Creative Modeling Exercise

Exploring Thomas Piketty's Hypotheses on Wealth Inequality in a Dynamic Open-Ended Macroeconomic Model

EXERCISE 10: TEAM/INDIVIDUAL (24 Points Total) **L. Tesfatsion**
DUE: 11:00am, Tuesday, December 8, 2015 **Econ 502/Fall 2015**

Note 1: Students are permitted to work on this exercise *either* individually *or* as a self-organized team consisting of at most three students. Teams should turn in only one exercise answer, and the point score awarded for a team exercise will be individually awarded to each team member.

Note 2: Each individual/team should make an **EXTRA** copy of their exercise answer to bring to class on the due date for use in class discussion.

Relevant Background Materials:

- * [1] Entire Piketty Symposium, *J. Econ. Perspectives* 29(1), 2015 **ON-LINE**
  http://www2.econ.iastate.edu/classes/econ502/tesfatsion/PikettySymposium.JEP2015.pdf
- ** [2] Syllabus I.C, Packet 4 (appendix): Intro to Walrasian General Equilibrium
- ** [3] Syllabus III.B, Elements of Dynamic Economic Modeling, **ON-LINE**:
  http://www2.econ.iastate.edu/tesfatsi/DynamicEconomicModelingBasics.WPVersion.pdf
- ** [4] Required readings, Syllabus VI (Macro Modeling of Endogenous Coordination)
  http://www2.econ.iastate.edu/classes/econ502/tesfatsion/syl502.htm#Coord
- * [5] Recommended readings, Syllabus VI

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Resolution used in our Piketty-Acemoglu/Robinson debate held on November 12 (Paraphrased from Acemoglu/Robinson in Ref. [1]):
The theoretical core of Piketty's book *Capital in the Twenty-First Century* is that, if $r - g$ is positive (or sufficiently large), it will lead to a divergence of wealth between the very rich and the rest of the population. Upon this basis, Piketty builds a bold and sweeping theory of growth and inequality applicable to all capitalist economies. However, this theory is misguided because it ignores the key forces shaping how an economy functions: namely, the endogenous evolution of technology and of the institutions and the political equilibrium that influence not only technology but also how markets function and how the gains from various different economic arrangements are distributed.

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EXERCISE GOALS:
This creative modeling exercise asks you to develop a dynamic macroeconomic model that can be used to address the above resolution (either in opposition or in the affirmative) in a more compelling manner than a one-sector optimal growth model that assumes a single representative consumer and an aggregate production function. Ideally, your model should satisfy the following three goals:

- Your model should generate *over time* a distribution of wealth across multiple modeled people in a mathematically consistent and economically reasonable manner.
- Your model should permit you to explore conditions under which wealth *inequality* over time will (or will not) arise, will (or will not) persist, and will (or will not) diverge.
- Your model should permit you to explore Piketty's hypotheses regarding the relationship between $r - g$ and wealth inequality.

EXERCISE METHOD:
Take as your starting point for this creative modeling exercise the particular Walrasian General Equilibrium (WGE) model developed in detailed analytic form in Ref. [2], i.e., in the *appendix* to Packet 4 ("Introduction to Walrasian General Equilibrium Modeling").

As detailed in Ref. [2], this WGE model is a one-period model of an economy consisting of two corporate firms, X and Y, and n consumers ($n > 1$). The corporate firms X and Y produce two distinct goods x and y using labor and capital services ℓ and k supplied by the n consumers. Each consumer i is endowed at the beginning of the period with an initial labor endowment l_i^o, an initial capital stock K_i^o, and initial percentage ownership shares θ_{xi}^o and θ_{yi}^o in firms X and Y. Firms X and Y have different production technologies $F_X(\ell, k)$ and $F_Y(\ell, k)$, and each consumer i has a distinct utility function $u_i(x_i, y_i, \ell e_i)$ over bundles $(x_i, y_i, \ell e_i)$ consisting of possible consumed amounts (x_i, y_i) of produced goods as well as possible amounts $\ell e_i = \ell_i^o - \ell_{xi} - \ell_{yi}$ of leisure.

Keeping in mind the above exercise goals, carry out the following four steps as best you can.

Step 1: [12 Points]

- Remove all equilibrium conditions from the WGE model in the Packet 4 appendix. In particular, remove all market clearing conditions and fulfilled expectations conditions. This breaks the circular flow between the consumers and firms.
- Establish a dynamic circular flow between the consumers and firms by introducing economic processes undertaken by the consumers and firms in each successive time period $t = 1, 2, \ldots$ that can feasibly be carried out, given reasonable assumptions regarding what the consumers and firms can observe and know about their economy as time passes. Be sure that your resulting model is able to generate over time a *distribution of wealth* across the n consumers in a manner that is both mathematically consistent and economically reasonable.

Step 2: [**4 Point**] Construct a specific metric for measuring wealth inequality across the n consumers at the beginning of any given period $t = 1, 2, \ldots$.

Step 3: [**4 Point**] Carefully discuss which factors in your model determine the extent of wealth inequality in your economy in any period t, as measured by the metric you have constructed in Step 4.

Step 4: [**4 Point**] Carefully discuss the extent to which Piketty's hypotheses concerning the relationship between $r - g$ and wealth inequality can be explored within your model.

IMPORTANT ADDITIONAL CLARIFICATIONS:

In Step 1 of this exercise you are, in effect, being asked to develop a *discrete-time state-space model* for a dynamic economy with two firms and n consumers that exists over discrete time periods $t = 1, 2, \ldots$; see Ref. [3]. This state-space model should permit your dynamic economy to move in a cause-effect (historical) fashion from any given state at the beginning of period t to a new state at the beginning of period $t + 1$, for $t = 1, 2, \ldots$.

Note that "state" in the above paragraph can refer either to a *scalar* state variable or to a state *vector*.

The movement of the state over time should be determined by behavioral specifications for the consumers and firms as constrained by institutional arrangements and physical conditions, where the latter might include realizations for various types of exogenous random shocks. The behavioral specifications should determine how the consumers and firms in your dynamic economy collect and exchange information, how the firms determine what to produce, how prices are determined, how consumers and firms engage in trades, how these trades are settled, and so forth.

Your state-space model should *not* include the imposition of any equilibrium or coordination assumptions that you (the modeler) impose on the consumers and firms; for example, markets somehow clear and/or expectations are somehow correct even though no explanation for this coordination is provided within your model. All events in your modeled world over periods $t = 1, 2, \ldots$ should arise from the actions and interactions of the consumers and firms over time as constrained by institutional arrangements and physical conditions, starting from an exogenously given initial state at the beginning of period 1 that you (the modeler) set.

As detailed in Ref. [3], you (the modeler) should provide precise specifications for all modeled relationships, both those occurring within a period ("simultaneous relationships") and those determining the change in the state from one period to the next ("dynamic relationships"). You (the modeler) should also provide a classification of variables together with any admissibility conditions to be imposed on exogenous variables and functional forms. You (the modeler) get to specify these modeled relationships, variable classifications, and admissibility conditions in any way you want as long as these specifications are in accordance with the model requirements and goals appearing in the statement of the exercise.

The simultaneous and dynamic relationships for your model can be in analytic equation form or, if you want to be more adventuresome, they can be in "agent-based" form; see

Refs. [3,4,5]. Your exercise will be evaluated in terms of how well your constructed model satisfies the goals of the exercise, not on your precise modeling approach.

For any discrete-time state-space model, an excellent way to check that the model has been correctly developed as a cause-effect (historical) process is to try to construct a *flow diagram* (or *box-arrow diagram*). A complete flow diagram would depict how, starting from any particular state at the beginning of an arbitrarily selected period t, the actions and interactions of the consumers and firms occurring during period t (plus any shock realizations during period t) lead to the determination of a new state at the beginning of period $t + 1$.