The Emergence of a Price System from Decentralized Bilateral Exchange

Herbert Gintis*

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Abstract

This paper derives the price system in a model of decentralized bilateral exchange. Out of equilibrium, prices are individual subjective estimates of payoff-maximizing exchange ratios, and hence are the private information of individual agents. We show that a system of public prices emerges in the long run, and these public prices are equilibria of the underlying general equilibrium model. We thus provide, for the first time, a general, decentralized disequilibrium adjustment mechanism that renders market equilibrium dynamically stable in a highly simplified production and exchange economy.

Journal of Economic Literature Classifications:
D51—Exchange and Production Economies
D58—Computable and Other Applied General Equilibrium Economies
D82—Asymmetric and Private Information

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> The problem with much of the complexity work is that it doesn't seem to lead anywhere.
>
> *Kenneth Arrow, in Colander et al. (2004), p. 293.*

1 Introduction

This paper studies an economy in which agents produce, exchange, and consume a number of goods in each of many periods. Rather than using analytically tractable but empirically implausible adjustment mechanisms and informational assumptions, we treat this economy as complex, dynamic, nonlinear system in which agents have extremely limited information and there is no aggregate price-adjustment mechanism institution (such as Walrasian tâtonnement) out of equilibrium.

The behavior of this economy can be studied through agent-based modeling. We assume that each agent has a private subjective estimate of payoff-maximizing prices, and the distribution of private prices moves through time according to an adaptive evolutionary dynamic. We show that from an initial random seeding of private prices, the economy evolves rapidly towards what may be termed a *private but common* price structure, in that the variance of private prices across agents becomes relatively small. In the long run, this private but common price structure evolves toward the market clearing price system for the underlying general equilibrium system. Thus, the economy enjoys price stability, which is not the case for the tâtonnement adjustment processes studied by Walras and his followers in modern general equilibrium theory (Fisher, 1983;Saari 1985,1995).

The economy begins each period with a production phase, in which each agent produces a fixed amount of a single good, which he does not consume. followed by an exchange phase, in which agents seek exchange partners. and finally a consumption phase. After a certain number of periods, generally twenty in our simulations, there is a reproduction phase in which a fraction (usually 5%) of low scoring agents copy the strategies (the private prices) of high-scoring agents. The cycle of production, exchange, consumption, and reproduction is repeated for a fixed but generally very large number of periods.

A strategy for an agent is a price vector for the various goods he produces or consumes. We assume an agent will only give a quantity of his production good in exchange for a quantity of the production good of his exchange partner, and an agent will agree to an exchange only if the value of what he receives in an exchange is at least as great as the value of what he gives in exchange, according to his private price vector.

Learning in this economy occurs only during a reproduction period, and takes the form of a small subset of agents each observing a single other agent, and switching to the strategy of the other agent if he is doing better with a probability that is increasing in the difference between the payoffs. This is a monotone dynamic, which is equivalent in its dynamical properties to the famous replicator dynamic (Taylor and Jonker 1978, Samuelson and Zhang 1992). The system of private prices therefore follows an evolutionary adaptive dynamic.

Agents in this economy have no knowledge of excess demand or supply for any good. Nor is there an ‘auctioneer’ (Walras 1954 [1874]) calling out prices, collecting information concerning aggregate demand, and dynamically ‘correcting’ the price structure, with the aim of moving the system towards market clearing. Of course, that such a tâtonnement process not only is completely infeasible and grossly violates the conditions of decentralized exchange, but also the induced dynamical adjustment mechanism in general generates chaotic price movements.

2 The Structure of a Simple Market Economy

Suppose there are n goods and $N \gg n$ agents. Each agent produces a single good at zero cost whenever his supply of this good is zero, or at the beginning of a new production period. Each agent also has a utility function that he maximizes subject to the income constraint that his expenditure cannot exceed the market value of his production good. In general equilibrium theory, its is assumed that there is a vector of prices that is known to all agents, and equilibrium is defined by a vector of prices for which demand equals supply in all markets. Well-known theorems assert the existence of such an equilibrium price vector under appropriate conditions. (Debreu 1959, Arrow and Hahn 1971).

In our economy, however, prices as public information do not exist. We can investigate the properties of this economy using an agent-based model whose properties are revealed by computer simulation.

Several general conclusions flow from such an investigation. First, the economy converges rather quickly to a system of *private but common* prices, in the sense that there is a rapid decline (fewer that 2000 periods are generally required) in the variance of the price of individual goods across individuals, to an stationary level that is maintained thereafter. Second, the economy becomes increasingly efficient as the variance of prices approaches this stationary value. Third, these private but common prices are generally far from their market equilibrium values at this point, but the deviation of prices from their market-clearing values declines gradually to a low level in the course of time. As private but common prices move towards their market-clearing values, the economy moves even closer to an efficient allocation, and the payoffs to the various types of producers tend to their market-clearing values as well.

3 Short-Term Convergence to Private but Common Prices

Suppose each agent produces a certain quantity of one good but consumes a subset of k goods in each period, the particular goods being respecified randomly at the start of each period, except that the agent does not consume his own production good. For simplicity, we will assume each agent i consumes in fixed proportions $(o_{i1}, \ldots, o_{in})$ such that the utility $u_t^i(x_1, \ldots, x_n)$ in period t is given by

$$u_t^i(x_1, \ldots, x_n) = \min_{t_j} \frac{x_{t_j}}{o_{it_j}}, \tag{1}$$

where $x_{t_1}, \ldots, x_{t_k}$ are the $k < n$ goods the agent consumes in that period. If agent i produces good g, uses prices $p_i = (p_{i1}, \ldots, p_{in})$ and has an inventory x_{ik}^o of his production good, he will maximize utility subject to the income constraint

$$p_g x_{ig}^o = p_{i1}x_{i1} + \ldots + p_{in}x_{in}. \tag{2}$$

The optimum is given by

$$x_{it_j}^* = \lambda^* o_{t_j}, \tag{3}$$

where

$$\lambda^* = \frac{p_{ig}x_{ig}^o}{\sum_j p_{it_j}o_{it_j}}. \tag{4}$$

At the start of each period, each agent has an inventory consisting only of an amount x_{ig}^o of his production good. Once in each period, agent i "shops," choosing a random producer of each of his non-produced consumption goods and attempt a trade with each. When he is not shopping, agent i is "selling," by which is meant that i waits to be approached by the current shopper, and all sellers of a particular good are equally likely to be approached. When shopper i contacts seller i', i offers to trade good g (his production good) for good j (the production good of seller i') in proportion p_{ig}/p_{ij}. Such a trade will be acceptable to seller i' if, according to his own prices $p_{i'}$, the trade is profitable. The amount traded will then be greatest amount acceptable to both parties. After a successful trade, both parties consume any portion of their inventories that gives positive utility, and if an agent's production good is depleted, the agent produces a new supply of the production good. When each agent has completed his turn as a shopper, the period is ended.

After a fixed number of periods, usually twenty in our models, a "reproduction period" is added, in which a fraction (usually 5%) of low-performing agents of each producer type are permitted to imitate the behavior of high-performing types by switching to the price structure used by the higher-performing types.

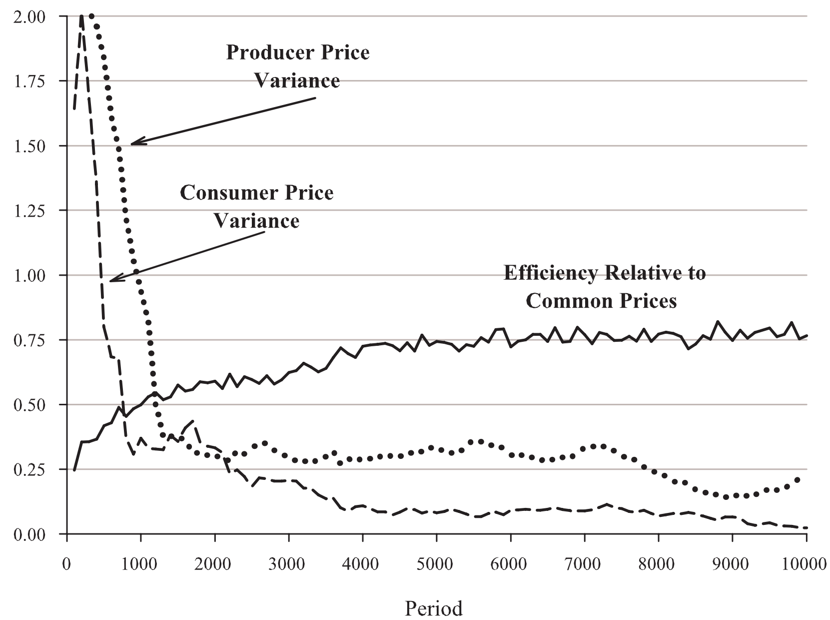

Figure 1: Short-Term Convergence to Private Common Prices

A run of this agent-based model begins by assigning each agent the utility function parameters o_{ij} and prices p_{ij} drawn from the uniform distribution on the unit interval. Figure 1 show the first 10,000 periods of a typical run of this agent-based model, assuming $n = 3$, $k = 2$, $N = 300$ (three goods, each agent consumes both goods he does not produce, 300 agents). Note than in the first 1200 periods the variance of individual prices falls precipitously from quite high to quite low levels. We measure producer and consumer prices separately because agents tend to have lower prices for the goods they sell than for the goods they buy, a practice that emerges endogenously but has a clear efficiency effect: agents are thereby predisposed to trade even when their private price ratios are somewhat discrepant. The curve labeled "Efficiency Relative to Common Prices" is determined as follows. First, we impose an arbitrary set of public prices on all agents and measure the resulting scores (utility levels) after one round of production and trade. We call this "common price efficiency." This curve measures the average payoff with private prices to this common price standard. This schedule begins at about 25% and as the variance of prices falls, rises to about 80%.

It might be argued that the shape and position of the Efficiency Relative to Common

Prices curve should depend on the particular structure of common prices. In fact, the particular configuration of prices does not matter that much. To ascertain this, we calculated the common price efficiency of trade for 100 different, randomly generated, configurations of relative prices, drawn uniformly from the interval [1,2]. Relative to the maximum common price efficiency, average efficiency was 88.6%, the standard deviation was 6.2%, and the lowest efficiency was about 80%.

4 Long-run Movement to Market Equilibrium

We have seen that in an economy with private prices, the price system undergoes an evolutionary dynamic that leads to what we termed "private but common prices," by which is meant a system of prices that are private information, but for which the variance of prices for the same good across agents is sufficiently low that a comparatively high level of efficiency in trade is attained.

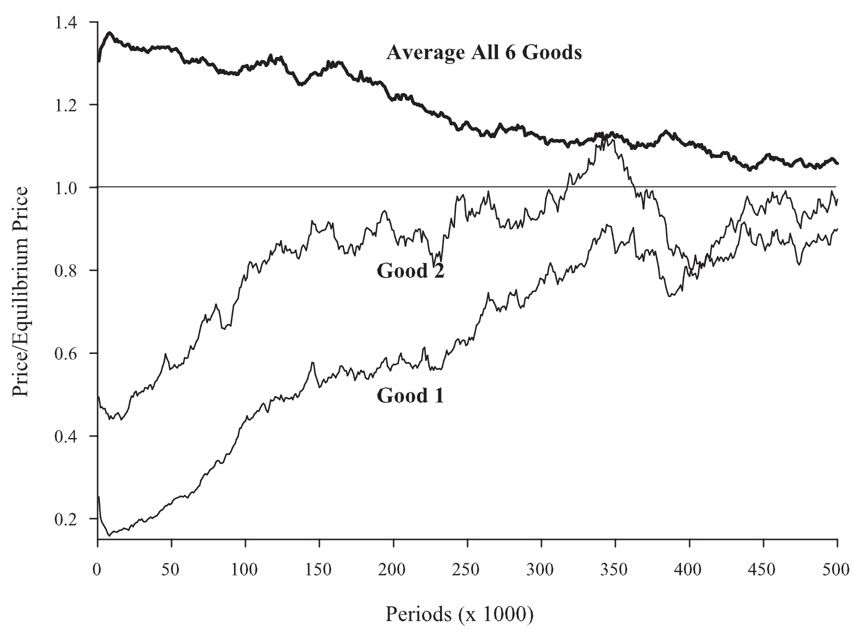

Figure 2: Long-Term Convergence of Private Common to Market Equilibrium Prices

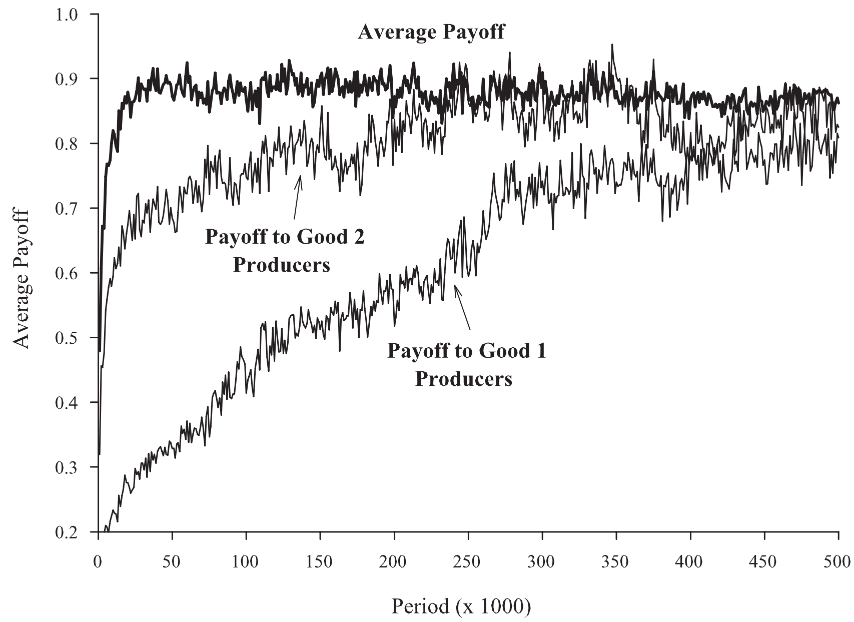

Figure 3: Long-Term Convergence of Payoffs to Market Equilibrium Payoffs

The next question is whether such a system of prices converges in the long run to what would be market-clearing prices in a general equilibrium system with public prices and the same underlying conditions of production. To address this question it is useful to know what the equilibrium prices would be in such a general equilibrium system. Such prices can be calculated explicitly only for specialized case, one of which is as follows. Suppose all agents have the same utility function

$$u(x_1, \ldots, x_n) = \min_j \frac{x_j}{o_j}. \tag{5}$$

for a common set $(o_1, \ldots, o_n)$ of proportionality coefficients. Suppose, further, that there are equal numbers of producers of each good, and each producer of good i produces an amount no_i of good i. Then, it is easy to check that if the price of good i satisfies $p_i = 1/o_i$, then (3) and (4) imply excess demand for each good is zero, each agents consumes a bundle equal to $(o_1, \ldots, o_n)$, and each has utility one. Since we know the equilibrium prices for such a system, we can the long-run behavior of

private common prices with market equilibrium prices. The result for the case of six goods, assuming $n = 6$, $k = 5$, with $(o_1, \ldots, o_6) = (1, 2, 3, 4, 5, 6)$ is shown in Figure 2. In this simulation there are six goods, and as in all our simulations, there are 100 agents per good. Prices are randomly generated independently for each price and each agent from the uniform distribution on the unit interval. Thus, whereas market equilibrium prices would be (6,5,4,3,2,1), initial prices have equal means, so there is a strong tendency for goods one and two to be underpriced and goods five and six to be overpriced. Figure 2 shows that the convergence of private common prices to market equilibrium prices takes place almost monotonically over 500,000 periods, and Figure 3 shows that the payoffs to the producers of the initially underpriced goods converge to the mean over this sequence of periods.
This strong movement of prices towards market equilibrium has nothing in common with the usual price adjustment mechanism in general equilibrium theory (Fisher 1983), in which prices changes are a function of aggregate excess supply. Rather, when a good is underpriced compared to its equilibrium price, a shopper who is willing to pay more and a seller who is willing to charge more will both do better, on average, than other agents whose private prices are nearer the reigning private common price structure.

As a check on this argument, we alter the initial conditions so that good 1 is produced at half its previous rate, we assume there are three goods, and all other aspects of the simulation are the same as previously. There is thus an excess demand for good 1 in each period. Figure 4 shows the long-run movement of the relative payoffs of the producers of the three types of goods. In the long-run, the price of the goods in excess supply tends to zero, so all of the social product accrues to the producers of the good for which there is excess demand.

5 The Effect of Flexible Supply Curves

The preceding models assumed that the conditions of supply of each good are fixed. If we allow agents to switch from producing one good to another, we would expect that once private but common prices obtain, agents will shift production so that the returns to all producer types are roughly equal. This will lead to a disequilibrium in the size of the various production sectors. Then, as prices approach market equilibrium, the production sectors will approach their equilibrium size, which with the above parameters, is equal size for all sectors. Figure 5 shows that is in fact what occurs. In this run, only sectoral share are shown. The movement of prices and payoffs is as previously described.

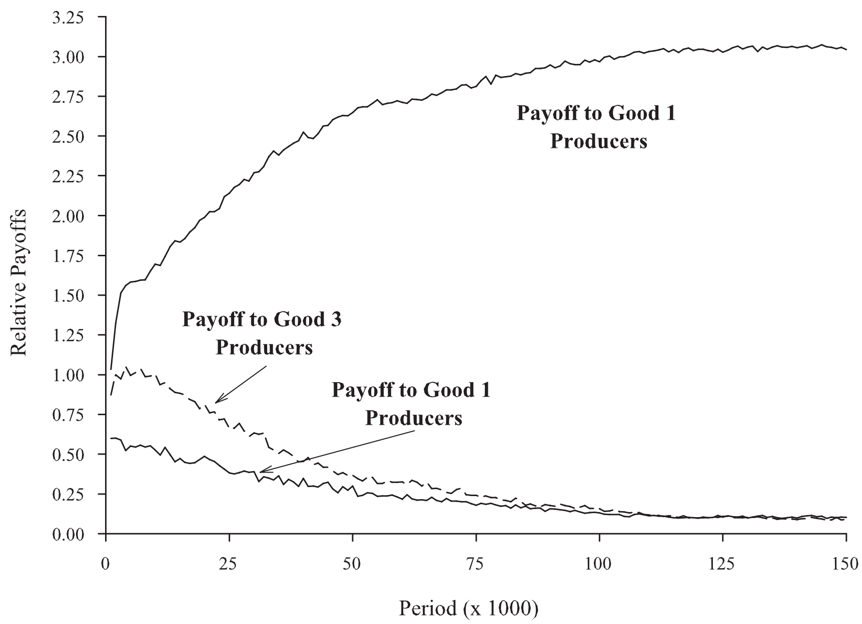

Figure 4: Convergence of Price to Zero when a Good is in Excess Supply. The figure shows that the payoffs to agents whose production good is in excess supply tends to zero.

6 Private Prices and Money

Our results extend to a monetary economy. By money we mean an exchangeable good that is in fixed supply, being neither produced or consumed. Each agent is endowed with a share of the total money supply at the beginning of a simulation. We interpret the evolutionary dynamic as poorly performing agents dying and being replaced by copies of the well performing agents. An agent who reproduced inherits the money supply of the agent who died, and shares this equally with its offspring. Agents now consider money the numeraire, so they continue to have n private exchange ratios, this time for each good versus money. Each agent is given a stock of 10 units of money at the start of the run. For this simulation we assume there are seven goods. of which each producer consumes three in each period. We choose $(o_1, \ldots, o_7) = (1, 2, 3, 4, 5, 6, 7)$, and an individual who produces

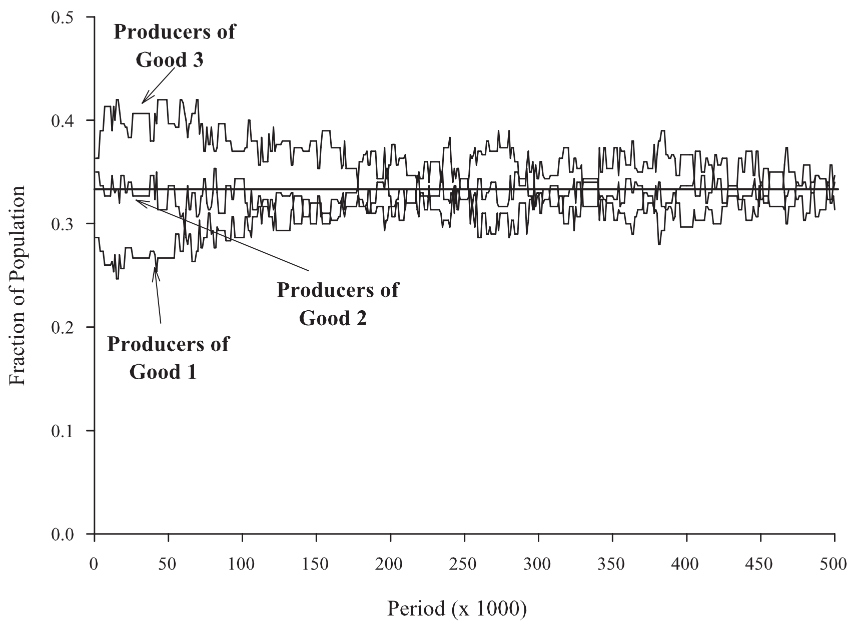

Figure 5: The Movement of Sectoral Shares when Supply is Flexible.

good i produces ko_i units per period. In market equilibrium, prices are $p_i = (1/o_1, \ldots, 1/o_n)$, a producer receives k money units for his production good, and spends one money unit each on his k consumption goods, receiving an amount o_c of each consumption good c, so his consumption utility is unity. With these production conditions, supply equals demand in all markets, at least on average (since the consumption goods assigned agents in each period are randomly assigned, the law of large numbers applies). Figure 6 shows the evolution of the percent deviation of prices from market equilibrium prices, as well as the evolution of average payoffs for this monetary economy. Once again, we have long-term movement towards the market equilibrium levels, including nearly 90% efficiency towards the end of the simulation.

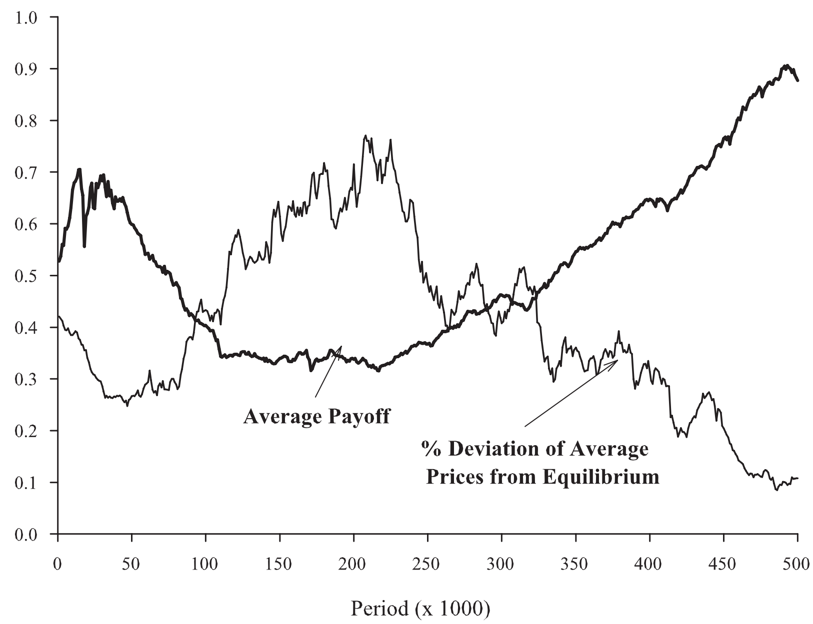

Figure 6: The Movement of Prices and Payoffs in a Monetary Economy.

7 Conclusion

Our agent-based models of economic dynamics have revealed a fact that analytical models have systematically failed to recognize: the price system in a market economy has strong elements of a complex adaptive system, the evolution of which tends to stabilize a market economy. Public prices, by contrast, lead all agents to adjust in the same direction in a given period, thus creating a situation in which instability and chaos are likely outcomes.

We have shown that the informational requirements for a successful economic system are quite modest, and among the most important are the ability to ascertain the relative success of other agents, and the capacity to imitate the strategic behavior of the relatively successful.

The range of economies studied in this paper are extremely simple, without firms, production processes, capital or intermediate goods, labor, or any of the many institutional characteristics of a real economy. In a companion paper, Gintis (2005) approaches the problem from the other direction, starting from a fairly complete implementation of a Walrasian general equilibrium model with public prices but no aggregate price adjustment mechanism, with results that are similar to those of this paper.

Our results should be considered empirical rather than theoretical: we have created a class of economies and investigated their properties for a range of parameters. It remains to develop analytical models of private pricing, the passage to private but common prices, and the subsequent passages to something akin to market clearing prices. The data we have generated in this paper should open the way for such analytical models.

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