DC Optimal Power Flow Formulation in AMES

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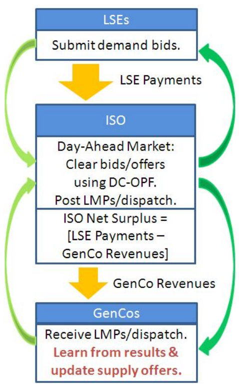

Fig. 1. AMES day-ahead energy market activities during each day D.

***Abstract*—This study reviews the form of the ISO's DC Optimal Power Flow (OPF) problem in the AMES (V2.05) Wholesale Power Market Testbed.**

I. AMES Testbed

The latest version of AMES (*A*gent-based *M*odeling of *E*lectricity *S*ystems) can be freely downloaded either at [3] or [4]. Section I-A summarizes the key features of Version 2.05 of AMES, used in this study. These key features reflect, in simplified form, day-ahead energy market operations in the MISO ([5], [6]) and ISO-NE [7]; cf. Fig. 1. Section I-B provides quantitative definitions for the net surplus amounts collected by the AMES LSEs, GenCos, and ISO, and for market efficiency measured in terms of total net surplus.

A. Overview of Key AMES Features

The AMES(V2.05) wholesale power market operates over an AC transmission grid starting with hour 00 of day 1 and continuing through hour 23 of a user-specified maximum day. AMES includes an *Independent System Operator (ISO)* and a collection of energy traders consisting of J *Load-Serving Entities (LSEs)* and I *Generation Companies (GenCos)* distributed across the buses of the transmission grid.

The objective of the not-for-profit ISO is the maximization of *Total Net Surplus (TNS)* subject to transmission constraints and GenCo operating capacity limits. In an attempt to attain this objective, the ISO operates a day-ahead energy market settled by means of LMP.

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The welfare of each LSE j is measured by the net earnings it secures for itself through the purchase of power in the day-ahead market and the resale of this power to its retail customers. During the morning of each day D, each LSE j reports a *demand bid* to the ISO for the day-ahead market for day D+1. Each demand bid consists of two parts: fixed demand (i.e., a 24-hour load profile) to be sold downstream at a regulated price r to its retail customers with fixed-price contracts; and 24 price-sensitive inverse demand functions, one for each hour, reflecting the price-sensitive demand (willingness to pay) of its retail customers with dynamic-price contracts.[1]

The objective of each GenCo i is to secure for itself the highest possible net earnings each day through the sale of power in the day-ahead market. During the morning of each day D, each GenCo i uses its current action choice probabilities to choose a supply offer from its action domain AD_i to report to the ISO for use in all 24 hours of the day-ahead market for day D+1.[2] Each supply offer in AD_i consists of a linear marginal cost function defined over an operating capacity interval. GenCo i's ability to vary its choice of a supply offer from AD_i permits it to adjust the ordinate/slope of its reported marginal cost function and/or the upper limit of its reported operating capacity interval in an attempt to increase its daily net earnings.

After receiving demand bids from LSEs and supply offers from GenCos during the morning of day D, the ISO determines and publicly posts hourly bus LMP levels as well as LSE cleared demands and GenCo dispatch levels for the day-ahead market for day D+1. These hourly outcomes are determined via *Security-Constrained Economic Dispatch (SCED)* formulated as bid/offer-based *DC Optimal Power Flow (DC-OPF)* problems with approximated TNS objective functions based

[1]The LSEs in AMES(V2.05) have no learning capabilities; LSE demand bids are user-specified at the beginning of each simulation run. However, as explained more carefully in [8], AMES(V2.05) includes a learning module, JReLM, that can be used to implement a wide variety of stochastic reinforcement learning methods for decision-making agents. Extension to include LSE learning is planned for future AMES releases.

[2]Whether GenCos are permitted to report only one supply offer or 24 supply offers for use in the day-ahead energy market varies from one energy region to another. For example, the ISO-NE permits only one supply offer whereas MISO permits 24 separate supply offers. Baldick and Hogan [9, pp. 18-20] conjecture that imposing limits on the ability of GenCos to report distinct hourly supply offers could reduce their ability to exercise market power.

on reported rather than true GenCo costs.[3]

At the end of each day D the ISO settles the day-ahead market for day D+1 by receiving all purchase payments from LSEs and making all sale payments to GenCos based on the LMPs for the day-ahead market for day D+1, collecting any difference as *ISO net surplus*. As will be clarified in Section I-B, this ISO net surplus is guaranteed to be nonnegative and, under congested grid conditions, will typically be strictly positive due to the separation of bus LMPs and the dispersion of the GenCos and LSEs across the various bus locations.

Each GenCo i at the end of each day D uses a stochastic reinforcement learning algorithm to update the action choice probabilities currently assigned to the supply offers in its action domain AD_i, taking into account its day-D settlement payment ("reward"). In particular, if GenCo i's supply offer on day D results in a relatively good reward, GenCo i increases the probability it will choose to report this same supply offer on day D+1, and conversely.

There are no system disturbances (e.g., weather changes) or shocks (e.g., line outages). Consequently, the dispatch levels determined on each day D for the day-ahead energy market for day D+1 are carried out as planned without need for settlement of differences in the real-time energy market for day D+1.

B. Total Net Surplus and Market Efficiency

In AMES(V2.05), total net surplus (TNS) is the sum of LSE, GenCo, and ISO net surplus. As detailed in Section II, for each hour H of the day-ahead energy market the ISO attempts to solve a standard bid/offer-based DC-OPF problem involving the maximization of TNS subject to power-flow balance constraints, transmission branch limits, and GenCo capacity constraints.[4] However, in GenCo learning treatments the ISO has to construct its TNS objective function using reported rather than true GenCo costs.

This subsection presents the general parameterized AMES(V2.05) formulations for LSE demand bids and GenCo supply offers as well as the LSE, GenCo, and ISO total net surplus amounts realized during each day D.

For each day D, LSE j's *demand bid* for hour H of the day-ahead market for day D+1 consists of a fixed demand for power, p^F_{Lj}(H,D), to be sold downstream at a regulated price r ($/MWh) to its retail customers with fixed-price contracts, and a linear price-sensitive inverse demand function

$$\mathrm{F}_{jHD}(p^S_{Lj}) \;=\; \mathrm{c}_j(H,D) - 2\mathrm{d}_j(H,D)p^S_{Lj} \quad (\$/MWh) \quad (1)$$

defined over a *power purchase interval*

$$0 \;\leq\; p^S_{Lj} \;\leq\; \mathrm{SLMax}_j(H,D) \quad (MW) \quad (2)$$

The expression $\mathrm{F}_{jHD}(\mathrm{p}^S_{Lj})$ in (1) denotes LSE j's *purchase reservation value* for energy evaluated at p^S_{Lj}, i.e., the maximum dollar amount it is truly willing to pay per MWh, which in turn reflects the willingness-to-pay of its retail customers with dynamic-price contracts.

Suppose LSE j, located at bus $k(j)$, is cleared at a total demand level p_{Lj}(H,D) = [p^F_{Lj}(H,D)+p^S_{Lj}(H,D)] at price $\mathrm{LMP}_{k(j)}$(H,D) for hour H of the day-ahead market for day D+1. The *payments* of LSE j for all 24 hours of day D+1, settled at the end of day D, are

$$\mathrm{Pay}_j(D) = \sum_{H=00}^{23} LMP_{k(j)}(H,D)\cdot p_{Lj}(H,D) \quad (\$) \quad (3)$$

Using standard market efficiency analysis [1], the net surplus accruing to the "last" MW of power sold by LSE j to its dynamic-price retail customers, evaluated at any total sale quantity p, is given by $[F_{jHD}(p)-\pi]$ + $[\pi - LMP_{k(j)}(H,D)]$, where π denotes the price charged by LSE j for this last MW. The first bracketed term is the net surplus portion accruing to the retail customers and the second bracketed term is the net surplus portion accruing to LSE j. For simplicity, it will hereafter be supposed that LSE j is able to extract all net surplus from its dynamic-price retail customers by charging these customers their maximum willingness to pay for each purchased MW, i.e., by setting $\pi = F_{jHD}(p)$ at each power level p.[5] It follows that the gross surplus for LSE j realized on day D is given by the revenue ($) amount

$$GS_j(D) = \sum_{H=00}^{23}[r\cdot p^F_{Lj}(H,D) + \int_0^{p^S_{Lj}(H,D)} \mathrm{F}_{jHD}(p)dp] \quad (4)$$

The *LSE net surplus* realized on day D is then

$$\mathrm{LSENetSur}(D) = \sum_{j=1}^{J}\left[\mathrm{GS}_j(D) - \mathrm{Pay}_j(D)\right] \quad (\$) \quad (5)$$

For each day D, the *supply offer* chosen by GenCo i to report to the ISO for use in each hour H of the day-ahead market for day D+1 consists of a linear *reported marginal cost function*

$$\mathrm{MC}^R_{iD}(p_{Gi}) \;=\; \mathrm{a}^R_i(D) + 2\mathrm{b}^R_i(D)p_{Gi} \quad (\$/MWh) \quad (6)$$

defined over an *operating capacity interval*

$$\mathrm{Cap}^L_i \;\leq\; p_{Gi} \;\leq\; \mathrm{Cap}^U_i \quad (MW) \quad (7)$$

for the generation of power p_{Gi}. The expression $\mathrm{MC}^R_{iD}(\mathrm{p}_{Gi})$ in (6) denotes GenCo i's *reported sale reservation value* for energy evaluated at p_{Gi}, i.e., the minimum dollar amount it reports it is willing to accept per MWh. The reported marginal cost functions (6) can lie either on or above GenCo i's *true marginal cost function*

$$\mathrm{MC}_i(p_{Gi}) \;=\; \mathrm{a}_i + 2\mathrm{b}_i p_{Gi} \quad (\$/MWh) \quad (8)$$

[3] A technical presentation of the bid/offer-based DC-OPF problem formulation for the ISO in AMES(V2.05) is provided in Section II. The solutions to these DC-OPF problems takes the form of "supply function equilibria" rather than market clearing outcomes based on single-point bids and offers; see [10]. As will be seen in Section I-B, the GenCos do not incur start-up/shut-down or no-load costs and do not face ramp rate constraints. Consequently, the ISO in AMES(V2.05) does not undertake *Security-Constrained Unit Commitment (SCUC)*. In future AMES versions the user will be able to specify these types of unit commitment costs and constraints for GenCos and to have the ISO undertake SCUC and SCED in tandem to determine GenCo commitments and dispatch levels.

[4] As will be seen below, when all demand is fixed (i.e., price insensitive), the maximization of TNS is equivalent to the minimization of GenCo total avoidable costs.

[5] At the other extreme, a dynamic-price contract with $\pi = LMP_{k(j)}(H,D)$ would award all of the net surplus to the retail customers.

At the beginning of any planning period, a GenCo's *avoidable costs* consist of the operational costs that it can avoid by shutting down production together with the portion of its fixed (non-operational) costs that it can avoid by taking appropriate additional actions such as asset re-use or re-sale. In order for production to proceed, revenues from production should at least cover avoidable costs. In the present study the GenCos do not incur start-up/shut-down or no-load costs, and all of their fixed costs are assumed to be *sunk*, i.e., non-avoidable. Consequently, the *avoidable cost function* $\mathrm{C}_i^a(\mathrm{p}_{Gi})$ for each GenCo i for any hour H is given by the integral of its true hourly marginal cost function:

$$\mathrm{C}_i^a(p_{Gi}) = \int_0^{p_{Gi}} \mathrm{MC}_i(p)dp = a_i p_{Gi} + b_i[p_{Gi}]^2 \quad (\$/h) \quad (9)$$

where p_{Gi} satisfies (7).

Suppose GenCo i, located at bus $k(i)$, is dispatched at level p_{Gi}(H,D) at price $\mathrm{LMP}_{k(i)}$(H,D) for hour H of the day-ahead market for day D+1. The *revenues* due to GenCo i for all 24 hours of day D+1, settled at the end of day D, are

$$\mathrm{Rev}_i(D) = \sum_{H=00}^{23} LMP_{k(i)}(H,D) \cdot p_{Gi}(H,D) \quad (\$) \quad (10)$$

Net earnings are defined as revenues minus avoidable costs. Let the avoidable costs incurred by GenCo i on day D for any hour H of day D+1 based on its day-D dispatch p_{Gi}(H,D) be denoted by C_i^a(H,D). Then the net earnings of GenCo i for all 24 hours of day D+1, realized on day D, are

$$\mathrm{NE}_i(D) \;=\; \mathrm{Rev}_i(D) - \sum_{H=00}^{23} \mathrm{C}_i^a(H,D) \quad (\$) \quad (11)$$

Using standard market efficiency analysis [1], the *GenCo net surplus* realized on day D is then

$$\mathrm{GenNetSur}(D) = \sum_{i=1}^{I} \mathrm{NE}_i(D) \quad (\$) \quad (12)$$

The *ISO net surplus* realized on day D is the difference between LSE payments and GenCo revenues for the day-ahead market for day D+1 that are settled at the end of day D. More precisely,

$$\mathrm{ISONetSur}(D) = \sum_{j=1}^{J} \mathrm{Pay}_j(D) - \sum_{i=1}^{I} \mathrm{Rev}_i(D) \quad (\$) \quad (13)$$

Figure 2 provides a simple example of ISO net surplus collection for a 2-bus system during a particular hour H. The LSE at bus 2 pays LMP_2 to the ISO for each MW of its cleared fixed demand p_L^F. A portion M of this demand is supplied by GenCo G1 at bus 1, who receives LMP_1 per MW from the ISO. The remaining portion [p_L^F-M] of this demand is supplied by GenCo G2 at bus 2, who receives $\mathrm{LMP}_2 > \mathrm{LMP}_1$ per MW from the ISO. The ISO net surplus for hour H is then calculated to be M $\times$ [LMP_2 - LMP_1].

Figure 2 illustrates several important general properties of ISO net surplus under LMP. As established in [2, Prop. 2.1], the ISO net surplus generated in any hour under a standard DC-OPF formulation, such as used in this study, is guaranteed

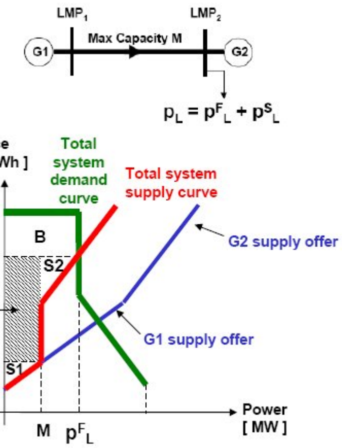

Fig. 2. Illustration of ISO net surplus collection for a simple 2-bus system with a branch limit M restricting power flow from the cheaper GenCo G1 at bus 1 to the load at bus 2. (Figure adapted from [11])

to be nonnegative. On the other hand, congestion arising anywhere on a transmission grid necessarily results in the separation of LMPs at two or more bus locations [12]. Moreover, the day-ahead energy purchases of each LSE and the day-ahead energy sales of each GenCo are settled each hour in accordance with the LMP determined at its own particular bus location. Consequently, under congested grid conditions, ISO net surplus will typically be strictly positive.[6]

The *total net surplus TNS(D)* realized on day D is given by the sum of component net surpluses as follows:

$$\mathrm{LSENetSur}(D) + \mathrm{GenNetSur}(D) + \mathrm{ISONetSur}(D) \quad (\$) \quad (14)$$

For example, TNS in Fig. 2 is the sum of the LSE net surplus B, the GenCo G1 net surplus S1, the GenCo G2 net surplus S2, and the ISO net surplus.

Finally, *market efficiency* is said to hold for day D if energy has been dispatched during day D in such a way that the maximum feasible amount of total net surplus TNS(D) defined in (14) has been extracted, conditional on existing physical conditions. For present purposes, these existing physical conditions include branch reactances, branch flow limits, LSE fixed demands, LSE reservation values, GenCo reservation values (true marginal cost functions), and true GenCo operating capacity limits.

II. DC-OPF Problem Formulation

The standard hourly *bid/offer-based DC optimal power flow (DC-OPF)* problem formulation for an ISO-managed day-ahead energy market involves the maximization on day D of reported total net surplus TNS^R for a particular hour H of day D+1 subject to transmission and generation capacity constraints in approximate linear form [13]. Total net surplus refers to the sum of LSE, GenCo, and ISO net surplus. The qualifier "reported" indicates that the ISO must base its total net surplus calculation on LSE demand bids and GenCo supply

[6] The qualifier "typically" is needed because, in special circumstances, a shadow price can vanish even though its corresponding inequality constraint holds with equality; the standard KKT-conditions do not rule this out.

offers rather than on their true purchase and sale reservation values, which are not directly observable by the ISO.

As detailed in [15], AMES(V2.05) solves this standard DC-OPF problem via *Extended DCOPFJ*, a highly accurate and efficient DC-OPF module. The Extended DCOPFJ solver wraps a SI/pu data conversion shell around *QuadProgJ*, a quadratic programming (QP) solver that implements the well-known Goldfarb-Idnani dual active set QP algorithm.

The SI form of the standard DC-OPF problem implemented in the current study is outlined below making using of the notation and concepts introduced in Section I. In all treatments, the LSEs in AMES(V2.05) report their true purchase reservation values (1). Consequently, for no-learning treatments, the objective function TNS^R coincides with true total net surplus (14) based on true purchase and sale reservation values. However, for GenCo learning treatments, TNS^R is based on reported GenCo sale reservation values (i.e., reported marginal costs) as given in (6) rather than on true GenCo sale reservation values (i.e., true marginal costs) as given in (8).

DC Optimal Power Flow Problem:

$$\max \quad \text{TNS}^R \tag{15}$$

with respect to LSE real-power price-sensitive demands, GenCo real-power generation levels, and voltage angles

$$p^S_{Lj},\ j=1,...,J;\ p_{Gi},\ i=1,...,I;\ \delta_k,\ k=1,...,K \tag{16}$$

subject to

(i) a real-power balance constraint for each bus k=1,...,K:

$$\sum_{i\in I_k} p_{Gi} \ - \ \sum_{j\in J_k} p^S_{Lj} \ - \ \sum_{km} P_{km} \ = \ \sum_{j\in J_k} p^F_{Lj} \tag{17}$$

where, letting x_{km} (ohms) denote reactance for branch km, and V_o denote the base voltage (in line-to-line kV),

$$P_{km} \ = \ [V_o]^2 \cdot [1/x_{km}] \cdot [\delta_k - \delta_m]$$

(ii) a limit on real-power flow for each branch km:

$$|P_{km}| \ \ \leq \ \ P^U_{km} \tag{18}$$

(iii) a real-power operating capacity interval for each GenCo i = 1,...,I:

$$\text{Cap}^L_i \ \ \leq \ \ p_{Gi} \ \ \leq \ \ \text{Cap}^U_i \tag{19}$$

(iv) a real-power purchase capacity interval for price-sensitive demand for each LSE j = 1,...,J:

$$0 \ \ \leq \ \ p^S_{Lj} \ \ \leq \ \ \text{SLMax}_j \tag{20}$$

(v) and a voltage angle setting at angle reference bus 1:

$$\delta_1 \ \ = \ \ 0 \tag{21}$$

This DC-OPF problem can be solved as a strictly concave quadratic programming problem either by using the bus balance constraints (17) to substitute out for voltage angles [13, Section 3.2] or by using an augmented Lagrangian method [14, p. 288] in which the objective function TNS^R in (15) is augmented with a physically meaningful quadratic penalty term for the sum of squared voltage-angle differences to produce a strictly concave objective function with respect to all of the choice variables (16). The latter augmented Lagrangian approach is taken in AMES(V2.05).[7]

The shadow price (Lagrange multiplier) solution for the real power balance constraint (17) at bus k, denoted by LMP_k, constitutes the *locational marginal price for bus k*. By the well-known envelope theorem, LMP_k (\$/MWh) measures the change in the maximized DC-OPF objective function (\$/h) with respect to a change in fixed demand (MW) at bus k; see [13] for a rigorous discussion. Stated less formally, LMP_k essentially measures the cost of efficiently servicing an additional MW of fixed demand at bus k.

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