

Recognition-Based Logic and Social Conflict: Toward a Topos Model

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ABSTRACT

Category theory and, more specifically, topos theory provide a more expressive type of mathematical modeling and, thus, open the door to social models that are both expressive and rigorous. The present analysis draws upon four views of topos theory to construct a rich model of recognition-based logic. The four aspects provide support for: 1) set theory (and classical logic), 2) topological regions, 3) a local intuitionist logic that can vary by actor types, and 4) the specification regions of finely-differentiated classifiers. These tools are used to define an integrated topos category of recognition that gives rise to diverse forms of local logic. The latter are then explored as a contributory basis of social conflict.

INTRODUCTION

For centuries, mathematical tools have arguably not been sufficiently expressive to support social models. Accordingly, rigorous social analysis has focused on counting and measuring, including the application of ever more powerful statistical techniques, while continuing to lack the mathematical capacity to represent dynamic interaction, evolving relations, and the pervasive fluidity of social processes. During the past century, however, dramatic progress has been made in abstract mathematics. However, the range, complexity and very abstraction of these advances have made it challenging to discern and apply the new insights and techniques to the modeling of social phenomena.

Mathematical activity constitutes a formal discourse that has grown rapidly, and continues to expand at a prodigious rate. In addition to being abstract, the areas in which recent advances have been achieved are diverse, and their prospective implications are not easy for researchers from other disciplines to identify. This is unfortunate, since various innovations have the potential to support new and productive research strategies in a variety of substantive disciplines. The subject of the present paper is to illustrate the way in which an emerging richness of mathematical formalisms provides innovative capabilities for theoretical modeling in the social sciences.

Potential contributions of category theory in the social sciences have been addressed in previous papers. The focus has been on the use of adjoint functors to represent social structures in classical social theory (Sallach 2012a), prospective categorical contributions to social methods, including the mapping of qualitative concepts to equivalence classes (Sallach 2012b), and the propagation of cross-scale social propensities (Sallach 2012c).

The present paper is more specialized and detailed. From within the extensive range of categories that have been defined, topos theory is quite expressive, and seems

likely to be especially productive in the social sciences. The focus of the present discussion is on summarizing what topos theory may be able provide to social analysis, and to illustrate ways that these capabilities can be applied.

CATEGORY THEORY

During the last 70 years, category theory has arisen as a rich formalism capable of tying many research domains together. It first emerged as a means of integrating topological surfaces with algebraic operators. However, soon it was being applied to other areas, first within mathematics and later to logic, computer science, physics, biology and neurology.

Whereas set theory has 1) focused on mathematical *objects* (sets), 2) privileged the member relation above other potential relations of interest, and 3) emphasized functions that manipulate a single object type (e.g., union, difference, intersection), category theory focuses on relations and morphisms that transform mathematical objects of widely varying types. It has developed a framework by which to shift to ever-higher levels of abstraction (e.g., to functors, natural transformations and higher categories).

Among the strengths of category theory that make it particularly relevant to social modeling are, first, that support for equivalence relations allows an *expressiveness* of qualitative concepts to be retained while translating them into a more *precise* form (Sallach 2012a). Second, categorical models apply duality analysis in a way that provides a means of integrating coupled processes. Together, such capabilities make rigorous theoretical integration feasible.

Topos Theory. Topoi¹ are categories that combine the strengths of topology and set theory. This blending makes them a powerful source of rich and variegated representations. Joyal and Johnstone provide a comprehensive set of descriptions of topoi that have emerged over the decades,² but explicating and/or elaborating them is outside of the scope of the present discussion. To begin, we will draw upon the definitions provided by Borceux (1994) and Johnstone (2002). Borceux defines topoi as “the categorical framework for studying those structures [that] behave like sets (1994:288). Somewhat more technically, Johnstone (2002:68) defines a topos as “a properly Cartesian closed category with a subobject classifier.”

Borceux identifies three representative examples (1994:288-289): 1) a topos of sets (the ‘classical’ formulation); 2) a topos of sheaves on a locale, “where elements exist at various levels and can be glued or restricted to produce elements at other levels”; and 3) a topos of G-sets, for a Group G, where “the sets of elements are provided with some

¹ Toposes and topoi are both used as the plural of topos. In this discussion, we use the latter.

² To show how mathematically expressive definitions of topoi have become, Johnstone (2002:vii-viii) lists the seven descriptions of topoidal categories first assembled by André Joyal): 1) a category of sheaves on a site, 2) a category with finite limits and power objects, 3) an intuitionistic higher-order theory, 4) a first-order (infinitary) geometric theory, 5) a totally cocomplete object in the meta-2-category of Cartesian categories, 6) a generalized space, and 7) a semantics for intuitionistic formal systems. He then notes that six additional definitions have been formulated since the initial list was formulated: 8) a Morita equivalence class of continuous groupoids, 9) the category of maps of a power allegory, 10) a category whose canonical indexing over itself is complete and well-powered, 11) the spatial manifestation of a Giraud frame, 12) a setting for synthetic differential geometry, and 13) a setting for synthetic domain theory. Formally, these definitions overlap each other to some extent, but they also illustrate how diverse are the contributions that topoi can make.

structure, namely, an action of the group G .” The first two Borceux examples are of particular interest because they illustrate the constructive interaction between the set-theoretic and topological aspects of topos theory.

In moving from generic definitions to topoidal capabilities that can readily contribute to social science representation and modeling, the present discussion will focus on four topos characteristics: 1) determinate objects and the sets they form, including set-theoretic functions and the (classical) predicate logic with which they are associated; 2) topological spaces, including stalks, presheaves, sheaves, locales, écales and, ultimately, topoi; 3) intuitionist logic (Bell 1988:162-219), a ‘local’ logic that does not assume the principle of the excluded middle obtains; and 4) subobject classification, a structured way of providing graduated, indexed and/or spectral distinctions within a stable set of values. The first characteristic makes available all of the familiar concrete entities that are measured, counted and subjected to statistical analysis. The second can be used to define synthetic geometries, but also create cultural and ideational spaces.

By providing a local logic, the third example allows the introduction of a social reference frame, relative to which social actors can draw distinctive inferences. Bell (1988:239-242) notes an analogy with relativity theory, which also has reference frames (coordinate systems) that define local patterns. Since it does not assume the ‘law’ of the excluded middle (cf., Godel 2004; Dummett 1977:17-21), it has the potential of supporting partial, qualified and probabilistic inferences, as well as varying types of inference in diverse relationships or scenarios. Finally, the fourth characteristic allows either an analyst or a simulative agent to assess the effects of incremental differences in spaces or structures of interest.

Since these four characteristics are alternative ways of viewing a common model, and may interact with each other in structured ways, these core mechanisms can contribute to represent complex and subtle dynamics. The expressiveness of the topos category, including the additional capabilities implicit in footnote one, have the potential to provide the underpinnings of a new generation of social science methodology.

Point-Set Topologies. Regarding the interaction of multiple topoidal aspects, point (and set) distributions are used to define topological regions, while spatial locations can influence actor characteristics. Figure 1 provides a generic example³ of one form that such codefinition (and coevolution) can take, and the way that point distribution and spatial shapes operate together to address ambiguities. While both patterns are defined mathematically, their joint presence provides the basis for an interpretive process.

When topos consists of sheaves on a locale, it is possible for elements to exist at various levels, in which case they can be *glued* or *restricted* to produce newly integrated elements, or processes that are coupled across levels. The next two figures show simple ways that more complex configurations can be generated. Figure 2 shows a system of two sheaves that have been joined at two structurally similar sites.

³ The pattern depicted in Figure 1 is based on arbitrary test data, and is shown only for illustrative purposes. Other than the recognition theory examples, none of the figures in this paper are intended to imply an empirical social process.

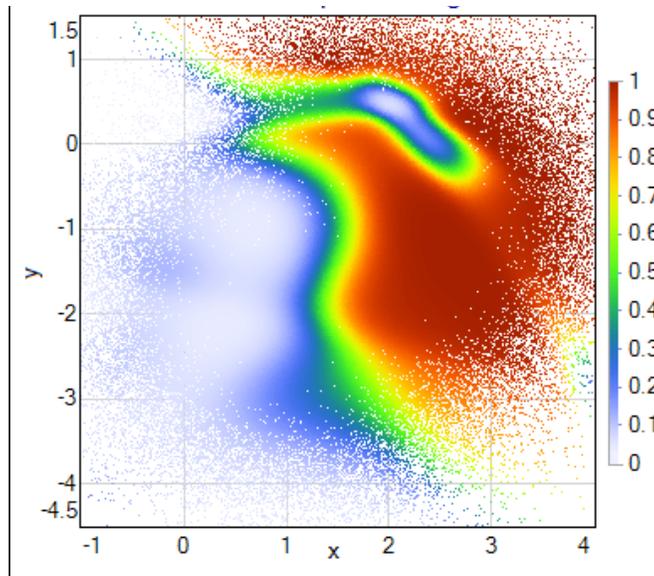


Figure 1. Points, Sets and Spaces Codefine Each Other

As categories, topoi naturally express such reciprocal part-whole influences (Goldblatt 1984; Bell 1988; McLarty 1996; Marquis 2009), which is one reason why they are of interest in social modeling.

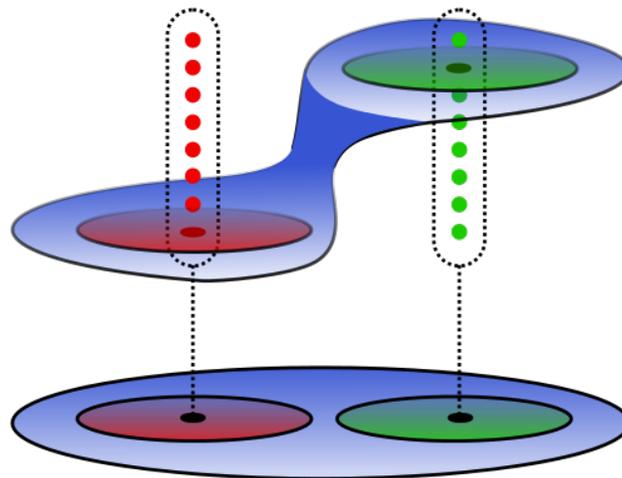


Figure 2. Gluing Sheaves at Two Points

Cross-Scale Influences. In topoi, categories with morphisms that are entirely invertible (and, thus, groupoids) can “send moving frames to moving frames according to the underlying structure ...”, making sure that it is done coherently (Marquis 2009:258). Sewell (2005:168-172) notes that different cultures often have a logic of their own, and this is what the dual cross-scale relations inherent in topoi provide. In particular, Sewell (2005:339) describes how discourses are *jointed* or *sutured* together. This can emerge on

an intuitive basis but, in other cases, there is an intentional process, sometimes involving multiple actors in order to achieve coupled discourse.

As an example, he (2005:340) describes professional basketball as a game that simultaneously integrates semiotic conventions regarding on-floor physical performance, the technical analysis of coaches and players, the physical codes of urban honor, media attentional priorities, the advertising focus on sports celebrity, the financial strategies of owners and investors, the substrate of legal hermeneutics, and many more. This example illustrates a case for which Sewell considers *suturing* as the appropriate term.

Actor Reference Frames. Set-theoretic models of socio-cultural systems have been limited by a lack of expressiveness. An exclusive focus on sets, as opposed to their relations (including transformations) has provided a spare, impoverished foundation for the representation of complex social dynamics. However, as Bell (1988:49) notes, “a topos is a ‘generalization’ of a set”, in which its (sheaf-based) locales provide much greater expressive power.

In sheaf categories, both the axiom of choice and the axiom of well-pointedness generally fail which, as Bell (2006:14) observes, shows that “both principles are incompatible with continuous variation.” However, in addition to being located within complex settings, actors in social conflicts continuously adapt and adjust their position and responses and, therefore, require the expressiveness that topos theory provides.

Nor is expressiveness the only contribution to social modeling that topos theory makes. Like relativity theory, topos theory can be understood in terms of reference frames (Bell 1988:239-245). That is, depending upon its specific axioms, each sheaf within a topos, is defined by a local logic as well. If local axioms are regarded as held by social actors, whether explicitly or implicitly, they can provide a formal basis for diverse forms of inference and, ultimately distinct universes of discourse (cf., Sewell 2005). Among other characteristics, their granularity can range from binary (polar), through discretely graduated (indexed), and continuous, to ‘smooth’ (Bell 2006:15). Such universes of discourse and inferential practices can be implemented computationally, and further refined, using *type classes* as provided by Haskell (O’Sullivan, Goerzen & Stewart, 2009:135-164; Lipovača 2011:109-152), and other computer languages.

Social actors in their reference frames exist across many scales. Such influences may arise from positive or negative affect (Heise 1979; Collins 1993; Sallach 2008), common interests (Collins 1998), shared strategies (Sallach, North & Tatara 2011), the elicitation of cooperation (Sallach 2012c) or persuasion (Perloff 2010).

RECOGNITION THEORY

The relationship among cross-scale actors is pertinent to a number of social theories and models. Recognition theory provides an important example. Honneth (1996) distinguishes among three patterns of recognition: love, rights and solidarity. The affinity relation that Honneth calls ‘love’ gives rise to beneficent strategies, calibrated by a selected response to the particular situation. The legal recognition relationship (‘rights’) is primarily instrumental in nature, where various actors agree to support legal rights for all as a means of securing their own (1996:109).

Groups based on solidarity (and its diverse bases) may draw upon multiple strategies, depending on their historic and current relationships. Groups can view each

other as: 1) allies (with whom there is a mutual affinity), 2) competitors and/or actors with asynchronous dependencies (cf. Emerson 1962) (who engage each other in a pragmatic or instrumental way), or 3) enemies (toward whom they attribute the necessity to be coerced, or threatened with coercion). In each case, the calibration of particular moves will depend upon the specific situation, and the actors' comprehension of it.

Recognition theory can be formulated categorically. As Figure 3 illustrates, each of the poles of the theory (affinity, solidarity and rights) can be treated as ideal types, invariant within the system, approached but never fully realized. They codefine an adjoint system by providing the means of binding (up to isomorphism) the affinity, homophily or generalized groups in question.

Broadening and narrowing are defined in terms of the scope of the group, with affinity groups being the narrowest, and generalized groups being the broadest. They define progressions that consist of a net balance between two coupled orders.

The left hand system (LHS) seems to imply that the specific ideal must be (nearly) realized before a reversal or alteration of a progression becomes possible. In contrast, for the right hand system (RHS), 'collars' define the size of regions in which

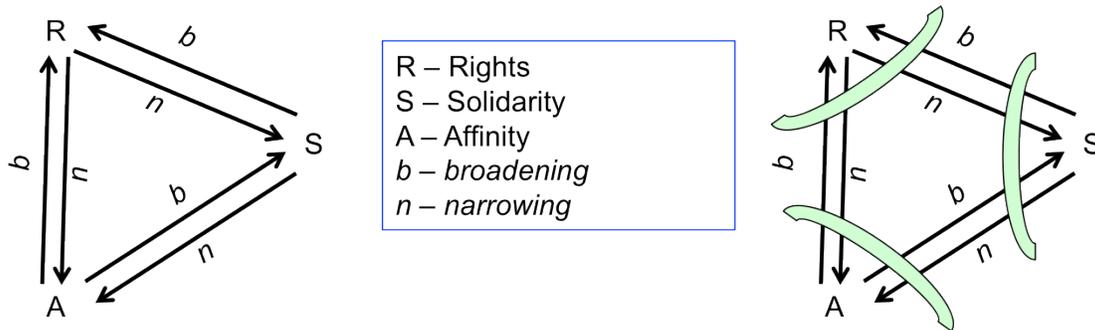


Figure 3. Adjoint Recognition Systems with (RHS) and without (LHS) Collars

progressions can reverse, or shift to an alternate coupling. It is posited that, over time, there is an alternation among the three focal poles. Overall, recognition dynamics will be shaped by the exploitation, and/or exploration of the current or possible forms of binding.

Figure 4 and the immediately preceding paragraphs illustrate how a category-theoretic analysis provides an approach to modeling that is both rigorous and expressive. However, the focus of the present paper is to (also) to demonstrate how topos theory can be applied within the social sciences. To provide the basis for this analysis, we will focus on the attribution of truth, or falsehood, toward third-party actors who are sources of information, albeit mediated by their recognition level.

Now, prior to such an analysis, what should be noted about recognition level is that the binding effect for each type of recognition is phenomenologically distinct. Affinity binding is based on extensive personal knowledge and interaction. Solidarity binding is based on similarity *and* the salience of that type of similarity. Generalized binding is based on an ideational (theory, theology or ideology) perspective or calculation that the recognition of rights is either normatively required, in the long-term interests of the actor, or both.

Reviewing the distinct bases of the bindings shows that, in practice, there are distinct differences in their bases. Therefore, individuals, groups and multiscale actors may have divergent bases for the attribution of truth (or falsehood) to differently situated actors. Figure 4 shows one way of representing these differences. The unit circle is used

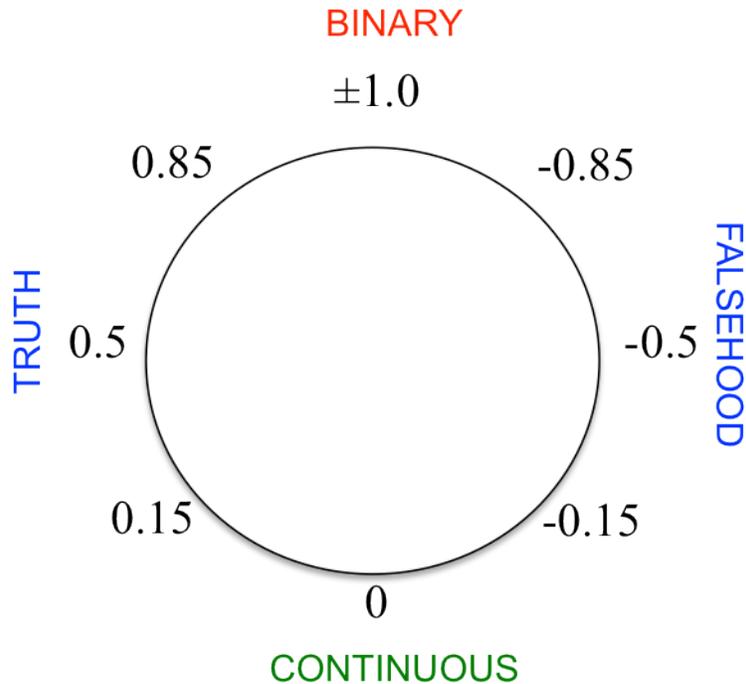


Figure 4. Representing Truth-Attribution with a Unit Circle

to combine two dimensions: 1) the distinction between the attribution of truth and the attribution of falsehood, and 2) the distinction between a binary (polar) definition of truth, as in the law of the excluded middle, and a graduated, continuous and/or probabilistic notion of truth.

Because, within a topos, the concept of truth is defined locally, each social actor of whatever scale can have a private method of attributing truth. Moreover, it may differ by recognition level, power differentials, in-group/out-group status, or a variety of other contingencies. For present purposes, the discussion will confine itself to differential practices in the logical style of the actor attributions based on role and recognition.

Table 1 shows one way that truth-attribution factors can be summarized. Specifically, the three styles of truth-attribution are (potentially) applied differentially

	Affinity	Solidarity	Rights
Enemy	75% Binary; 25% Indexed	Binary	Binary
Neutral	75% Smooth; 25% Indexed	25% Smooth; 75% Indexed	Indexed
Friend	Smooth	50% Smooth; 50% Indexed	Indexed

Table 1. Notional Truth-Attribution Style by Recognition Level

based on the role and recognition level of an alter. Here it is presumed that smooth attribution is preferable to binary, with indexed or incremental in an intermediate position. Therefore, is a continuous or probabilistic attribution of truth more likely based upon personal contact, similarity or an inclusive worldview? And, similarly, is a continuous or probabilistic attribution of truth more likely based upon the actor's characterization of an alter's role (friend, neutral or enemy)? Historically, of course, a wide range of patterns has been empirically effective. What the application of topos theory allows us to do, in a systematic way, is to explore, via simulation or extended analysis, the effects of diverse truth-attribution practices of a collection of actors, regarding these practices as an actor reference frame that is sometimes shared.

Figure 5 shows the same dimensions as Table 1, with various assumptions regarding truth-attribution styles and/or practices. As the truth-attribution key indicates, binary logic (e.g., true or false, but also, in particular circumstances: "all or nothing", "life or death", "kill or be killed", "fish or cut bait", "win or lose", "laugh now or cry later") is indicated by the color red. The finest-grained attribution of truth or falsehood (e.g., in the conundrum expressed by Oscar Hammerstein's lyric in the *King and I*: "Some things *nearly* so, others *nearly* not ...". [Asch 2008:348]) is indicated by the color blue, with green representing the indexed or incremental position. The two actors on the

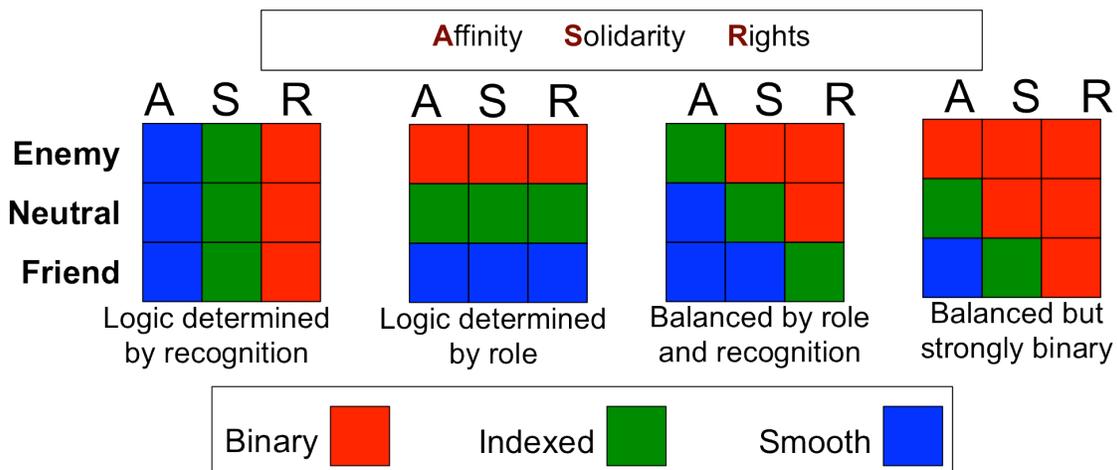


Figure 5. Truth Granularity by Role and Recognition Levels

left attribute truth strictly by recognition or role, respectively, while the third actor has a balanced style and the fourth, on the far right, is overwhelmingly binary. The examples are hypothetical, but the representation of diverse forms of logical practices will be a great benefit for those constructing computational models of social processes under varying assumptions. Of course, it will also be possible to shape distributions of truth-attribution practices based upon additional considerations.

THEORIES OF CONFLICT

During the decade following World War II, a single sociological theory came to define the dominant paradigm of the discipline. Referred to as functionalism, or structural-functionalism (Parsons 1953; Parsons & Smelser 1956), the theory, which focused on the sources of social order, and synthesized a number of classical social theories, became regarded as definitive.

In the later 1950s, however, the theory was subject to extensive criticism. The basic objection was that functionalism failed to adequately address social conflict and change (Lockwood 1956; Dahrendorf 1958; 1959; Horton 1966). Functional theorists attempted to respond by showing how the theory could account for change (Merton 1957; Cancian 1960) and/or incorporate conflict (Coser 1964), but they addressed only limited conflict and continuous change and, thus, did not fully respond to the critics.

Several efforts were made to synthesize consensual and conflictual processes (van den Berghe 1963; Johnson 1966; Buckley 1967), but the results were not deemed to be compelling. Rather than producing an integrated paradigm, social theory continued to fragment into ever more specialized forms including separate theories of conflict (Collins 1975; 1986). While a variety of scholars made significant progress in conflict theory, *per se* (see, Collins 1994:47-120), these advances have not produced a general theory. Not even Collins' unique prediction of the collapse of the USSR (1986) brought conflict models to the forefront of social theorizing. More generally, the failure to define an integrated theoretical paradigm weakened the explanatory power of subsequent social science.

Social Model of War. More recently, Clausewitz's theory of war is serving as a catalyst for the rearticulation of social conflict theories, in this case emerging from the disciplines of political science and international relations (cf., Echevarria 2007; Herberg-Rothe 2007; Strachan 2007). Several characteristics of this theory contribute to its centrality to a more general theory of conflict. First, Clausewitz's analysis of war is inherently multi-scale. Its focus ranges from the most detailed historical cases to the most abstract theory. Because social phenomena are high-dimensional and manifest great fluidity, their scope makes the identification of unambiguous generalization difficult (as Clausewitz acknowledged). When it is achieved, the ability to effectively model multi-scale interactions will greatly strengthen social theory and analytical tools based on it.

Second, Clausewitz recognized the need to break down false distinctions. Perhaps, the best-known example is his characterization of war as a continuation of politics by additional means. Rather than trying to limit the complexity of his theory by treating war as a separate, *sui generis* phenomena, he accepted the more challenging task of conceptualizing how politics and war co-define (and co-refine) each other. Sharma (2008) calls this a "social theory of war", which seems apt, as long as we also recognize that the social theory of war (and conflict) continues to evolve.

Sharma suggests that, in a social theory of war, no distinction should be made between international and civil wars. Certainly, throughout history, each type of war has repeatedly transmogrified into the other. Thus, a second potentially misleading distinction has been identified, with the focus shifting to more inclusive forms of interaction.

Third, as a means of identifying historical reference points, Clausewitz reasons by using pure or ideal concepts (which he often calls extreme or absolute). Examples include absolute war, in which pure violence (as a reference point) has become detached from political purpose. As Sharma (2008) suggests, patterns of empirical violence can be assessed relative to a state of pure violence that has no logical limit. While Clausewitz is not as methodologically rigorous in the formulation of ideal types as Weber (1949; Burger 1987), he nonetheless lays a foundation for a more systematic analysis. Ideal types, such as those introduced by Clausewitz, point toward the identification of social invariants, relative to which theories can be cogently formulated.

Sharma's social theory of war advances the prospective Clausewitzian contribution to a theory of social conflict by asserting that the scale and intensity of warfare, whether international or domestic, is related to the amount of institutional upheaval that is sought or implied. Sharma summarizes this as the difference between 'who rules' (suggesting limited restructuring) versus 'what rules' are appropriate (implying increasing levels of transformation). This formulation can be further extended by observing that, when empirical patterns approach absolute violence (as occurs in genocidal situations), the question becomes 'who lives'. In each case, the scale of conflict is defined by observed patterns of violence and the degree of mobilization.

The 'what rules' question can be addressed in fine-grain logic that topos theory makes available. The established power will have a particular configuration of logics based on affinity, group and generalized criteria.

CONCLUSION

The innovations of abstract mathematics have many prospective applications in social analysis. The present paper introduces and illustrates that potential, especially through the use of topos theory in the representation of social theory with applications of computational models.

When topoi are used to model social phenomena, they provide the expressiveness to address subtle structures and dynamic processes. The area of social recognition theory has been used to show that topos theory can provide rich articulation of complex social theories that include various kinds of subtlety.

Such representational capabilities have the potential to increase the effectiveness of computational social modeling. The fact that these representations provide a basis for formal validation is an additional advantage for the use of category-theoretical methods in the social sciences.

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