then the trader constructs its market order with the quoted trading quantity $Q_m^{(N)}[t]$ and submits the order to XAMM.

Trader N realizes the trading price $P_X[t]$ and its transaction volume $Z_X^{(N)}[t]$ after XAMM determines the trading price and the trading volume in the price determination phase. Then the trader completes with XAMM the payment for its transaction.

After trading the risky asset, trader N attains from the risk-free asset market the shares of the risk-free asset $y^{(N)}[t] = -P_X[t] \cdot Z_X^{(N)}[t]$. The portfolio holding that the trader acquires after trading in period t is $(y_0^{(N)}[t] + y^{(N)}[t], Z_0^{(N)}[t] + Z_X^{(N)}[t])$ and the trader's initial endowment for the next trading period $t+1$ is

$$\begin{cases} y_0^{(N)}[t+1] & = R(y_0^{(N)}[t] + y^{(N)}[t]), \\ Z_0^{(N)}[t+1] & = Z_0^{(N)}[t] + Z_X^{(N)}[t]. \end{cases} \tag{11}$$

The decision making process of trader N is illustrated in Figure 7. □

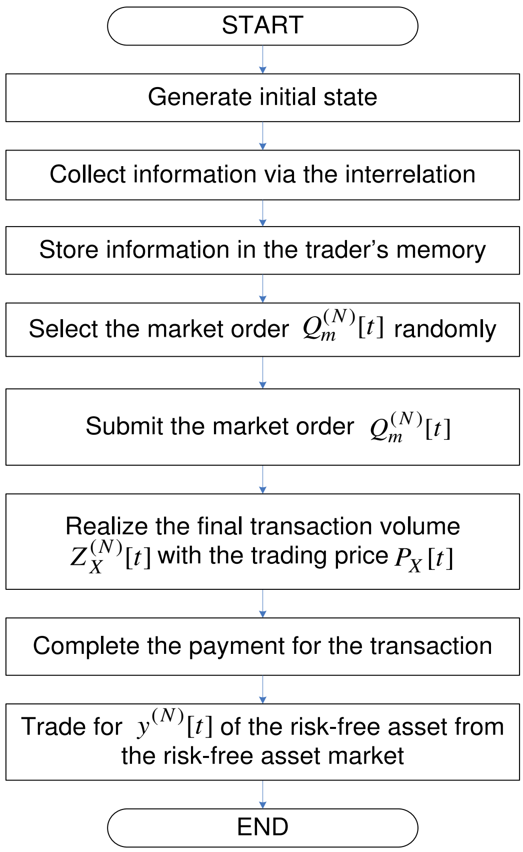

Figure 7: The decision making process of trader N in Example 4.

XAMM is another type of active economic agent considered in the ACE model. Its objective is to determine Xetra auction price and the final transaction volume according to the central order book.

Example 5 (XAMM). At the beginning of the trading period t, XAMM has historical trading prices $P_X[i]$ for trading period $i \in \{-K_{\text{XAMM}}+1,\dots,0\}$ with the memory span $K_{\text{XAMM}} > 0$. It contains trading information $(\overline{\mathfrak{I}_0[i]}, P_X[i], Z_X[i])$ for trading period $i \in \{1,\dots,t-1\}$ where $\overline{\mathfrak{I}_0[i]}$ is the order book data set at the end of the call phase, $P_X[i]$ is the Xetra auction price, and $Z_X[i] = \{Z_X^{(1)}[i],\dots,Z_X^{(N)}[i]\}$ is the collection of the final transaction volume $Z_X^{(j)}[i]$ for each trader $j \in \{1,\dots,N\}$. In summary, XAMM at the beginning of the trading period t is with the historical trading information set

$$\begin{aligned} Infor[t-1] \;=\{ \;& (\overline{\mathfrak{I}_0[t-1]}, P_X[t-1], Z_X[t-1]),\dots,(\overline{\mathfrak{I}_0[1]}, P_X[1], Z_X[1]), \\ & P_X[0],\dots,P_X[-K_{\text{XAMM}}+1] \;\}. \end{aligned} \tag{12}$$

During the call phase, XAMM works through the submodule of information acquirement to collect order specifications $\{Q_m^{(1)}[t],\dots,(P_X^{e(j)}[t], Q_l^{(j)}[t]),\dots,Q_m^{(N)}[t]\}$ submitted by traders and stores this information via the submodule of storage. It simultaneously disseminates the real-time order book data set $\mathfrak{I}_0[t]$.

The objective of XAMM is to determine in the price determination phase the Xetra auction price $P_X[t]$ and the final transaction volumes $Z_X[t] = \{Z_X^{(1)}[t],\dots, Z_X^{(N)}[t]\}$ by applying Xetra auction trading rules stated in Gruppe Deutsche Börse (2003), see the formulation in Appendix A. After determining Xetra auction price and the trading volume, XAMM cancels the unexecuted part of order specifications and conducts the settlement process to complete the payment for each transaction. Then XAMM closes the market until the next trading period $t+1$. The updating rules of XAMM are depicted in Figure 8. □

The numeraire, the risky asset, and the system clock are passive economic agents considered in the ACE model. They act as information providers to provide on request the information about the currency employed in the market, the security traded in the market, and the time considered in the model. As the environment of the ACE model, the risk-free asset market provides the trading on the risk-free asset.

The last step in the integrative framework is to explicitly present the interactions among agents in the ACE model. We consider dynamics of XAMS with T trading periods. XAMM starts the call phase at the beginning of the trading period. It disseminates to traders the real-time trading information of the central order book and simultaneously collects order specifications submitted by traders. We assume that traders submit their order specifications in a random sequence during the call phase. To simplify our analysis, we further assume that price takers submit limit orders prior to the price setter and noise trader submit their market orders.

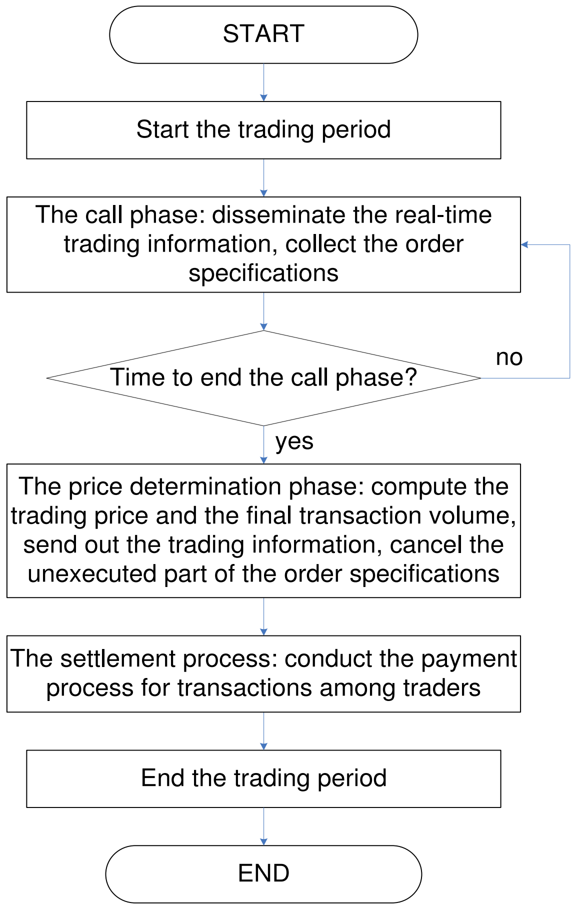

Figure 8: The updating rules of XAMM.

The call phase stops randomly after a fixed time span and is followed by the price determination phase. XAMM determines in the price determination phase the Xetra auction price and the final transaction volume. Then it cancels the unexecuted part of the orders and conducts the settlement process to complete the payment for each transaction. After trading in the Xetra auction market, traders obtain the risk-free asset holdings via the interaction with the risk-free asset market. XAMS iterates to the next trading period until it reaches the last trading period T. The diagram of the interaction in Figure 9 explicitly illustrates the workflows of activities among agents in the market dynamics.

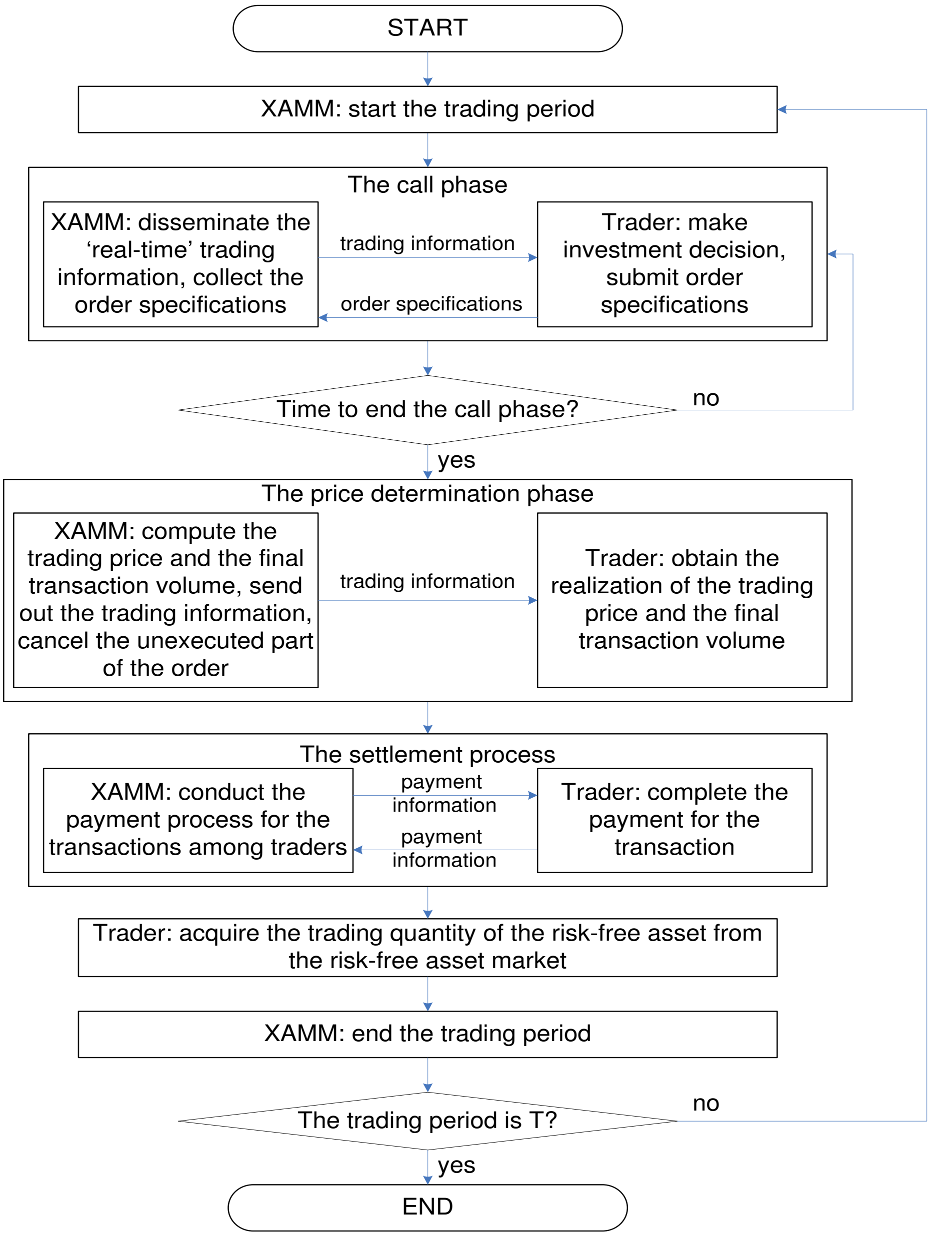

Figure 9: The diagram of the interaction for the ACE model of XAMS.

4 Market Experiment

We implement the ACE model of XAMS by applying the computer language Groovy/Java with the database backend of Microsoft ACCESS/MySQL. Then we construct the market experiment and conduct the computer simulation of XAMS. The focus of the market experiment is on the generated dynamics of Xetra auction price. We specifically concern with:

1. whether the generated Xetra auction price is generically non market-clearing.
2. the impact of the price setter on the non market-clearing property and on the volatility of the Xetra auction price in the market.

4.1 Experimental Setup

We setup the simulation profile for the Xetra auction market experiment by initializing the parameters and by specifying the forecasting methods employed by agents.

Model's Parameters to Initialize

- $T = 250\ldots$ time horizon. The time horizon approximates the time span of one year when considering one auction for each trading day and 255 trading days for Frankfurt Stock Exchange in the year of 2009.
- $N = 22\ldots$ number of traders. Three types of traders are considered in the model with 1 price setter, 20 price takers, and 1 noise trader.
- $r\ldots$ the interest rate of the risk-free asset. The interest rate is assumed to be constant in each profile. According to the Eurostat[5], the 3-months interest rate in European Union (27 countries) for the period of October 2008 to September 2009 is in the range of $[1.04\%, 5.52\%]$. We choose r randomly from this range.

XAMM's Parameters to Initialize

- $\{P_X[0], \ldots, P_X[-K_{\text{XAMM}} + 1]\}\ldots$ the historical trading prices. We consider $P_X[0], \ldots, P_X[-K_{\text{XAMM}} + 1]$ as historical auction prices of the stock "Deutsche Börse AG" listed in Xetra for the period from August 27, 2009 to November 04, 2009, with the memory span $K_{\text{XAMM}} = 100$.[6]

[5] See http://epp.eurostat.ec.europa.eu.

[6] This historical data of the stock trading price is provided online by Deutsche Börse, see http://deutsche-boerse.com.

- $\text{range}_\text{p} = 10\%$ ... the percentage of the price range. The Xetra platform requires transactions executed under certain price range from the last traded price P_{ref}. While it does not publicly provide the information of the percentage of the price range, another electronic trading platform Euronext requires the percentage of $\pm 10\%$. We employ the setting in Euronext and choose $\text{range}_\text{p} = 10\%$. Thus, XAMM considers Xetra auction price in the range of $[P_{\text{ref}}(1-10\%), P_{\text{ref}}(1+10\%)]$.

Trader's Parameters to Initialize

- $y_0^{(j)}[1]$ ... trader j's initial risk-free asset holding at the beginning of the trading period 1. We take $y_0^{(j)}[1]$ as a random positive number.
- $Z_0^{(j)}[1]$ ... trader j's initial risky asset holding at the beginning of the trading period 1. We take $Z_0^{(j)}[1]$ as a random integer number. To simplify the analysis, we assume that the aggregated volume in the market is constant with $\sum_{j=1}^{22} Z_0^{(j)}[1] = 1000$.
- $\alpha^{(j)}$ ... the measure of absolute risk aversion in the trader's utility function with the linear mean-variance preference. $\alpha^{(j)}$ is assumed to be constant in each profile and is selected randomly from the range of $(0, 2]$.
- Q_{range} ... the set of the trading behavior for the noise trader $j = 22$ depicted in Example 4.

  We assume that the noise trader randomly chooses for each trading period the trading behavior from the set of $Q_{\text{range}} = \{$ selling 1 unit, buying 1 unit $\}$. We keep the noise trader's random choice of trading behavior for each period the same in the benchmark market experiment as in the Xetra auction market experiment.

Trader's Forecasting Methods to Initialize

1. Forecasting Methods in Common

   For each period t, trader $j = 1$ depicted in Example 2 and price takers $j \in \{2, \ldots, 21\}$ depicted in Example 3 compute the expected mean value $q^{e(j)}[t]$ of the risky asset price for the next trading period $t+1$ and its associated variance $V^{e(j)}[t]$. We assume two types of forecasting: the stabilizer and the chartist. The forecasting type remains unchanged in each profile after trader j randomly chooses between these two types with equal probability.

(a) The Stabilizer

- $q^{e(j)}[t]\ldots$ The stabilizer computes $q^{e(j)}[t]$ as the mean value of the historical trading prices $P_X[0],\ldots,P_X[-K_{\text{XAMM}}+1]$ with

$$q^{e(j)}[t] = \frac{1}{K_{\text{XAMM}}}\sum_{n=1}^{K_{\text{XAMM}}} P_X[1-n].$$

- $V^{e(j)}[t]\ldots$ The stabilizer computes $V^{e(j)}[t]$ as the associated variance of the historical trading prices $P_X[0],\ldots,P_X[-K_{\text{XAMM}}+1]$ with

$$V^{e(j)}[t] = \frac{1}{K_{\text{XAMM}}-1}\sum_{n=1}^{K_{\text{XAMM}}} (P_X[1-n]-q^{e(j)}[t])^2.$$

Thus the stabilizer has constant forecast of $q^{e(j)}[t]$ and $V^{e(j)}[t]$ for each trading period.

(b) The Chartist

- $q^{e(j)}[t]\ldots$ The chartist expects the trend of the price movement based on the historical price movement. There are two types of chartists in our simulation: the trend trader and the anti-trend trader. The trend trader expects the trading price will increase (decrease) given the trading price increased (decreased) in the last trading period while the anti-trend trader expects the opposite. Let the indicator $\text{id}_\text{c} = 1$ for trend trader and $\text{id}_\text{c} = -1$ for anti-trend trader. With historical trading prices $P_X[t-1]$ and $P_X[t-2]$, the forecasting of the chartist at period t is

$$q^{e(j)}[t] = \begin{cases} P_X[t-1](1+\text{id}_\text{c}\cdot\omega^{(j)}) & \text{if} \qquad P_X[t-1] > P_X[t-2], \\ P_X[t-1] & \text{if} \qquad P_X[t-1] = P_X[t-2], \\ P_X[t-1](1-\text{id}_\text{c}\cdot\omega^{(j)}) & \text{if} \qquad P_X[t-1] < P_X[t-2]; \end{cases}$$

  where $\omega^{(j)}$ measures how aggressive of the price movement that the trader expects.
  id_c and $\omega^{(j)}$ are assumed to be constant after id_c is chosen randomly from $\{-1,1\}$ with equal probability and $\omega^{(j)}$ is chosen randomly from the range of $(0,\text{range}_\text{p}]$.
- $V^{e(j)}[t]\ldots$ It is assumed that the chartist keeps $V^{e(j)}[t]$ constant in the profile after it is chosen randomly from the range $(0,5]$.

2. Forecasting Method for Price Setter $j = 1$ depicted in Example 2

   - $P_X^{e(1)}[t] \ldots$ the forecast on Xetra auction price in the current trading period t. By applying formulation (16) in Appendix B, the price setter computes the forecast $P_X^{e(1)}[t](Q_m^{(1)}[t])$ that is a function of the quoted trading quantity $Q_m^{(1)}[t]$ in its market order.
   - $Z_X^{e(1)}[t] \ldots$ the forecast on the trader's final transaction volume in the current trading period t. By applying formulation (17) in Appendix B, the price setter computes $Z_X^{e(1)}[t](Q_m^{(1)}[t])$ that is a function of the quoted trading quantity $Q_m^{(1)}[t]$ in its market order.

3. Forecasting Method for Price Taker $j = 2, \ldots, 21$ depicted in Example 3

   - $P_X^{e(j)}[t] \ldots$ the forecast on Xetra auction price in the current trading period t. It is assumed that $P_X^{e(j)}[t]$ is randomly chosen from the price range $[P_{\text{ref}}(1 - 10\%), P_{\text{ref}}(1 + 10\%)]$ stipulated in XAMM.

4.2 Experimental Procedure

To investigate the market dynamics and the impact of the price setter in Xetra auction market, we construct along with the Xetra auction market experiment a benchmark market experiment. The benchmark market experiment has the same setup as the Xetra auction market experiment except for the price setter $j = 1$ depicted in Example 2. The benchmark market experiment replaces the price setter $j = 1$ with the benchmark trader which follows the same setup as depicted in Example 2 except for adopting the forecast $Z_X^{e(1)}[t](Q_m) = Q_m$ and $P_X^{e(1)}[t](Q_m) = P_{\text{ref}}$ where P_{ref} is the last traded price in Xetra auction market. Thus, the benchmark trader presents its objective in the submodule of objectives as the portfolio selection problem:

$$\begin{aligned}\max_{Q_m \in \mathbb{Z}} \quad & \{q^{e(1)}[t] - R \cdot P_{\text{ref}}\} \cdot Q_m + q^{e(1)}[t] \cdot Z_0^{(1)}[t] + R y_0^{(1)}[t] \\ & - \frac{\alpha_1}{2}\{Z_0^{(1)}[t] + Q_m\}^2 \cdot V^{e(1)}[t], \qquad (13)\end{aligned}$$

where α_1 is a constant measure of absolute risk aversion. As observed in (13), the trader considers in its portfolio selection problem that the current trading price and the trader's final trading volume are independent of Q_m. The benchmark trader is thus regressed to a price taker who has no manipulation on the current trading price and on the trading volume by its market order.

We consider 50 rounds of market experiments. It starts with initiating 50 simulation profiles for each round of the market experiment with the index $s \in \{1, \ldots, 50\}$. Then for each profile s, we conduct the benchmark market experiment and the Xetra auction market experiment with 250 trading periods of simulations.

4.3 Experimental Results

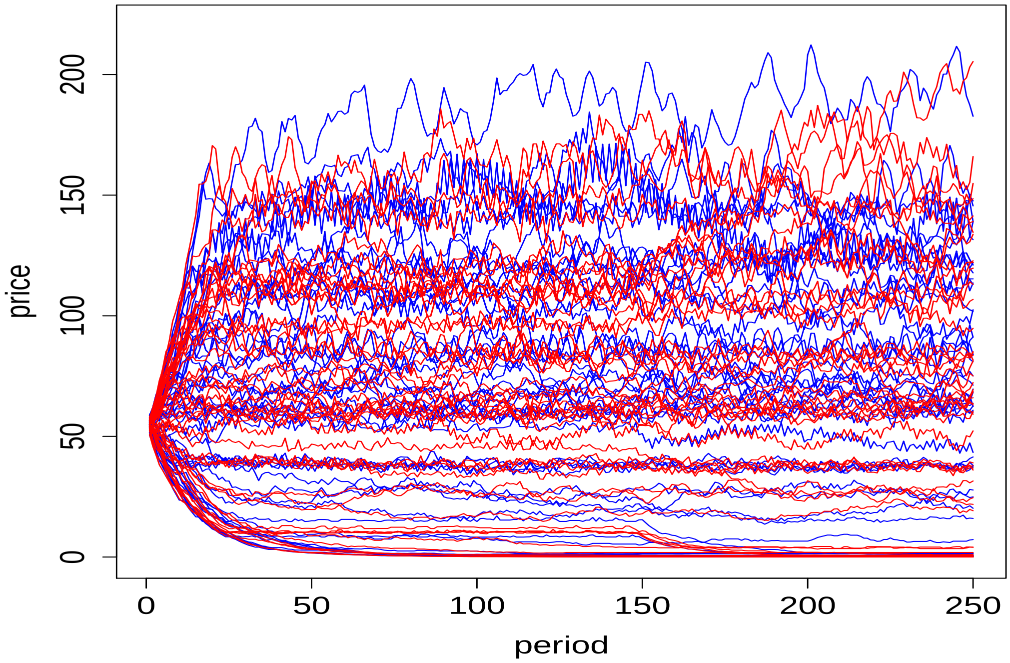

Figure 10: The price dynamics in 50 profiles.

Price Dynamics. We consider the price dynamics for 250 trading periods generated in the market experiment. Figure 10 illustrates the price dynamics generated by all 50 simulation profiles. The blue color in this figure as well as in the following figures is for the benchmark market experiment and the red color is for the Xetra auction market experiment.

Figure 10 demonstrates the divergence of the price dynamics in the market experiment. Consider the end-of-period trading price as the market price in the last trading period $t = 250$. It ranges $[0,\ 182.67]$ for all 50 benchmark market

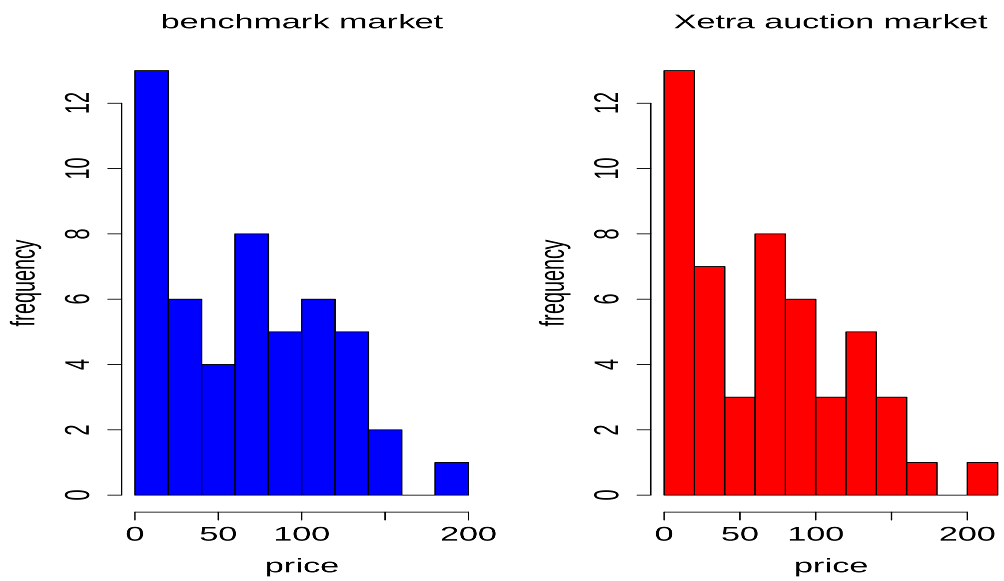

Figure 11: The histogram of the end-of-period trading price in the market experiments.

experiments and [0, 205.39] for all 50 Xetra auction market experiments in the simulation. The histogram of the end-of-period trading price depicted in Figure 11 illustrates the divergence emerges in both market experiments.

Property of Non Market-Clearing Trading Price. We employ the percentage of the non market-clearing price $percent^{(s)}_{non}$ to investigate the non market-clearing property of the trading price for each profile $s \in \{1, \ldots, 50\}$ with

$$percent^{(s)}_{non} = \frac{\sharp\{\text{trading periods with non market-clearing trading price}\}}{\sharp\{\text{trading periods with trading price}\}}.$$

Figure 12 shows the percentage for the benchmark market experiment and for the Xetra auction market experiment in each profile. The lowest percentage of non market-clearing price is 41.82% and the highest percentage is 96.58% in our simulation results, which implies that Xetra auction price generated in the market experiments is generically non market-clearing. This property can be easily observed in Figure 12 where the blue line and the red line are far above the x-axis.

We conduct statistical test to investigate the impact of the price setter on the percentage of the non market-clearing price. We specifically test whether $percent^{(s)}_{non}$

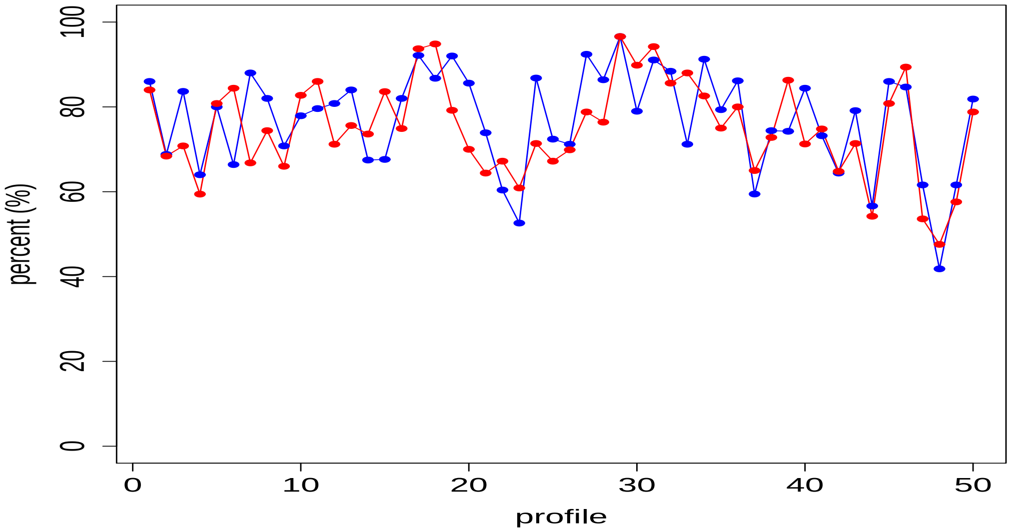

Figure 12: The percentage of non market-clearing price.

associated with the benchmark market experiment is greater than that associated with the Xetra auction market experiment. We apply the nonparametric statistical test – the Wilcoxon signed ranks test – and verbally present the null hypothesis as: $percent^{(s)}_{non}$ in the benchmark market experiment is no greater than that in the Xetra auction market experiment. The test result has $\mathrm{p-value} = 0.0659 < 0.1$. Thus, we reject the null hypothesis with 90% level of confidence and accept that $percent^{(s)}_{non}$ in the benchmark market experiment is greater than that in the Xetra auction market experiment. Thus, the Xetra auction market experiment where the price setter participates has a higher percentage of market equilibrium than the benchmark market experiment. This implies that the introduction of the price setter increases the possibility of market equilibrium and thus increases the market efficiency in Xetra auction market.

Intuitively, when there exists a surplus in Xetra auction market the price setter could submit a market order to accept the surplus without affecting the trading price in the market. When the price setter would submit a market order exceeding the surplus, the trading price would jump to an inferior position such that the trading price would fall down when the price setter would submit a market order in the sell side and vice versa. Thus the price setter has the incentive to meet but not outnumber the surplus in the market. When the price setter submits its market order to fully accept the surplus, Xetra auction market drives to equilibrium. The introduction of the price setter thus increases the market efficiency in Xetra auction market.

Price Volatility. We apply in this work the variance of the price dynamics $\{P_X^{(s)}[1], \ldots, P_X^{(s)}[250]\}$ for each profile s to measure price volatility in the market experiment. The variance of the price dynamics is formulated as:

$$Var^{(s)} = \frac{1}{249} \sum_{n=1}^{250} (P_X^{(s)}[n] - \overline{P_X^{(s)}})^2,$$

where $\overline{P_X^{(s)}}$ is the mean value of $\{P_X^{(s)}[1], \ldots, P_X^{(s)}[250]\}$. Figure 13 shows the variance for the benchmark market experiment and for the Xetra auction market experiment.

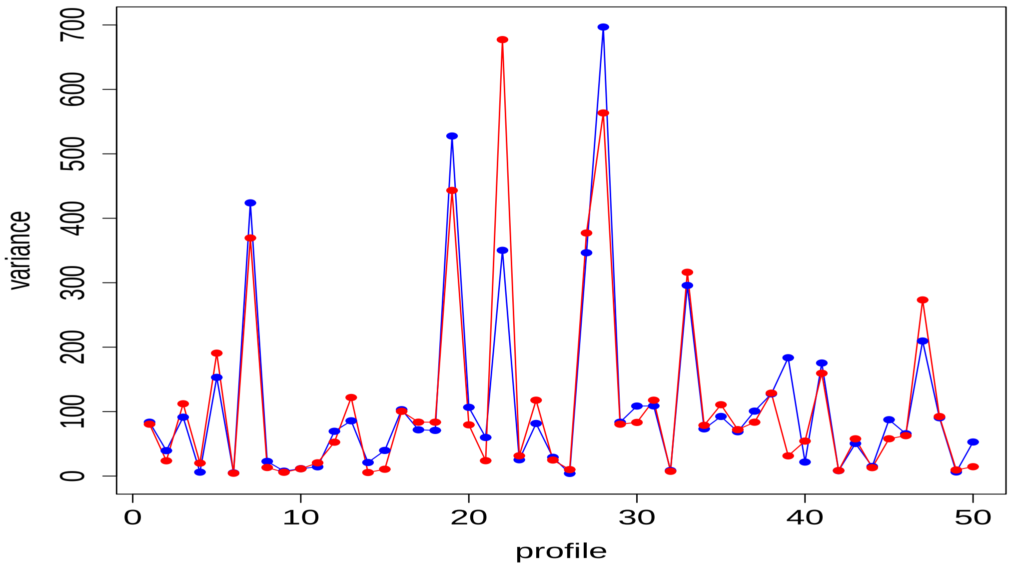

Figure 13: The variance of the trading price dynamics.

We use the Wilcoxon signed ranks test to investigate the impact of the price setter on the price volatility. The null hypothesis that we testify is verbally presented as: the variance on the price dynamics for the benchmark market experiment has the same measure as that for the Xetra auction market experiment. The test result has $\mathrm{p-value} = 0.8545$. We can not reject the null hypothesis to accept that the variance on the price dynamics for the benchmark market experiment is significantly different from that for the Xetra auction market experiment. It seems that the introduction of the price setter does not significantly impact the price volatility in Xetra auction market.

Intuitively the price setter influences the price volatility of Xetra auction market in two different directions. The price setter exploits the market for a profit by its aggressive trading behavior, which would increase the price volatility in Xetra auction market. On the other hand, the introduction of the price setter increases the possibility of market equilibrium. The increase in market efficiency would allow traders to efficiently adjust their trading behavior to stabilize the market in equilibrium, which implies a decrease in the price volatility of the market. These two opposite impacts offset against each other so that the participation of the price setter would not significantly influence the price volatility in Xetra auction market .

5 Concluding Remarks

We have developed in this work the integrative framework for ACE modelling as general guidance for analyzing economic systems from 'bottom-up' and for constructing the corresponding agent-based models. By applying this integrative framework, we have constructed the agent-based model of XAMS with the formulation of XAMM and of the price setter. The success of developing the agent-based model of XAMS by applying the integrative framework demonstrates that the modelling procedure and the modelling templates applied in this framework are sufficiently applicable for analyzing the economic system and for developing the corresponding agent-based model in a step-by-step manner.

We have implemented the agent-based model into computer software system and conducted the computer simulation for the market experiment. The investigation of simulation results on market dynamics has validated the property of non market-clearing trading price in Xetra auction market. This finding reveals the flexibility of the agent-based model in depicting non equilibrium market dynamics. Another flexibility of the agent-based modelling demonstrated in our work is on the aspect of modelling active economic agents. We have applied MAEA to model heterogenous traders in Xetra auction market with the mean-variance preference in the submodule of objectives. Essentially the submodule of objectives in MAEA presents the mechanism of selection that the agent employs to choose its action plan. By applying MAEA, one can easily extend the model of traders by replacing the mean-variance preference with new criteria of selection that allow more flavor of nonoptimizing and adaptive behavioral rules. One possibility is to introduce psychological patterns of decision making in the trader's submodule of objectives, which is open for the future work.

References

BERTALANFFY, L. V. (1993): *General system theory*. Braziller, New York, revised edition, 11. print. edn.

BLAHA, M. & J. RUMBAUGH (2004): *Object-Oriented Modeling and Design with UML*. Prentice Hall, second edition edn.

DAS, S. (2003): "Intelligent market-making in artificial financial markets", Master's thesis, Massachusetts Institute of Technology.

GRUPPE DEUTSCHE BÖRSE (2003): "The Market Model Stock Trading for *Xetra*", Frankfurt a. M.

LEBARON, B. (2006): "Agent-based computational finance", in *Handbook of Computational Economics, Vol. 2: Agent-Based Computational Economics.*, ed. by L. S. Tesfatsion & K. L. Judd, Handbooks in Economics Series, chap. 9. North-Holland.

LI, X. (2010): "Microeconomic Foundation of Investment Decisions for Electronic Security Trading Systems", Ph.D. thesis, Bielefeld Graduate School of Economics and Management, Bielefeld University, forthcoming.

LI, X. & J. WENZELBURGER (2005): "Price Formation and Asset Allocations of the Electronic Trading System *Xetra*", Discussion Paper No.540, Bielefeld University, Department of Economics.

MERTON, R. C. (1992): *Continuous-Time Finance*. Blackwell.

NYSE EURONEXT (2010): *Euronext Rule Book - Book I.*

PINDYCK, R. & D. RUBINFELD (2001): *Microeconomics*. Prentice Hall, 5th edition edn.

POTTS, J. (2000): *The new evolutionary microeconomics*, New horizons in institutional and evolutionary economics. Elgar.

Russell, S. J. & P. Norvig (2003): *Artificial intelligence, a Modern Approach*, Prentice Hall series in artificial intelligence. Prentice Hall, Upper Saddle River, 2. ed. edn.

Sharpe, W. (1964): "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk", *Journal of Finance*, 19(3), 425–442.

Tesfatsion, L. S. (2006): "Agent-Based Computational Economics: A Contructive Approach to Economic Theory", in *Handbook of Computational Economics, Vol. 2: Agent-Based Computational Economics*, ed. by L. S. Tesfatsion & K. L. Judd. North-Holland.

A Xetra Auction Market Mechanism

The Xetra auction market mechanism is composed of the Xetra auction price mechanism and the allocation mechanism to determine the Xetra auction price and the final trading volume respectively. Based on the description of the Xetra auction trading rules in Gruppe Deutsche Börse (2003) and the formalization proposed in Li & Wenzelburger (2005), Figure 14 illustrates the Xetra auction price mechanism in which the subprocess Xetra-Auction-PDA($\mathcal{I}_0$) is depicted in Figure 15. The formulation of the Xetra auction allocation mechanism is illustrated in Figure 16.

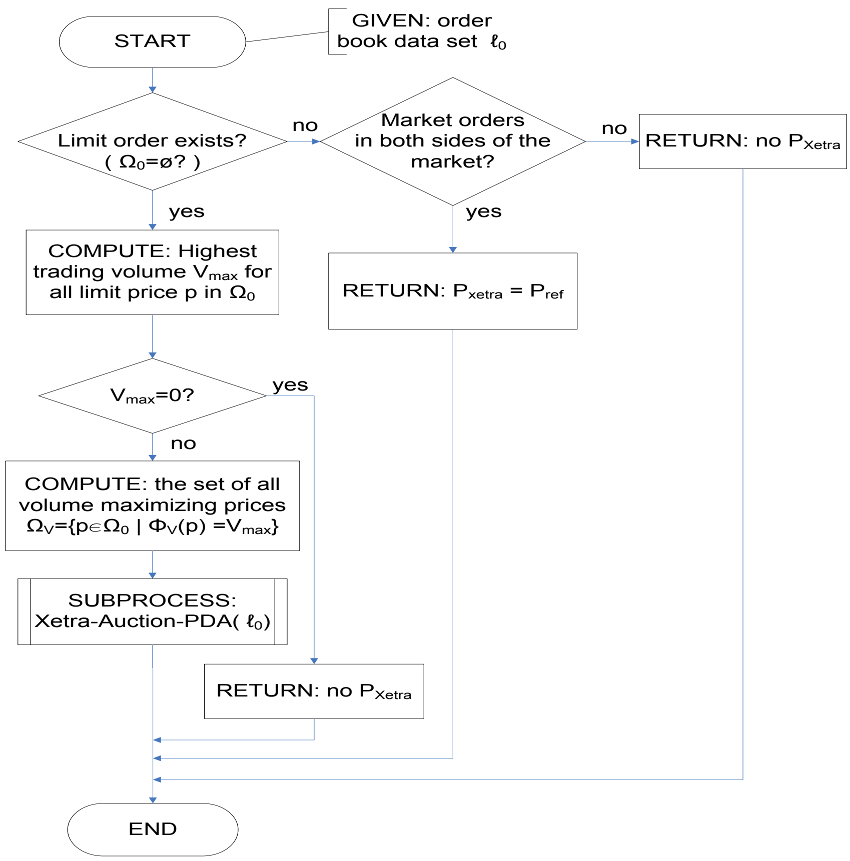

Figure 14: The flowchart of the Xetra auction price mechanism.

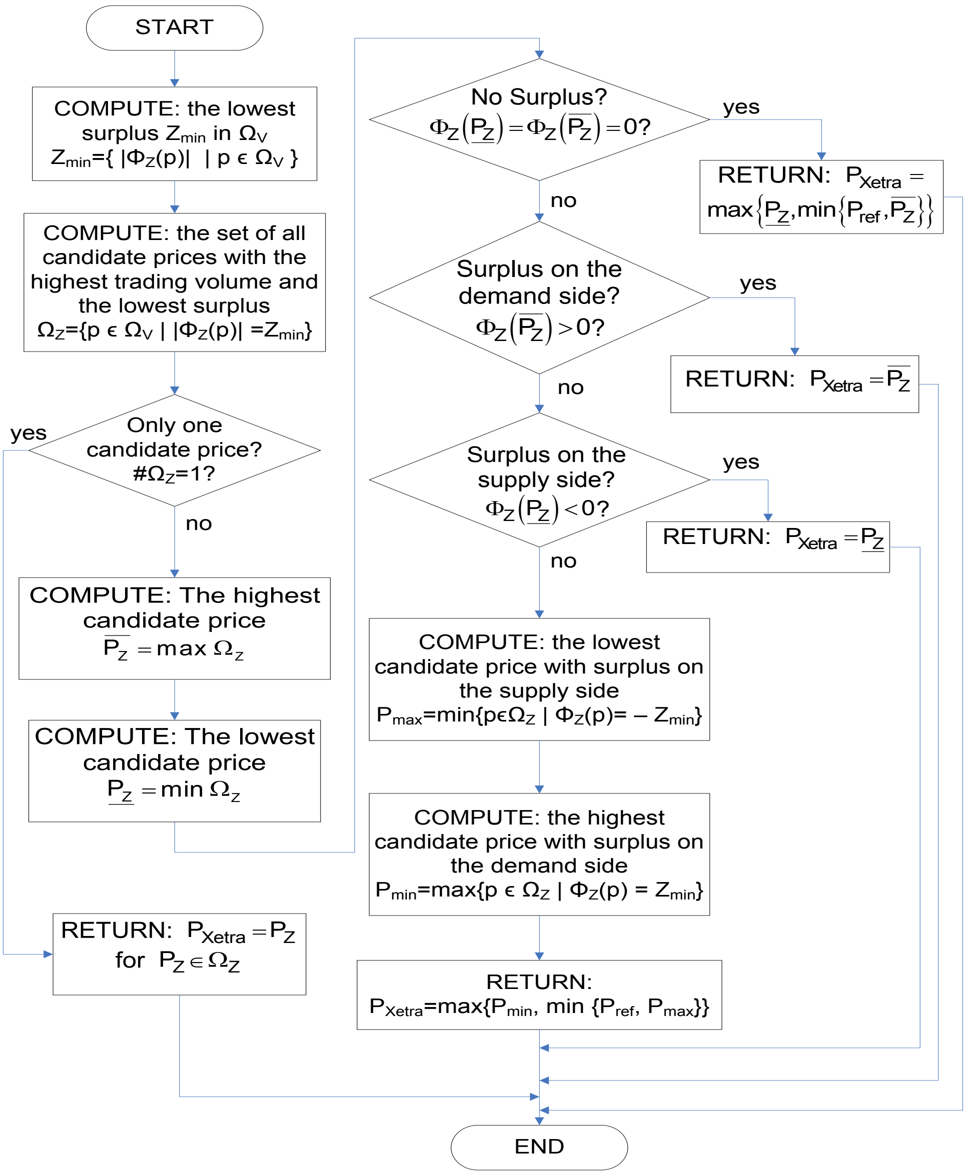

Figure 15: The flowchart of subprocess: Xetra-Auction-PDA($\mathfrak{I}_0$).

The function $QV(Ord)$ in Figure 16 returns the quoted trading quantity of the order Ord and the function $RV(Ord)$ denotes the realized trading volume for the order Ord.

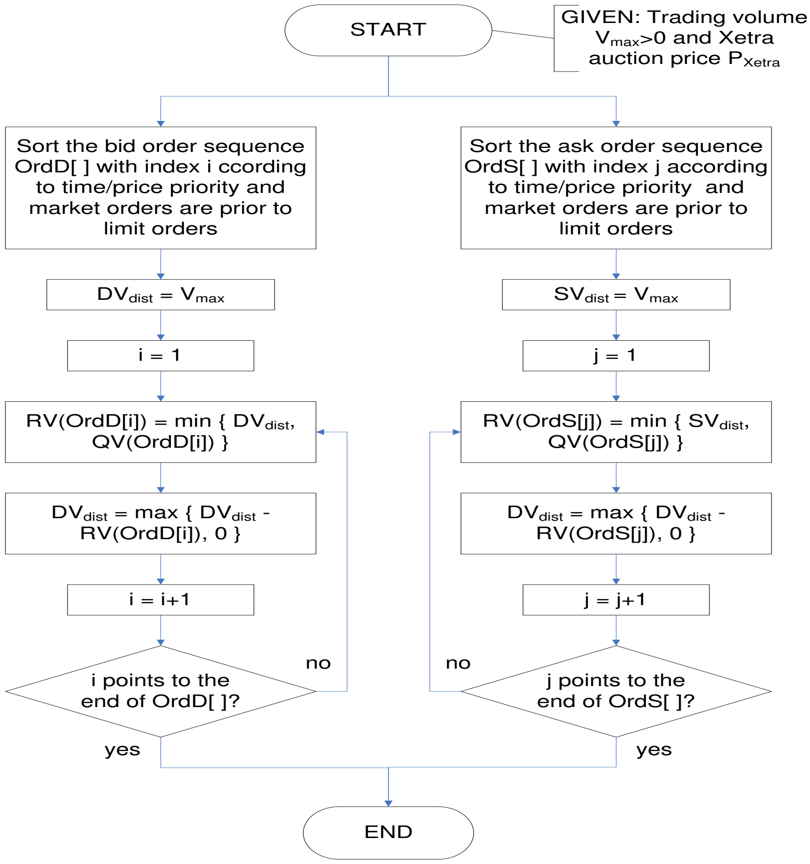

Figure 16: The flowchart of the Xetra auction allocation mechanism.

B Subjective Forecast of Price Setter

We present the price setter's forecasting on the Xetra auction price and the final transaction volume.[7] Introduce a new notation "I" to represent a closed half or an open half of an interval. For example, for any real numbers a and b with $a < b$, $\mathrm{I}\, a, b\, \mathrm{I} := [a, b]$, $[a, b)$, $(a, b]$, or (a, b). For trading period t, consider the central order book data set $\mathcal{J}_0[t]$ in Xetra auction contains a series of limit prices $P_1 \leq P_2 \leq \cdots \leq P_{N_l}$ and the reference price P_{ref} that is the last trading price. The excess demand function for the order book $\mathcal{J}_0[t]$ is a step function with

$$\Phi_Z(p; \mathcal{J}_0[t]) = \sum_{n=0}^{N_l} \phi_n^Z 1_{A_n^Z}(p), \tag{14}$$

[7]See Chapter 3 in Li (2010) for comprehensive description.

where ϕ_n^Z is a constant for $n = 0, \dots, N_l$ and $\{A_0^Z, A_1^Z, \dots, A_{N_l}^Z\}$ is a partition of $\mathbb{R}_+$ with $A_0^Z := [0, P_1 \,[$; $A_n^Z :=] \, P_n, P_{n+1} \,[$ for $n = 1, \dots, N_l - 1$; and $A_{N_l}^Z :=]$ $P_{N_l}, +\infty)$.

The price setter is to submit the market order $Q_m^{(1)}[t]$ with $Q_m^{(1)}[t] > 0$ denoting a market order on the demand side and $Q_m^{(1)}[t] < 0$ denoting that on the supply side. After the price setter submit its market order, the excess demand function updates to the form

$$\begin{aligned} \Phi_Z'(Q_m^{(1)}[t], p; \mathcal{I}_0[t]) &= \Phi_Z(p; \mathcal{I}_0[t]) + Q_m^{(1)}[t] \\ &= \sum_{n=0}^{N_l} \phi_n^Z 1_{A_n^Z}(p) + Q_m^{(1)}[t]. \end{aligned} \tag{15}$$

The price setter expects to be the last trader submitting order to the market. Then the price setter's forecast $P_X^e(Q_m^{(1)}[t])$ on the upcoming Xetra auction price is the step function:

$$P_X^e(Q_m^{(1)}[t]) = \begin{cases} P_1 & \text{when } Q_m^{(1)}[t] \in (-\infty, -\phi_1^Z); \\ P_n^* & \text{when } Q_m^{(1)}[t] = -\phi_n^Z, \quad n \in \{1, 2, \dots, N_l - 1\}; \\ P_n & \text{when } Q_m^{(1)}[t] \in (-\phi_{n-1}^Z, -\phi_n^Z), \quad n \in \{2, 3, \dots, N_l - 1\}; \\ P_{N_l} & \text{when } Q_m^{(1)}[t] \in (-\phi_{N_l-1}^Z, +\infty); \end{cases} \tag{16}$$

where for any $n \in \{1, 2, \cdots, N_l - 1\}$

$$P_n^* = \begin{cases} P_n & \text{CASE 1;} \\ P_{n+1} & \text{CASE 2;} \\ \max\{P_n, \min\{P_{\text{ref}}, P_{n+1}\}\} & \text{CASE 3;} \end{cases}$$

for **CASE 1:** either $A_n^Z = [P_n, P_{n+1})$ or $A_n^Z = (P_n, P_{n+1})$ with $|\, \Phi_Z'(-\phi_n^Z, P_n; \mathcal{I}_0[t]) \,|<|\, \Phi_Z'(-\phi_n^Z, P_{n+1}; \mathcal{I}_0[t]) \,|$;

for **CASE 2:** either $A_n^Z = (P_n, P_{n+1}]$ or $A_n^Z = (P_n, P_{n+1})$ with $|\, \Phi_Z'(-\phi_n^Z, P_n; \mathcal{I}_0[t]) \,|>|\, \Phi_Z'(-\phi_n^Z, P_{n+1}; \mathcal{I}_0[t]) \,|$;

for **CASE 3:** either $A_n^Z = [P_n, P_{n+1}]$ or $A_n^Z = (P_n, P_{n+1})$ with $|\, \Phi_Z'(-\phi_n^Z, P_n; \mathcal{I}_0[t]) \,|=|\, \Phi_Z'(-\phi_n^Z, P_{n+1}; \mathcal{I}_0[t]) \,|$.

Consider the notation $a^+ := \max\{0, a\}$ and $a^- := \min\{0, a\}$ for any $a \in \mathbb{R}$. The price setter's forecast $Z_X^e(Q_m^{(1)}[t])$ on the its final transaction volume is as:

$$Z_X^e(Q_m^{(1)}[t]) = \max\{(-\phi_0^Z)^-, \min\{Q_m^{(1)}[t], (-\phi_{I+J}^Z)^+\}\}. \tag{17}$$