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An Objective Function for Simulation Based Inference on Exchange Rate Data

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Empirical Validation of Agent-Based Models

1. Introduction
2. The principle of indirect estimation/validation
3. Benchmark characteristics of FX data
4. The objective function
5. Examples
6. Outlook: Optimization

Simulation of Agent Based Models

- Explicit aggregation of individual decisions
- Heterogeneity
- (Social) Interaction
- Feedback from aggregates
(e.g. rational expectations, market outcomes etc.)

Modelling the Agents

- Based on econometric estimates
- Based on experimental evidence
- Assuming endogenous behaviour (interaction, learning, etc.)

Modelling the Markets

- Implicit market clearing
(Kirman 1991, 1993)
- Market maker with triggered adjustment
(Lux 1998, Lux and Marchesi 1999)
- Explicit demand and supply functions
→ market clearing
- ...

The Validation Issue

“For microsimulation models even the simplest form of aggregate validation is either difficult or impossible.”

Stoker (1993)

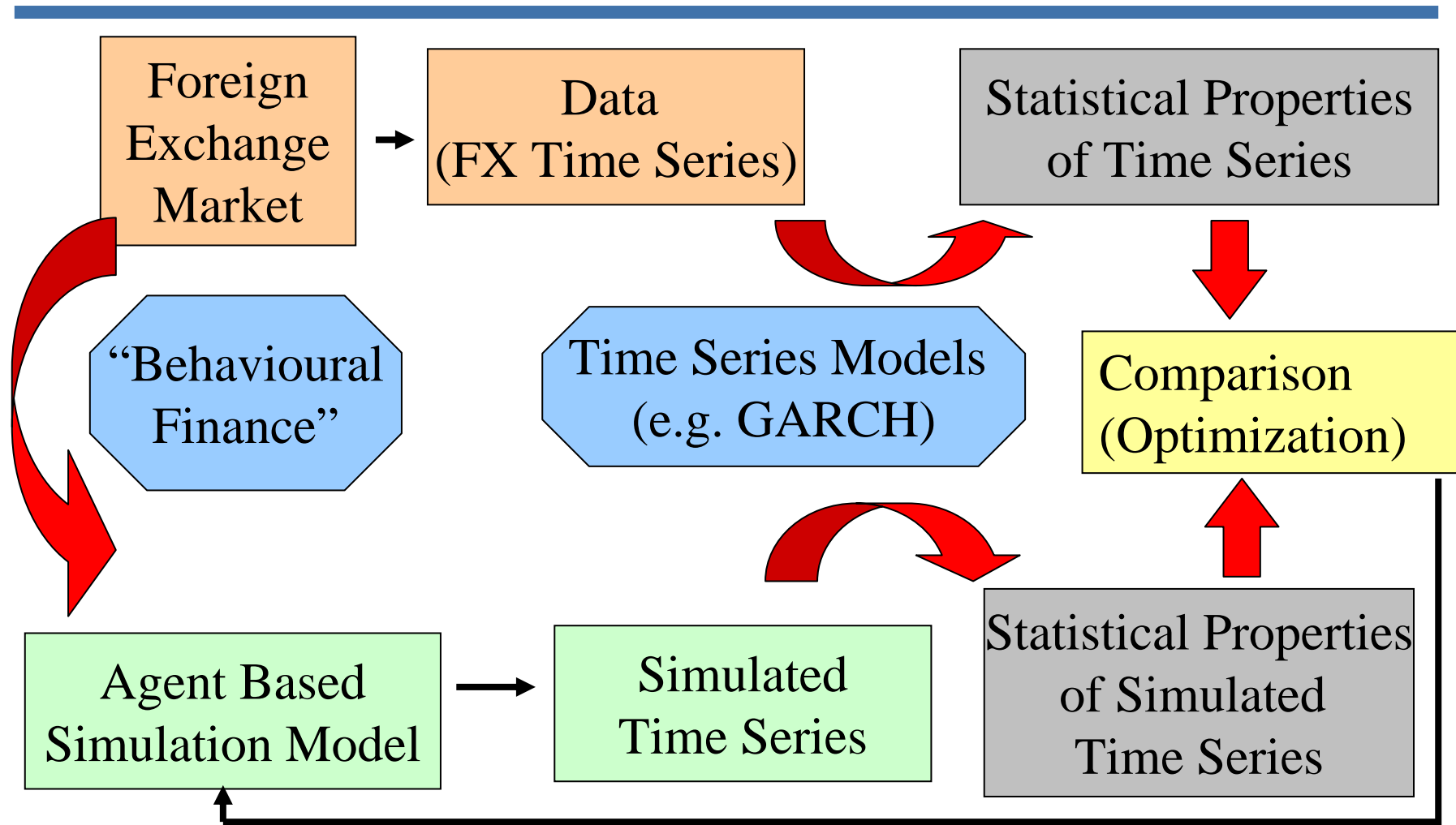
“Validation [...] remains a very weak area for the class of models described here.”

LeBaron (2000)

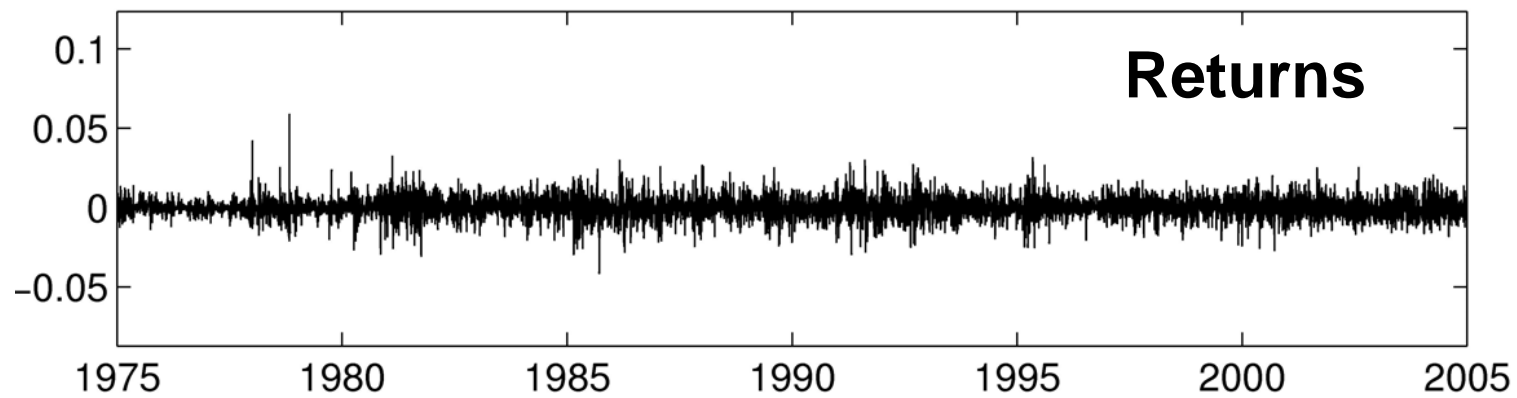
First Approaches

- Gilli and Winker (2003)
- Alfarano, Wagner and Lux (2004)
(Model allowing for a closed form solution)

2. The Principle of Indirect Estimation/Validation



3. Benchmark Characteristics of FX Data



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- a. Unconditional return distribution
- b. Conditional distribution(s) of returns
- c. Robustness of the characteristics

3. Benchmark Characteristics of FX Data

- a. Unconditional return distribution
 - Empirical moments
 - Tests for normality/comparison with empirical distribution (Kolmogorov-Smirnov)
 - Fat tails
- b. Conditional distribution(s) of returns
- c. Robustness of the characteristics

3. Benchmark Characteristics of FX Data

- a. Unconditional return distribution
- b. Conditional distribution(s) of returns
 - BDS-test
 - Autocorrelation of $|r_t|^d$
 - ARCH, GARCH, EGARCH,...
 - Long memory, Stationarity
- c. Robustness of the characteristics

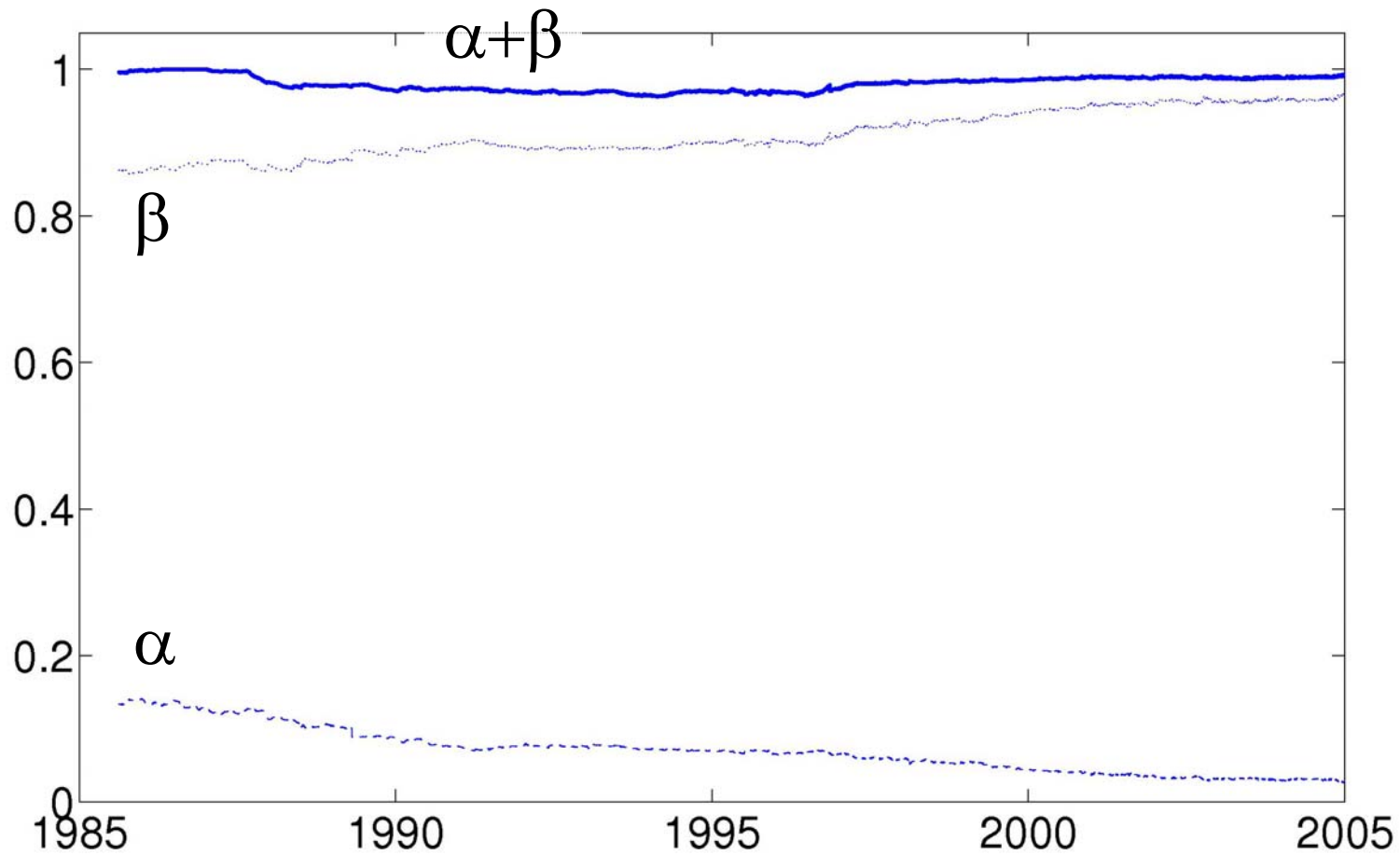
3. Benchmark Characteristics of FX Data

- a. Unconditional return distribution
- b. Conditional distribution(s) of returns
- c. Robustness of the characteristics
 - Subsamples
 - Bootstrap
 - Time aggregation

Robustness

- Only „stylized“ facts should be considered
- Changes over time:
 - Rolling windows
 - Sample split
- Variance of stylized facts:
 - (Block) Bootstrap
- Properties under temporal aggregation

Rolling Windows: GARCH(1,1)



Bootstrap

Statistic	DM/US	Mean	5%	50%	95%	Std.dev.
Mean	-0.68E-4	-0.71E-4	-1.98E-4	-0.72E-4	0.49E-4	0.75E-4
Std.dev.	0.0065	0.0065	0.0064	0.0065	0.0066	0.0001
Skewness	0.0314	0.0255	-0.1592	0.0165	0.2241	0.1217
Kurtosis	5.9866	5.9503	4.9149	5.8387	7.4826	0.7985
JB-Statistic	2806	2957	1175	2534	6351	1673
KS-Statistic	0.0490	0.0513	0.0465	0.0512	0.0566	0.0031
Tail index (left)	3.6208	3.6300	3.3709	3.6272	3.9091	0.1656
Tail index (right)	3.5409	3.5602	3.2598	3.5607	3.8545	0.1809

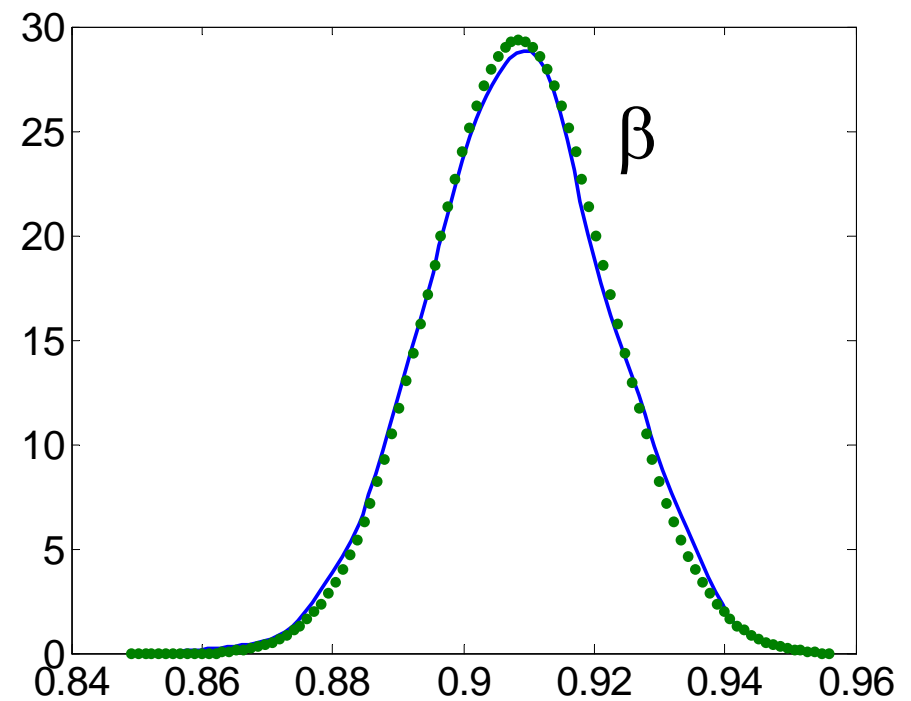
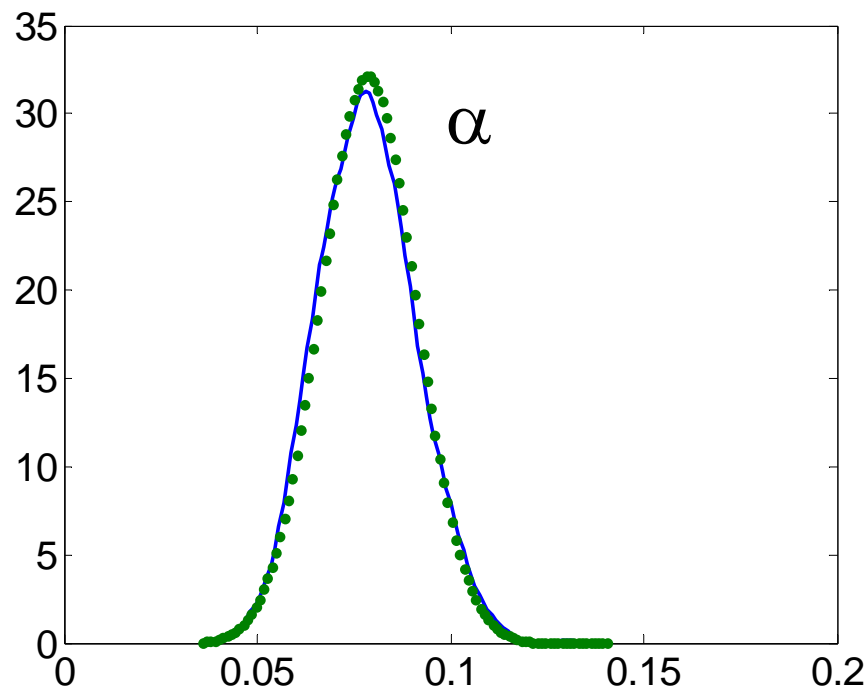
Block Bootstrap

Statistic	DM/US	Mean	5%	50%	95%	Std.dev.
BDS (dim. 2)	8.441	8.518	5.337	8.502	11.686	1.917
BDS (dim. 3)	11.425	11.425	7.836	11.426	14.978	2.191
BDS (dim. 4)	14.274	14.161	9.989	14.112	18.260	2.470
GARCH(1,1) α	0.071	0.078	0.059	0.078	0.099	0.012
GARCH(1,1) β	0.921	0.908	0.886	0.909	0.931	0.014
GARCH(1,1) $\alpha + \beta$	0.992	0.987	0.972	0.988	0.997	0.008
GPH ($m = 0.55$) r_{DM}	0.091	0.044	-0.041	0.044	0.128	0.051
GPH ($m = 0.55$) $ r_{DM} $	0.386	0.360	0.255	0.361	0.447	0.058
GPH ($m = 0.55$) r_{DM}^2	0.323	0.266	0.163	0.269	0.366	0.062
ADF (no drift) (DM)	-1.087	-1.546	-4.001	-1.406	0.404	1.393
ADF (with drift) (DM)	-1.297	2.026	0.061	2.039	3.910	1.146

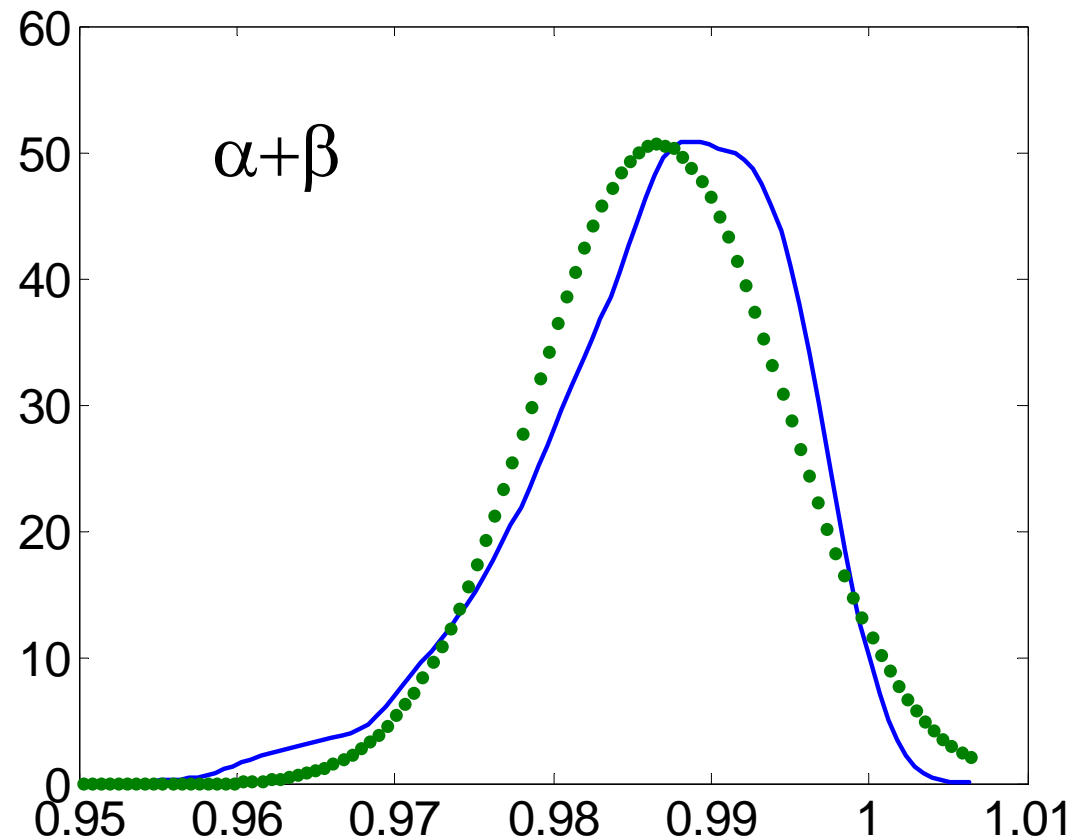
4. The Objective Function

- Estimation/validation should be based on
 - Relevant characteristics
 - Robust characteristics
 - Discriminating characteristics
- Use several characteristics
- Weighting of the characteristics?

Block Bootstrap: GARCH(1,1)



Block Bootstrap: GARCH(1,1)



A First Selection

- Unconditional distribution:
 - Mean
 - Standard deviation
 - Tail index
 - Kolmogorov-Smirnov statistic
(empirical return distribution as benchmark)
- Conditional distribution:
 - GARCH(1,1): $\alpha + \beta$
 - GPH (m=0.5) estimate of degree of fractional integration
 - ADF-Test (with drift, no lags)

Moments and Variances

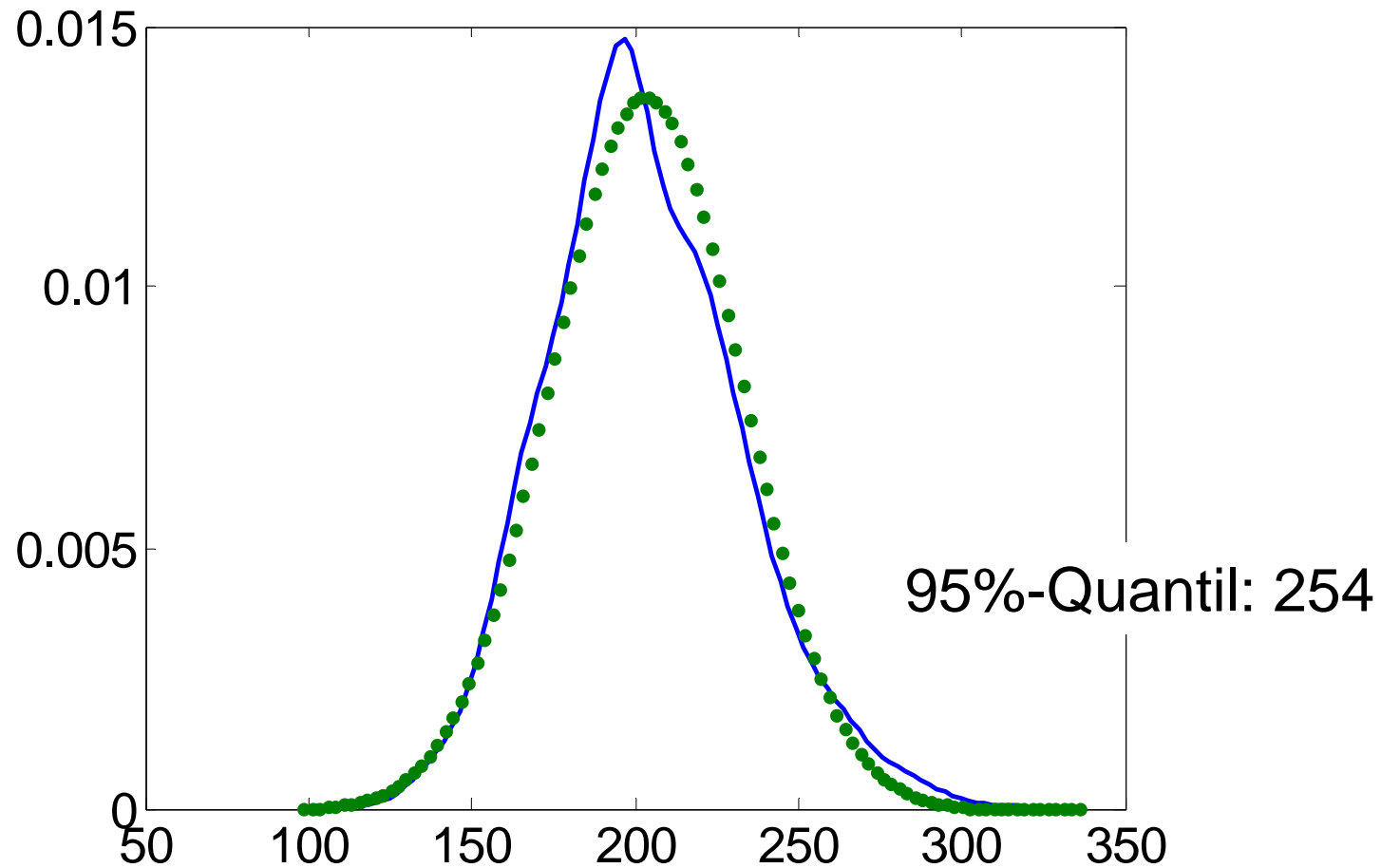
- $\mathbf{m} = [m_1, \dots, m_k]$ vector of „moments“
- \mathbf{m}^e estimates for real data
- Estimate of variance-covariance matrix of \mathbf{m}^e by block bootstrap (250): Σ_{BB}
- \mathbf{m} is approximated through simulation of the agent based model (mean or median): \mathbf{m}^s
- Monte-Carlo variance of \mathbf{m}^s can be controlled by number of replications.

The Objective Function

- $f(\mathbf{m}^s) = (\mathbf{m}^s - \mathbf{m}^e)' \hat{\Sigma}_{BB}^{-1} (\mathbf{m}^s - \mathbf{m}^e)$
- Where $\hat{\Sigma}_{BB}$:

Mean	Std.dev.	KS-stat.	Hill (5%-10%)	GARCH $\alpha + \beta$	GPH (m=0.55)	ADF
$7.9 \cdot 10^{-9}$	$-8.7 \cdot 10^{-10}$	$-4.2 \cdot 10^{-8}$	$6.4 \cdot 10^{-6}$	$-3.5 \cdot 10^{-8}$	$5.2 \cdot 10^{-7}$	0.00005
$-8.7 \cdot 10^{-10}$	$6.1 \cdot 10^{-8}$	$-1.3 \cdot 10^{-7}$	$2.2 \cdot 10^{-6}$	$-4.2 \cdot 10^{-7}$	$-3.9 \cdot 10^{-6}$	$-1.1 \cdot 10^{-6}$
$-4.2 \cdot 10^{-8}$	$-1.3 \cdot 10^{-7}$	$4.7 \cdot 10^{-5}$	$4.1 \cdot 10^{-6}$	$1.2 \cdot 10^{-6}$	0.00002	0.00141
$6.4 \cdot 10^{-6}$	$2.2 \cdot 10^{-6}$	$4.1 \cdot 10^{-6}$	0.04113	-0.00026	-0.00189	0.04890
$-3.5 \cdot 10^{-8}$	$-4.2 \cdot 10^{-7}$	$1.2 \cdot 10^{-6}$	-0.00026	0.00006	0.00035	-0.00006
$5.2 \cdot 10^{-7}$	$-3.9 \cdot 10^{-6}$	0.00002	-0.00189	0.00035	0.00570	0.00491
0.00005	$-1.1 \cdot 10^{-6}$	0.00141	0.04890	-0.00006	0.00491	0.97167

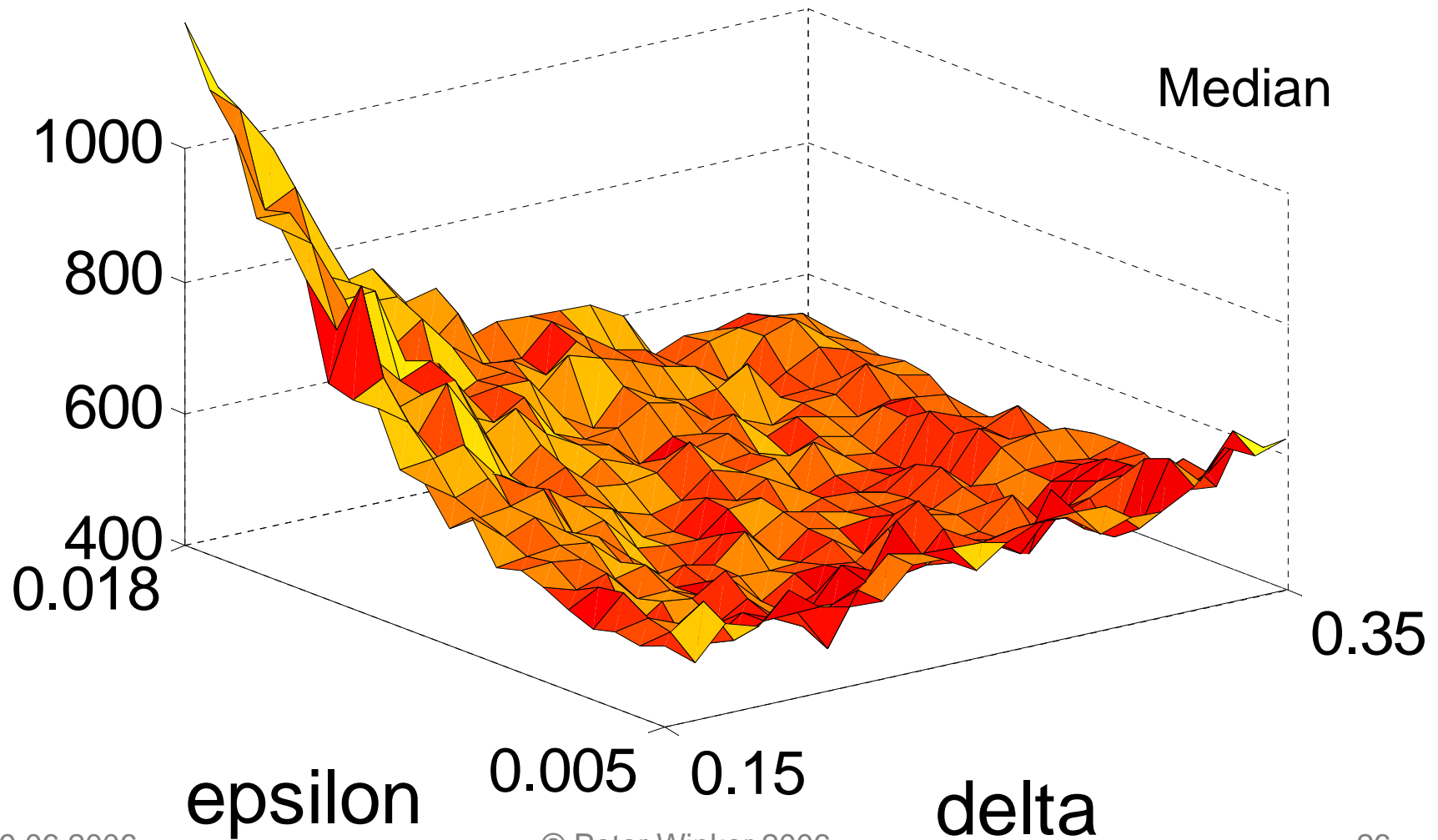
Block Bootstrap Results for f



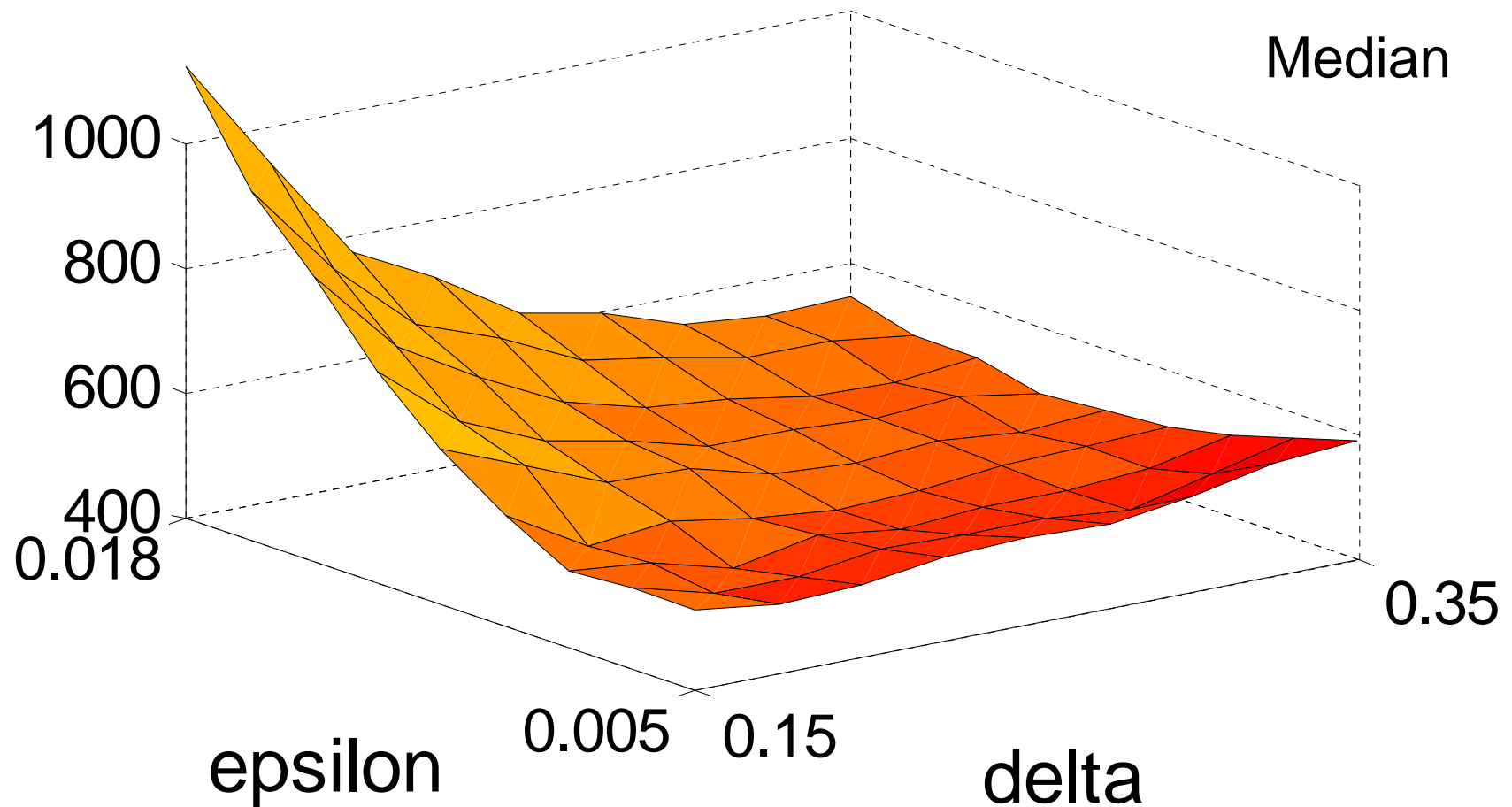
5. Examples

- Kirman (1993) (see Gilli and Winker, 2003)
 - Two types of agents
 - Switching random
- Lux (1998), Lux and Marchesi (1999, 2000)
 - Three types of agents
 - Switching depends on past success
 - Stepwise market adjustment
- Important parameters: Switching rates

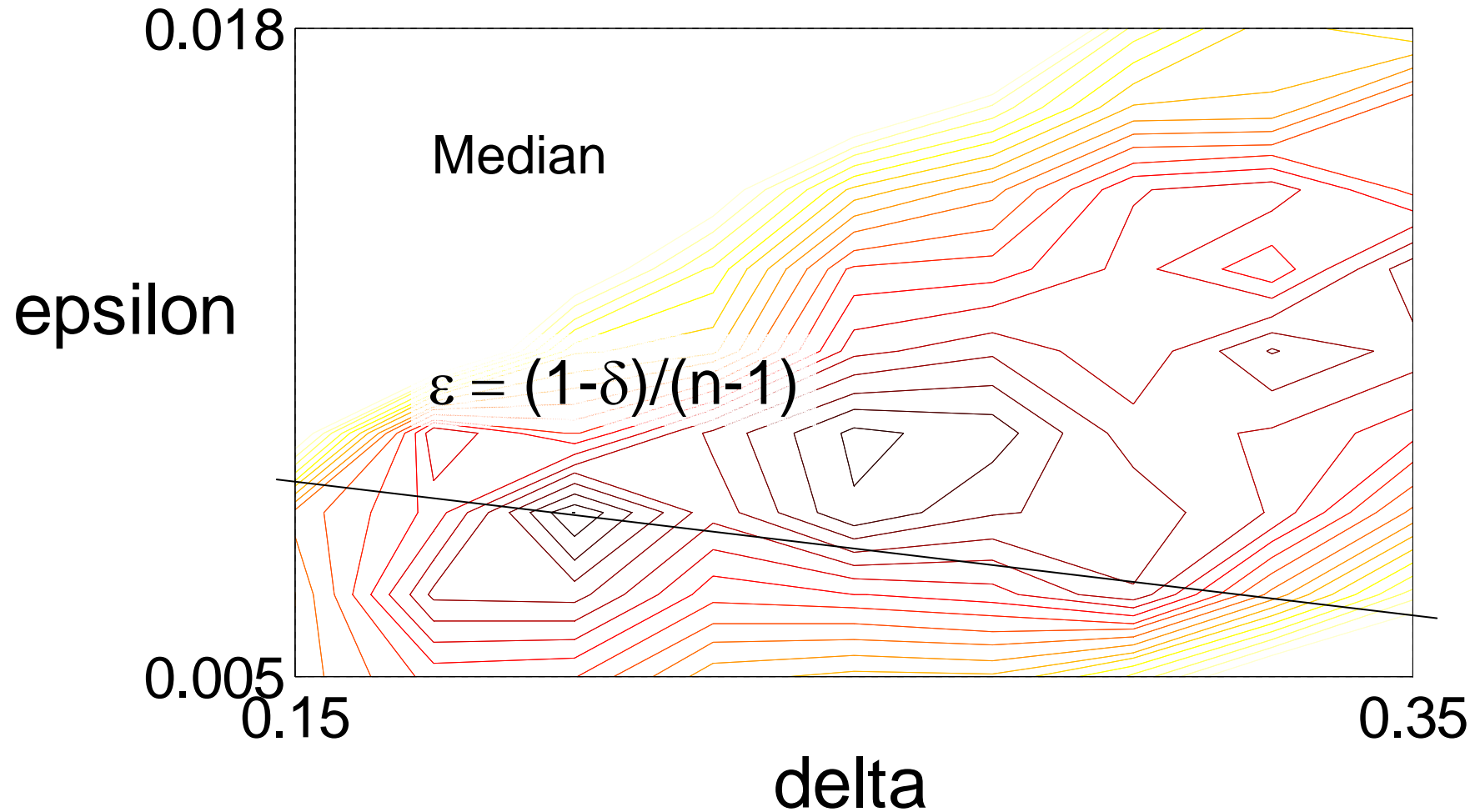
Kirman: ε and δ (200 MC-Repl.)



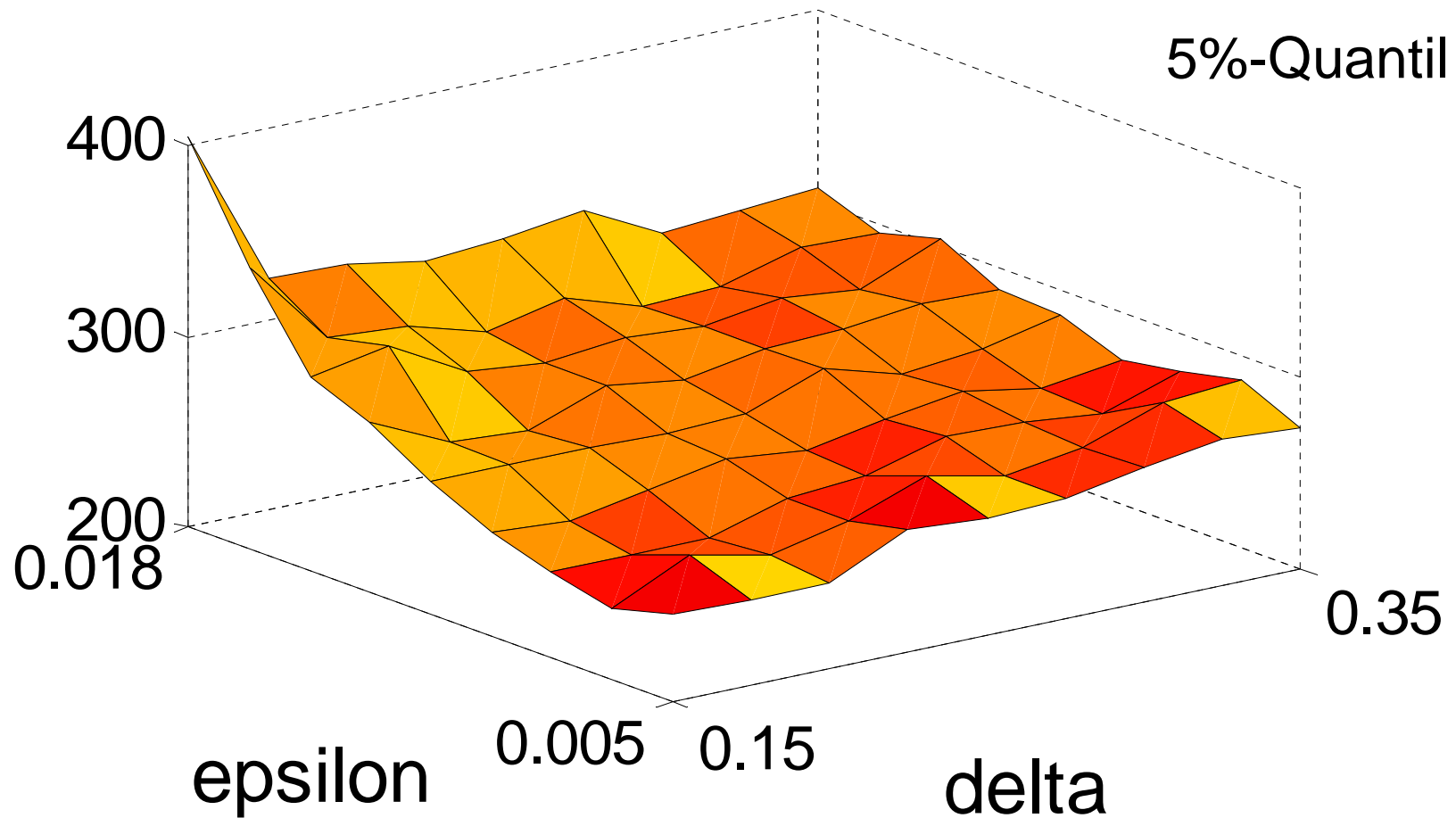
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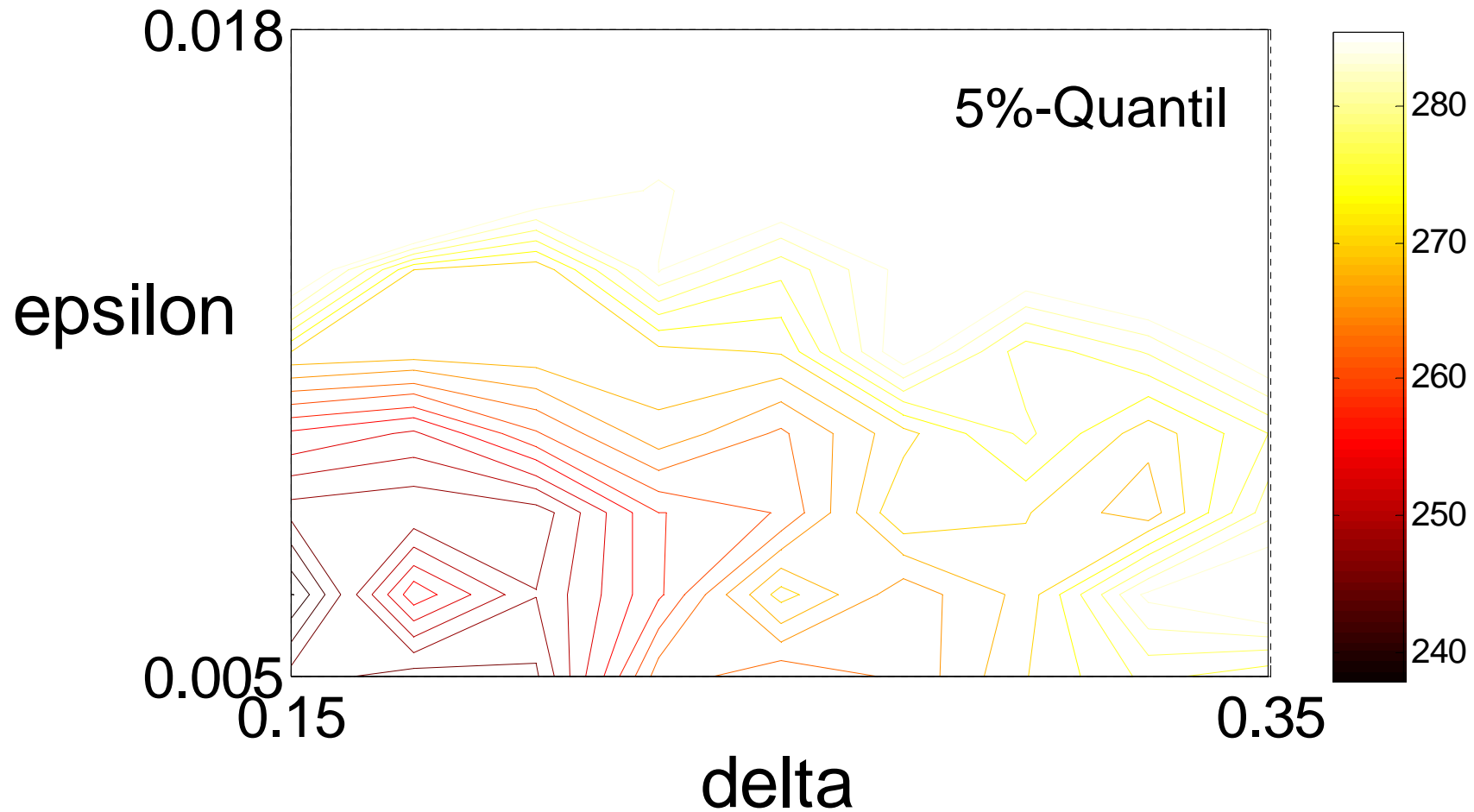
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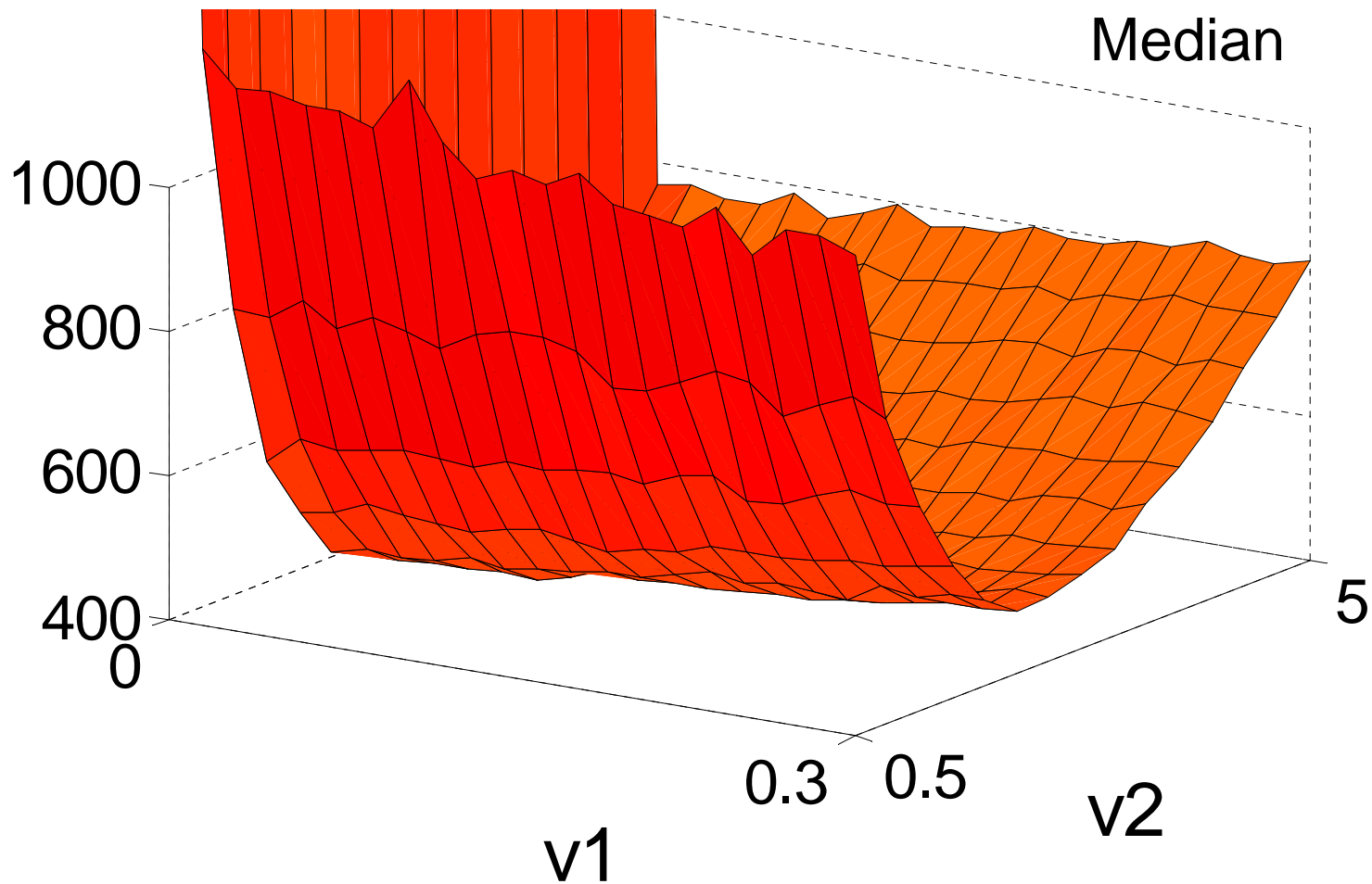
Kirman: ε and δ (1000 MC-Repl.)



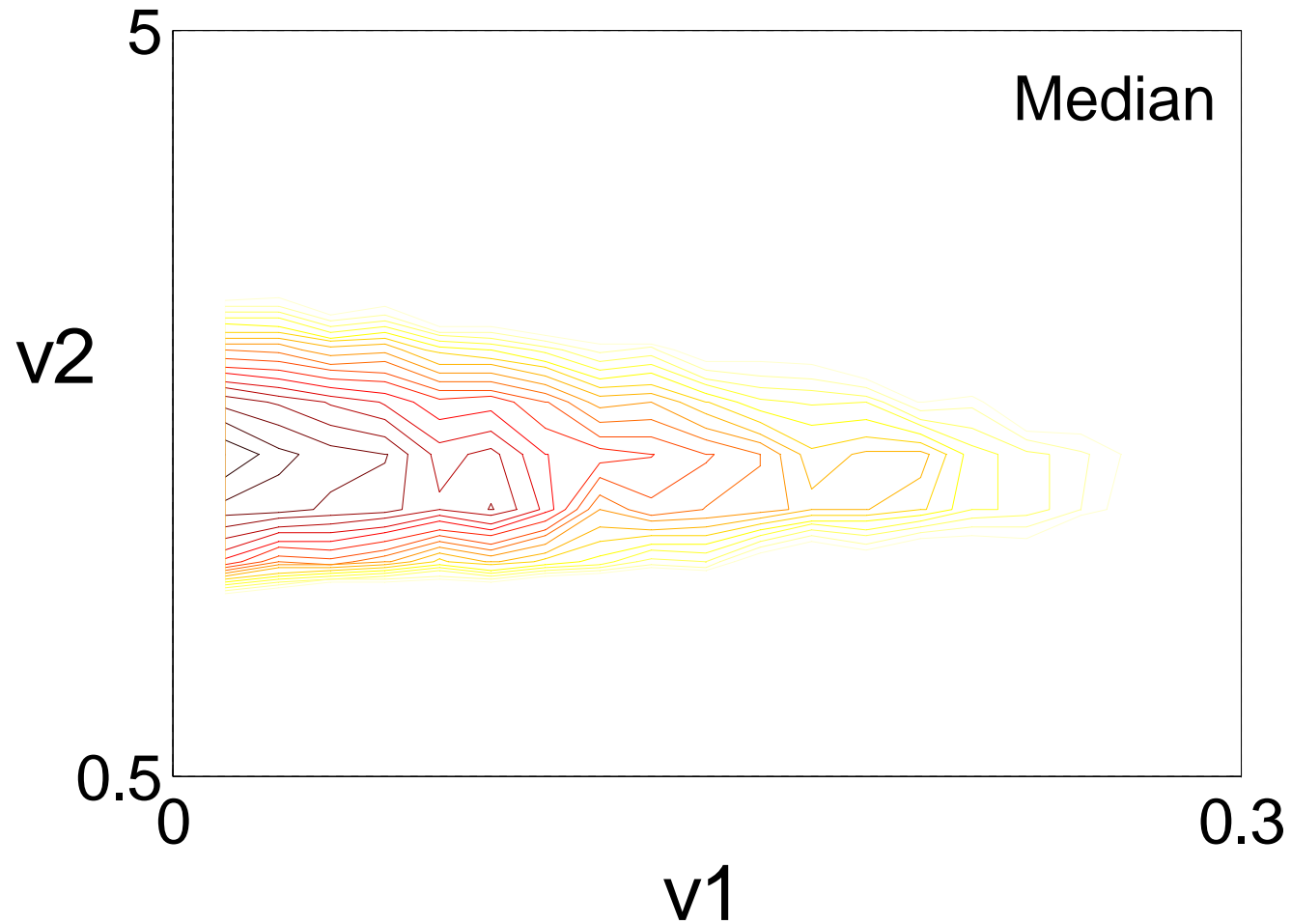
Summary for Kirman's model

- Other parameters fixed at „promising values“
- „Optimal“ combination of ε and δ is close to satisfy $\varepsilon = (1-\delta)/(n-1)$
- Values of objective function for „optimal“ parameter values are not significantly worse than for real data
- „Model cannot be rejected“

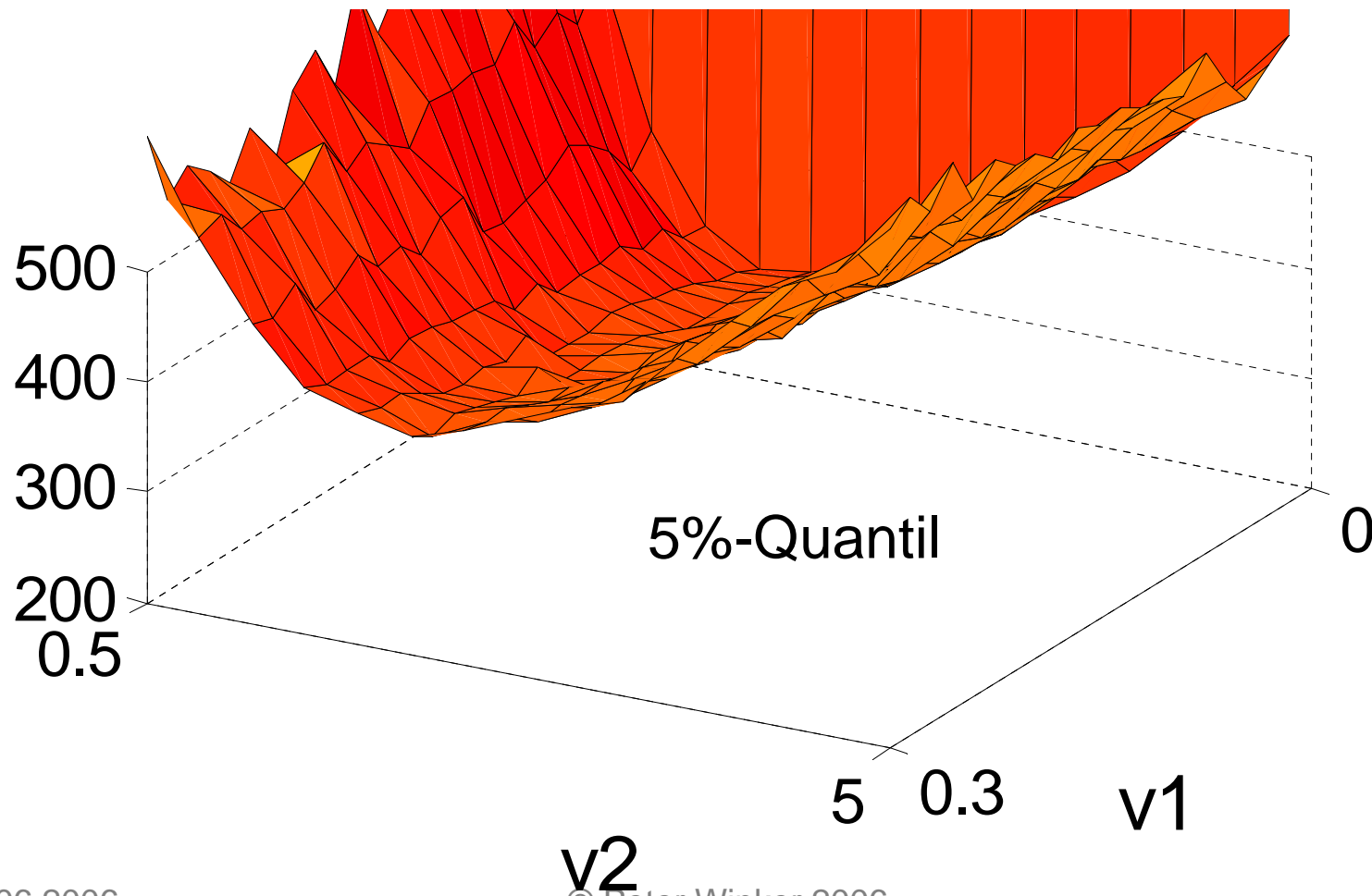
Lux: v_1 und v_2 (200 MC-Repl.)



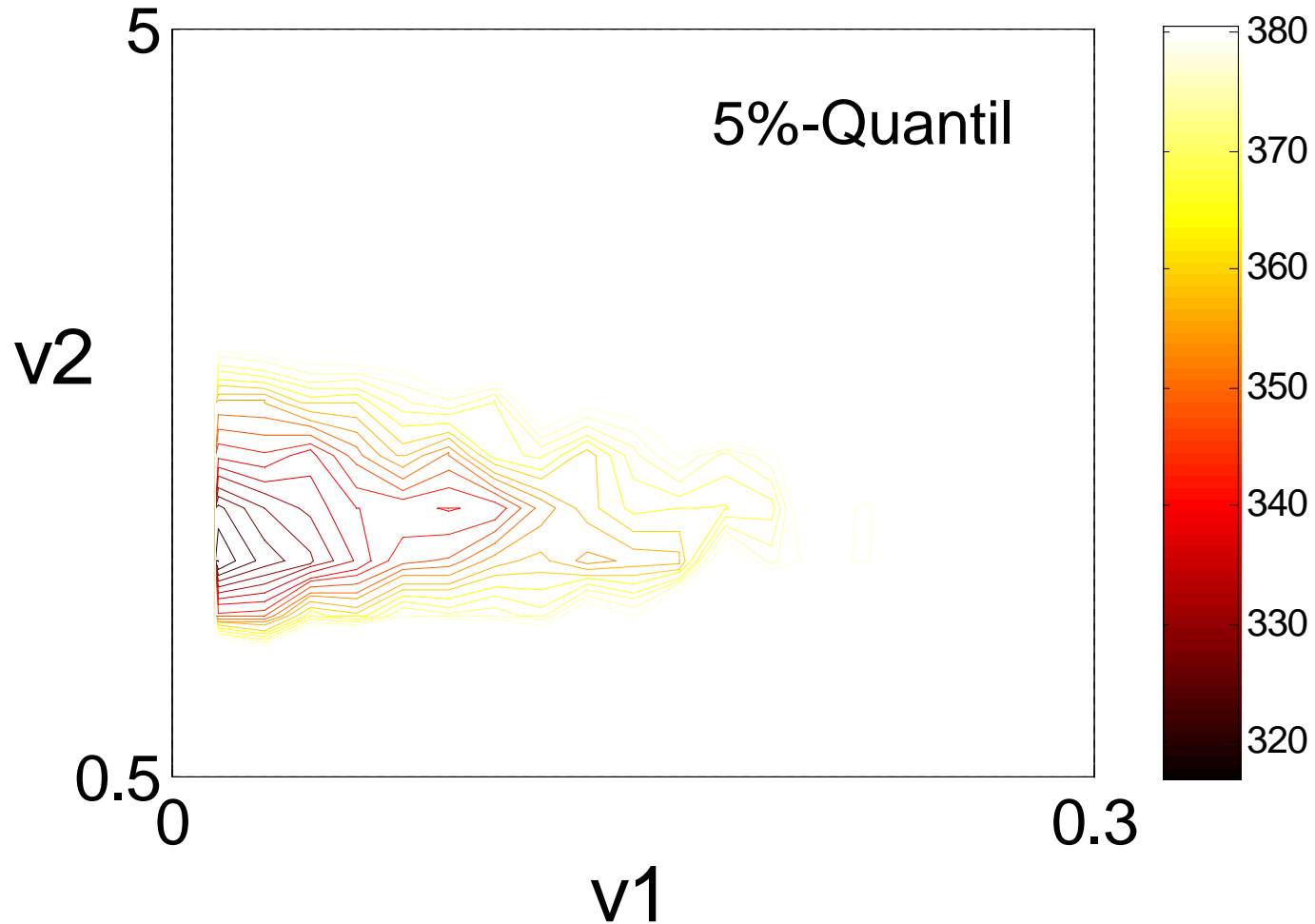
Lux: v_1 und v_2 (200 MC-Repl.)



Lux: v_1 und v_2 (200 MC-Repl.)



Lux: v_1 und v_2 (200 MC-Repl.)



Summary for Lux's model

- Other parameters fixed at „promising values“
- Identification of v_1 appears to be difficult
- Values of objective function for „optimal“ parameter values are significantly worse than for real data
- Optimization required to find optimal values for all parameters!

Summary

- 95%-Quantil of objective function for data (from block bootstrap): 254
- 5%-Quantil of objective function for
 - Kirman model (ε, δ „optimal“): ~240
 - Lux model (v_1, v_2 „optimal“): ~ 320
 - Mod. Lux model (v_1, v_2 „optimal“): >> 100000
- Note: All other parameters are fixed (to promising, but probably suboptimal values)!

6. Outlook

- Validation of agent based models is important
- Simulation based inference appears feasible
- Selection of relevant/robust properties is crucial; additional statistics will be considered
- Parameters of models might be estimated

Optimization Methods

- Minimize $f(\mathbf{m})$ over parameter space Ω
- Problems:
 - $f(\mathbf{m})$ has to be approximated by $f(\mathbf{m}^s)$;
how many replications to control MC error?
 - Objective function not globally convex?
- Approaches:
 - Response surface methods
 - Heuristic optimization (TA with Direct Search)