Agent-Based Modeling of Non-Walrasian Markets with Entrance and Exit of Agents*

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Abstract

Housing is a widely-studied but ultimately poorly-understood sector of the national economy. In particular, the process by which prices are created and the market-outcome is determined is of interest, but very difficult to model mathematically. Previous work has focused on attempting to analytically determine what assumptions are strictly necessary to achieve a competitive equilibrium, but has required perfect information even in the cases where the agents themselves are only boundedly rational.

In this paper I use agent-based modeling to explore not only the effects of bounded rationality, but also the effects of asymmetric information on markets. I find that even small departures from perfect information cause large changes in both the total surplus and the distribution of that surplus. Furthermore, the magnitude of these effects suggests that very few actual markets are likely to have outcomes anywhere near the competitive equilibrium.

*This work submitted in fulfillment of the requirements for a degree with honors at Williams College, Spring 2004

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Introduction

Housing markets represent an interesting area of study for economists. Housing represents an important sector of the national economy, there is data available to test hypotheses, and even more appealingly, many of the phenomena associated with housing markets are still poorly understood. Finally, housing markets also have a strong parallel with another area of great interest: labor markets.

In either a housing market or a labor market, conditions very often give one side a great deal more market experience than the other, leading to information asymmetries. This dimension of housing and labor markets has not been fully integrated into the literature because of the difficulty of mathematically modeling such information states and their impacts. This is unfortunate because the implications of information asymmetry in terms of the total quantity and distribution of economic surplus can be quite significant.

In a much-cited book *Strategic Foundations of General Equilibrium*, Douglas Gale [4] addresses analytically the issues of what assumptions need to be made in order for a market with boundedly rational agents to reach equilibrium. My model very naturally satisfies most of these prerequisite assumptions while relaxing a couple of other key ones, allowing us to very directly test the robustness of his analytical results. While testing the sensitivity of Gale's results to changes in every assumption is not within the scope of what I hope to accomplish, I can hope to provide some insight into how actual markets might compare to these predictions.

As a practical example of the kind of market that I will be exploring, imagine a college town filled with students looking for housing and landlords willing to provide it. Over a number of years, if no new participants enter the market and none of the current participants leave, one might imagine that the market would eventually approximate a Walrasian equilibrium as everyone eventually gains perfect information about the market. This is an unrealistic scenario, however, as very few college students rent housing for more than a couple of years, while landlords may well be in the market for many times that. Rather, new and relatively naive students come in every year while other, more 'experienced,' students graduate. If new students have approximately the same distribution of reservation prices as outgoing students, then the Walrasian equilibrium price and quantity are not affected by this churning in the market. In a non-Walrasian environment, however, changes in the relative amount of information each participant has can be quite significant; here, landlords would gain relatively high levels of knowledge compared to the students who likely only have vague secondhand anecdotes.

A solution to the difficulty of analytically modeling housing markets with asymmetric information is to use agent-based computing, a method of inquiry that allows us to relax assumptions about perfect information by modeling markets from the point of view of each individual agent. This paper makes use of such a simulation to investigate the effects of information asymmetries in a market where agents gain experience over time and change their strategies accordingly.

Section 1 reviews the idea of Walrasian and non-Walrasian markets, the literature on agent-based systems, and the game-theoretical predictions of the impact of information disparities. Section 2 sets out my model in terms of such a game, and Section 3 discusses the outcome of various kinds of models as I vary the degree of asymmetry. Finally, Section 4 discusses the theoretical implications of this work.

1 Review of the Literature

1.1 Walrasian and Non–Walrasian Markets

The structure of a so-called Walrasian market, developed by Leon Walras, relies upon an auctioneer to act as the intermediary between buyers and sellers. The auctioneer calls out a price, and buyers and sellers are allowed to make contracts at that price. If there are more buyers than sellers willing to trade at that price (or vice-versa), all of the contracts are voided and the participants are allowed to costlessly re-contract at the next price the auctioneer chooses. Another equivalent formulation of this process is that of the profit-maximizing middleman. In typical analyses of competitive markets, the price reached by either of these processes is assumed to be the equilibrium price. In this market, then, neither renters unwilling to pay at least this price nor landlords unwilling to sell for only this price get to trade. For a market with eleven renters and eleven landlords whose reservation prices range from one to eleven at each integer, the Walrasian Equilibrium price would be six with a total of six trades.[1]

Many markets of interest, however, do not have a mechanism that ensures that all transactions occur at a single, market-clearing price. In particular, the markets I will be looking at replace the auctioneer with a random matching and bargaining mechanism. In such a

[1] For convenience, I assume that the last trade, between a renter of reservation price six and a landlord of reservation price six, happens.

market, depending on the bargaining model, agents who would not get to trade in a Walrasian market may well get to trade some or even most of the time. Trades involving these agents are called 'extramarginal trades' because they involve an agent whose marginal cost is greater than the market price or whose marginal valuation is lower than the market price. One effect of these extramarginal trades is to lower the total amount of surplus received in the market; in the case with eleven renters and eleven landlords described above, for example, allowing landlord seven and renter five to trade as well results in a 6.6% drop in total surplus from 30 to 28.

1.2 Agent–Based Modeling

Agent-based computing has been defined as "the computational study of economics modeled as evolving systems of autonomous interacting agents" [8]. This approach is not a new one, but is rapidly gaining popularity in any number of different fields of study (including economics). Gilbert and Terna [5] argue that this approach is fundamentally different from other traditional means of investigation, or rather that it represents a third way of conducting analysis apart from verbal argumentation or mathematics.

Robert Axtell [2] points to four distinct advantages associated with approaching problems in this manner. First, this approach makes it easy to limit agent rationality. Second, even if one did wish to use perfectly rational agents, doing so would not be difficult. Third, because of the way in which this kind of simulation works, one can study not only the end state(s) of a market, but also every step along the way. Fourth, traditional mathematics has difficulty modeling either space or social networks–ideas often integral to the way in which we perceive the world to work.

These advantages are not all equally relevant to my model, but they do suggest why a variety of disciplines have come to embrace agent-based computing as a way to solve complex problems. Of them, the first and the third are likely the most important to this particular inquiry. The first is important because one of the most difficult rationality assumptions to relax is that of perfect information; the third is relevant because I am interested not only in the final state of the market, but also in the process by which the market approaches it and the length of time it takes to do so. As Gale [4] points out, it seems self-defeating to use a method of analysis that considers only equlibria when attempting to model limited rationality, because there is no guarantee that even one such equilibrium exists.

Axtell [2] lists a single disadvantage to the agent-based method of analysis: the very nature of simulating market conditions in order to see the outcome means that a single run through the market provides very little statistically significant information. It is becoming increasingly easier to compensate for this, however, by using the ever-expanding speed of computers to run such an experiment over and over in order to calculate what outcomes can be expected.

1.3 Game Theory

The simulation presented in this paper aims to explore much the same kind of questions as Gale does in his book *Strategic Foundations of General Equilibrium* [4]. At the heart of the issue is the question of where prices come from. In 1874 Leon Walras developed a price-determination process in markets that requires an 'auctioneer' to set and reset prices until the competitive outcome is achieved. As an approximation of markets in which each agent has perfect information, this kind of model continues to serve us relatively well, despite the fact that very few markets actually have someone to explicitly or implicitly play the role of auctioneer. In most markets, however, it is participants' relative knowledge about the market and their experiences that leads to prices, but developing an analytical model that uses this mechanism and therefore has the power to explain markets without perfect information has proven to be very difficult.

I, like Gale, am engaged in trying to understand the processes that generate prices and market outcomes. Gale's approach is to find a minimal set of assumptions that allows a market to reach a competitive equilibrium as time extends out to infinity. The first model Gale presents is comprised purely of a large but finite set of consumers who are randomly paired in a Dynamic Matching and Bargaining Game. Here, he finds it necessary to introduce several assumptions, including anonymity, Markov strategies, and continuity, in order to make progress.[2] His second model tries to justify these assumptions in terms of bounded rationality and simplicity, and it succeeds in simplifying the analysis quite a bit. Nevertheless, both assume a possibly unreasonable knowledge about equilibrium strategies. In his third model, Gale introduces a process of random searching for a better strategy rather than relying on agents to simply intuit it. Here, it is also possible to find a competitive equilibrium, but, in order to make the analysis mathematically tractable, he moves back to a Walrasian-style auctioneer to guarantee the greatest possible surplus contingent on the agents' chosen strategies.

[2] A discussion of the particulars of these assumptions as well as their relation to my own model can be found at the end of Section 3.

I, on the other hand, want to consider not only Walrasian markets, but also those in which the outcome either systematically deviates from or does not agree at all with what a Walrasian auctioneer produces. The agent-based simulation approach allows us to study not only the final state of the market but the entirety of the market progress toward whatever result develops.

2 Description of the Model

What follows is a description of the simulation model. In the introduction I very briefly described an example relating to a college town, an example that is a good real-world example of the model I am about to present. College towns have a relatively constant number of students looking for off-campus lodging and a number of landlords with units dedicated to pretty much just that. The market opens every spring when students have to begin thinking about the next year, and closes every fall when college actually starts. In between, students wander about looking at housing, but the order in which they encounter landlords is essentially random. Finally, while students may say for a couple of years, they are likely, on average, to be in the market for fewer years than the landlords.

2.1 Agents

There are two distinct types of agents: renters and landlords. Each class of agents is heterogeneous and each individual agent is fully characterizable by reservation price, defined as the price at which an agent receives no surplus from an exchange. No restriction is made on the number of agents of each type in the market, and any number of each can be created at any reservation price. Each agent wishes to make at most one exchange, and will refuse an exchange if it does not result in at least as much surplus as the agent has come to expect in the market if he/she rejects the exchange. Each agent maximizes over a one period horizon. All agents have utility functions into which housing enters linearly.

2.2 Commodities

There are only two commodities in this market: housing and money. All housing units are identical and I assume, for simplicity's sake, that the difference in reservation prices for landlords is directly tied to differences in their costs of keeping their housing in the residential market.

2.3 Time

In order to narrow the scope of this investigation, I assume that there are no costs associated with searching for a trade. As this is the case, and because a market does not close until there are no possible trades left, I mark the passage of time within a market simulation solely by counting the number of transactions that have already occurred. This number, called the 'transaction count,' is known to all agents, and agents' expectations regarding the surplus they can expect to obtain if they reject a feasible trade are conditioned on the transaction count at which that feasible bargain occurs.

Two other ideas are important in terms of the passage of time. First, agents do not update their expectations in between every market simulation, but rather do so only at the conclusion of some pre-defined number of market simulations. This is necessary because it is virtually certain that at least some individual market simulations will result in improbable outcomes that, if used as the sole basis for updating information, would cause very erratic swings in agents' behaviors. In my modeling this period is called a 'stage' and typically consists of 30,000 market simulations. Each of these market simulations with constant expectations is called an 'iteration'.

2.4 Market Interactions

Renters and landlords are placed into separate pools, and pairs are drawn at random. If a trade can be negotiated between the chosen agents, it is made and the agents are removed from their respective pools, and the specifics of the trade are recorded. Otherwise, agents are put back in their respective pools, from which they may or may not be drawn again for a bargaining encounter. A market ends if all possible trades have been exhausted, or if there have been a predetermined number of successive bargaining encounters without a successful trade. There is no possibility of 'reserving' a bargain while an agent continues to search. As it is, the particular order of the random encounters between renters and landlords can have a large effect upon the outcome of a market.

2.5 Information

Agents acquire information for themselves through direct experience in the market. Each agent keeps track of the amount of surplus he/she expects to receive at every transaction count from beginning to end, which is to say that in a market with eleven agents on each side (hence eleven possible trades), each agent keeps eleven different expectations. Agents keep track of the points in the market at which they reject trades, and at the end of each stage (typically comprises 30,000 simulations), they compute the average amount of surplus they actually received at the various transaction counts in the market and they update their expectations accordingly.

Intuitively, one expects that everyone entering a housing market would have at least some sort of knowledge or expectations about the way in which that market works. In order to give agents some degree of knowledge about the market they are entering, the first stage of every simulation is a 'bootstrap' stage in which half of the trades that otherwise would have happened are randomly rejected allowing them to learn how much surplus they can expect if they reject a feasible bargain at any point. Over several thousand runs through the market, this provides a solid base of information for the agents, though it remains far short of perfect information.[3]

Agents' expectations for surplus, also known as disagreement-points, reflect the information that they have about the market. As agents' expectations rise, fewer and fewer trades are possible, which means that there are fewer and fewer ways for the market to play out. It is entirely feasible, for instance, to have an outcome identical to the Walrasian equilibrium in a market with entirely uninformed renters, but such an outcome would only occur by chance. The only guarantee that a market will have the Walrasian outcome is for the expectations of all renters (landlords) to be such that they insist on paying no more (receiving no less) than

[3] Increasing this number has relatively little effect on the amount of information agents acquire, as there is only so much that can be learned from a market in which every agent is taking any deal that nets positive surplus.

the Walrasian equilibrium price. Indeed, if when I simulate a basic version of this model (see section 3.1), expectations do rise enough to rule out many possible trades, cutting down the number of outcomes. They do not, however, rise enough to bring about the Walrasian equilibrium.

For most agents expectations for the surplus they will receive if they refuse a feasible bargain declines as the transaction count rises. This makes intuitive sense, because both renters with very high reservation prices and landlords with very low reservation prices (the people with whom it is most attractive to trade) are able to make deals with a greater number of other agents and are therefore likely to exit the market earlier. Interestingly, for those agents who only rarely get to trade, however, expectations initially fall but then take a large jump as the transaction count rises high enough to indicate that the market should already have closed; even knowledgeable agents are inexperienced at high transaction counts, and at least one now-desperate low (high) reservation price landlord (renter) must remain, as at least one more trade is still possible.

2.6 Negotiations

When agents are randomly matched and a trade is possible given their reservation prices, I set the exchange price to be equal to that which would occur if the agents were to engage in a bargaining process based on a Rubinstein alternating-offers game to determine a price for the rental unit [6].[4] A great deal of this bargaining happens when buying or selling a housing unit, but this bargaining approach might at first seem inappropriate for rental

[4] Much of this section describing the bargaining model is based upon a working paper by Bradburd and Sheppard.

housing markets where, more often than not, the rental rate is set in advance by the landlord. There are, however, a few reasons why such a bargaining model is not unreasonable. First, many housing units may be rented through less formal listing mechanisms in which explicit bargaining over the rental rate is common, or by landlords who are willing to haggle. Second, even in the case of rental units in larger multi-unit buildings, some elements of bargaining do come in as renters and landlords bargain over the initial duration of the rental agreement, over the nature of improvements/repairs that the landlord will make prior to occupation, over the terms of subsequent extensions to the initial lease, and over subsequent improvements and the rent increases that should attach to them.

The Nash bargaining solution is the unique subgame perfect equilibrium for a Rubinstein alternating-offers game in this context. Each agent has not only a reservation price (as described above) but also a set of expectations about future opportunities. These expectations determine the agent's disagreement point. If a trade is possible given both agents. reservation prices and disagreement points, the trade price is such that each agent gains the same amount relative to his/her respective disagreement point. Thus,

$$P_{i,j} = \frac{(P_{R_i} - REB_i + P_{L_j} + LEB_j)}{2} \tag{1}$$

where

$P_{i,j}$ = the price obtained in an exchange between renter i and landlord j;

P_{R_t} = the reservation price of the ith renter;

REB_i = the ith renter's expected gain if the current bargain is rejected;

P_{L_j} = the reservation price (= marginal cost) of the jth landlord;

LEB_j = the jth landlord's expected gain if the current bargain is rejected.

This bargaining model is fairly standard; the challenge is in finding a reasonable approach for determining the agents' respective disagreement points. The appropriate disagreement point for each agent is the amount of surplus (s)he would expect to receive if (s)he rejected the current bargaining opportunity. In a scenario with perfect information and perfectly rational behavior, such expected surplus would be a function of the numbers and reservation prices of all opposite-type and same-type agents remaining in the market, of the nature of the bargaining process that determines prices in bargaining encounters, and of the possibilities for strategic bargaining strategies. However, attempting to take all of these factors into account is analytically intractable and computationally impractical. Furthermore, it requires us to assume that agents have both exceptional analytical abilities and perfect information– assumptions I explicitly want to avoid. I therefore use experience from previous stages to set expectations for the current stage, and begin with a special 'bootstrap' stage designed to give agents initial information about what they can expect from this market.

In the first stage of every simulation, agents acquire knowledge that they can then employ in their bargaining in every subsequent stage. This information continues to evolve as the agents gain experience in subsequent stages. Ideally, the amount of knowledge gained during the bootstrap stage should be close to the amount available to traders in actual housing markets, rather than unreasonably extensive or limited. The challenge, therefore, is to give agents enough simulated information so that they have reasonable guidance while still allowing for trades at prices other than the Walrasian market clearing price.

My objective–to study what happens to markets as renters are replaced with more naive counterparts–would be compromised if the information available to renters as they entered the market were near-perfect. I have chosen to assume that during bargaining, both renters and landlords know two things: how many transactions have already occurred (the transaction count) and how much surplus an agent of his/her type (renter/landlord) and reservation price can expect to receive in the market if he/she rejects a feasible bargain at that particular transaction count. This is very much like those markets that seasonally close, such as summer rental properties or college housing, and therefore seems quite reasonable.[5]

The bootstrap stage is intended to give agents experience about the surplus they can expect to receive if they reject a possible trade at any given transaction count. But in the bootstrap stage itself, agents have no prior experience that could inform their judgment of possible trades. By assumption, I give them disagreement points of zero in the bootstrap stage, implying that they would accept any trade that gave them positive surplus. In effect, I assume that each agent begins this learning process with the naive belief that the alternative to accepting a feasible bargain is to receive no surplus at all. Of course, under this assumption, agents would never acquire experience about rejecting feasible trades because none of them would ever reject such a trade in the first place. A solution to this is to provide this experience for the agents by randomly selecting some of their proposed trades and canceling them, returning both agents to their respective pools. Each agent whose trade opportunity is canceled in this way records the transaction count at which it occurred as well as the

[5] Even in the market for single-family homes, very few such units are sold between Thanksgiving and the middle of January, creating an effect much like the closing of a market, even though the opportunity for surplus is not permanently lost.

level of surplus eventually attained in that market run. These data are averaged over the 30,000 iterations of the bootstrap stage to arrive at each agent's expectations regarding the amount of surplus they can expect to receive if he/she rejects a feasible trade at any given transaction count. While these trades could be rejected with any probability greater than zero and less than one, I chose to reject them with probability one-half.[6]

This knowledge informs the agents' respective disagreement points in every stage after the first. It is worth noting that the amount of information agents have is constrained not only by the limits to their experience, but also by the evolving nature of the market. Information gathered in the previous stage cannot fully inform the present stage because agents update their information (and hence their strategies) between stages. Even with perfect information from the previous stage, agents may well not use an optimal strategy because of the changes in others' expectations. Of course, once in equilibrium, agents' strategies cease to change because their experiences simply reinforce their previous strategy: any deviation is punished either by leaving the agent without a trade (if expectations are excessive) or with less surplus than (s)he could have gotten (if expectations are too low). Note that bounded rationality–a result of market evolution–does not necessarily rule out an equilibrium, because changes in strategy slow down as the market moves closer.

Agents with different reservation prices will naturally have different experiences. Furthermore, an agent's expected surplus after rejecting a feasible trade will vary with the transaction count. At any given point in the market, agents who are more favorably situated (renters with high reservation prices and landlords with low reservation prices) are more

[6] Previous experiments by Bradburd and Sheppard have found that moving this number to either one quarter or three quarters has no effect.

likely to be matched with an opposite type agent with whom a bargain is feasible than are less favorably situated agents.[7] Their expected surplus conditional on rejecting a feasible trade at that transaction count will thus be higher. At the same time, because these more favorably situated agents are withdrawn from the pools from which agents are selected in the matching process when they complete a bargain, the pools of renters and landlords from which agents are drawn becomes increasingly dominated by low reservation price renters and high reservation price landlords over the course of a market run. The result is that for the typical agent, the expected surplus received after rejecting a feasible bargain falls as the transaction count increases. Thus the agents' disagreement points in their bargaining encounters, which are derived from experience in previous stages, will vary both as a function of their reservation prices and the transaction count.

This system of using previous experience as well as a specially tailored first stage creates disagreement points for agents in their bargaining throughout the entirety of the simulation. Any trade from which both agents would gain surplus at least equal to his/her expectations for surplus (at that transaction count) will take place, and the market proceeds until no further trades are possible. Again, in order to make sure that the data are representative of all of the possible outcomes (as well as their relative probabilities), each stage is run for 30,000 iterations.

[7] This advantage disappears as the market begins to converge toward equilibrium as favorably situated agents increase their expectations and thereby rule out some agents of the opposite type.

2.7 Information Asymmetry

The focus of my work is to investigate how market outcomes–output, total surplus, and its distribution between renters and landlords–are affected by the exit and entry of renters. In order to simulate such a flow of old renters out of the market and new renters into the market, at the end of each stage a number of renters are selected at random and are replaced with new agents of identical type and reservation price but equipped with only the knowledge an identical agent had at the end of the bootstrap stage. Because each agent consists only of a reservation price, type, and experience, this is effectively identical to creating an entirely new renter with some experience. Furthermore, this also effectively limits the maximum amount of information that any renter can have about the market, as, on average, renters are replaced every n/R stages, where n is the number of renters and R is the number replaced each stage.

3 Results

3.1 Basic Simulations

All of the simulations in this paper hold constant a number of parameters unless otherwise specified. I have chosen to use a market with eleven renters and eleven landlords with one at each integer reservation price from one to eleven, because the Walrasian equilibrium price and quantity of such a market are, conveniently, each six, and the total market surplus is 30. As indicated above, agents' information is updated only at certain intervals rather than continuously. Because the information each agent possesses is only updated at the end

Average Price	Total Surplus	Renter Surplus	Landlord Surplus	Transaction Ct.
6.0003	26.9483	13.474	13.474	6.3861

Table 1: Basic Simulation Results

of each stage, each iteration of the market within a particular stage has identical starting parameters. In order to obtain results that are representative of the average outcome of the market, each stage consists of the same market run being played out 30,000 times with the results averaged before information is updated. [8]

The most basic kind of simulation exercise I perform contains only two stages: the bootstrap stage (where agents gain information), and a single trading stage producing 'results' for analysis. Table 1 shows the results of a typical simulation of this type for the first post-bootstrap stage.

The Walrasian equilibrium for this market would be six trades at a price of six, with a total surplus of 30 divided evenly between renters and landlords. As can be seen from Table 1, while the average trade price is almost exactly six, a surplus of 30 is not nearly realized–the total surplus is in fact more than 10% less. This loss is divided evenly over the renters and landlords, and caused by a number of agents getting to trade who would not ordinarily get to trade in the Walrasian equilibrium.

Table 2 shows the overall surplus expectations for each of the renters and landlords before the bootstrap, after the bootstrap, and after the first stage. Each agent comes into the bootstrap with no expectations at all: that is to say, with the expectation that the

[8] Section 2.5 talks about the necessity of such averaging, and Section 3.7 gives some insights as to what happens without it.

Renter	Pre-Boot	Bootstrap	1stStage	Landlord	Pre-Boot	Bootstrap	1stStage
1	0	0	0	1	0	1.4586	1.5502
2	0	0.0534	0.0415	2	0	1.2884	1.3805
3	0	0.1371	0.1084	3	0	1.1226	1.2087
4	0	0.2496	0.2147	4	0	0.9428	1.0069
5	0	0.3955	0.3659	5	0	0.7551	0.7815
6	0	0.5691	0.5597	6	0	0.5658	0.5574
7	0	0.7566	0.7830	7	0	0.3972	0.3675
8	0	0.9433	1.0066	8	0	0.2512	0.2162
9	0	1.1225	1.2090	9	0	0.1370	0.1083
10	0	1.2940	1.3865	10	0	0.0532	0.0412
11	0	1.4537	1.5452	11	0	0	0

Table 2: Renters' and Landlords' Expectations

alternative to accepting a feasible trade (at any transaction count) is to receive no surplus at all. Though they gain great deal of experience during the 30,000-iteration bootstrap stage, agents have far from what might be considered 'perfect' information in the first post-bootstrap stage. For instance, Renter 11 would expect a surplus of 5 in a Walrasian market, but after the bootstrap, that renter has an expectation of 1.45.[9] This is hardly surprising because of the way in which bargaining is done: in the bootstrap, Renter 11 would trade with literally anyone, gaining a surplus of 0 from trading with Landlord 11, a surplus of .5 from Landlord 10, and so on, gaining a surplus of five only from trades with Landlord 1. As one would expect, information is symmetric between the renters and the landlords; for instance, Renter 1 and Landlord 11 are positioned to get equal amounts of surplus from this market and have almost exactly the same expectations.

In the first stage *after* the bootstrap, both the renters and the landlords gain additional information. Those traders who get to trade increase their expectations, and those for whom

[9] Expectations are used to calculate the disagreement point, so, in the next stage, the worst trade that this renter will accept is one that nets a surplus of at least 1.45

trading is a rarity scale back what they expect,[10] but again, information between the renters and the landlords is almost perfectly symmetric.

3.2 Fifty Stage Simulations

A two-stage simulation exercise, however, does not tell us anything about how the market evolves as agents gain more information. In order to see what happens as both sides gather information at about the same rate, simulations were run out to 50 stages (49 stages after the bootstrap). Table 3 shows the averaged results of 30 such 50 stage simulation runs, where each stage was comprised of 30,000 iterations.[11]

Over the course of 48 additional stages, both the renters and the landlords gain a great deal of information. By the end of the last stage, total surplus reaches just over 29, or about 97% of the Walrasian equilibrium, and is still split evenly between renters and landlords. The average trade price is almost exactly six and the total number of transactions has fallen to 5.8.a reflection of the fact that there are many fewer extramarginal trades and that the last trade (between Renter and Landlord 6, at a price of six, for no total surplus) does not always happen. Figure 1 shows total surplus over these stages.

It is important to note here that even after 50 stages of 30,000 market runs apiece, the market is still a fair distance from the Walrasian equilibrium–a deadweight loss of 3% in fact.

[10] Those agents who have expectations that are too high end up rejecting viable trades and either end up with less later on or, in some instances, do not get to trade at all, which results in their having a zero averaged in.

[11] Whereas with the two stage run where averaging was not strictly necessary (because every run was within a very small distance from the average), it became much more important to average the longer runs. Here, each stage affects the one afterwards, meaning that in a single run, one stage with atypical results could profoundly affect several stages thereafter.

Stage	Trade Price	Total Surplus	Renter Surplus	Landlord Surplus	Transaction Ct.
B	6.000	20.008	10.004	10.004	7.941
1	5.999	26.965	13.486	13.478	6.386
2	6.000	27.203	13.605	13.597	6.303
3	5.999	27.299	13.652	13.648	6.289
4	6.000	27.520	13.764	13.756	6.234
5	5.999	27.595	13.800	13.794	6.204
6	6.000	27.649	13.828	13.821	6.195
7	5.999	27.654	13.830	13.824	6.193
8	6.000	27.848	13.929	13.919	6.146
9	6.000	27.926	13.967	13.960	6.133
10	6.000	28.106	14.055	14.051	6.080
11	5.999	28.213	14.109	14.103	6.056
12	5.999	28.347	14.177	14.170	6.026
13	5.999	28.383	14.193	14.190	6.010
14	6.000	28.428	14.218	14.210	5.991
15	6.000	28.455	14.232	14.223	5.973
16	6.000	28.567	14.288	14.279	5.953
17	6.000	28.649	14.330	14.319	5.944
18	5.999	28.728	14.365	14.363	5.926
19	5.999	28.758	14.383	14.375	5.915
20	5.999	28.755	14.381	14.375	5.915
21	5.999	28.755	14.381	14.374	5.915
22	6.000	28.757	14.382	14.375	5.915
23	5.999	28.758	14.382	14.376	5.914
24	5.999	28.759	14.382	14.377	5.915
25	6.000	28.768	14.388	14.380	5.912
26	6.000	28.848	14.426	14.421	5.893
27	5.999	28.930	14.465	14.465	5.864
28	5.999	28.965	14.484	14.481	5.852
29	5.999	28.967	14.486	14.481	5.851
30	6.000	28.967	14.486	14.481	5.852
31	5.999	28.967	14.486	14.481	5.851
32	6.000	28.974	14.490	14.484	5.849
33	6.000	28.978	14.492	14.487	5.846
34	6.000	28.979	14.492	14.487	5.846
35	6.000	28.982	14.493	14.489	5.842
36	6.000	28.984	14.495	14.489	5.841
37	6.000	28.986	14.495	14.491	5.838
38	6.000	28.985	14.495	14.490	5.837
39	6.000	28.995	14.501	14.494	5.835
40	6.000	29.035	14.519	14.516	5.821
41	6.000	29.064	14.535	14.529	5.810
42	6.000	29.065	14.535	14.530	5.816
43	5.999	29.065	14.536	14.530	5.819
44	6.000	29.063	14.533	14.529	5.823
45	6.000	29.063	14.535	14.529	5.824
46	6.000	29.063	14.534	14.529	5.824
47	6.000	29.064	14.535	14.529	5.824
48	6.000	29.066	14.535	14.531	5.824
49	5.999	29.076	14.540	14.536	5.820

Table 3: Summary Statistics for a 50 Stage Simulation

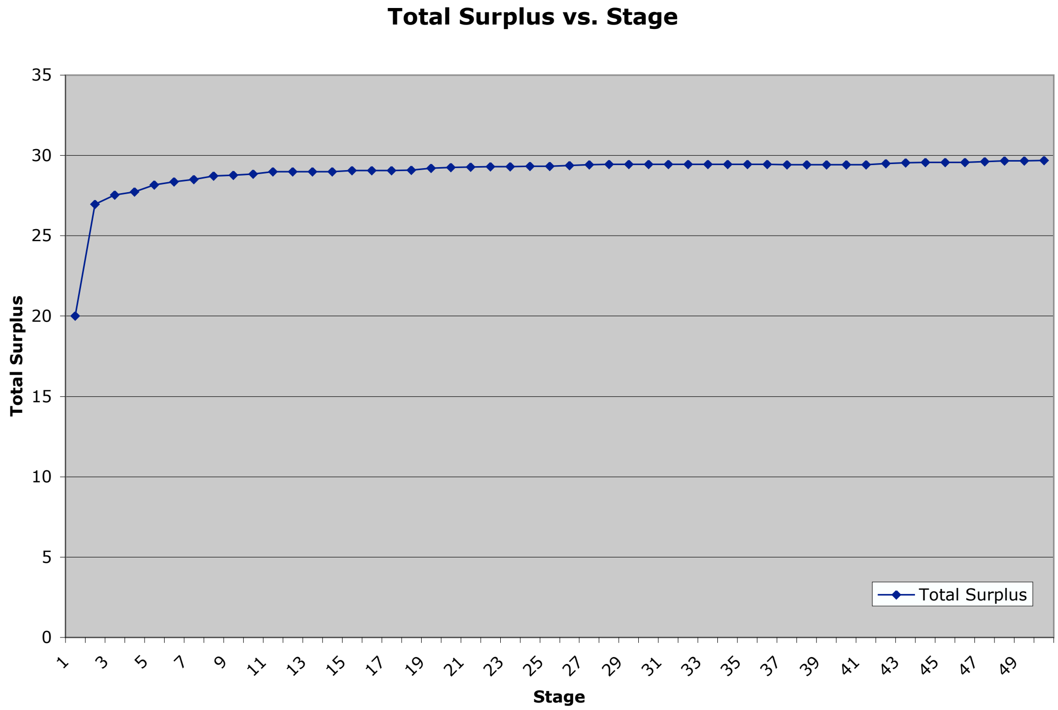

Figure 1: Total Surplus by Stage

Much longer runs under similar conditions (1,000 stages), show an increase in Total Surplus of only .3 over the 50 stage runs, making it unclear as to whether or not this is, in fact, asymptotic convergence to the Walrasian equilibrium. On the other hand, even information that is far from perfect can create results fairly close to the Walrasian outcome.

Table 4 shows the average expectations for renters and landlords after the first and 49th stages. Again, there are large changes in the manner we would expect: those agents who should not get to trade in a Walrasian world have their expectations decreased, while those who can reliably expect to trade (renters with high reservation prices and landlords with low reservation prices) see their expectations increase.[12]

[12] Expectations only change when an agent actually rejects a deal, however, so for those agents who find themselves pushed out of the market, their continued demand for some surplus is a remnant of very early good fortune that has never been wiped out, because

Renter	1st Stage	49th Stage	Landlord	1st Stage	49th Stage
1	0	0	1	1.5502	2.5561
2	0.0415	0.0167	2	1.3805	2.2521
3	0.1084	0.0386	3	1.2087	1.9180
4	0.2147	0.0718	4	1.0069	1.3912
5	0.3659	0.1576	5	0.7815	0.8135
6	0.5597	0.3762	6	0.5574	0.3734
7	0.7830	0.8148	7	0.3675	0.1593
8	1.0066	1.3922	8	0.2162	0.0734
9	1.2090	1.9168	9	0.1083	0.0388
10	1.3865	2.2570	10	0.04128	0.0165
11	1.5452	2.5510	11	0	0

Table 4: Agents' Expectations for a 50 Stage Simulation

Expectations for each agent are not limited to a single number, but rather contain a data point for each possible transaction count in the market. For example, in a market with 11 renters and 11 landlords, each agent would have 11 different expectations, telling that agent how well he/she can expect to do if he/she rejects a trade at each transaction count. Table 5 shows the full expectations for the renters and for the landlords in the first post-bootstrap stage and the 49th. For those agents who always get to trade, expectations fall as the transaction count increases, but increase across the board from the first to the 49th stage. Likewise, for those agents who very rarely get to trade, expectations either fall or stay the same (because these agents never reject a trade).

Expectations are more interesting for agents at the margin, however. For Renter 5 and Landlord 7 (who should never get to trade in the Walrasian equilibrium, but who, as we will see, get to trade about half the time even late in the late stages of the simulation runs), expectations initially decrease as the transaction count increases (the best deals are most

those agents simply never get the chance to reject a trade anymore.

	TC 0	TC 1	TC 2	TC 3	TC 4	TC 5	TC 6	TC 7	TC 8	TC 9	TC 10
Stage 1											
Renter 1	0	0	0	0	0	0	0	0	0	0	0
Renter 2	0.040	0.038	0.038	0.037	0.035	0.036	0.041	0.047	0.053	0.051	0
Renter 3	0.126	0.118	0.113	0.111	0.096	0.084	0.086	0.094	0.103	0.153	0
Renter 4	0.303	0.279	0.262	0.237	0.198	0.157	0.153	0.160	0.190	0.210	0
Renter 5	0.548	0.510	0.475	0.426	0.351	0.274	0.262	0.249	0.286	0.277	0
Renter 6	0.843	0.797	0.742	0.671	0.576	0.479	0.417	0.361	0.372	0.339	0
Renter 7	1.160	1.112	1.039	0.962	0.868	0.762	0.618	0.487	0.457	0.366	0
Renter 8	1.465	1.417	1.319	1.243	1.164	1.048	0.870	0.638	0.538	0.364	0
Renter 9	1.743	1.693	1.577	1.493	1.407	1.289	1.090	0.820	0.608	0.372	0
Renter 10	2.003	1.942	1.833	1.712	1.606	1.460	1.237	0.973	0.718	0.382	0
Renter 11	2.249	2.176	2.067	1.903	1.776	1.605	1.365	1.097	0.825	0.391	0
Stage 49											
Renter 1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
Renter 2	0.002	0.002	0.002	0.002	0.002	0.003	0.011	0.042	0.053	0.051	0
Renter 3	0.007	0.006	0.006	0.005	0.005	0.005	0.016	0.081	0.103	0.153	0
Renter 4	0.030	0.027	0.024	0.020	0.016	0.011	0.033	0.158	0.190	0.210	0
Renter 5	0.154	0.140	0.117	0.091	0.058	0.035	0.171	0.249	0.286	0.277	0
Renter 6	0.533	0.487	0.429	0.359	0.245	0.242	0.395	0.361	0.372	0.339	0
Renter 7	1.209	1.154	1.083	0.991	0.832	0.835	0.735	0.487	0.457	0.366	0
Renter 8	2.034	1.988	1.928	1.851	1.725	1.542	1.311	0.641	0.538	0.364	0
Renter 9	2.765	2.728	2.679	2.597	2.507	2.354	1.717	0.842	0.608	0.372	0
Renter 10	3.353	3.302	3.233	3.125	2.941	2.693	1.801	1.021	0.718	0.382	0
Renter 11	3.934	3.845	3.747	3.597	3.356	2.896	1.803	1.117	0.825	0.391	0
Stage 1											
Landlord 1	2.252	2.176	2.069	1.903	1.778	1.605	1.362	1.100	0.811	0.447	0
Landlord 2	1.996	1.935	1.826	1.707	1.605	1.458	1.231	0.977	0.721	0.350	0
Landlord 3	1.741	1.686	1.572	1.494	1.405	1.286	1.088	0.817	0.612	0.387	0
Landlord 4	1.465	1.418	1.328	1.244	1.164	1.042	0.871	0.636	0.533	0.367	0
Landlord 5	1.159	1.112	1.035	0.962	0.868	0.760	0.618	0.486	0.460	0.355	0
Landlord 6	0.844	0.799	0.739	0.673	0.576	0.481	0.414	0.362	0.371	0.315	0
Landlord 7	0.544	0.511	0.478	0.423	0.353	0.273	0.262	0.251	0.286	0.294	0
Landlord 8	0.304	0.277	0.260	0.238	0.198	0.157	0.152	0.161	0.189	0.227	0
Landlord 9	0.127	0.116	0.113	0.110	0.097	0.085	0.085	0.095	0.103	0.155	0
Landlord 10	0.040	0.038	0.038	0.037	0.035	0.035	0.041	0.047	0.052	0.049	0
Landlord 11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
Stage 49											
Landlord 1	3.934	3.846	3.749	3.598	3.357	2.897	1.802	1.120	0.811	0.447	0
Landlord 2	3.350	3.299	3.231	3.123	2.938	2.690	1.795	1.023	0.721	0.350	0
Landlord 3	2.763	2.727	2.678	2.596	2.506	2.354	1.716	0.841	0.612	0.387	0
Landlord 4	2.034	1.987	1.927	1.850	1.724	1.541	1.311	0.639	0.533	0.367	0
Landlord 5	1.208	1.153	1.082	0.991	0.831	0.835	0.735	0.487	0.460	0.355	0
Landlord 6	0.533	0.487	0.428	0.359	0.245	0.242	0.393	0.362	0.371	0.315	0
Landlord 7	0.153	0.140	0.116	0.090	0.058	0.035	0.171	0.250	0.286	0.294	0
Landlord 8	0.030	0.027	0.024	0.020	0.016	0.011	0.033	0.159	0.189	0.227	0
Landlord 9	0.007	0.006	0.006	0.005	0.005	0.005	0.015	0.081	0.103	0.155	0
Landlord 10	0.002	0.002	0.002	0.002	0.002	0.003	0.011	0.042	0.052	0.049	0
Landlord 11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0

Table 5: Expectations By Transaction Count

Renter	Stage1	Stage49	Landlord	Stage1	Stage49
1	0.000	0.000	1	3.286	4.307
2	0.000	0.000	2	2.859	3.627
3	0.021	0.000	3	2.429	2.939
4	0.169	0.002	4	1.956	2.073
5	0.458	0.059	5	1.414	1.129
6	0.884	0.400	6	0.887	0.400
7	1.418	1.129	7	0.458	0.059
8	1.955	2.073	8	0.168	0.002
9	2.430	2.939	9	0.021	0.000
10	2.865	3.630	10	0.000	0.000
11	3.287	4.307	11	0.000	0.000

Table 6: Realizations for Renters and Landlords by Reservation Price

likely to leave the market early), but these expectations shoot back up when the transaction count rises over the expected number of trades. In order for the market to even reach such a high transaction count, an unusual trade must have happened earlier in the market run, leaving available at least one agent who would normally trade in a Walrasian setting. As all of those agents have strictly downward-sloping expectation curves, these agents seem more and more 'desperate' as the transaction count increases, thus increasing both the probability of trading, and the return from doing so for Renter 5 and Landlord 7.

Table 6 shows the average surplus each renter and landlord earned in the first stage after the bootstrap, as well as the 49th. As we would expect, the renters willing to pay more and the landlords willing to sell for less get more total surplus, though the difference in surplus between agents that actually trade is about half the difference in their reservation prices. Again, there exists almost a perfect symmetry between the high reservation price renters and their corresponding low reservation price landlords.

3.3 Two-Replacement

To get a very basic look at the effects of asymmetric information, I began by simply replacing a fraction of the renters with more naive counterparts of identical reservation price at the end of each update period. This is essentially the same as a natural churning in the market, where some renters leave for other places and new, less experienced renters take their places. As it seems intuitive that everyone has at least some knowledge of the market, the starting knowledge for the replacement agents comes from the beginning of the stage after the bootstrap, rather than from before the bootstrap.

Table 7 shows the results from randomly replacing two of the eleven renters each stage. As we would expect, landlords begin to do significantly better than renters and landlords' expectations continue to rise. The number of transactions moves quickly up above the Walrasian equilibrium, increasing to about 5% over the same stage of a no-replacement simulation. Overall, the total surplus drops about 5%; though the landlords are about 3% better off than they were before, the renters are only getting about 85% as much surplus.

An interesting effect of this replacement is to bring high reservation price landlords back into the market at least some of the time. When a high reservation price renter is replaced with a more naive counterpart, it may suddenly be possible for one of the landlords with a cost over six to do a deal, increasing the average number of trades in that stage.

One of the problems with this particular data set is that the market outcome does not converge, but rather bounces around somewhat erratically, even at late stages. This is hardly surprising, because at every stage I continue to introduce random elements by way of replacing two of the renters with less experienced counterparts. In order to see if there is

Stage	Trade Price	Total Surplus	Renter Surplus	Landlord Surplus	Transaction Count
1	5.9995	20.0324	10.0162	10.0162	7.9365
2	6.0014	27.0710	13.5532	13.5178	6.3892
3	5.9960	27.5807	13.8337	13.7470	6.1960
4	5.9909	27.7613	13.9789	13.7825	6.1381
5	6.0669	27.8278	13.6384	14.1894	6.1193
6	6.0510	28.2964	13.9896	14.3068	6.0126
7	6.0863	28.2201	13.8126	14.4075	6.0135
8	6.0524	28.5438	14.1268	14.4170	5.9368
9	6.1387	28.3015	13.5663	14.7353	6.1107
10	6.1504	28.4668	13.5104	14.9564	6.0910
11	6.1355	28.5507	13.6689	14.8817	6.0472
12	6.0900	28.7177	13.9650	14.7527	5.9525
13	6.0635	29.0131	14.2839	14.7292	5.8375
14	6.1218	28.7365	13.8249	14.9116	5.9480
15	6.1511	28.7386	13.6878	15.0508	6.0303
16	6.1295	28.7771	13.7883	14.9887	5.9688
17	6.3854	27.8434	12.1519	15.6916	6.1219
18	6.3738	27.9653	12.3007	15.6646	6.0345
19	6.3557	27.9538	12.4595	15.4943	6.0157
20	6.2868	28.3166	13.0124	15.3043	5.9005
21	6.2806	28.4309	13.0395	15.3914	5.9328
22	6.2182	28.7489	13.4498	15.2991	5.8533
23	6.2691	28.4308	13.1036	15.3272	5.9196
24	6.3523	28.1564	12.5911	15.5654	5.9393
25	6.3601	28.1685	12.5290	15.6395	5.9396
26	6.3051	28.3612	12.9111	15.4501	5.9486
27	6.2922	28.3545	12.9775	15.3771	5.9779
28	6.3055	28.3781	12.9103	15.4677	5.9922
29	6.3566	28.3219	12.7175	15.6044	5.9527
30	6.3158	28.4968	12.9957	15.5011	5.8839
31	6.3758	28.1611	12.4822	15.6789	6.0060
32	6.3664	28.1746	12.5636	15.6109	5.9810
33	6.3228	28.3863	12.8130	15.5733	5.9381
34	6.2625	28.6567	13.1963	15.4604	5.8949
35	6.2560	28.7040	13.2521	15.4519	5.8703
36	6.2670	28.6337	13.1649	15.4689	5.8977
37	6.3628	28.2897	12.5934	15.6963	5.9104
38	6.3037	28.5366	12.9782	15.5584	5.9194
39	6.3093	28.5022	12.9203	15.5819	5.9694
40	6.2579	28.5865	13.2385	15.3481	5.9194
41	6.2569	28.7850	13.3988	15.3862	5.8601
42	6.2276	28.8762	13.5352	15.3409	5.8575
43	6.3082	28.5598	13.0481	15.5117	5.8830
44	6.2954	28.6277	13.1486	15.4791	5.8333
45	6.3804	28.2401	12.5843	15.6558	5.8829
46	6.3714	28.3427	12.6184	15.7244	5.9193
47	6.3226	28.5399	12.9341	15.6059	5.8733
48	6.3191	28.6246	12.9495	15.6751	5.8957
49	6.2586	28.8741	13.4107	15.4634	5.7930
50	6.2398	28.8439	13.4540	15.3899	5.8914

Table 7: Summary of a 50 Stage Simulation with Two-Replacement

Multiple R	0.620681
R Square	0.385246
Adjusted R Square	0.364047
Standard Error	.239883
Observations	31

	Coefficients	σ	tStat	P-value	Lower 95%	Upper 95%
Intercept	13.7087	.1752	78.217	2.74E-35	13.350	14.067
X Variable	-.08944	0.0209	-4.263	0.0002	-0.132	-0.0465

Table 8: Regression Statistics

a correlation between the renters replaced and the outcome of the next stage, I estimated a simple linear regression of the renters' surplus versus the highest reservation price of a renter replaced in the previous stage, predicting that the relationship would look like Equation 2. Figure 1 shows the results of this regression.

$$B = aR^{t-1} + b \tag{2}$$

B = The surplus for renters at time t

R = The highest RP of a renter replaced in stage t-1

This yields a coefficient for the dependent variable of -.0894 with a standard deviation of 0.021, a result that is statistically significant well past the 95% confidence interval. This regression explains almost 40% of the total variation, which suggests that the total surplus received by renters is very highly correlated with which renters have been recently replaced. The relationship is very likely affected by not only one but both of the renters replaced and

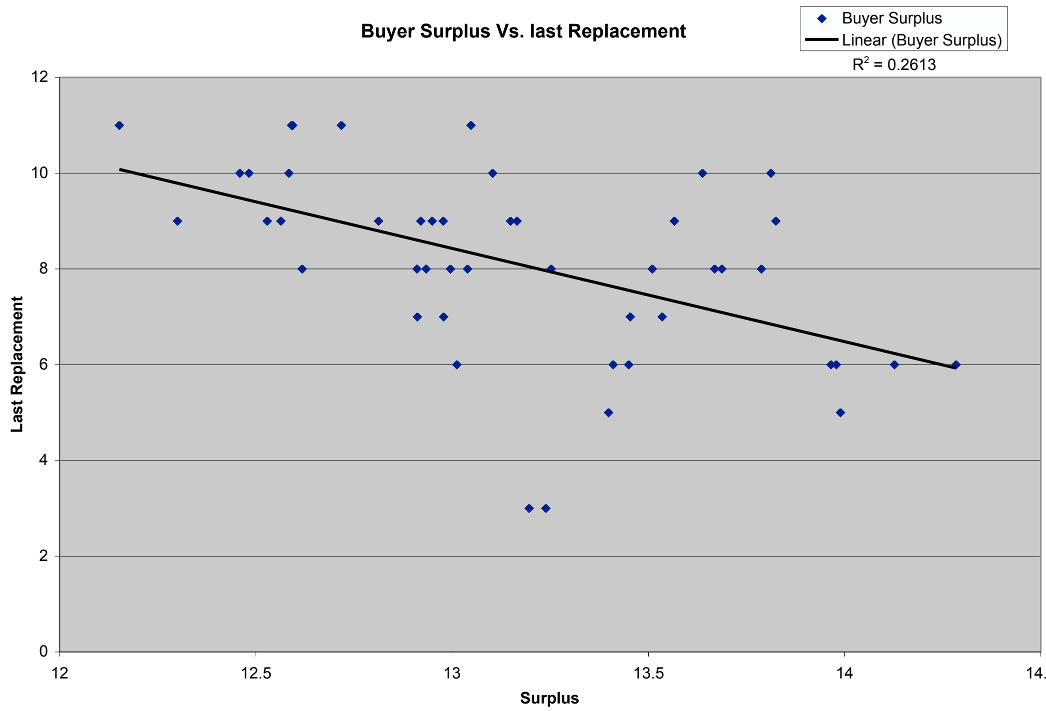

Figure 2: Renter Surplus vs. Highest RP Renter Replaced at t-1

is exponential in shape, meaning that the replacement in previous stages is also a factor. Nevertheless, this basic regression seems sufficient to show that while there may be some kind of 'average' stage in the long term, any actual stage will deviate from that based upon which renters have recently been replaced.

In order to find such an average, two possibilities suggest themselves: either increase the number of renters and landlords in the market so that a single replacement does not affect all of the renters of one reservation price, or average together multiple simulations with the same starting parameters. Both options seem equally valid, but the second approach was chosen because it was comparatively easier to work with a smaller number of agents and average the results afterwards. Table 9 shows the results of 30 such simulation runs averaged together. Most of the uncertainty disappears and the correlation with the identities of the agents who

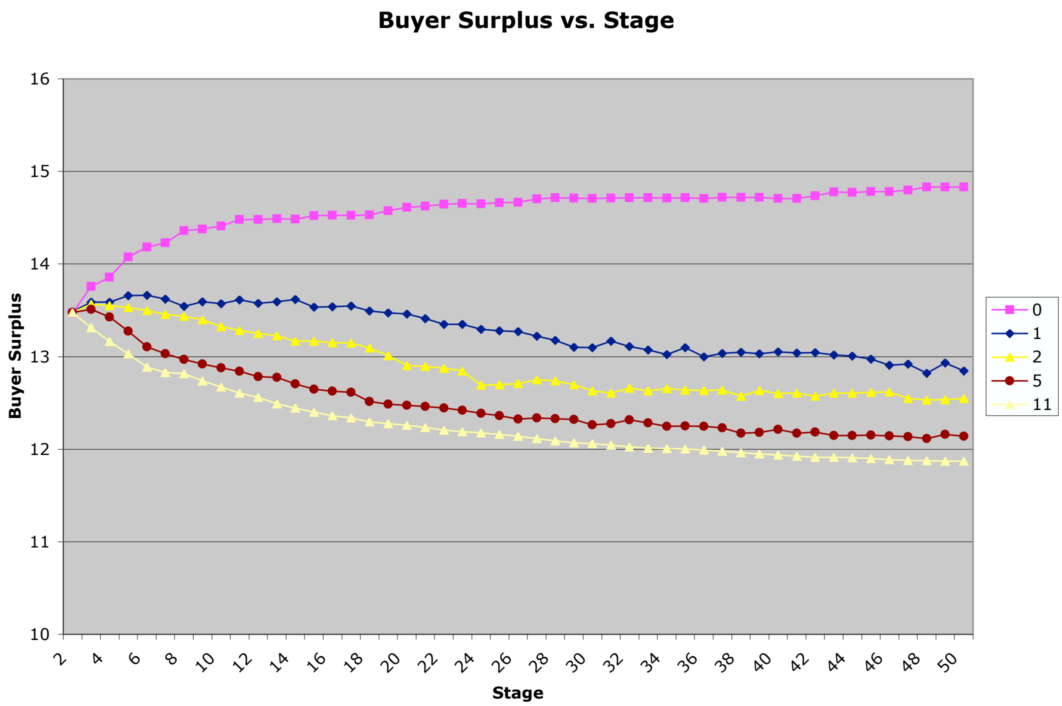

Figure 3:

were replaced disappears entirely.

Figures 3-7 show the effect of different levels of churning on the outcome of the market. As we would expect, a greater level of replacement on the renter's side (and henceforth less experience for the average renter) leads to a higher average price, lower surplus for the renters, higher surplus for the landlords, and an overall loss of total surplus–almost an 8% drop from the Walrasian case, and a 7% drop from the no-replacement case. One interesting effect of replacement is that the average number of transactions steadily increases with the amount of replacement, a trend caused by an increasing willingness on the part of renters to take all kinds of deals that they would reject with better information.

Another interesting effect of replacement is that over the first 15 stages of trading, landlord surplus actually rises more slowly with some replacement than it does in the no-

Stage	Trade Price	Total Surplus	Renter Surplus	Landlord Surplus	Transaction Count
1	6.0003	20.0067	10.0034	10.0034	7.9412
2	5.9992	26.9799	13.4891	13.4908	6.3827
3	6.0160	27.4781	13.6664	13.8116	6.2438
4	6.0439	27.6564	13.6489	14.0075	6.1960
5	6.0629	28.0009	13.7352	14.2656	6.0976
6	6.1054	28.0878	13.5916	14.4962	6.0839
7	6.1225	28.2144	13.5709	14.6435	6.0645
8	6.1433	28.2908	13.5070	14.7838	6.0490
9	6.1603	28.3173	13.4497	14.8676	6.0393
10	6.1803	28.3637	13.3858	14.9779	6.0252
11	6.1931	28.3733	13.3306	15.0428	6.0207
12	6.2346	28.2609	13.0826	15.1783	6.0347
13	6.2325	28.3338	13.1651	15.1688	5.9986
14	6.2437	28.2930	13.1002	15.1928	5.9860
15	6.2421	28.3681	13.1444	15.2237	5.9750
16	6.2443	28.3858	13.1550	15.2308	5.9619
17	6.2566	28.3782	13.1022	15.2760	5.9695
18	6.2717	28.3792	13.0478	15.3315	5.9496
19	6.2510	28.5269	13.2239	15.3030	5.9088
20	6.2845	28.3882	13.0100	15.3783	5.9307
21	6.2921	28.4080	12.9953	15.4128	5.9417
22	6.2987	28.3697	12.9550	15.4147	5.9380
23	6.2864	28.4612	13.0546	15.4066	5.9190
24	6.3017	28.3965	12.9553	15.4412	5.9214
25	6.2957	28.4506	12.9970	15.4536	5.9263
26	6.3181	28.3420	12.8517	15.4903	5.9403
27	6.3056	28.4065	12.9402	15.4663	5.9312
28	6.2977	28.4692	13.0142	15.4550	5.9008
29	6.3041	28.4635	12.9651	15.4984	5.9100
30	6.3016	28.4779	12.9853	15.4927	5.9026
31	6.3003	28.4826	12.9989	15.4837	5.9055
32	6.2992	28.4820	13.0044	15.4776	5.9191
33	6.2875	28.5319	13.0869	15.4450	5.9048
34	6.2951	28.5309	13.0507	15.4802	5.8894
35	6.3116	28.4782	12.9403	15.5379	5.9041
36	6.3205	28.4698	12.9057	15.5641	5.9000
37	6.3016	28.5528	13.0391	15.5137	5.8900
38	6.3016	28.5444	13.0278	15.5166	5.8910
39	6.3093	28.5051	12.9852	15.5200	5.8975
40	6.2933	28.5681	13.0845	15.4836	5.8980
41	6.3028	28.5534	13.0333	15.5201	5.8930
42	6.3182	28.4943	12.9357	15.5587	5.9002
43	6.3260	28.4736	12.8936	15.5800	5.9054
44	6.3178	28.5220	12.9485	15.5735	5.8907
45	6.3171	28.5240	12.9580	15.5660	5.8963
46	6.3188	28.5188	12.9573	15.5616	5.8948
47	6.3085	28.5634	13.0192	15.5442	5.8918
48	6.3216	28.5044	12.9370	15.5674	5.9022
49	6.3263	28.5012	12.9117	15.5895	5.8975
50	6.3213	28.5172	12.9444	15.5728	5.8892

Table 9: Summary of 30 Two-Replacement Simulations

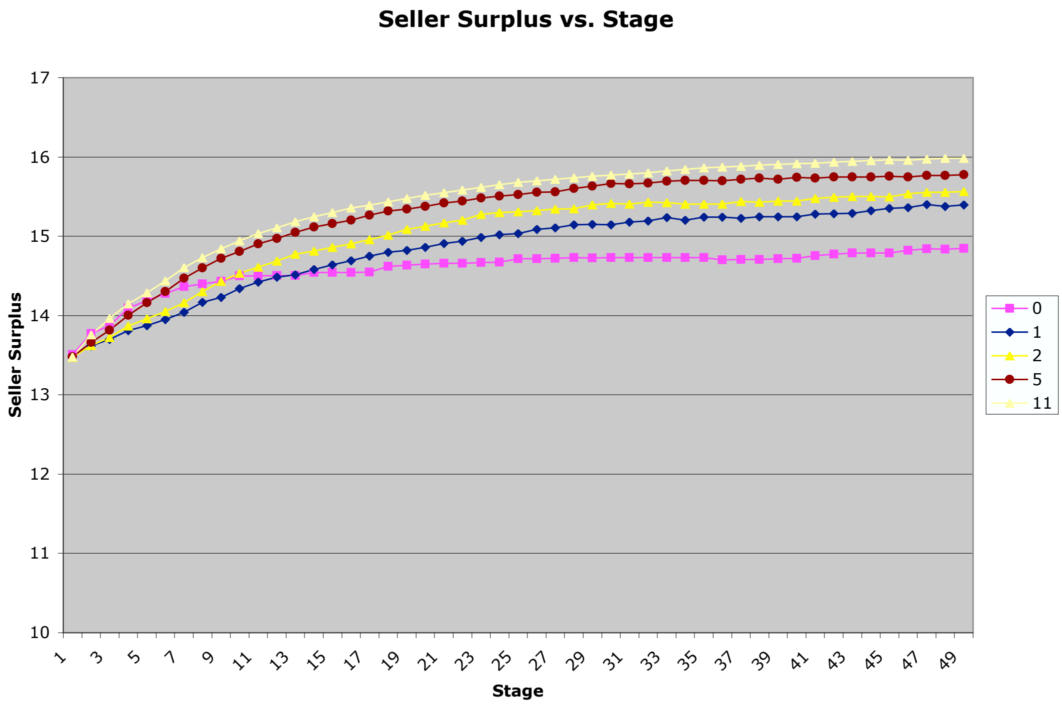

Figure 4:

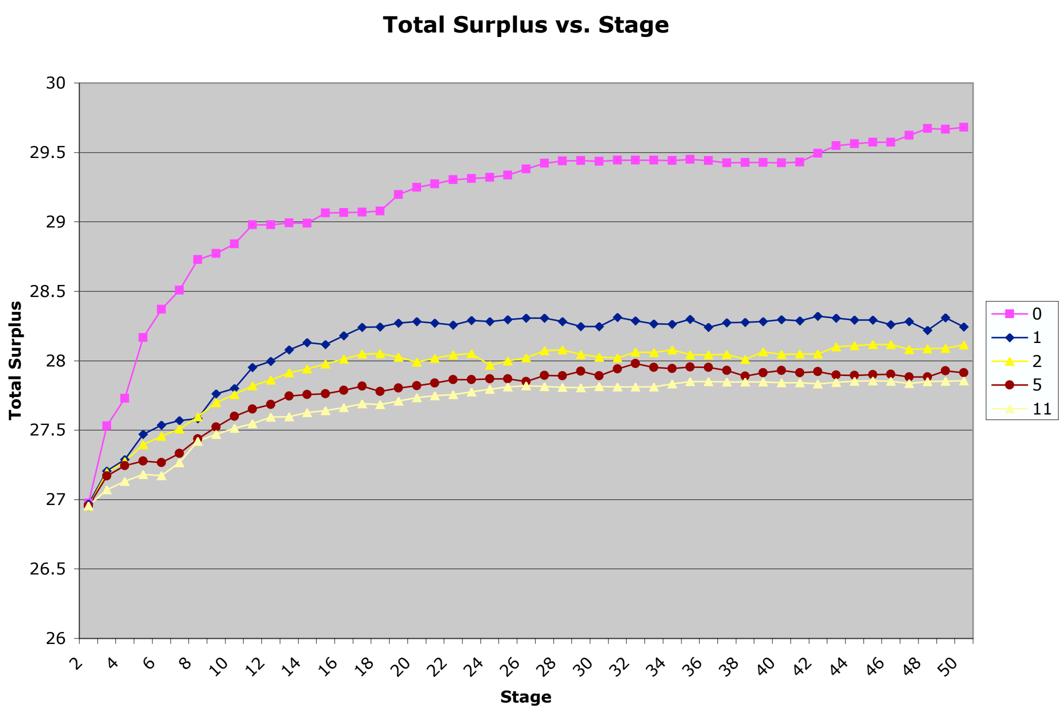

Figure 5:

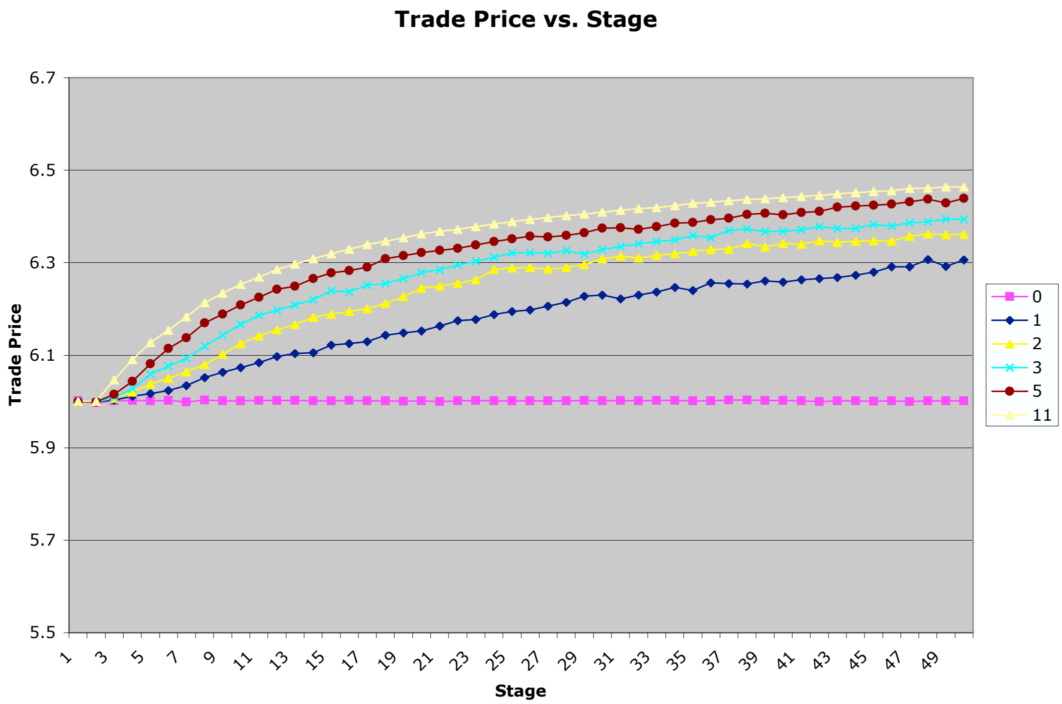

Figure 6:

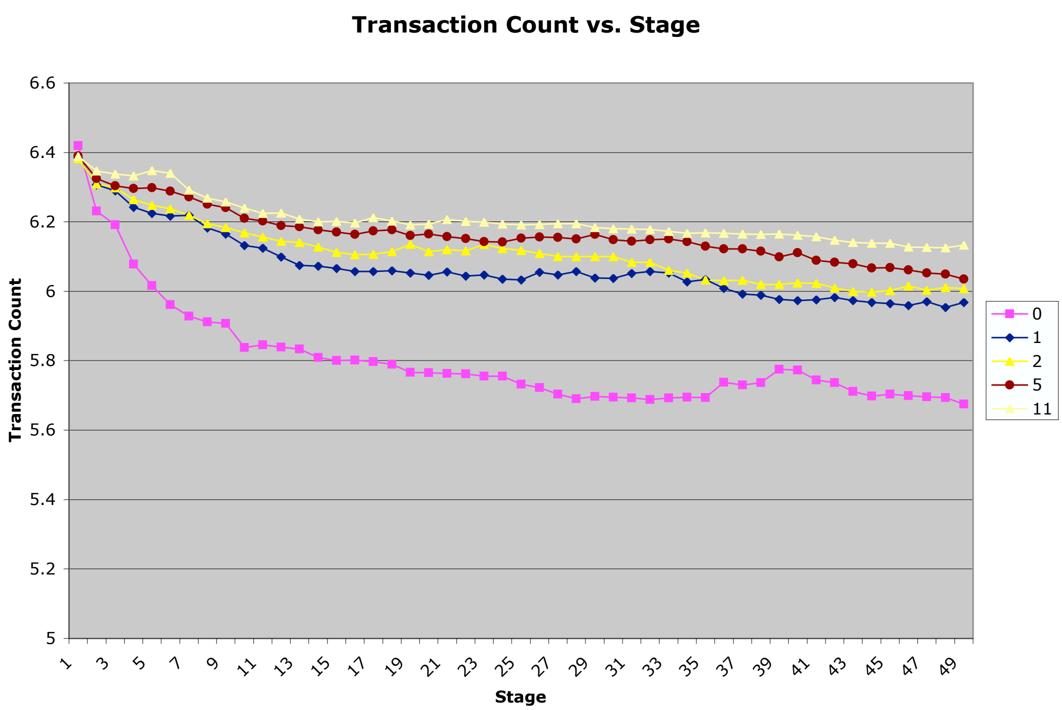

Figure 7:

replacement case, despite the fact that the amount of surplus that landlords receive is actually greater in later stages. Some level of replacement among the renters clearly creates confusion for the landlords. This is because at least one trade in ten is with a renter who has only bootstrap stage expectations.

Tables 10 and 11 show the expectations after the 50th stage for both renters and landlords in 2, 6, and 11-replacement markets. Much like with the overall surplus, we can see that a majority of the effect of replacement on the renters happens as soon as any replacement is begun.[13] Even with just two renters per stage replaced, expectations for those renters with high reservation prices drop by 25% or more. Near the margins, however, where renters only sometimes (or only rarely) get to trade, expectations do not change very much. Part of this is no doubt due to the fact that it is more difficult to take advantage of someone when a trade is only barely possible in the first place, but a great deal of it is due to the way in which the landlords. expectations change.

As we can see from Table 11, those landlords who always get to trade do indeed come to expect a small bit more surplus–Landlord 1 expects almost three% more at the beginning of the 11 replacement scenario compared to the 0 replacement one. Nevertheless, compared to differences in the distribution of surplus between renters and landlords, this is tiny. For landlords closer to the margin, however, this is not the case at all. At transaction count one, Landlord 6 expects more than 30% more surplus in the two renter-replacement case than the zero-replacement case, a figure that rises to over 40% in the 11 replacement case. At

[13] Again, this is unsurprising because in the no-replacement scenario, renters have 50 stages of experience, whereas in the one replacement scenario, they have an average of just over five stages of experience.

# Rep	RP	TC 1	TC 2	TC 3	TC 4	TC 5	TC 6	TC 7	TC 8	TC 9	TC 10	TC 11
0	1	0	0	0	0	0	0	0	0	0	0	0
0	2	0.002	0.002	0.002	0.002	0.002	0.003	0.011	0.042	0.053	0.051	0
0	3	0.007	0.006	0.006	0.005	0.005	0.005	0.016	0.081	0.103	0.153	0
0	4	0.030	0.027	0.024	0.020	0.016	0.011	0.033	0.158	0.190	0.210	0
0	5	0.154	0.140	0.117	0.091	0.058	0.035	0.171	0.249	0.286	0.277	0
0	6	0.533	0.487	0.429	0.359	0.245	0.242	0.395	0.361	0.372	0.339	0
0	7	1.209	1.154	1.083	0.991	0.832	0.835	0.735	0.487	0.457	0.366	0
0	8	2.034	1.988	1.928	1.851	1.725	1.542	1.311	0.641	0.538	0.364	0
0	9	2.765	2.728	2.679	2.597	2.507	2.354	1.717	0.842	0.608	0.372	0
0	10	3.353	3.302	3.233	3.125	2.941	2.693	1.801	1.021	0.718	0.382	0
0	11	3.934	3.845	3.747	3.597	3.356	2.896	1.803	1.117	0.825	0.391	0
2	1	0	0	0	0	0	0	0	0	0	0	0
2	2	0.019	0.018	0.017	0.016	0.014	0.015	0.024	0.046	0.050	0.062	0
2	3	0.061	0.055	0.049	0.042	0.035	0.032	0.057	0.097	0.105	0.154	0
2	4	0.161	0.143	0.129	0.109	0.087	0.073	0.126	0.162	0.189	0.215	0
2	5	0.244	0.226	0.208	0.182	0.146	0.112	0.290	0.248	0.285	0.278	0
2	6	0.699	0.669	0.635	0.588	0.506	0.546	0.433	0.360	0.376	0.323	0
2	7	1.206	1.177	1.135	1.076	1.011	0.908	0.627	0.484	0.457	0.334	0
2	8	1.629	1.589	1.540	1.489	1.432	1.270	0.855	0.633	0.533	0.341	0
2	9	1.847	1.792	1.729	1.661	1.561	1.362	1.049	0.810	0.609	0.380	0
2	10	2.255	2.187	2.105	2.008	1.853	1.571	1.205	0.959	0.716	0.392	0
2	11	2.588	2.511	2.392	2.238	2.004	1.672	1.338	1.097	0.808	0.407	0
6	1	0	0	0	0	0	0	0	0	0	0	0
6	2	0.031	0.029	0.027	0.024	0.022	0.022	0.030	0.047	0.052	0.061	0
6	3	0.095	0.086	0.077	0.066	0.053	0.047	0.070	0.098	0.104	0.143	0
6	4	0.186	0.169	0.150	0.126	0.098	0.080	0.130	0.160	0.186	0.250	0
6	5	0.364	0.335	0.307	0.272	0.220	0.181	0.295	0.251	0.284	0.279	0
6	6	0.760	0.726	0.694	0.645	0.569	0.574	0.427	0.362	0.380	0.321	0
6	7	1.150	1.119	1.070	1.002	0.946	0.800	0.604	0.487	0.458	0.340	0
6	8	1.485	1.434	1.371	1.314	1.246	1.068	0.816	0.635	0.529	0.385	0
6	9	1.745	1.681	1.621	1.549	1.451	1.270	1.011	0.816	0.609	0.406	0
6	10	2.013	1.947	1.865	1.764	1.642	1.423	1.174	0.960	0.712	0.416	0
6	11	2.329	2.250	2.114	1.984	1.817	1.573	1.318	1.091	0.811	0.471	0
11	1	0	0	0	0	0	0	0	0	0	0	0
11	2	0.039	0.036	0.033	0.029	0.026	0.026	0.034	0.048	0.055	0.066	0
11	3	0.114	0.104	0.092	0.079	0.063	0.055	0.068	0.098	0.106	0.139	0
11	4	0.229	0.205	0.182	0.154	0.119	0.098	0.130	0.161	0.190	0.225	0
11	5	0.417	0.385	0.352	0.312	0.256	0.218	0.289	0.250	0.283	0.289	0
11	6	0.786	0.751	0.712	0.663	0.594	0.571	0.423	0.360	0.370	0.340	0
11	7	1.135	1.101	1.051	0.987	0.928	0.775	0.602	0.487	0.458	0.365	0
11	8	1.434	1.369	1.317	1.255	1.175	0.993	0.807	0.637	0.530	0.351	0
11	9	1.684	1.633	1.568	1.500	1.399	1.221	1.000	0.817	0.616	0.363	0
11	10	1.954	1.879	1.807	1.712	1.587	1.386	1.161	0.962	0.725	0.388	0
11	11	2.246	2.171	2.033	1.913	1.760	1.541	1.318	1.091	0.809	0.443	0

Table 10: Renters' Expectations in Different Replacement Simulations

# Rep	RP	TC 1	TC 2	TC 3	TC 4	TC 5	TC 6	TC 7	TC 8	TC 9	TC 10	TC 11
0	1	3.934	3.846	3.748	3.598	3.357	2.896	1.802	1.119	0.811	0.446	0
0	2	3.349	3.298	3.230	3.123	2.938	2.690	1.795	1.022	0.721	0.350	0
0	3	2.763	2.726	2.677	2.596	2.505	2.353	1.715	0.840	0.611	0.387	0
0	4	2.034	1.986	1.926	1.849	1.723	1.540	1.310	0.638	0.533	0.367	0
0	5	1.207	1.152	1.081	0.990	0.831	0.834	0.734	0.486	0.459	0.354	0
0	6	0.532	0.487	0.428	0.359	0.245	0.241	0.392	0.361	0.371	0.314	0
0	7	0.153	0.139	0.116	0.090	0.057	0.034	0.170	0.250	0.285	0.293	0
0	8	0.029	0.026	0.023	0.019	0.015	0.010	0.032	0.158	0.189	0.226	0
0	9	0.006	0.006	0.005	0.005	0.005	0.005	0.015	0.081	0.103	0.154	0
0	10	0.001	0.001	0.001	0.001	0.001	0.002	0.010	0.041	0.051	0.049	0
0	11	0	0	0	0	0	0	0	0	0	0	0
2	1	4.003	3.903	3.767	3.609	3.390	2.983	2.156	1.162	0.823	0.474	0
2	2	3.446	3.361	3.257	3.135	2.965	2.687	2.121	1.126	0.720	0.347	0
2	3	2.818	2.742	2.649	2.529	2.362	2.099	1.774	0.895	0.612	0.391	0
2	4	2.069	1.986	1.881	1.755	1.585	1.316	1.290	0.642	0.536	0.349	0
2	5	1.270	1.179	1.064	0.915	0.709	0.555	0.570	0.487	0.455	0.355	0
2	6	0.699	0.626	0.534	0.416	0.257	0.166	0.292	0.357	0.374	0.309	0
2	7	0.315	0.272	0.220	0.155	0.079	0.042	0.114	0.248	0.280	0.278	0
2	8	0.096	0.084	0.067	0.047	0.028	0.017	0.036	0.157	0.187	0.221	0
2	9	0.011	0.011	0.010	0.009	0.008	0.009	0.020	0.081	0.105	0.154	0
2	10	0.002	0.001	0.001	0.002	0.002	0.003	0.012	0.040	0.051	0.050	0
2	11	0	0	0	0	0	0	0	0	0	0	0
6	1	4.041	3.935	3.792	3.623	3.406	3.018	2.278	1.219	0.806	0.481	0
6	2	3.493	3.400	3.274	3.148	2.978	2.678	2.190	1.214	0.719	0.403	0
6	3	2.862	2.778	2.674	2.549	2.380	2.127	1.721	0.921	0.613	0.358	0
6	4	2.071	1.973	1.855	1.713	1.532	1.241	1.169	0.643	0.531	0.392	0
6	5	1.295	1.192	1.065	0.908	0.692	0.502	0.491	0.485	0.455	0.356	0
6	6	0.744	0.665	0.566	0.434	0.264	0.154	0.245	0.358	0.372	0.341	0
6	7	0.362	0.313	0.250	0.171	0.081	0.044	0.115	0.246	0.281	0.295	0
6	8	0.128	0.110	0.086	0.057	0.030	0.019	0.039	0.157	0.188	0.233	0
6	9	0.015	0.015	0.014	0.012	0.009	0.010	0.023	0.080	0.105	0.167	0
6	10	0.002	0.001	0.002	0.002	0.002	0.004	0.013	0.041	0.051	0.051	0
6	11	0	0	0	0	0	0	0	0	0	0	0
11	1	4.051	3.942	3.807	3.637	3.414	3.026	2.309	1.242	0.805	0.441	0
11	2	3.507	3.409	3.285	3.157	2.985	2.665	2.201	1.257	0.732	0.364	0
11	3	2.892	2.808	2.710	2.591	2.432	2.192	1.709	0.925	0.609	0.384	0
11	4	2.070	1.964	1.845	1.703	1.516	1.213	1.100	0.642	0.529	0.362	0
11	5	1.302	1.196	1.067	0.909	0.689	0.480	0.441	0.487	0.461	0.344	0
11	6	0.758	0.678	0.571	0.440	0.266	0.148	0.227	0.357	0.374	0.331	0
11	7	0.377	0.325	0.255	0.173	0.080	0.046	0.117	0.247	0.280	0.289	0
11	8	0.139	0.120	0.091	0.059	0.030	0.020	0.041	0.158	0.190	0.231	0
11	9	0.018	0.018	0.017	0.013	0.009	0.010	0.024	0.083	0.103	0.138	0
11	10	0.002	0.002	0.002	0.002	0.002	0.004	0.014	0.041	0.052	0.058	0
11	11	0	0	0	0	0	0	0	0	0	0	0

Table 11: Landlords' Expectations in Different Replacement Simulations

low transaction counts, Landlords 7, 8, and even 9 see large percentage increases in their expectations. This makes a great deal of sense, because the replacement sometimes puts their naive high reservation price renters into the market.easy targets even for high reservation price landlords. An interesting effect of this is that it effectively increases the competition for the high-end renters, providing downward pressure on the expectations of those landlords with low reservation prices. This effect dwindles as the transaction count increases (the naive renters are likely to exit the market early) but it never entirely fades away. Even with the higher transaction count in the with replacement simulations, however, expectations beyond transaction count 7 tell us very little because the market very rarely gets to such a point after the first few stages.

Another interesting effect of replacement is that landlords with reservation price 3 and above begin to expect less in the with-replacement scenarios than they do in the no-replacement scenarios, while the expectations held by renters with reservation price less than 7 actually go up. Although unexpected and in some ways an artifact of the way in which the model works, this is a very interesting result. As landlords' expectations increase, they can find themselves in a situation late in a market where they simply will not trade, despite possible deals, because they want more than the renters are willing to give up. When this happens, neither side receives any surplus, and because the landlords were the ones unwilling to compromise, they get a zero averaged into their expectations, dropping these expectations radically and making the landlords take very weak bargaining positions late in the market. As the renters who tend to be left near the end of a market learn this, their expectations increase and they actually manage to do slightly better. While in some ways it seems strange for landlords to

RP	Renter0	Landlord0	Renter2	Landlord2	Renter6	Landlord6	Renter11	Landlord11
1	0.000	4.307	0.000	4.454	0.000	4.489	0.000	4.510
2	0.000	3.627	0.000	3.799	0.000	3.842	0.000	3.861
3	0.000	2.939	0.000	2.961	0.000	2.972	0.000	3.096
4	0.002	2.073	0.006	2.126	0.008	2.117	0.009	2.111
5	0.059	1.129	0.104	1.234	0.124	1.255	0.145	1.260
6	0.400	0.400	0.610	0.649	0.669	0.692	0.685	0.702
7	1.129	0.059	1.294	0.289	1.262	0.324	1.244	0.335
8	2.073	0.002	1.904	0.081	1.783	0.109	1.739	0.117
9	2.939	0.000	2.307	0.004	2.248	0.009	2.215	0.011
10	3.630	0.000	2.928	0.000	2.688	0.000	2.675	0.000
11	4.307	0.000	3.326	0.000	3.246	0.000	3.158	0.000

Table 12: Distribution of Surplus By Reservation Price

back off far enough to weaken their positions relative to the renters, this does make some sense if one considers that the penalty for not renting a unit at all is to obtain zero surplus.

Table 12 shows the actual distribution of surplus in the 0, 2, 6, and 11 replacement cases. Again, those landlords who always get to trade do not fare very much better than they did before, gaining about 5%. The big losers are clearly the high reservation price renters who now take many sub-optimal deals, though again the difference between a small amount of replacement and total replacement is only 5%. The extra-marginal or nearly extra-marginal landlords are the ones who gain the most, at least in percentage terms, because they had so few (or no) opportunities for trade before but can now expect to run into a relatively naive renter with at least some probability. Finally, while the very low reservation price renters gain not at all because they still never get to trade, the with reservation prices near six actually show some small gains, because, as I discussed above, several landlords are conditioned into taking weak bargaining stances as the transaction count increases.

Table 13 shows the gains by renters divided by their reservation price. If we take reser-

RP	2 Renters	6 Renters	11 Renters
1	0.000	0.000	0.000
2	0.000	0.000	0.000
3	0.000	0.000	0.000
4	0.001	0.002	0.002
5	0.009	0.013	0.017
6	0.035	0.045	0.048
7	0.024	0.019	0.016
8	-0.021	-0.036	-0.042
9	-0.070	-0.077	-0.080
10	-0.070	-0.094	-0.096
11	-0.089	-0.096	-0.104

Table 13: Difference in Renter's Surplus Divided by Reservation Price

vation price as a reasonable proxy for income, an assumption that seems reasonable because people tend to spend about one-third of their income on housing, then it gives us an idea of how replacement affects renters proportional to their income.

3.4 Implications of Replacement

As the Walrasian outcome has the same price and quantity as the competitive outcome, my comparison of markets with replacement to those without is essentially a comparison of the deviation of markets with asymmetric information from the competitive equilibrium.

The effect of this asymmetry in a market is, with one exception, strikingly similar to the effect of monopolistic behavior. As replacement increases, renter surplus falls, landlord surplus increases, the average trade price rises, and total surplus falls (as what the landlords gain does not quite make up for what the renters lose). In a classical monopoly, we would expect these effects to be more exaggerated than they are in my market, but increased replacement clearly takes us a long way in this direction. One big difference between my market and one dominated by a monopoly, however, is that the number of trades actually increases as the number of replacements goes up. With a greater number of replacements, it is increasingly likely that in any given stage, the high reservation price renters have been replaced; it therefore becomes easier for high reservation price landlords to edge in and make trades. If even one such trade happens early on in a market iteration as the market plays out, it becomes increasingly clear to the remaining landlords that such a trade must have occurred, and these landlords lower expectations at high transaction counts allow renters who might otherwise not have gotten to trade to do so, increasing the number of trades.

Even with a great deal of replacement, however, renters are still much better off than they would be facing a perfectly price-discriminating monopolist, in which case we would still expect a total of six trades, but with the entire surplus of 30 going to the landlord.

3.5 Extramarginal Trading

With reductions in the amount of replacement, the market moves closer to the Walrasian outcome, and the number of trades involving at least one extramarginal agent falls, eventually reaching about 10% when there is no turnover in the market. With some replacement, however, the extent of extramarginal trading rises substantially, just a single renter out of eleven every stage results in a 60% jump in the proportion of trades that are extramarginal to about 16 %. This result seems quite intuitive; after all, the less knowledge renters have, the more likely they are to take trades that would, with better information, appear suboptimal. Even so, the size of this increase is not as large as one might expect. With one replacement per stage, a renter gets, on average, about 10 stages of experience before being removed from

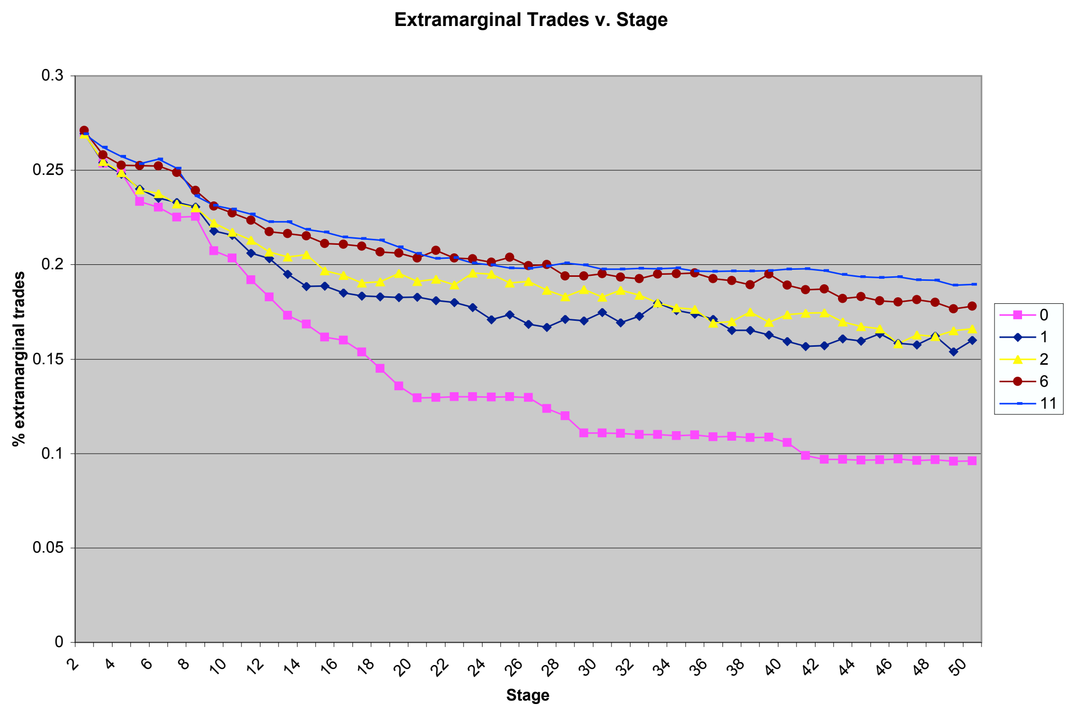

Figure 8: Percentage of Trades in a Stage that are Extramarginal

the market, so the average renter in a market has about 5 stages of experience. In a situation where all agents have only 5 stages of experience, however, such as the no-replacement market after 5 stages, the number of extramarginal trades is over 23%. Again, this is a direct result of the ability of high reservation price landlords to get lucky and edge into the market.

3.6 Decomposition

Even from all of this, it the question remains as to what leads to the changes: is it increased expectations on the part of landlords, decreased expectations on the part of renters, or some of both? A unique way of testing this is available because of the modeling approach I have adopted: renters used to one kind of market can be paired against landlords used to another kind of market and the results observed.

Type	Av. Price	Total Surplus	Renter Surplus	Landlord Surplus	Transaction Ct.
No-replacement	5.999	29.076	14.540	14.536	5.820
Replacement	6.240	28.844	13.454	15.390	5.891
Decomposition	5.990	29.313	14.659	14.653	5.843

Table 14: No-Replacement Renters and Replacement Landlords

Type	Av. Price	Total Surplus	Renter Surplus	Landlord Surplus	Transaction Ct.
No-replacement	5.999	29.076	14.540	14.536	5.820
Replacement	6.240	28.844	13.454	15.390	5.891
Decomposition	6.343	27.971	12.634	15.338	5.852

Table 15: Replacement Renters and No-Replacement Landlords

Table 14 shows the result of running such a simulation with the usual parameters and no replacement, but beginning with renters who already have 50 stages of experience in a market with no replacement matched with landlords who already have 50 stages of experience in a market with two out of eleven renters replaced each stage.[14]

As we can see, increased expectations alone provide no benefit to the landlords, as they experience no gain in surplus when compared to landlords in the 50th stage of a simple no-replacement scenario. The average number of trades is ever so slightly higher than it was before, due mostly to more trading by renter and Landlord 6.

Table 15 shows the opposite experiment from the one described above, using landlords with experience in a no-replacement market and renters who have experience in a market with two out of eleven renters replaced each stage.

[14] In this case, because the renters and landlords both already have experience, it is not necessary (nor desirable) to run a bootstrap stage. In both this and the next simulation, the expectations fed in were the result of averaging the final expectations for the appropriate agents from 30 different simulations run out to 50 stages. Without such averaging, the results of one of the decomposition simulations would have been heavily dependent on which renters happened to be replaced in the last stage of the first simulation.

Here, the effect is huge. Even without the increased expectations on the part of the landlords, the renters do almost as badly as they were doing against landlords who had learned to take advantage of them. This decomposition is interesting because it shows that in a Nash-bargaining environment like this, an agent basically has to agree to be taken advantage of in order to get less than he/she should expect. Even if all of the landlords increase their demands, hard bargaining by the renters can keep the distribution of surplus even.

3.7 Speed of Convergence

In grounding this kind of simulation in the real world, an issue that I have not yet considered is the speed at which agents learn compared to actual renters or landlords. After all, most of my simulations consist of 50 stages, each of which consists of 30,000 iterations: a number of 'experiences' much larger than any actual person ever actually accrues.

Because agents in my simulation can only learn through their own experience, that is, their knowledge can only come from market simulations, an additional number of experiences is needed simply to compensate for the fact that actual renters or landlords have additional sources of information available to them. For instance, an actual renter can learn a great deal about what is currently available simply by looking at the daily classifieds or by taking a couple of hours to view a selection of different units, and many renters inquire with friends about their experiences (as, perhaps, do landlords).

Another factor making the numbers involved less significant is the limit to how much can be learned about a market during any particular stage. Because agents update their expectations between stages, the market will not be exactly the same during any given stage as it was during the previous stage, and as a result, the optimal strategy for an agent actually changes as the market evolves. This kind of social learning limits the amount of information that any agent can collect in a given stage, regardless of how many iterations are run.

Nevertheless, in order to make sure that my results are not overly sensitive to changes in the number of iterations or the number of stages, I ran several simulations with a different number of iterations at each stage, as represented in Figure 9. Even between the simulations with 10,000 and 100,000 iterations a stage, there is virtually no difference in the rate at which agents learn (judged by the total amount of surplus gained at each stage in the market).

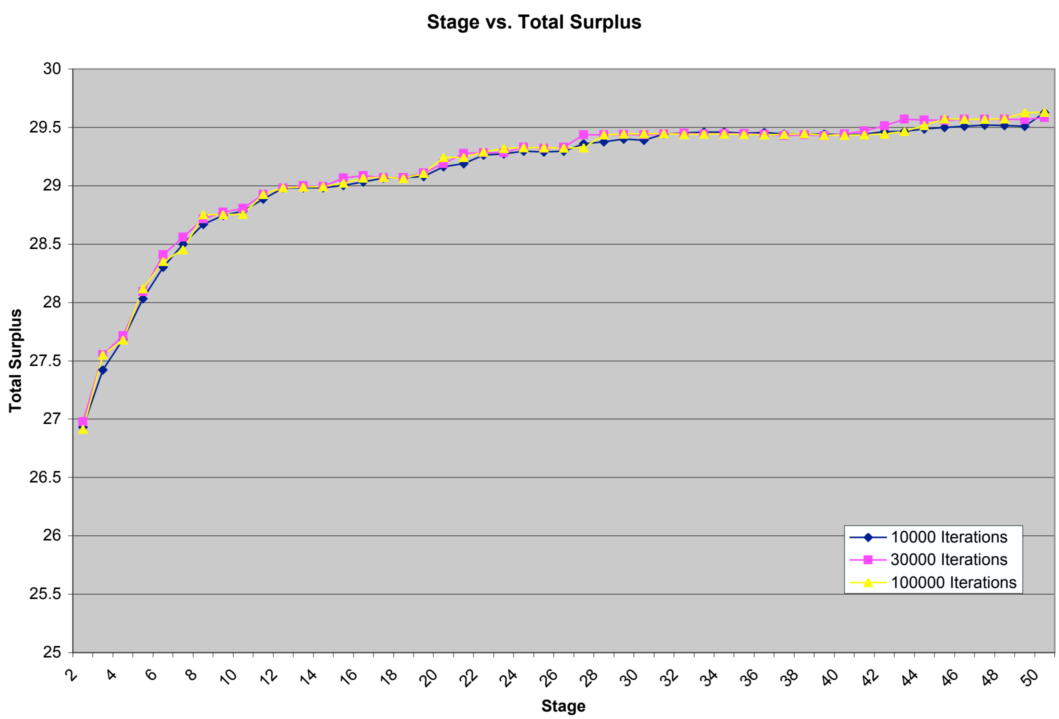

Figure 9: Total Surplus by Stage

Figure 10 shows the same graph with the addition of a simulation using just 10 iterations per stage. With so few experiences, however, agents learn very little and even a couple of unusual iterations are enough to cause large swings in the total surplus. As a result, while total surplus is still generally moving higher, the result of any particular stage is very unpredictable.

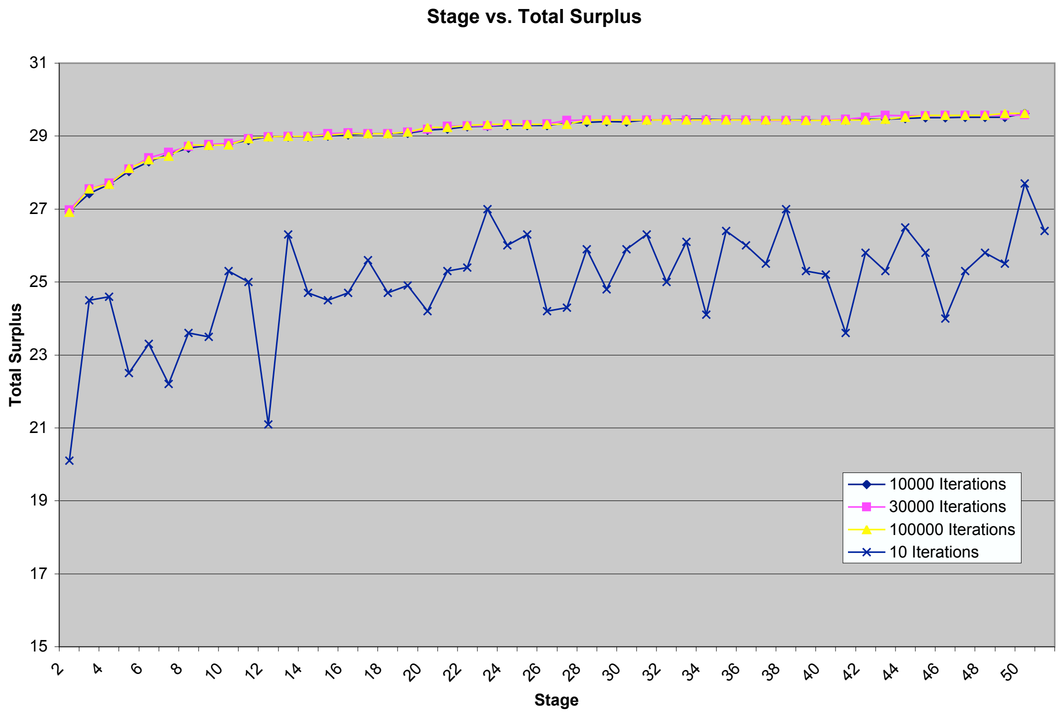

Figure 10: Total Surplus by Stage

From this we can relatively reasonably conclude that while agents are clearly learning as much as is possible from each stage, my particular choice for the number of iterations is not badly biasing the outcome. One might argue that, if we equate a stage with a year, this pace of learning seems unrealistically fast. On the other hand, 5 stages of learning without replacement is sufficient only to get within about 90% of the Walrasian outcome: a figure that seems a little low if anything.

If we indeed wish to somehow equate stages with years, then one might argue that the

50th stage is the wrong one to analyze because no actual market ever gets there.[15] Because of the fact that there are inevitably changes in the national economy and other factors exogenous to the market that nevertheless have important consequences for its outcome, we should perhaps instead analyze these markets after only the first few stages. Figure 11 shows the total surplus versus stage graph from earlier, but out to only five stages.

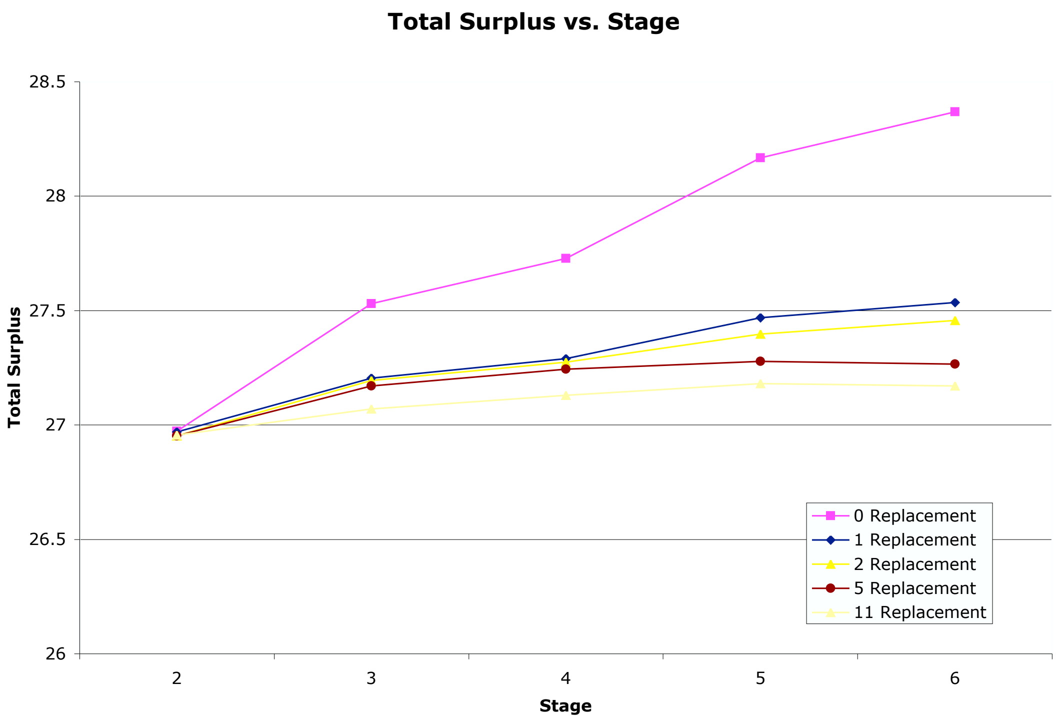

Figure 11: Total Surplus by Stage

While, after 5 stages, none of the markets are as close to the Walrasian outcome as they are after 50 stages, the fundamental analysis does not change very much. Even in the early stages, renters in a replacement simulation suffer significant losses compared to their counterparts in a no-replacement scenario, losses that are not made up for in gains by the

[15] It does not seem clear that a one-to-one correlation is the only possible choice here, but it serves to illustrate the point and the argument would work much the same with a two-to-one or a three-to-one correlation.

landlords. Total surplus in the 5th stage is 3% lower in the one-replacement case than in the no-replacement case, and is 5% lower in the eleven-replacement case. Even after the 5th stage, these markets show significant deadweight loss.

4 Conclusion

In this paper I have used agent-based modeling to explore the impact of asymmetric information upon housing markets. My most significant findings are twofold. First, in real-world terms, the impact of asymmetric information looks a great deal like the distortion caused by monopolistic influence: the greater the renter-replacement rate in a market, the greater the deadweight loss and the greater the imbalance between the amount of surplus received by renters and amount received by landlords. One important difference, however, is that the loss is almost exclusively suffered by the renters with the highest reservation prices; because of the increased extramarginal trading that occurs with renter-replacement, those renters with reservation prices just above or below the Walrasian market-clearing price actually manage to do better with replacement than they do without it.

Second, the magnitude of the difference between the perfectly competitive outcome and my results suggests strongly that Gale's [4] results are not robust to even relatively modest changes in his assumptions. Comparing his third model (which includes bounded rationality) to my model, for instance, the only differences are that my model contains a slightly more rational learning model and that it lacks a profit-maximizing auctioneer, changes that intuitively seem to make the model more realistic. While this does not at all detract from the analytical beauty of Gale's results, it does suggest that very few actual markets are likely to have outcomes near the Walrasian equilibrium.

There are a number of other very interesting results to come out of my analysis. First, that market turnover (and therefore the possibility of very naive high reservation-price renters) allows extramarginal landlords to edge back into the market, thereby limiting the gains of low reservation-price landlords. Second, that in a bargaining environment like the one I chose, participants can only be taken advantage of when they lack knowledge about the market: landlords with very aggressive bargaining positions fared no better than those who simply expected the Walrasian outcome when paired with renters who had good information.

There are a number of promising avenues for applications of agent-based modeling to the problem of how market equilibria are affected by asymmetric information. The most immediately interesting would be a slight change to my replacement mechanism so that only those renters who are actually managing to participate in the trading are replaced. As it is, in some stages of the replacement simulations, only very low reservation-price renters end up being replaced, effectively meaning that no replacement happens that stage.

This paper has used random replacement in order to study the effects of asymmetric information. However, the ability to replace agents between stages gives us the ability not only to do it randomly and without changing the overall balance of the market, and also to do it according to some feedback mechanism, changing the number or landlords or renters at each reservation price. Effectively, this allows us to create a market that evolves in the manner that real markets do.

In a more general sense, other possibilities for this kind of modeling are practically unlimited. For instance, incorporating different quality housing units is something that seems both entirely reasonable and also very useful. Social networks are another topic that is very hard to consider analytically but that could be tackled in a relatively straightforward manner by agent-based modeling.

Finally, while I have not pursued it, these results also have implications in labor markets where large companies very likely have the same kind of informational advantage over individual workers and where a large union might have this kind of advantage over many small companies.

My results suggest that even modest departures from the assumption of perfect information yield significant departures from equilibrium outcomes. The ability to not only quantify the extent of these deviations but also to examine the process by which they come to be and the distributional impacts on each agent is an indication of the power of this agent-based approach. In our attempt to understand how prices and market outcomes are generated, this represents a significant step forward not only because it shows just how important it is to relax the assumption of perfect information, but also because it demonstrates the power of this kind of modeling to help us do so.

Acknowledgments

I would like to thank professors Ralph Bradburd and Steven Sheppard for their invaluable help: they both made an incredible amount of time throughout the year to help me and I never could have come this far without them. I would also like to thank professor Al Roth

from Harvard for taking the time out of his trip to talk with me about this project and for his very insightful comments: along with many other things, Section 3.7 came almost directly from answering his questions.

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Appendix I: Glossary

Agent–A participant in a market: in this model specifically, a renter or a landlord.

Bootstrap Stage–A special kind of stage designed to give initial information to agents. Because totally naive agents would accept any deal presented to them, I randomly reject half of the proposed trades in this stage in order to give them information about what kind of surplus they can expect if they reject a feasible bargain at any point.

Expectation–An agent's belief about how much surplus he/she can expect to receive after rejecting a feasible bargain. A separate such belief is kept for each possible value of the transaction count.

Extramarginal–An agent that would not have gotten to trade in a Walrasian outcome. Alternatively, any trade involving such an agent.

Iteration–A single market simulation, continued until no more trades are possible, or until some predefined number of possible successive trades have been unsuccessful.

Stage–A series of market simulations, between which agents do not update their expectations. This typically consists of 30,000 iterations.

Transaction Count–In any given market, the number of deals that have already been made. As there is no other way to mark the passage of time in my model, agents keep a separate expectation for each possible value so as to be able to differentiate based on the progress of the market.

Appendix II: Simulation Summaries

Stage	Trade Price	Total Surplus	Buyer Surplus	Seller Surplus	Transaction Ct.
1	6.000	20.008	10.004	10.004	7.941
2	5.999	26.965	13.486	13.478	6.386
3	6.000	27.203	13.605	13.597	6.303
4	5.999	27.299	13.652	13.648	6.289
5	6.000	27.520	13.764	13.756	6.234
6	5.999	27.595	13.800	13.794	6.204
7	6.000	27.649	13.828	13.821	6.195
8	5.999	27.654	13.830	13.824	6.193
9	6.000	27.848	13.929	13.919	6.146
10	6.000	27.926	13.967	13.960	6.133
11	6.000	28.106	14.055	14.051	6.080
12	5.999	28.213	14.109	14.103	6.056
13	5.999	28.347	14.177	14.170	6.026
14	5.999	28.383	14.193	14.190	6.010
15	6.000	28.428	14.218	14.210	5.991
16	6.000	28.455	14.232	14.223	5.973
17	6.000	28.567	14.288	14.279	5.953
18	6.000	28.649	14.330	14.319	5.944
19	5.999	28.728	14.365	14.363	5.926
20	5.999	28.758	14.383	14.375	5.915
21	5.999	28.755	14.381	14.375	5.915
22	5.999	28.755	14.381	14.374	5.915
23	6.000	28.757	14.382	14.375	5.915
24	5.999	28.758	14.382	14.376	5.914
25	5.999	28.759	14.382	14.377	5.915
26	6.000	28.768	14.388	14.380	5.912
27	6.000	28.848	14.426	14.421	5.893
28	5.999	28.930	14.465	14.465	5.864
29	5.999	28.965	14.484	14.481	5.852
30	5.999	28.967	14.486	14.481	5.851
31	6.000	28.967	14.486	14.481	5.852
32	5.999	28.967	14.486	14.481	5.851
33	6.000	28.974	14.490	14.484	5.849
34	6.000	28.978	14.492	14.487	5.846
35	6.000	28.979	14.492	14.487	5.846
36	6.000	28.982	14.493	14.489	5.842
37	6.000	28.984	14.495	14.489	5.841
38	6.000	28.986	14.495	14.491	5.838
39	6.000	28.985	14.495	14.490	5.837
40	6.000	28.995	14.501	14.494	5.835
41	6.000	29.035	14.519	14.516	5.821
42	6.000	29.064	14.535	14.529	5.810
43	6.000	29.065	14.535	14.530	5.816
44	5.999	29.065	14.536	14.530	5.819
45	6.000	29.063	14.533	14.529	5.823
46	6.000	29.063	14.535	14.529	5.824
47	6.000	29.063	14.534	14.529	5.824
48	6.000	29.064	14.535	14.529	5.824
49	6.000	29.066	14.535	14.531	5.824
50	5.999	29.076	14.540	14.536	5.820

Table 1: Zero-Replacement

Stage	Trade Price	Total Surplus	Buyer Surplus	Seller Surplus	Transaction Ct.
1	5.999	20.009	10.004	10.004	7.940
2	5.999	26.969	13.484	13.485	6.384
3	6.003	27.205	13.588	13.616	6.307
4	6.011	27.289	13.588	13.700	6.289
5	6.017	27.469	13.660	13.809	6.242
6	6.023	27.536	13.663	13.872	6.224
7	6.034	27.570	13.620	13.949	6.216
8	6.051	27.583	13.543	14.039	6.218
9	6.063	27.760	13.591	14.169	6.182
10	6.073	27.801	13.573	14.228	6.164
11	6.083	27.952	13.612	14.339	6.131
12	6.096	27.995	13.574	14.421	6.124
13	6.103	28.077	13.593	14.483	6.098
14	6.105	28.131	13.617	14.513	6.075
15	6.122	28.117	13.534	14.583	6.072
16	6.124	28.179	13.540	14.639	6.066
17	6.129	28.239	13.548	14.690	6.057
18	6.143	28.243	13.492	14.750	6.056
19	6.148	28.270	13.471	14.799	6.059
20	6.152	28.283	13.462	14.820	6.053
21	6.163	28.271	13.409	14.862	6.045
22	6.174	28.258	13.347	14.910	6.056
23	6.177	28.289	13.349	14.939	6.043
24	6.188	28.281	13.295	14.986	6.047
25	6.194	28.297	13.277	15.019	6.035
26	6.197	28.306	13.270	15.035	6.033
27	6.206	28.307	13.219	15.088	6.055
28	6.214	28.282	13.177	15.105	6.046
29	6.227	28.245	13.099	15.146	6.057
30	6.230	28.246	13.096	15.149	6.038
31	6.221	28.312	13.166	15.146	6.037
32	6.230	28.288	13.111	15.177	6.051
33	6.237	28.264	13.071	15.192	6.057
34	6.246	28.262	13.024	15.237	6.052
35	6.239	28.298	13.096	15.201	6.027
36	6.256	28.239	12.997	15.242	6.034
37	6.255	28.273	13.033	15.240	6.008
38	6.254	28.275	13.049	15.226	5.992
39	6.260	28.281	13.032	15.248	5.988
40	6.258	28.297	13.052	15.244	5.976
41	6.263	28.287	13.040	15.246	5.973
42	6.265	28.320	13.042	15.278	5.975
43	6.268	28.306	13.019	15.286	5.981
44	6.273	28.293	13.005	15.288	5.973
45	6.279	28.293	12.971	15.322	5.967
46	6.291	28.259	12.908	15.350	5.964
47	6.290	28.281	12.918	15.362	5.959
48	6.306	28.219	12.819	15.400	5.970
49	6.291	28.308	12.931	15.376	5.953
50	6.305	28.244	12.847	15.397	5.968

Table 2: One-Replacement

Stage	Trade Price	Total Surplus	Buyer Surplus	Seller Surplus	Transaction Ct.
1	5.999	20.010	10.005	10.005	7.940
2	5.999	26.952	13.478	13.474	6.381
3	6.006	27.193	13.569	13.624	6.309
4	6.020	27.275	13.551	13.723	6.297
5	6.037	27.396	13.530	13.866	6.263
6	6.049	27.456	13.498	13.958	6.248
7	6.064	27.509	13.457	14.051	6.237
8	6.079	27.596	13.435	14.160	6.219
9	6.102	27.699	13.396	14.302	6.194
10	6.125	27.757	13.325	14.431	6.183
11	6.141	27.820	13.284	14.535	6.168
12	6.154	27.860	13.250	14.610	6.156
13	6.166	27.912	13.224	14.687	6.144
14	6.182	27.940	13.166	14.773	6.140
15	6.189	27.976	13.165	14.811	6.127
16	6.194	28.012	13.151	14.861	6.112
17	6.200	28.048	13.146	14.902	6.105
18	6.211	28.049	13.091	14.958	6.106
19	6.227	28.025	13.009	15.015	6.114
20	6.244	27.991	12.903	15.087	6.135
21	6.250	28.018	12.893	15.125	6.114
22	6.255	28.041	12.873	15.168	6.119
23	6.262	28.050	12.846	15.204	6.116
24	6.285	27.969	12.692	15.276	6.134
25	6.288	27.998	12.698	15.300	6.123
26	6.289	28.020	12.709	15.310	6.117
27	6.286	28.073	12.751	15.322	6.108
28	6.289	28.078	12.736	15.341	6.099
29	6.296	28.045	12.698	15.347	6.100
30	6.308	28.026	12.632	15.394	6.100
31	6.314	28.020	12.607	15.413	6.100
32	6.309	28.062	12.659	15.403	6.083
33	6.316	28.058	12.630	15.427	6.083
34	6.319	28.078	12.655	15.422	6.060
35	6.324	28.042	12.636	15.405	6.052
36	6.328	28.043	12.635	15.407	6.033
37	6.329	28.045	12.638	15.406	6.030
38	6.341	28.009	12.571	15.438	6.031
39	6.334	28.065	12.636	15.429	6.019
40	6.341	28.044	12.599	15.444	6.020
41	6.339	28.048	12.605	15.443	6.024
42	6.347	28.048	12.573	15.475	6.022
43	6.344	28.100	12.606	15.494	6.010
44	6.346	28.108	12.607	15.500	5.999
45	6.347	28.116	12.613	15.502	5.997
46	6.346	28.116	12.618	15.497	6.001
47	6.357	28.081	12.548	15.533	6.015
48	6.361	28.087	12.532	15.555	6.003
49	6.360	28.090	12.536	15.553	6.011
50	6.361	28.113	12.548	15.565	6.007

Table 3: Two-Replacement

Stage	Trade Price	Total Surplus	Buyer Surplus	Seller Surplus	Transaction Ct.
1	6.000	20.005	10.002	10.002	7.941
2	5.999	26.957	13.478	13.478	6.382
3	6.007	27.188	13.557	13.630	6.310
4	6.027	27.265	13.510	13.755	6.300
5	6.058	27.343	13.417	13.926	6.274
6	6.076	27.401	13.351	14.050	6.269
7	6.093	27.449	13.288	14.160	6.259
8	6.119	27.525	13.224	14.300	6.240
9	6.143	27.612	13.172	14.439	6.223
10	6.166	27.652	13.075	14.576	6.214
11	6.186	27.712	13.035	14.676	6.193
12	6.197	27.789	13.035	14.753	6.172
13	6.208	27.828	12.994	14.833	6.166
14	6.219	27.859	12.965	14.894	6.157
15	6.238	27.839	12.868	14.970	6.152
16	6.236	27.909	12.909	14.999	6.144
17	6.251	27.912	12.849	15.063	6.144
18	6.255	27.920	12.826	15.094	6.146
19	6.265	27.935	12.779	15.156	6.138
20	6.278	27.911	12.718	15.193	6.141
21	6.284	27.938	12.692	15.245	6.137
22	6.295	27.942	12.651	15.290	6.132
23	6.303	27.919	12.606	15.313	6.143
24	6.311	27.938	12.571	15.367	6.140
25	6.321	27.937	12.536	15.400	6.132
26	6.321	27.954	12.541	15.413	6.128
27	6.320	27.974	12.551	15.423	6.127
28	6.326	27.963	12.519	15.444	6.128
29	6.318	28.033	12.584	15.448	6.114
30	6.328	28.036	12.542	15.494	6.108
31	6.334	28.023	12.520	15.502	6.104
32	6.340	27.999	12.504	15.494	6.096
33	6.345	27.999	12.486	15.513	6.087
34	6.349	28.014	12.491	15.522	6.080
35	6.358	28.003	12.452	15.551	6.072
36	6.353	28.021	12.498	15.522	6.068
37	6.369	27.957	12.412	15.545	6.065
38	6.373	27.956	12.409	15.547	6.058
39	6.367	27.987	12.453	15.533	6.053
40	6.368	27.990	12.450	15.539	6.049
41	6.371	27.980	12.435	15.544	6.054
42	6.377	27.970	12.401	15.568	6.048
43	6.374	28.014	12.434	15.579	6.036
44	6.373	28.042	12.447	15.595	6.037
45	6.382	28.002	12.395	15.607	6.044
46	6.379	28.043	12.443	15.600	6.031
47	6.386	28.022	12.396	15.626	6.041
48	6.388	28.027	12.385	15.642	6.037
49	6.394	28.008	12.354	15.653	6.041
50	6.393	28.030	12.364	15.665	6.043

Table 4: Three-Replacement

Stage	Trade Price	Total Surplus	Buyer Surplus	Seller Surplus	Transaction Ct.
1	5.999	20.008	10.004	10.004	7.941
2	5.999	26.949	13.473	13.475	6.378
3	6.013	27.171	13.519	13.652	6.317
4	6.035	27.255	13.471	13.784	6.296
5	6.070	27.316	13.346	13.970	6.285
6	6.097	27.329	13.219	14.109	6.284
7	6.124	27.349	13.093	14.255	6.281
8	6.143	27.509	13.112	14.396	6.254
9	6.167	27.575	13.039	14.536	6.238
10	6.197	27.603	12.933	14.669	6.224
11	6.208	27.685	12.930	14.755	6.199
12	6.223	27.747	12.888	14.859	6.189
13	6.238	27.744	12.820	14.924	6.187
14	6.251	27.772	12.776	14.996	6.177
15	6.262	27.799	12.735	15.064	6.163
16	6.274	27.807	12.678	15.129	6.165
17	6.284	27.815	12.634	15.181	6.167
18	6.286	27.840	12.630	15.210	6.166
19	6.287	27.870	12.647	15.222	6.163
20	6.296	27.878	12.611	15.267	6.154
21	6.305	27.904	12.589	15.315	6.154
22	6.316	27.895	12.529	15.365	6.149
23	6.326	27.870	12.481	15.389	6.153
24	6.329	27.903	12.473	15.430	6.141
25	6.333	27.911	12.465	15.445	6.138
26	6.338	27.921	12.442	15.479	6.140
27	6.342	27.913	12.412	15.501	6.148
28	6.340	27.951	12.439	15.512	6.140
29	6.354	27.903	12.360	15.542	6.140
30	6.359	27.924	12.353	15.570	6.133
31	6.360	27.937	12.348	15.588	6.139
32	6.358	27.989	12.378	15.610	6.139
33	6.371	27.952	12.312	15.639	6.136
34	6.375	27.950	12.321	15.629	6.125
35	6.376	27.973	12.343	15.629	6.115
36	6.383	27.926	12.307	15.619	6.104
37	6.387	27.936	12.295	15.641	6.103
38	6.390	27.924	12.282	15.641	6.094
39	6.407	27.852	12.170	15.682	6.107
40	6.405	27.874	12.215	15.659	6.085
41	6.405	27.902	12.231	15.671	6.073
42	6.410	27.872	12.192	15.679	6.080
43	6.404	27.919	12.246	15.672	6.072
44	6.406	27.922	12.238	15.684	6.064
45	6.406	27.946	12.263	15.683	6.053
46	6.403	27.981	12.290	15.690	6.046
47	6.413	27.938	12.236	15.701	6.053
48	6.420	27.919	12.192	15.726	6.059
49	6.423	27.909	12.178	15.731	6.059
50	6.418	27.952	12.218	15.734	6.047

Table 5: Four-Replacement

Stage	Trade Price	Total Surplus	Buyer Surplus	Seller Surplus	Transaction Ct.
1	5.999	20.008	10.004	10.004	7.940
2	5.998	26.952	13.476	13.476	6.390
3	6.015	27.171	13.509	13.661	6.325
4	6.043	27.244	13.429	13.815	6.303
5	6.081	27.278	13.273	14.005	6.296
6	6.114	27.266	13.106	14.160	6.298
7	6.137	27.332	13.029	14.302	6.288
8	6.169	27.436	12.967	14.469	6.271
9	6.188	27.522	12.919	14.603	6.251
10	6.208	27.598	12.876	14.721	6.240
11	6.225	27.652	12.841	14.810	6.210
12	6.242	27.685	12.783	14.902	6.202
13	6.249	27.745	12.776	14.969	6.188
14	6.265	27.755	12.704	15.050	6.186
15	6.278	27.763	12.647	15.116	6.176
16	6.283	27.786	12.625	15.161	6.170
17	6.290	27.816	12.612	15.204	6.163
18	6.308	27.777	12.513	15.263	6.173
19	6.314	27.803	12.486	15.316	6.176
20	6.321	27.820	12.475	15.345	6.160
21	6.326	27.840	12.462	15.378	6.164
22	6.330	27.864	12.446	15.418	6.156
23	6.338	27.862	12.418	15.444	6.151
24	6.345	27.868	12.385	15.483	6.143
25	6.351	27.870	12.362	15.508	6.141
26	6.357	27.849	12.324	15.524	6.152
27	6.356	27.893	12.338	15.555	6.155
28	6.359	27.890	12.330	15.559	6.155
29	6.364	27.925	12.321	15.603	6.150
30	6.374	27.891	12.260	15.630	6.164
31	6.375	27.941	12.274	15.666	6.148
32	6.371	27.978	12.316	15.662	6.144
33	6.377	27.951	12.282	15.669	6.148
34	6.385	27.944	12.248	15.696	6.150
35	6.386	27.953	12.251	15.702	6.143
36	6.392	27.952	12.247	15.705	6.129
37	6.396	27.929	12.230	15.698	6.121
38	6.403	27.888	12.170	15.718	6.122
39	6.406	27.912	12.178	15.733	6.115
40	6.403	27.928	12.212	15.716	6.098
41	6.408	27.913	12.170	15.743	6.110
42	6.410	27.921	12.186	15.735	6.088
43	6.420	27.897	12.148	15.749	6.084
44	6.422	27.895	12.146	15.748	6.079
45	6.423	27.900	12.152	15.747	6.066
46	6.426	27.900	12.144	15.756	6.067
47	6.431	27.882	12.135	15.747	6.061
48	6.437	27.883	12.114	15.768	6.052
49	6.429	27.925	12.160	15.765	6.049
50	6.439	27.912	12.137	15.774	6.034

Table 6: Five-Replacement

Stage	Trade Price	Total Surplus	Buyer Surplus	Seller Surplus	Transaction Ct.
1	6.000	20.005	10.002	10.002	7.940
2	6.000	26.954	13.475	13.478	6.379
3	6.018	27.158	13.496	13.662	6.324
4	6.053	27.214	13.368	13.845	6.307
5	6.097	27.236	13.183	14.052	6.298
6	6.127	27.237	13.041	14.196	6.315
7	6.153	27.292	12.951	14.340	6.298
8	6.181	27.418	12.914	14.503	6.275
9	6.202	27.494	12.862	14.632	6.256
10	6.225	27.559	12.794	14.764	6.239
11	6.238	27.620	12.770	14.850	6.220
12	6.255	27.667	12.719	14.947	6.206
13	6.267	27.685	12.656	15.029	6.206
14	6.279	27.716	12.618	15.097	6.189
15	6.289	27.751	12.590	15.160	6.177
16	6.297	27.755	12.550	15.205	6.182
17	6.311	27.754	12.488	15.265	6.180
18	6.314	27.770	12.479	15.291	6.183
19	6.321	27.806	12.461	15.345	6.176
20	6.333	27.786	12.408	15.377	6.172
21	6.343	27.789	12.363	15.425	6.170
22	6.346	27.814	12.357	15.456	6.157
23	6.351	27.813	12.335	15.478	6.158
24	6.359	27.810	12.294	15.515	6.163
25	6.362	27.831	12.280	15.550	6.162
26	6.366	27.855	12.280	15.574	6.161
27	6.370	27.864	12.264	15.599	6.171
28	6.368	27.908	12.285	15.622	6.174
29	6.375	27.893	12.251	15.641	6.177
30	6.383	27.886	12.206	15.679	6.176
31	6.384	27.917	12.223	15.694	6.166
32	6.387	27.940	12.211	15.729	6.162
33	6.392	27.912	12.184	15.727	6.167
34	6.396	27.896	12.162	15.733	6.168
35	6.396	27.924	12.171	15.753	6.161
36	6.400	27.928	12.157	15.770	6.159
37	6.402	27.925	12.142	15.782	6.156
38	6.413	27.891	12.110	15.781	6.140
39	6.420	27.859	12.073	15.786	6.132
40	6.418	27.877	12.089	15.788	6.135
41	6.419	27.911	12.107	15.804	6.113
42	6.425	27.862	12.082	15.780	6.111
43	6.428	27.868	12.085	15.783	6.100
44	6.440	27.800	12.026	15.773	6.093
45	6.437	27.850	12.063	15.787	6.082
46	6.442	27.851	12.048	15.803	6.072
47	6.445	27.828	12.042	15.786	6.076
48	6.445	27.837	12.029	15.807	6.077
49	6.447	27.844	12.052	15.792	6.059
50	6.448	27.851	12.032	15.819	6.063

Table 7: Six-Replacement

Stage	Trade Price	Total Surplus	Buyer Surplus	Seller Surplus	Transaction Ct.
1	6.000	20.001	10.000	10.000	7.941
2	6.000	26.952	13.476	13.476	6.380
3	6.020	27.154	13.485	13.669	6.325
4	6.069	27.160	13.267	13.893	6.318
5	6.106	27.195	13.125	14.069	6.310
6	6.134	27.217	13.002	14.215	6.318
7	6.162	27.285	12.900	14.385	6.319
8	6.183	27.455	12.931	14.523	6.276
9	6.206	27.537	12.883	14.654	6.250
10	6.230	27.548	12.774	14.773	6.248
11	6.238	27.634	12.777	14.857	6.222
12	6.258	27.676	12.715	14.961	6.206
13	6.275	27.666	12.626	15.039	6.205
14	6.285	27.696	12.584	15.112	6.196
15	6.300	27.705	12.514	15.190	6.189
16	6.307	27.713	12.479	15.233	6.188
17	6.314	27.742	12.462	15.280	6.185
18	6.325	27.737	12.410	15.326	6.187
19	6.332	27.766	12.387	15.378	6.184
20	6.339	27.775	12.381	15.394	6.171
21	6.346	27.793	12.359	15.434	6.161
22	6.353	27.779	12.313	15.466	6.170
23	6.357	27.811	12.305	15.505	6.160
24	6.367	27.790	12.246	15.544	6.175
25	6.366	27.834	12.268	15.565	6.173
26	6.372	27.833	12.249	15.584	6.171
27	6.376	27.832	12.224	15.608	6.182
28	6.378	27.873	12.232	15.641	6.176
29	6.382	27.860	12.204	15.656	6.186
30	6.386	27.882	12.201	15.681	6.173
31	6.393	27.871	12.161	15.710	6.174
32	6.397	27.854	12.135	15.718	6.177
33	6.405	27.857	12.095	15.761	6.176
34	6.402	27.899	12.145	15.753	6.160
35	6.406	27.906	12.130	15.775	6.162
36	6.402	27.962	12.164	15.797	6.147
37	6.417	27.860	12.052	15.808	6.163
38	6.414	27.925	12.094	15.830	6.147
39	6.420	27.898	12.073	15.825	6.143
40	6.421	27.888	12.064	15.824	6.138
41	6.427	27.872	12.040	15.832	6.139
42	6.426	27.895	12.058	15.836	6.128
43	6.433	27.870	12.028	15.841	6.124
44	6.436	27.873	12.012	15.861	6.125
45	6.440	27.865	12.025	15.839	6.102
46	6.443	27.863	12.011	15.852	6.097
47	6.448	27.833	12.004	15.828	6.081
48	6.455	27.830	11.959	15.870	6.088
49	6.450	27.881	12.007	15.874	6.079
50	6.458	27.837	11.978	15.859	6.068

Table 8: Seven-Replacement

Stage	Trade Price	Total Surplus	Buyer Surplus	Seller Surplus	Transaction Ct.
1	5.999	20.007	10.003	10.003	7.941
2	6.000	26.944	13.472	13.471	6.379
3	6.024	27.140	13.455	13.684	6.331
4	6.075	27.156	13.238	13.917	6.320
5	6.109	27.228	13.125	14.102	6.315
6	6.141	27.197	12.959	14.237	6.334
7	6.168	27.277	12.879	14.398	6.324
8	6.193	27.424	12.881	14.543	6.281
9	6.219	27.498	12.813	14.684	6.263
10	6.237	27.547	12.758	14.789	6.246
11	6.251	27.590	12.701	14.889	6.226
12	6.267	27.626	12.648	14.977	6.219
13	6.280	27.639	12.580	15.059	6.215
14	6.294	27.649	12.526	15.122	6.204
15	6.305	27.688	12.483	15.204	6.194
16	6.310	27.718	12.469	15.248	6.186
17	6.326	27.720	12.407	15.312	6.189
18	6.333	27.707	12.361	15.346	6.193
19	6.338	27.749	12.358	15.391	6.188
20	6.347	27.755	12.332	15.423	6.176
21	6.351	27.803	12.337	15.465	6.162
22	6.361	27.760	12.269	15.491	6.178
23	6.364	27.806	12.276	15.530	6.166
24	6.370	27.799	12.242	15.557	6.182
25	6.375	27.811	12.207	15.603	6.192
26	6.380	27.824	12.200	15.624	6.187
27	6.378	27.858	12.218	15.639	6.193
28	6.387	27.841	12.169	15.671	6.189
29	6.392	27.844	12.154	15.689	6.187
30	6.395	27.858	12.145	15.712	6.181
31	6.400	27.846	12.122	15.723	6.176
32	6.404	27.840	12.093	15.746	6.183
33	6.405	27.859	12.104	15.754	6.175
34	6.409	27.872	12.089	15.783	6.174
35	6.413	27.884	12.085	15.798	6.166
36	6.417	27.865	12.056	15.808	6.169
37	6.419	27.890	12.049	15.841	6.166
38	6.420	27.886	12.049	15.836	6.157
39	6.425	27.888	12.029	15.859	6.154
40	6.425	27.896	12.034	15.862	6.153
41	6.429	27.888	12.016	15.871	6.146
42	6.431	27.897	12.018	15.879	6.141
43	6.440	27.853	11.974	15.879	6.134
44	6.444	27.847	11.976	15.870	6.116
45	6.443	27.871	11.977	15.893	6.122
46	6.449	27.850	11.968	15.882	6.106
47	6.454	27.842	11.954	15.887	6.097
48	6.457	27.820	11.939	15.880	6.095
49	6.456	27.871	11.970	15.901	6.082
50	6.460	27.828	11.929	15.899	6.092

Table 9: Eight-Replacement

Stage	Trade Price	Total Surplus	Buyer Surplus	Seller Surplus	Transaction Ct.
1	6.000	20.008	10.004	10.004	7.941
2	6.000	26.945	13.472	13.472	6.383
3	6.026	27.137	13.445	13.692	6.333
4	6.081	27.151	13.210	13.940	6.330
5	6.117	27.210	13.084	14.125	6.327
6	6.146	27.195	12.937	14.258	6.340
7	6.169	27.295	12.887	14.408	6.324
8	6.201	27.426	12.861	14.565	6.291
9	6.223	27.506	12.807	14.699	6.260
10	6.242	27.544	12.732	14.811	6.251
11	6.259	27.562	12.649	14.913	6.233
12	6.274	27.616	12.610	15.005	6.220
13	6.285	27.634	12.557	15.076	6.217
14	6.301	27.632	12.482	15.150	6.209
15	6.309	27.680	12.459	15.221	6.194
16	6.319	27.687	12.419	15.267	6.192
17	6.332	27.691	12.362	15.328	6.197
18	6.335	27.710	12.349	15.360	6.200
19	6.344	27.729	12.328	15.401	6.193
20	6.352	27.745	12.303	15.442	6.185
21	6.359	27.753	12.275	15.478	6.179
22	6.363	27.770	12.264	15.505	6.178
23	6.366	27.785	12.252	15.533	6.183
24	6.374	27.793	12.215	15.578	6.186
25	6.377	27.808	12.200	15.607	6.198
26	6.384	27.817	12.175	15.642	6.193
27	6.387	27.823	12.166	15.657	6.192
28	6.392	27.817	12.143	15.673	6.193
29	6.398	27.809	12.112	15.697	6.193
30	6.400	27.822	12.103	15.718	6.189
31	6.402	27.853	12.113	15.740	6.176
32	6.408	27.835	12.075	15.760	6.178
33	6.411	27.828	12.058	15.770	6.178
34	6.414	27.842	12.058	15.784	6.173
35	6.419	27.855	12.044	15.810	6.171
36	6.419	27.870	12.048	15.822	6.169
37	6.424	27.860	12.019	15.840	6.166
38	6.424	27.873	12.026	15.846	6.162
39	6.428	27.872	12.006	15.866	6.158
40	6.431	27.876	11.991	15.885	6.158
41	6.433	27.882	11.986	15.895	6.154
42	6.437	27.876	11.968	15.908	6.152
43	6.440	27.882	11.962	15.919	6.146
44	6.443	27.878	11.956	15.922	6.140
45	6.447	27.853	11.937	15.916	6.134
46	6.449	27.861	11.935	15.925	6.129
47	6.450	27.870	11.931	15.938	6.130
48	6.450	27.893	11.938	15.954	6.125
49	6.455	27.889	11.937	15.951	6.108
50	6.457	27.884	11.927	15.957	6.110

Table 10: Nine-Replacement

Stage	Trade Price	Total Surplus	Buyer Surplus	Seller Surplus	Transaction Ct.
1	6.000	20.004	10.002	10.002	7.941
2	5.999	26.949	13.472	13.476	6.386
3	6.031	27.121	13.411	13.710	6.340
4	6.085	27.121	13.178	13.943	6.327
5	6.123	27.184	13.050	14.134	6.320
6	6.150	27.172	12.901	14.271	6.335
7	6.177	27.275	12.847	14.427	6.331
8	6.206	27.418	12.837	14.580	6.288
9	6.229	27.487	12.766	14.720	6.267
10	6.248	27.524	12.696	14.827	6.257
11	6.262	27.553	12.636	14.917	6.236
12	6.281	27.598	12.575	15.023	6.225
13	6.294	27.604	12.504	15.100	6.223
14	6.304	27.641	12.469	15.171	6.204
15	6.314	27.651	12.425	15.225	6.195
16	6.325	27.667	12.377	15.289	6.195
17	6.333	27.694	12.363	15.331	6.197
18	6.342	27.695	12.317	15.377	6.206
19	6.347	27.731	12.313	15.418	6.196
20	6.358	27.731	12.271	15.459	6.190
21	6.362	27.767	12.268	15.498	6.180
22	6.368	27.763	12.229	15.534	6.196
23	6.372	27.787	12.222	15.565	6.197
24	6.381	27.785	12.182	15.602	6.195
25	6.382	27.814	12.189	15.625	6.194
26	6.389	27.816	12.155	15.660	6.192
27	6.393	27.823	12.137	15.685	6.191
28	6.399	27.808	12.105	15.702	6.193
29	6.400	27.825	12.103	15.722	6.191
30	6.405	27.816	12.081	15.734	6.185
31	6.407	27.824	12.080	15.743	6.179
32	6.412	27.810	12.045	15.765	6.182
33	6.414	27.818	12.038	15.779	6.180
34	6.420	27.829	12.021	15.807	6.177
35	6.423	27.843	12.020	15.822	6.172
36	6.425	27.860	12.018	15.842	6.166
37	6.428	27.856	12.002	15.853	6.165
38	6.431	27.853	11.986	15.866	6.164
39	6.434	27.845	11.969	15.876	6.164
40	6.436	27.857	11.965	15.891	6.160
41	6.440	27.835	11.942	15.893	6.160
42	6.441	27.841	11.943	15.898	6.154
43	6.445	27.850	11.938	15.912	6.142
44	6.448	27.844	11.925	15.918	6.141
45	6.449	27.850	11.917	15.933	6.141
46	6.451	27.849	11.911	15.937	6.137
47	6.455	27.852	11.908	15.943	6.125
48	6.457	27.860	11.897	15.963	6.129
49	6.461	27.852	11.893	15.958	6.115
50	6.464	27.860	11.892	15.968	6.110

Table 11: Ten-Replacement

Stage	Trade Price	Total Surplus	Buyer Surplus	Seller Surplus	Transaction Ct.
1	6.000	20.007	10.003	10.003	7.941
2	6.000	26.956	13.476	13.480	6.388
3	6.046	27.070	13.313	13.756	6.347
4	6.090	27.130	13.163	13.967	6.338
5	6.127	27.181	13.032	14.148	6.332
6	6.154	27.171	12.886	14.285	6.347
7	6.182	27.267	12.827	14.439	6.340
8	6.213	27.420	12.816	14.604	6.291
9	6.234	27.470	12.738	14.731	6.269
10	6.253	27.513	12.671	14.842	6.257
11	6.268	27.547	12.604	14.943	6.240
12	6.285	27.595	12.559	15.036	6.224
13	6.296	27.596	12.489	15.106	6.225
14	6.308	27.627	12.443	15.183	6.207
15	6.319	27.641	12.397	15.244	6.199
16	6.329	27.663	12.362	15.300	6.201
17	6.339	27.691	12.335	15.355	6.196
18	6.346	27.685	12.296	15.388	6.212
19	6.354	27.710	12.276	15.433	6.202
20	6.362	27.733	12.257	15.476	6.191
21	6.367	27.748	12.233	15.515	6.194
22	6.372	27.757	12.206	15.550	6.206
23	6.378	27.774	12.190	15.584	6.201
24	6.383	27.794	12.176	15.618	6.199
25	6.389	27.811	12.158	15.652	6.194
26	6.393	27.819	12.138	15.681	6.191
27	6.397	27.815	12.114	15.700	6.192
28	6.401	27.808	12.090	15.717	6.194
29	6.405	27.805	12.069	15.735	6.194
30	6.409	27.814	12.059	15.755	6.184
31	6.413	27.811	12.042	15.769	6.180
32	6.416	27.812	12.025	15.786	6.179
33	6.419	27.812	12.010	15.802	6.178
34	6.423	27.832	12.006	15.825	6.173
35	6.428	27.848	12.003	15.845	6.167
36	6.430	27.849	11.989	15.860	6.168
37	6.433	27.847	11.976	15.871	6.166
38	6.436	27.846	11.962	15.884	6.164
39	6.438	27.846	11.949	15.897	6.163
40	6.440	27.840	11.935	15.904	6.164
41	6.442	27.841	11.925	15.916	6.162
42	6.445	27.834	11.913	15.920	6.157
43	6.449	27.845	11.911	15.934	6.147
44	6.451	27.853	11.906	15.946	6.140
45	6.454	27.854	11.898	15.956	6.138
46	6.455	27.853	11.888	15.964	6.138
47	6.460	27.835	11.877	15.957	6.128
48	6.461	27.849	11.873	15.975	6.126
49	6.463	27.853	11.871	15.981	6.125
50	6.464	27.856	11.872	15.983	6.132

Table 12: Eleven-Replacement

Appendix III: Extramarginal Trades

Stage	Rep0	Rep1	Rep2	Rep3	Rep4	Rep5	Rep6	Rep7	Rep8	Rep9	Rep10	Rep11
1	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539	0.539
2	0.270	0.269	0.269	0.269	0.270	0.271	0.270	0.269	0.270	0.270	0.270	0.269
3	0.253	0.254	0.254	0.255	0.256	0.258	0.258	0.258	0.259	0.260	0.260	0.262
4	0.248	0.248	0.248	0.250	0.252	0.248	0.252	0.255	0.256	0.256	0.256	0.257
5	0.233	0.240	0.239	0.243	0.245	0.262	0.252	0.255	0.253	0.250	0.253	0.253
6	0.230	0.235	0.237	0.240	0.244	0.247	0.252	0.250	0.254	0.253	0.254	0.255
7	0.225	0.232	0.232	0.238	0.243	0.243	0.248	0.246	0.251	0.248	0.250	0.251
8	0.225	0.230	0.230	0.232	0.234	0.240	0.239	0.236	0.235	0.236	0.237	0.236
9	0.207	0.217	0.222	0.229	0.230	0.242	0.230	0.227	0.232	0.231	0.230	0.231
10	0.203	0.215	0.217	0.227	0.224	0.230	0.227	0.226	0.226	0.228	0.229	0.229
11	0.191	0.206	0.213	0.221	0.219	0.206	0.223	0.222	0.223	0.225	0.226	0.226
12	0.182	0.203	0.206	0.210	0.218	0.208	0.217	0.214	0.222	0.222	0.223	0.222
13	0.173	0.194	0.204	0.209	0.218	0.209	0.216	0.217	0.219	0.221	0.221	0.222
14	0.168	0.188	0.205	0.205	0.213	0.208	0.215	0.216	0.217	0.219	0.218	0.218
15	0.161	0.188	0.196	0.204	0.206	0.205	0.211	0.213	0.215	0.215	0.216	0.217
16	0.160	0.185	0.194	0.196	0.207	0.215	0.210	0.211	0.211	0.214	0.214	0.214
17	0.153	0.183	0.190	0.200	0.204	0.206	0.209	0.214	0.212	0.214	0.212	0.213
18	0.145	0.183	0.191	0.201	0.202	0.200	0.206	0.210	0.212	0.212	0.212	0.212
19	0.135	0.182	0.195	0.198	0.206	0.216	0.206	0.210	0.209	0.211	0.209	0.209
20	0.129	0.182	0.191	0.198	0.203	0.211	0.203	0.208	0.205	0.207	0.206	0.205
21	0.129	0.181	0.192	0.199	0.201	0.193	0.207	0.201	0.203	0.204	0.203	0.203
22	0.130	0.179	0.189	0.195	0.200	0.203	0.203	0.208	0.202	0.203	0.204	0.203
23	0.130	0.177	0.195	0.196	0.198	0.195	0.203	0.201	0.203	0.203	0.201	0.200
24	0.129	0.170	0.195	0.201	0.197	0.194	0.201	0.201	0.200	0.200	0.199	0.199
25	0.130	0.173	0.190	0.200	0.199	0.194	0.203	0.200	0.204	0.201	0.199	0.198
26	0.129	0.168	0.191	0.191	0.189	0.199	0.199	0.200	0.199	0.201	0.198	0.197
27	0.123	0.166	0.186	0.194	0.190	0.202	0.200	0.203	0.199	0.198	0.200	0.199
28	0.119	0.171	0.183	0.195	0.190	0.214	0.193	0.198	0.199	0.199	0.199	0.200
29	0.110	0.170	0.186	0.187	0.194	0.209	0.194	0.196	0.197	0.200	0.199	0.199
30	0.110	0.174	0.182	0.185	0.199	0.185	0.195	0.194	0.196	0.198	0.197	0.197
31	0.110	0.169	0.186	0.182	0.195	0.187	0.193	0.197	0.198	0.196	0.196	0.197
32	0.110	0.172	0.183	0.179	0.186	0.181	0.192	0.198	0.198	0.196	0.198	0.197
33	0.109	0.179	0.179	0.184	0.188	0.181	0.194	0.197	0.196	0.196	0.198	0.197
34	0.109	0.175	0.177	0.182	0.183	0.192	0.195	0.192	0.195	0.195	0.199	0.198
35	0.109	0.174	0.176	0.179	0.185	0.184	0.195	0.193	0.195	0.195	0.197	0.196
36	0.108	0.171	0.169	0.175	0.183	0.178	0.192	0.192	0.193	0.197	0.196	0.196
37	0.109	0.165	0.169	0.176	0.182	0.198	0.191	0.196	0.194	0.194	0.196	0.196
38	0.108	0.165	0.174	0.175	0.178	0.184	0.189	0.193	0.194	0.194	0.196	0.196
39	0.108	0.162	0.169	0.174	0.182	0.185	0.194	0.193	0.194	0.195	0.196	0.196
40	0.105	0.159	0.173	0.170	0.179	0.181	0.189	0.189	0.192	0.195	0.196	0.197
41	0.098	0.156	0.174	0.169	0.180	0.189	0.186	0.191	0.192	0.194	0.198	0.197
42	0.096	0.157	0.174	0.171	0.178	0.189	0.187	0.189	0.191	0.195	0.196	0.196
43	0.096	0.160	0.169	0.167	0.177	0.180	0.181	0.187	0.191	0.193	0.195	0.194
44	0.096	0.159	0.167	0.164	0.177	0.176	0.183	0.187	0.189	0.191	0.194	0.193
45	0.096	0.163	0.166	0.170	0.169	0.153	0.180	0.185	0.192	0.192	0.195	0.193
46	0.097	0.158	0.158	0.164	0.166	0.159	0.180	0.180	0.187	0.192	0.194	0.193
47	0.096	0.157	0.162	0.170	0.169	0.173	0.181	0.180	0.188	0.190	0.191	0.192
48	0.096	0.162	0.161	0.169	0.171	0.171	0.180	0.185	0.182	0.189	0.191	0.191
49	0.095	0.153	0.165	0.170	0.175	0.169	0.176	0.179	0.183	0.186	0.191	0.189
50	0.096	0.159	0.166	0.167	0.169	0.177	0.178	0.176	0.185	0.186	0.189	0.189

Table 13: Extramarginal Trades By Replacement # and Stage