ENDOGENOUS NETWORK FORMATION AND THE EVOLUTION OF PREFERENCES

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ABSTRACT

We develop analytical and computational models to study the conditions for the stability of a population consisting of agents with heterogeneous preferences. The analytical models that utilize an indirect evolutionary approach show that the ability to detect others' types is critical for the evolution of reciprocal preferences. The computational models of this paper incorporate agents' memories and endogenously built social networks into the evolutionary dynamics. The simulations based on the computational models show that the strength of the social network is a critical factor for the success of nonselfish preferences. A fully heterogeneous population consisting of egoists, reciprocators, and altruists can be stable for a range of parameter conditions.

INTRODUCTION

There are many social situations that require cooperation among multiple individuals to achieve a common goal, but that benefit those who free-ride on others' efforts. If there are any biological or social selection mechanisms that favor those who gain by cheating, we would probably see societies mainly inhabited by selfish individuals. In both Economics and Political Science, the modern fashion of thinking has been to assume that everyone is selfish and to devise rules and institutions that still deliver tolerable social outcomes. However, self-reflection, careful observation of other human beings, and experimental evidence from the social sciences, indicate that our societies are not composed entirely of selfish, but rather of diverse types which can be schematically divided into three categories: those who are selfish, those who are fair, and those who are altruistic. Where does this heterogeneity come from? How do the non-selfish motivations survive?

While the question has been widely addressed by evolutionary game theorists (Axelrod and Hamilton, 1981; Axelrod, 1981, Bendor and Swistak, 1997, for example), their models often underestimate the cognitive capability of human agents and the flexibility of human behavior. Instead, in this paper, we extend the indirect evolutionary approach (Güth and Yaari, 1992; Güth and Kliemt, 1998; Güth, Kliemt, and Peleg, 2000, Ahn, 2001) to combine the features of standard non-cooperative game theory and standard evolutionary game theory. The agents in the indirect evolutionary models are rational in the sense that they have utility functions instead of

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fixed behavioral rules, and they make choices based on the utility maximization principle. In terms of motivations, agents are heterogeneous; some agents have utility functions that do not map the material payoffs into utilities in a linear manner. In other words, they care about the social consequences of their actions.

In an indirect evolutionary process, selection operates on material payoffs. Thus, the types that are more successful materially increase over time. We will use a mathematical formula of evolution that is consistent with both biological and cultural interpretations of the evolutionary process.

A variety of social interactions have the material payoff structure of the Prisoner's Dilemma in which individuals face the temptation to defect, cheat, or free-ride. But if all the individuals behave selfishly, everyone is worse off as a result than he/she would be in at least one other outcome in which some of the individuals cooperate. Suppose a public good provision problem involving two individuals shown in Figure 1. Each of the two individuals has an initial endowment of p ($0 < p < 0.5$) and makes a binary choice of whether to contribute (Cooperation) or not (Defection) for the provision of a public good. Contribution costs 1 to the contributor but returns $1-p$ to each of the two individuals. No matter what the other does, an individual is always better off when he or she does not contribute. Therefore, if both individuals are selfish neither will contribute. Then each receives a material payoff of p, which is smaller than $1-p$ that each of them obtains if they both contribute.

		Individual *j*	
		Cooperation	Defection
Individual *i*	Cooperation	$1-p, 1-p$	0, 1
	Defection	1, 0	p, p

FIGURE 1 Two-Person Public Good Provision Problem.

MODELING MOTIVATIONAL HETEROGENEITY AMONG RATIONAL AGENTS

Experimental evidence strongly supports the hypothesis that there is a significant proportion of individuals whose preference ordering over the four possible outcomes of the action situation is not linear to the amount of material payoff he or she obtains in each of the four outcomes shown in Figure 1 (Ahn, Ostrom, Walker, forthcoming; Ahn et al., 2001; Cho and Choi, 2000; Clark and Sefton, 1999; Hayashi et al., 1999). In particular, most of the nonselfish individuals seem to have an assurance preference with the following ordering over the four outcomes: u(C,C) > u(D,C) > u(D,D) > u(C,D). Those who have an assurance type preference are *reciprocators* in the sense that they cooperate if their partners cooperate but defect if their partners defect.

A relatively small proportion of individuals show a preference ordering of u(C,C) > u(D,C) > u(C,D) > u(D,D), which implies unconditional cooperation. They will be called *altruists*. In most of the experiments, about a half of individuals reveal a self-interested preference ordering of u(D,C) > u(C,C) > u(D,D) > u(C,D). They will be called *egoists*. Other possible types are empirically and analytically insignificant. Figure 2 is the utility payoff matrix that models the three prefernce types.

		Individual j	
		Cooperation	Defection
Individual i	Cooperation	$1-p$	$0+\alpha_i$
	Defection	$1-\beta_i$	p

$$0 \le \beta_i \le \alpha_i \le 1$$

FIGURE 2 Utility Payoff Matrix for Individual *i*.

In Figure 2, if α_i is greater than p, individual i prefers to cooperate when j also cooperates. If β_i is larger than p, individual i prefers to cooperate even when j defects. The restriction $\beta_i \le \alpha_i$ implies that no individual has a preference ordering by which he prefers to cooperate when the other defects, but prefers to defect when the other cooperates. Substantively, p can be interpreted as the relative magnitude of the material temptation to defect.

Notice that one's preference type (egoist, reciprocator, or altruist) is a joint function of one's *generic* type (α_i, β_i) and the material payoff parameter (p). For a given generic type, one is more likely to be an egoist when p is large. A population can be characterized by a probability distribution function $F(\alpha_i, \beta_i)$. For a given F, the proportion of behavioral reciprocators (δ) and that of (γ) are again functions of p.

INDIRECT EVOLUTION

In an indirect evolutionary process, agents interact in the action situation shown in Figure 1 based on their preferences shown in Figure 2. Evolution selects those who are more successful materially. The question is whether or not any non-selfish types can survive and, if so, which type would. In this section, we analyze the indirect evolutionary process under four different conditions. [1]. In the next section, we extend the simulation model to incorporate repeated interactions, memory, and social networks.

In this section, we assume that, at each evolutionary stage, each player plays the game only once with another player who is randomly drawn from a population of infinite size. There are four possible ways under which such a game can be played. The key factors are (1) whether the game is played under complete or incomplete information regarding players' types and (2) whether the game is played simultaneously or sequentially. From these two dichotomies result four different evolutionary conditions: simultaneous, complete information (SC), simultaneous, incomplete information (SI), sequential, complete information (QC), and sequential, incomplete information (QI).

The expected material payoff for an egoist (reciprocator, altruist) at time *t* will be denoted as $\pi_{e,t}$ ($\pi_{r,t}$ $\pi_{a,t}$). At each evolutionary stage, a reasonable solution concept of non-cooperative game theory is used to derive players' behavior[2]. When there exist multiple equilibria, we assume that a cooperative equilibrium, defined as one in which at least some players cooperate, is played.

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[1] For complete analyses of all the four conditions, see Ahn (2001).

[2] A Nash equilibirum for SC, a Bayesian equilibirum for SI, a subgame perfect equilibirum for QC, and a sequential equilibirum for QI.

To simplify mathematical analysis, we further assume that for a given behavioral type, the values of α and β are the same across players. This allows us to study the population dynamic of $F^{t}(\alpha,\beta) \rightarrow F^{t+1}(\alpha,\beta)$ in a simpler dynamic of $(\delta,\gamma)^{t} \rightarrow (\delta,\gamma)^{t+1}$ in which the proportion of reciprocators (δ) and that of altruists (γ) at time *t+1* are calculated by following time-independent replicator functions:

$$\delta_{t+1} = \frac{\delta_t \pi_{r,t}}{(1-\delta_t-\gamma_t)\pi_{e,t} + \delta_t \pi_{r,t} + \gamma_t \pi_{a,t}} \tag{2}$$

$$\gamma_{t+1} = \frac{\gamma_t \pi_{a,t}}{(1-\delta_t-\gamma_t)\pi_{e,t} + \delta_t \pi_{r,t} + \gamma_t \pi_{a,t}}. \tag{3}$$

It suffices to say that a type's relative proportion in an evolutionary stage is exactly proportional to its relative proportion in the immediately preceding stage times its relative success measured in terms of the obtained material payoffs. This evolutionary dynamic may occur either genetically or culturally. The entire evolutionary process, regardless of the original population condition $(\delta,\gamma)^{0}$ can be approximated by a continuous-time dynamic of which the vector derivatives are

$$[\dot{\delta} = \delta_{t+\Delta t} - \delta_t, \quad \dot{\gamma} = \gamma_{t+\Delta t} - \gamma_t] \quad . \tag{4}$$

Figure 3 illustrates the evolutionary dynamics of all of the four possible single-play environments. In this section only the evolutionary dynamics under the QI condition will be discussed in some more detail. We think that the sequential, incomplete condition is more common than other conditions in the real world. That is, agents in the real world can hardly be sure of the exact motivational types of others.

Under the QI condition, a player plays the game as a first mover with probability 0.5 and as a second mover with the same probability. Since agents are rational, their behavior is not deterministic. The utility maximizing behavior is a function of the material incentive, *p*, and the composition of types within a population $(\delta,\gamma)^{t}$. The lower-right panel of Figure 3 shows three different equilibrium zones under the QI condition as functions of p, δ (Rec), and γ (Alt).

In all the three zones, the behavior of second movers is a direct function of their types: egoists always defect, reciprocators copy the choice of the first mover, and altruists always cooperate. The difference across types is in their behavior as first movers. In zone I, all three types of first mover cooperate. Since the proportion of reciprocators is relatively large compared to that of altruists and egoists combined, it pays for the first-mover egoists to cooperate. In zone II, egoist first movers defect, but reciprocator first movers still cooperate. In zone III, there are too few reciprocators and altruists, thus there does not exist an equilibrium in which reciprocator first movers cooperate. In all three zones, egoists obtain the highest average payoff. Altruists decrease in all three zones. The relative proportion of reciprocators decreases in zone I and zone II, but increases in zone III. Therefore, stable states exist along the horizontal axis with the proportion of reciprocators smaller than $\frac{p}{1-p}$. However, if there is a constant stream of invasions by altruistic mutants, the stable states are absorbed into the attractor in which only egoists exist.[3]

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[3] Random mutations at these stable states imply that the relative proportion of reciprocators to egoists, $\frac{\delta}{1-\gamma-\delta}$, remains the same after an arbitrarily small invasion by altruists. The disturbance caused by this kind of mutuation

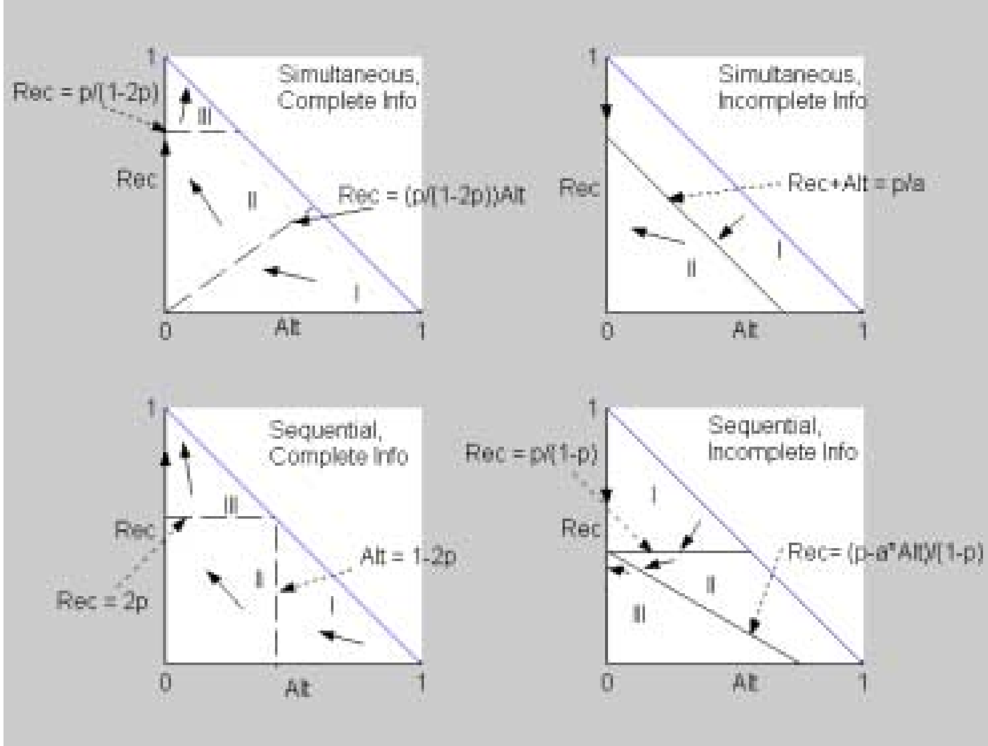

Figure 3 Information, Play Sequence, and Evolution of Preferences. Under the SC and QC conditions, stable states exist along the line $\delta+\gamma=1$. However, $\delta=1$ is the only attractor. In both incomplete information conditions, $\delta=\gamma=0$ is the only attractor. Rec = δ; Alt = γ; a = α (see Figure 2).

RATIONAL AGENTS IN SMALL WORLD NETWORKS

The indirect evolutionary models of the previous section used one-shot social dilemma games as the conditions of interaction. In this section, we study the effects of endogenous network formation on the evolution of preferences using computer simulations. We select the sequential incomplete information as the baseline interaction. Sequential interactions are by far more common than simultaneous interactions. However, when agents are assumed to be completely ignorant of another agent's type in a given interaction the evolutionary result is one in which egoists prevail. In real social settings, incomplete information is not necessarily the most common information condition. Any increase in agent's informational capacity favors the evolution of nonselfish preference types. For example, a model in Ahn (2001) assumes that, in an indirect evolutionary dynamics, agents know the type of another agent with probability *q*. This mixed-information assumption results in stable mixed population for a wide range of parameter values. [4]

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reaches back to another stable state which is only slightly removed from the original stable state. However, since the existence of altruists favors egoists, the recovered stable state inhabits a larger proportion of egoists than that of the original stable state. After a long sequence of disturbances and recoveries, the population converges to the attactor in which only egoists remain.

[4] Güth, Kliemt, and Peleg (2000) provide elegant analyses of the models in which agents develop informational capabilities.

In the simulations of this paper, we select the incomplete information condition, which favors egoists, as the baseline condition of interaction to highlight the effect of endogenous networks. For the network building to be possible, it is also necessary to allow agents to live more than one evolutionary stage and to have memory of past interactions. A strict single-play game situation would imply either a perfect anonymity or a perfect certainty regarding the future − players are perfectly sure that there would be no more interaction among currently interacting players. One-shot games are a useful approximation to account for specific cases in which the probability of a future encounter between a pair of players is very small. But there are also many social interactions characterized by ongoing relationships that are built on past interactions.

Specifically, we assume that agents have memory of past interactions and partly condition their interactions with others based on the memory. Thus, when an agent is in the position of playing the game as the first mover, it recalls past interactions and search for those who behaved in a trustworthy manner in the past and offer to play the game by taking the first move. It does not necessarily mean that agent always cooperate as first movers. An agent may not have any past incidents of cooperative interactions; in that case the agent has to play the game with another agent chosen randomly. In addition, if the agent is egoistic and he knows, from his past experience, an altruistic agent, the egoist first mover will choose to defect to the altruistic second mover.

Secondly, we assume that agents die with a probability after each evolutionary stage. When an agent dies it is replaced by its offspring whose type is probabilistically determined by the distribution of types at the moment of its birth.

The two added assumptions, that agents live for an uncertain length of time and that they have memory expands the definition of cooperation. Cooperation is not merely an outcome in the $\{0,1\}^2$ strategy space of a single-shot game. Cooperation cannot be dissociated from all the possible future worlds it brings about. For this potentiality to be effective, agents must have memory while the future must be uncertain. We will now build an extension of the sequential incomplete information game incorporating the two additional assumptions.

Memory, Endogenous Networks, and the Evolution of Preferences

Consider a population of N agents that play the basic social dilemma game under the QI condition. The agents play the game several times with different partners during their lifetime. After an agent plays a game − or multiple games − in an evolutionary stage, it dies with a probability 1-θ, giving each agent a mean life expectancy of $\frac{1}{1-\theta}$ evolutionary stages. When an agent dies, it is immediately replaced by a new agent. The type of the new agent is determined probabilistically in the manner specified in equations (2) and (3).

Each agent has a perfect memory of past interactions. This is done by maintaining "address book" where it writes the names of other agents who cooperated with it in the past. The agents whose names appear in an agent's address book are called the "relationships" of the agent. When an agent's name is in the address book of anther agent, the latter is a "friend" of the former. At each evolutionary stage, an agent randomly chooses a name from his/her address book and plays a sequential social dilemma game as a first mover. If an agent's address book is empty, it interacts as a first mover with another agent randomly chosen from the population. At each evolutionary stage, an agent plays only once as a first mover. However, he/she can play multiple times as a second mover depending on the number of “requests” it receives. This reflects the fact that being a first mover of an interaction usually takes much more time than just reacting to another’s initiative as a second mover.

This gives rise to the possibility of a lock-in by which a pair of players play the game for the entire duration of their lifetime, which is not very realistic. Instead, we will assume that an agent, even when his address book is not empty, interacts with someone outside his address book with a probability $e \in [0,1]$. In real life, when a pair of agents interacts too often and only between themselves, the returns from the interaction decrease. Therefore, to diversify their information and optimize their payoffs, agents have to go beyond their established relationships at times. Individuals can also be forced to interact with someone they do not know; a large e in this case reflects instability due to political and economics reasons. The parameter e reflects how often this voluntary or involuntary *exploration* occurs. For now, let us note that for a given configuration of other parameters, there is a value of e, which optimizes the expected payoff for an agent.

The model outlined above defines a directed adaptive network, endogenously built by the agents. It will evolve in time as agents live and die. Notice that setting $e = 1$ and $\theta=0$ results in the baseline single-play condition. In other words, the baseline single play condition is a special case of the more general model outlined here. If the address book of agent i is not empty, we will say i has some relations. Then, as a first mover, i interacts, with the probability $(1-e)$, with one of the agents in the address book. With a probability e it interacts with a randomly chosen agent from outside of its relationships. Table 1 summarizes the behavior of the agents. Notice that the agents make their decisions at a stage to maximize their expected utility. This is why the decisions by egoists and reciprocators, when they play the game as first movers with someone chosen outside of their address books are functions of the distribution of types in the current population.

TABLE 1. Behavior in the Presence of Relations and Friends

	As a First Mover		As a Second Mover
	From Address Book	Outside Address Book	
Egoists	Pick an Altruist and Defect. If there are no Altruists, Cooperate	Cooperate iff $\delta > \frac{p}{1-p}$	Always Defect
Reciprocators	Pick at Random and Cooperate	Cooperate iff $\delta > \frac{p-\alpha\gamma}{1-p}$	Copy first mover’s Behavior
Altruists	Pick at Random and Cooperate	Always Cooperate	Always Cooperate

Simulation Results

We have simulated this model in Java. For this series of simulations, we focus on examining the influence of the exploration parameter *e* and the initial distribution of types while keeping *p* = 3, α = 0.4, θ = 0.99, and N (the number of agents at any given evolutionary stage) = 1000 for all simulations. Table 2 shows the nine parameter conditions with which the simulations are run and reports the mean proportions of altruists and reciprocators, and their standard errors in parentheses. Each simulation is run for 5,000 evolutionary stages, which corresponds to about 50 generations given that the life expectancy of an agent is approximately 100 evolutionary stages. The results reported in Table 2 are best viewed by comparing three columns for each of the three rows. Alternatively, one can also compare three rows of a given column to see whether or not, for a given network strength, the population dynamics differ depending on the initial condition.

The three columns of the first row address the question of whether or not a small combined proportion of altruists and reciprocators can invade a population mostly inhabited by egoists. When e = 0.1 and, thus, an agent, as a first mover, interacts nine our of ten times with someone in its current address book whenever the address book is not empty, the non-selfish preference types successfully invade the egoistic population. In fact, the egoists are completely driven out of the population by the stage 5,000. Figure 4 visualizes the average evolutionary trajectory of the five simulations with initial conditions of the first row and first column in Table 2. Note that the rectangular space in each panel of Figure 4 is only a relevant subspace of the entire state space shown in the panels of Figure 3. The evolutionary age of the population is marked on the trajectory for each 1,000th stage. Starting from the initial population state with consists of 10% altruists, 10% reciprocators, and 80% of egoists, the population steadily evolves toward northeast, signifying that both the altruists and reciprocators increase. The graph shows both the direction and speed of the evolution. Once there are not many egoists left in the population, the evolution is slow; that is why the distance between the stage 4, 000 and the stage 5,000 in the graph is very short. The two smaller panels of Figure 4 show the standard errors of the proportions of altruists at a regular interval.

TABLE 2 Simulation Parameters and Results: Mean Proportions and their Standard Errors at 5,000th Evolutionary Stage.

		Network Strength		
		Strong Social Network: e = 0.1	Modest Social Network: e=0.5	Weak Social Network: e=0.9
Initial Population	Altruists :10 % Reciprocators: 10%	$\gamma = 0.70(0.11)$ $\delta = 0.29(0.12)$	$\gamma = 0.21(0.11)$ $\delta = 0.13(0.07)$	$\gamma = 0.0(0.00)$ $\delta = 0.11(0.06)$
	Altruists: 33.3 % Reciprocators: 33.3%	$\gamma = 0.45(0.16)$ $\delta = 0.55(0.16)$	$\gamma = 0.39(0.10)$ $\delta = 0.57(0.09)$	$\gamma = 0.03(0.03)$ $\delta = 0.32(0.11)$
	Altruists: 45 % Reciprocators: 45%	$\gamma = 0.49(0.05)$ $\delta = 0.51(0.05)$	$\gamma = 0.60(0.08)$ $\delta = 0.40(0.08)$	$\gamma = 0.13(0.07)$ $\delta = 0.40(0.11)$

The eight panels of Figure 5 show the average evolutionary trajectories of the remaining eight simulation conditions. Each panel has the initial population composition and the network strength at the upper-left corner. When networks are weak, the invasion by altruists and reciprocators into an egoistic population is either slow and incomplete (e = 0.5), or impossible (e = 0.9). With networks of modest strength, both the proportions of altruists and reciprocators increase slightly, but egoists maintain the majority at 5,000th evolutionary stage. Whether or not the non-selfish preference types will eventually take over the entire population cannot be answered within the simulation data. But, given the slow and tortuous evolutionary trajectory (see the left panel of the first row in Figure 5), our conjecture is that this is a range of parameter configuration in which a fully mixed population composed of all three types can be stable.

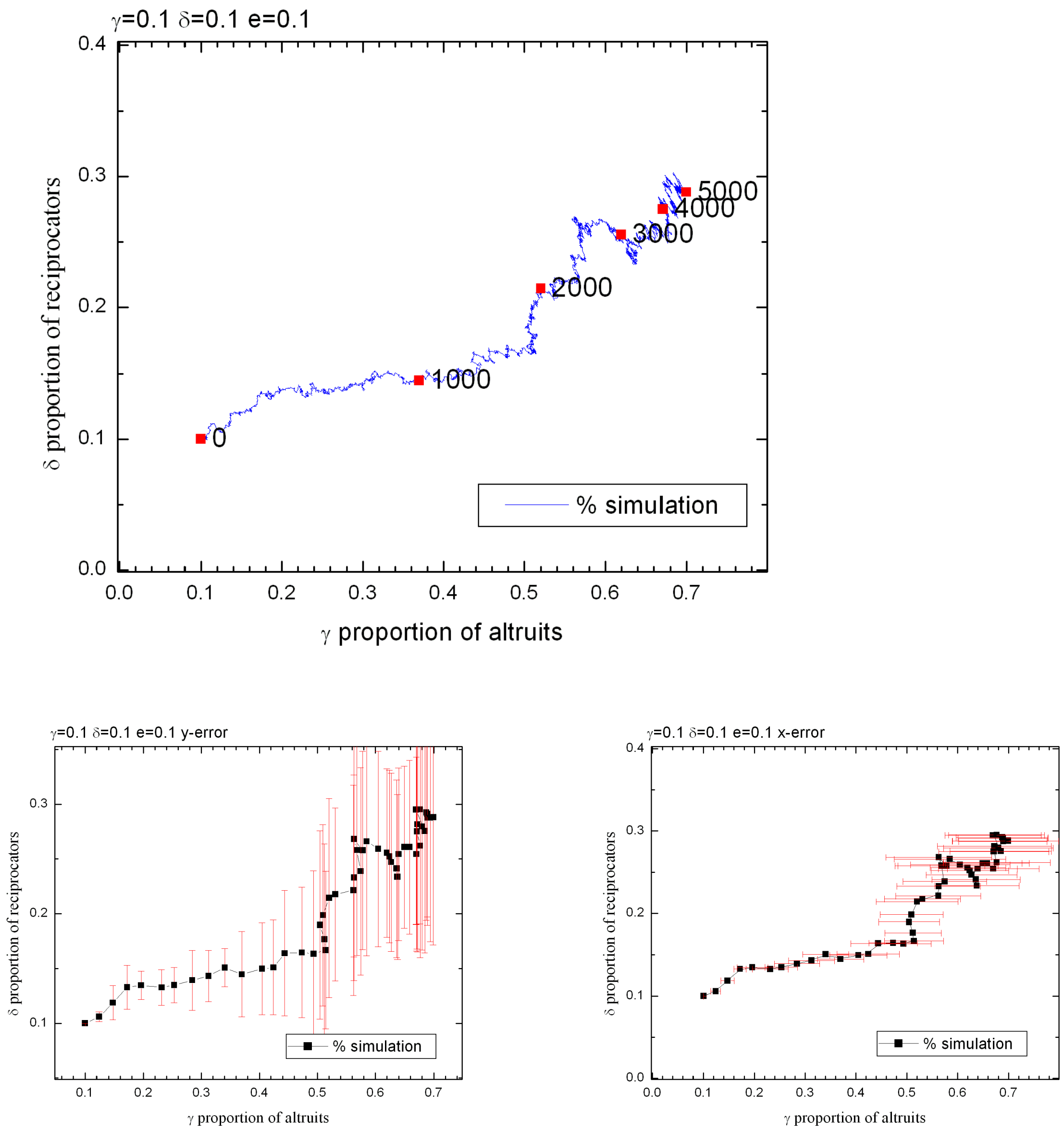

FIGURE 4 Strong Social Networks. Can Non-selfish Preferences Invade an Egoistic Population?

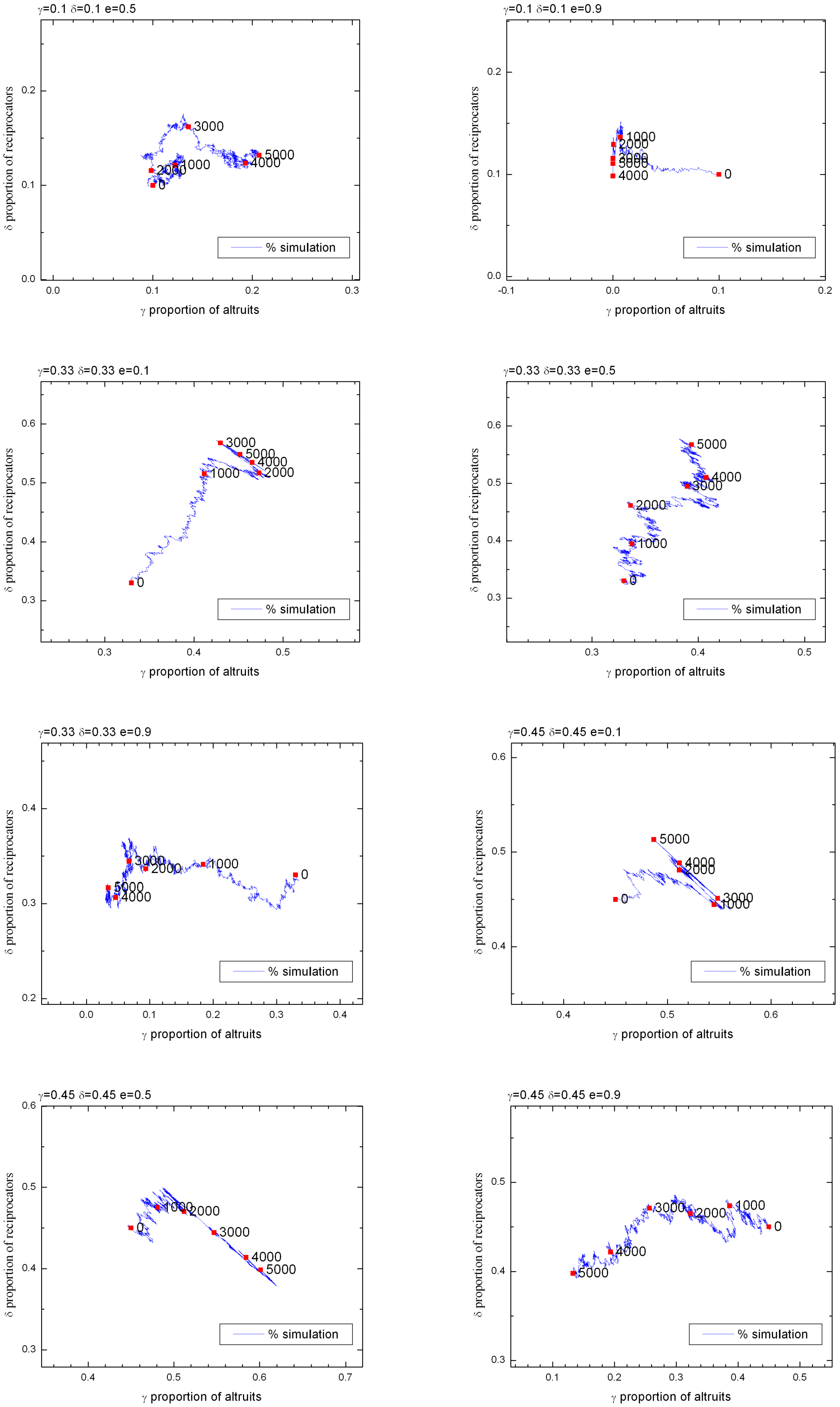

Figure 5 Average Evolutionary Trajectories from 5 Simulations

On the other hand, when the network is weak (e=0.9), meaning that agents interact with someone outside of their address book nine out of ten times, the invasion is not successful. Altruists are completely driven out of the population and reciprocators survive only because they remain as neutral mutants; that is, because of the large proportion of egoists, reciprocators do not cooperate as first movers and, thus, behave exactly the same as egoists.

When the three types are evenly distributed at the initial stage of an evolution, the evolutionary trajectories show two patterns depending on the network strength. This can be seen by comparing the three columns of the second row in Table 2, and corresponding panels in Figure 5. When networks are either strong or modest, egoists are driven out of the population. However, when the force of endogenous network formation is weak because of the high probability of exploration, altruists are driven out of the population and the reciprocators remain as neutral mutants.

The final question is whether or not egoists can invade a population consisting of altruists and reciprocators. The third row of Table 2 provides the answers to this question. When networks are strong or modest, egoists' invasion is unsuccessful. In fact, they are completely driven out the population. However, when networks are weak, egoists can successfully invade the population driving altruists and neutralizing reciprocators.

CONCLUSION

In this paper, we have developed indirect evolutionary models that explore the conditions for the evolution of different preference types. Within one-shot game settings, either reciprocators or egoists are favored evolutionarily depending on the information conditions. In our extended models with memory and endogenous social networks, we also examined the conditions for altruists and reciprocators to survive, and even invade, an egoistic population. While many authors assumed that every type of preference except an egoistic one is not evolutionarily viable, we have explored other possibilities. The computational models show that the presence of social networks, endogenously built by agents who have memories, can change the evolutionary dynamics. When agents have the cognitive capacity to classify their environment, social networks play an important role, and social cooperation emerges at a substantial scale, and non-selfish preferences can flourish.

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