Allocative Efficiency of Markets with Zero-Intelligence Traders: Market as a Partial Substitute for Individual Rationality

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We report market experiments in which human traders are replaced by "zero-intelligence" programs that submit random bids and offers. Imposing a budget constraint (i.e., not permitting traders to sell below their costs or buy above their values) is sufficient to raise the allocative efficiency of these auctions close to 100 percent. Allocative efficiency of a double auction derives largely from its structure, independent of traders' motivation, intelligence, or learning. Adam Smith's invisible hand may be more powerful than some may have thought; it can generate aggregate rationality not only from individual rationality but also from individual irrationality.

Becker (1962) proved that several basic features of economics such as downward-sloping demand functions and upward-sloping supply functions can be derived as market-level consequences of agents' random choice behavior subject to a budget constraint. He pointed out that "households may be irrational and yet markets quite rational"

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(p. 8) and that we should not impute all observed irrationalities of individuals to markets or impute all rationality of markets to their participants.

Smith (1962) presented evidence that Walrasian tâtonnement, conducted by a central auctioneer, is not necessary for market outcomes to closely approximate economic equilibrium, even with a handful of traders. Smith modeled his simple classroom markets after the double auction of stock and commodity exchanges. These markets are decentralized systems in which each contract is binding. Such markets allowed the traders—mostly students motivated by money or course credit—to extract virtually 100 percent of the maximum exploitable (i.e., equilibrium) surplus. Equilibrium predictions of economic theory can accurately describe the outcomes of many trading mechanisms, not just Walrasian tâtonnement, in a variety of environments (see Plott [1982] for a survey).

Performance of an economy is the joint result of its institutional structure, market environment, and agent behavior. Institutional structure is defined by the rules that govern exchange, market environment by agents' tastes and endowments of information and resources, and agents' behavior by their trading strategy (see Smith 1982). Standard economic theory is built on two specific assumptions: utility-maximizing behavior and the institution of Walrasian tâtonnement. Becker showed that the market-level predictions of economic theory are consistent with individual behaviors more general than utility maximization, whereas Smith showed that such predictions are consistent with trading mechanisms more general than Walrasian tâtonnement.

This paper is an attempt to synthesize these seminal generalizations by Becker and Smith. Becker assumed that the supply and demand functions yield equilibrium results through the traditional tâtonnement mechanism; Smith's subjects were motivated to seek trading profits. We show that a double auction, a non-Walrasian market mechanism, can sustain high levels of allocative efficiency even if agents do not maximize or seek profits. In its first-order magnitude, allocative efficiency seems to be a characteristic of the market structure and the environment; rationality of individual traders accounts for a relatively small fraction (second- or third-order magnitude) of the efficiency.

It is not possible to control the trading behavior of individuals. Human traders differ in their expectations, attitudes toward risk, preferences for money versus enjoyment of trading as a game, and many other respects. The problem of separating the joint effects of these variations, unobservable to the researcher, can be mitigated by studying market outcomes with participants who follow specified
rules of behavior. We therefore replaced human traders by computer programs.

To isolate the effect of market rules and agent behavior on market performance, we proceed in three steps. First, in Section I we select two types of market participants: profit-motivated human traders and "zero-intelligence" (ZI) machine traders.¹ Second, in Section II we observe the performance of a double auction with human traders and with ZI traders. In one set of ZI markets, the traders are subject to the budget constraint; in the second set of ZI markets, the budget constraint is absent. Third, in Sections III and IV we compare these observations to isolate the performance characteristics of the markets, which can be attributed to their structure. Section V contains a summary and some concluding remarks.

I. Traders

The performance of markets with human traders (graduate students of business) is compared to the performance of markets with machine traders. Each ZI trader generated random bids or offers (depending on whether it was a buyer or a seller) distributed independently, identically, and uniformly over the entire feasible range of trading prices from 1 to 200: [probability(bid = i) = 1/200; i = 1, 2, . . . , 200] and [probability(offer = j) = 1/200; j = 1, 2, . . . , 200]. It has no intelligence, does not seek or maximize profits, and does not observe, remember, or learn. It seems appropriate to label it as a zero-intelligence trader.

While human traders were unique, 12 identical ZI traders participated in each market. Experiments were conducted at Carnegie Mellon’s Laboratory for Market Design using Market-2001 software, which permits both human and machine traders to operate in the same environment.

II. Market Mechanism

A double auction is a multilateral process in which buyers as well as sellers can freely enter limit orders (bids or asks) and accept asks or

¹ These ZI traders are not intended as descriptive models of individual behavior. For strategic models of individual behavior, see Garcia (1980), Garcia and Zangwill (1981), and Wilson (1987); for nonstrategic models, see Easley and Ledyard (1992) and Friedman (1984, 1991); and for double-auction trading algorithms, see Rust, Palmer, and Miller (1992). In the sense that Friedman (1991) and Easley and Ledyard (1992) moved away from strategic modeling of individual behavior, the ZI model can be thought of as extending their idea to an extreme limit. See also Cason and Friedman (1992) for an empirical comparison of the predictions of the Wilson (1987) and Friedman (1991) models against the ZI benchmark.
bids entered by others. We chose this mechanism for two reasons. First, major stock, commodity, currency, and many other markets are organized as double auctions. Second, laboratory double auctions with human traders are known to yield data that approximate the equilibrium predictions of economic theory in a variety of environments.

In a double auction any buyer can enter a bid by stating his or her identity, unit price, and quantity. The same buyer or other buyers can subsequently raise the bid. Similarly, any seller can enter an ask by stating quantity and price. If bids and asks match or cross, a binding transaction occurs. Any buyer is free to accept the outstanding ask and any seller is free to accept the outstanding bid at any time to conclude a transaction.

There are several variations of the double auction. We made three choices to simplify our implementation of the double auction. Each bid, ask, and transaction was valid for a single unit. A transaction canceled any unaccepted bids and offers. Finally, when a bid and ask crossed, the transaction price was equal to the earlier of the two.

Each double auction consisted of 12 traders, equally divided into two groups: buyers and sellers. Smith's (1976) induced value mechanism was used to implement demand and supply in these markets. Each market was run for six periods of specified duration (4 minutes for human traders and 30 seconds for machine traders). We chose the duration of each period to allow sufficient time for trading. At the beginning of each period, each buyer was endowed with the right to buy one or more units of an unspecified commodity. The buyer was privately informed of the redemption value $v_i$ of each unit $i$, and the buyer's profit from buying this unit at price $p_i$ was given by $v_i - p_i$. Redemption values $v_i$, $i = 1, 2, \ldots, n$, defined the individual buyer's demand function for the commodity. Since redemption values of each buyer were private, the market demand function was unknown to the buyers. At the beginning of each period, sellers were endowed with the right to sell one or more units of the commodity, $c_i$ being the cost of the $i$th unit to the seller. The seller's profit from selling the $i$th unit at price $p_i$ was $p_i - c_i$. Sellers had no fixed costs and incurred costs only for units sold. The market supply function was unknown to the sellers. Every trader had to trade the $i$th unit before trading the $(i + 1)$th unit. Points earned by human traders were included in the grading scheme for a credit course.

Market rules impose a budget constraint on the participants by

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2 Double-auction markets with a single unit per trader also yield results similar to those reported here (see Gode and Sunder 1992b).
requiring traders to settle their accounts. As shown by Becker (1962), merely imposing a budget constraint on random agent behavior induces enough regularity in aggregate market behavior to yield downward-sloping demand and upward-sloping supply functions. To explore the consequences of the budget constraint or market discipline, we placed the ZI traders in two versions of the double auction.

In the first version, ZI traders were subject to the budget constraint: if they generated a bid (to buy) above their redemption value or an offer (to sell) below their cost, such actions were considered invalid and were ignored by the market. In other words, the market forbade traders to buy or sell at a loss because then they would not have been able to settle their accounts. Therefore, the support of the distribution that generated the uniform random bids was restricted between one and the redemption value of the bidder. Similarly, uniform distribution of random asks was restricted to the range between the seller's cost and 200. These markets are labeled "ZI with constraint" (ZI-C).

In the second version, the ZI traders were freed from the market discipline. The budget constraint was removed and random bids and offers were permitted over the entire range (1–200) without regard to the buyers' redemption values and sellers' costs. Buyers and sellers were free to engage in money-losing transactions that they could not settle. This version of markets is labeled "ZI unconstrained" (ZI-U).

The difference between the performance of the human markets and that of the ZI-C markets is attributable to systematic characteristics of human traders. If ZI-C traders can be considered to have zero rationality, this difference in performance would be a measure of the contribution of human rationality to market performance. On the other hand, the difference between the performance of markets that do impose a budget constraint on ZI traders and the performance of those that do not is attributable to the market discipline. Traders have no intelligence in either the ZI-U or the ZI-C market; the ZI-C market prevents the traders from engaging in transactions that they cannot settle. Consequently, we can attribute the differences in market outcomes to the discipline imposed by the double auction on traders.

III. Market Parameters

The demand and supply schedules are shown on the left side of figures 1–5, adjacent to the transaction price charts. The subjects received the same endowment of redemption values or costs for all the periods in a market. We chose the five sets of demand and supply schedules to yield a broad range of equilibrium prices (from 69 in
market 2 to 170 in market 4) and volumes (from 6 in market 3 to 24 in market 1). In all cases a unique equilibrium price existed. In mar-

3 The human trader sessions had an extra buyer in markets 1 and 2 and an extra seller in markets 3 and 4. As a result, the demand schedule for the human trader sessions of markets 1 and 2 and the supply schedules for the human trader sessions for markets 3 and 4 are slightly different from the supply schedules for the other sessions, as can be seen from figs. 1–4.
ket 5, costs and redemption values of all the units of several buyers and sellers were placed just beyond the equilibrium point, making it difficult to attain 100 percent efficiency in the double auction.⁴

⁴ We are grateful to Charles Plott for suggesting this design to us. Traders whose first units lie just beyond the margin are more likely to be able to displace some intramarginal units through aggressive trading in a double auction.
**IV. Results**

**Prices**

The first panel of figure 1 shows the transaction prices in market 1 with ZI-U traders (figs. 2–5 provide similar charts for the other four markets). Prices in this market exhibit little systematic pattern and no tendency to converge toward any specific level. This price series is the consequence of random individual behavior in the absence of the market discipline.
In contrast, the third panel of figure 1 shows the transaction prices for market 1 with human traders. After some initial adjustments, prices in human trader markets settle in the proximity of the equilibrium price (indicated by a solid horizontal line in all panels of fig. 1). These markets are characterized by the stability of price and volume. Both price and volume are close to the point of intersection of the supply and demand functions. This price series is the consequence
Fig. 5.—Demand and supply functions and transaction price time series (market 5)

of subjecting profit-motivated, intelligent human traders to market discipline.

The main question addressed in the article is, How much of the difference between the market outcomes with ZI-U traders and those with human traders is attributable to intelligence and profit motivation, and how much is attributable to the market discipline (see panels
1 and 3 of figs. 1–5)? The middle panels of figures 1–5 show the
time series of prices in markets 1–5 when they were populated by
the ZI-C traders.

Three features of the ZI-C price series should be noted. First, in
contrast to the human trader data in panel 3 and as in the ZI-U
trader data in panel 1, this series shows no signs of learning from
period to period; the series from every period are statistically identi-
cal. This is to be expected because the ZI traders cannot remember
or adapt. Second, the volatility of this price series is greater than the
volatility of the price series from human markets, but less than the
volatility of ZI-U markets. Imposing a budget constraint on ZI traders
is sufficient to shift the market performance toward the human mar-
et performance. Third, the ZI-C price series, though more volatile
than the human market price series, converges slowly toward equilib-
rium within each period. This can be confirmed from figure 6, which
plots the root mean squared deviation of transaction prices from the
equilibrium price averaged across the six periods of a market. Mar-
kets with ZI-U traders do not converge, markets with human traders
converge quickly, and markets with ZI-C traders converge slowly to
the equilibrium price. Regression of the root mean squared deviation
on the transaction sequence number yields significantly negative
slopes for the ZI-C markets but not for the ZI-U markets. By the end
of a period, the price series in ZI-C trader markets converges to the
equilibrium level almost as precisely as the price series from human
trader markets does. The regression results for ZI-C are shown in
table 1.

Since the ZI traders have no memory of events within the current
or the past periods, this convergence cannot be attributed to learning
from market participation. Instead, the difference between the top
and the middle panels of figures 1–5 is caused solely by the progres-
sive narrowing of the opportunity sets of ZI-C traders. The left end
of the market demand function represents units with higher redemption
values. Expected values of the bids generated for these units by
ZI-C traders are also higher. Therefore, these units are likely to be
traded earlier than units further down the market demand function.
As the higher-value units are traded, the upper end of the support
of ZI-C bids shifts down. Similarly, as the lower-cost units are sold
earlier in a period, the lower end of the support of ZI-C offers moves
up.

This means that the feasible range of transactions prices narrows
as more units are traded. Though individual units may not be traded
in the order in which they appear in the market demand and supply
functions, there is a greater probability that the last transaction repre-
Fig. 6.—Root mean squared deviation of prices from equilibrium
sents an exchange between the marginal buyer and the marginal seller. If the surplus associated with the marginal units is smaller, then the range of feasible transaction prices is also narrower, increasing the probability that the market prices will converge to the equilibrium price. If the marginal units involve a larger surplus, convergence will be correspondingly less precise. In either case, the convergence of transaction prices to the proximity of the theoretical equilibrium price in ZI-C markets is a consequence of the market discipline; traders’ attempts to maximize their profits, or even their ability to remember or learn about events of the market, are not necessary for such convergence.

**Efficiency**

Following Smith (1962), one can define allocative efficiency of markets as the total profit actually earned by all the traders divided by the maximum total profit that could have been earned by all the traders (i.e., the sum of producer and consumer surplus). Figure 7 and table 2 show the period-by-period efficiency of the five markets under the three modes of operation. The efficiency of markets with ZI-U traders is constant across periods (see the dotted line in Fig. 7). Since the budget constraint is absent, the maximum possible number of units (equal to the lower of the total units sellers are allowed to sell and the total units the buyers are allowed to buy) is always traded.

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5 The Spearman rank correlation between the actual and the efficient order of surplus extracted is, on average, highest for ZI-C traders (.74), lowest for ZI-U traders (.42), and in between the two for human traders (.52). The correlation for ZI-U traders (arising out of the rule that higher-value/lower-cost units of a given trader must be traded first) can be thought of as the baseline correlation for these markets. Higher correlations for ZI-C markets suggest that a greater part of their efficiency may be attributable to the statistical consequences of market rules. Human traders, acting strategically, may be able to extract the same surplus in spite of a lower correlation between the actual and the efficient order of trading.
Fig. 7.—Periodwise efficiency (percentage of total surplus extracted) of five markets. In contrast to human markets, the efficiency of the ZI-U and ZI-C markets in each period is an independently and identically distributed random variable with no change across periods since these traders do not learn.

Not only is the total positive surplus extracted, but the maximum possible negative surplus is also extracted. Negative surplus is extracted because extramarginal units are traded at a loss. Therefore, the efficiency of these markets depends on the shape of the demand and supply functions to the left as well as to the right of the equilibrium point. As can be seen in table 2, this baseline efficiency was chosen to lie in the range of 48.8 percent (for market 4) to 90 percent (for markets 1 and 2). When one allows the demand and supply functions to extend far enough to the right of the equilibrium point, it is possible to lower this efficiency without limit (including negative levels).

The efficiency of human markets is 100 percent in most periods, with an occasional shortfall of a few percentage points in all markets except market 5 (see the solid line in fig. 7). In most of these markets,

6 Gode and Sunder (1992b) present results to explain why and how the shape of the demand and supply functions to the right of the equilibrium point affects the expected efficiency of double auctions.

<table>
<thead>
<tr>
<th>Traders</th>
<th>Market 1</th>
<th>Market 2</th>
<th>Market 3</th>
<th>Market 4</th>
<th>Market 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZI-U</td>
<td>90.0</td>
<td>90.0</td>
<td>76.7</td>
<td>48.8</td>
<td>86.0</td>
</tr>
<tr>
<td>ZI-C</td>
<td>99.9</td>
<td>99.2</td>
<td>99.0</td>
<td>98.2</td>
<td>97.1</td>
</tr>
<tr>
<td>Human</td>
<td>99.7</td>
<td>99.1</td>
<td>100.0</td>
<td>99.1</td>
<td>90.2</td>
</tr>
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</table>
efficiency fell short of 100 percent in the first period and climbed to almost 100 percent in the later periods.\textsuperscript{7} The average efficiency across the 29 periods of the human markets 1–5 is 97.9 percent. This level of efficiency is consistent with other research into human double auctions. Human traders learn quickly (within one or two trading periods) and then stay virtually 100 percent efficient for the remaining periods. The efficiency of human markets is not sensitive to the baseline efficiency attained with ZI-U traders because profit-seeking human traders refuse to exchange extramarginal units at a loss.

Surprisingly, the efficiency of ZI-C markets is hardly distinguishable from the efficiency of human markets (see the dashed line in fig. 7). The market discipline seems to raise efficiency from the baseline level to the 96–100 percent range (average efficiency for the five markets is 98.7 percent). In the absence of these results, one might have attributed the high efficiency of the markets with human traders to their rationality, motivation, memory, or learning. Since our ZI traders, bereft of such faculties, exhibit comparable performance, the validity of such attribution is doubtful. Note that we do not wish to argue that the human traders behaved like our ZI traders in the market.\textsuperscript{8} Our point is that imposing market discipline on random, unintelligent behavior is sufficient to raise the efficiency from the baseline level to almost 100 percent in a double auction. The effect of human motivations and cognitive abilities has a second-order magnitude at best.

\textit{Distribution of Profits across Individuals}

Figure 8 and table 3 show the cross-sectional root mean squared difference between the actual profits and the equilibrium profits of individual traders.\textsuperscript{9} While there are no significant differences in the abilities of human and ZI-C traders to extract the total surplus in these double-auction markets, there are significant differences in the way this total surplus is distributed across the individual traders. The profit dispersion is greatest in the ZI-U markets (see the dotted line in fig. 8) and lowest in the human markets (see the solid line).\textsuperscript{10} Profit

\textsuperscript{7} In period 5 of market 2, one trader made an order-of-magnitude keyboard error in his bid, causing efficiency-reducing extramarginal units to be traded.

\textsuperscript{8} They obviously did not. Human markets exhibit a pattern of lower efficiency in the first period, followed by higher efficiency in the later periods; the performance of the ZI traders is, by design, statistically identical across all periods.

\textsuperscript{9} Let \( a_i \) and \( \pi_i \) be the actual and theoretical equilibrium profits of trader \( i, i = 1, \ldots, n \). Then profit dispersion is given by \( n^{-1} \sum (a_i - \pi_i)^2 \).\textsuperscript{15}

\textsuperscript{10} An occasional large value of this coefficient in human markets occurred because a keyboard error by a trader caused a transaction to take place at a price far removed from the equilibrium and adjacent transaction prices.
dispersion in ZI-C markets (see the dashed line) is close to the human markets but has a greater magnitude. Profit dispersion decreases in later periods in two out of five human markets. Without memory or learning, the ZI markets exhibit no such trend. These results suggest that, in contrast to aggregate efficiency, distributional aspects of market performance may be sensitive to human motivation and learning.

V. Concluding Remarks

The primary cause of the high allocative efficiency of double auctions is the market discipline imposed on traders; learning, intelligence, or profit motivation is not necessary. The same market discipline also plays an important role in the convergence of transaction prices to equilibrium levels.

We examined the behavior of markets with zero-intelligence traders that submit random bids and offers. In contrast to the evolutionary models of Alchian (1950) and Nelson and Winter (1982), natural selection plays no role in arriving at our conclusions; the surplus extraction property of double auctions is attained by an unchanged

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<th>Market 4</th>
<th>Market 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZI-U</td>
<td>225.48</td>
<td>253.12</td>
<td>90.54</td>
<td>363.80</td>
<td>156.28</td>
</tr>
<tr>
<td>ZI-C</td>
<td>28.53</td>
<td>49.81</td>
<td>15.90</td>
<td>60.47</td>
<td>19.07</td>
</tr>
<tr>
<td>Human</td>
<td>18.67</td>
<td>28.74</td>
<td>8.23</td>
<td>15.37</td>
<td>30.69</td>
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TABLE 3
Average of the Root Mean Squared Difference in Profits in Figure 8
population of ZI traders. The absence of rationality and motivation in the traders seems to be offset by the structure of the auction market through which they trade. The rules of this auction exert a powerful constraining force on individual behavior.

Becker (1962) showed that price changes alter the opportunity set of consumers in such a way that even if they choose randomly from this set, the expected demand function is downward sloping; the assumption of utility maximization is not necessary to generate a downward slope. Our results are analogous to Becker's in the sense that the convergence of price to equilibrium and the extraction of almost all the total surplus seem to be consequences of the double-auction rules.

Our findings have several interesting implications. First, the extraction of surplus appears to be a characteristic of this auction and the environment in which it is conducted; striving by individual participants to maximize their profits is not necessary for the extraction of surplus.\footnote{Gode and Sunder (1992a) examine a broader class of economic institutions (various types of sealed-bid and double auctions) and show that even sealed-bid markets can be highly efficient with budget-constrained ZI traders.}

Second, since stronger forms of individual rationality reduce the cross-sectional dispersion of the profits of traders, the maximization assumption may still be quite relevant to the equity considerations. Paradoxically, profit maximization seems to be associated with lowering, not raising, profit dispersion across individuals. In addition, a lower price variability in markets populated by human traders (who attempt to increase their profits) suggests that other aspects of market behavior may be sensitive to profit-maximizing behavior.

Third, in the experimental economics literature, the percentage of the maximum possible surplus extracted has often been used as an index of learning and rationality and of the control attained in an experimental economy. Such inferences may not be appropriate for market mechanisms that yield all their surplus to ZI traders.

Fourth, we already know that when double-auction markets aggregate and disseminate information about the state of the world, human traders can significantly improve their ability to extract surplus through learning (see Plott and Sunder 1982, 1988). When populated by ZI traders, such markets may be less efficient. More work is needed to separate the effects of the structure from profit-oriented trader behavior on market performance.

Finally, our results may help reconcile the predictions of neoclassical economic theory with its behavioral critique. Economic models assume utility-maximizing agents to derive market equilibria and
their welfare implications. Since such maximization is not always consistent with direct observations of individual behavior, some social scientists doubt the validity of the market-level implications of models based on the maximization assumption. Our results suggest that such maximization at the individual level is unnecessary for the extraction of surplus in aggregate. Adam Smith's invisible hand may be more powerful than some may have thought: when embodied in market mechanisms such as a double auction, it may generate aggregate rationality not only from individual rationality but also from individual irrationality.

References


Plott, Charles R., and Sunder, Shyam. “Efficiency of Experimental Security

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12 See Simon (1981, chap. 2) for a discussion of individual, market, and economy levels of analysis in economics.


