Analyzing Oligopolistic Electricity Market Using Coevolutionary Computation

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***Abstract*—This paper presents a new unified framework of electricity market analysis based on coevolutionary computation (CCEM) for both the one-shot and the repeated games of oligopolistic electricity markets. The standard Cournot model and the new Pareto improvement model are used. The linear and constant elasticity demand functions are considered. Case study shows that CCEM is highly efficient and can handle the nonlinear market models that are difficult to be handled by conventional methods. The framework presented in this paper can help to overcome the difficulties of demand function specification encountered by the Cournot models. CCEM is found to be an effective and powerful approach for electricity market analysis.**

***Index Terms*—Coevolutionary computation (CCEM), Cournot oligopoly, market simulation, Pareto improvement.**

I. Introduction

IN RECENT years, different equilibrium models have been used in the analysis of strategic interaction between participants in an electricity market, including the oligopoly models of Cournot, Betrand, Stackelberg, supply function equilibrium (SFE), and Collusion [1], [2]. Among them, the Cournot and SFE models are the most extensively used models for analyzing pool-based electricity markets.

The Cournot oligopoly model assumes that strategic firms employ quantity strategies: Each strategic firm decides its quantity to produce, while treating the output level of its competitors as a constant. Hogan and Cardell *et al.* [3], [4] apply the Cournot quantity approach to a single-period market trading. The market model developed exploits the standard approach to interpreting a market equilibrium as defining the first-order conditions for a related optimization problem. Borenstein and Bushnell [5] simulate the California electricity market after deregulation as a static Cournot market with a competitive fringe. The model indicates the potential for significant market power in high demand periods. They discuss the weaknesses of concentration measures as a viable measure of market power in the electricity industry and propose a market simulation approach based on the Cournot–Nash concept [6]. The Cournot equilibrium in a transmission-constrained network is investigated in [7].

The general SFE model was introduced by Klemperer and Meyer [8] and first applied to electricity market analysis by Green and Newbery [9], in which each firm chooses as its strategy a "supply function" relating its quantity to its price. Green and Newbery [9] assume that each firm submits a smooth supply schedule, relating amount supplied to marginal price and look for the noncooperative Nash equilibria of the spot market, which implies a high markup on marginal cost and substantial deadweight losses. Green [10] further models the effect of three policies that could increase the amount of competition in the electricity spot market in England and Wales by the SFE approach. Hobbs *et al.* [11] formulate the problem of calculating SFE in the presence of transmission constraints as a mathematical program with equilibrium constraints (MPEC). Rudkevich *et al.* [12] present an analysis that estimates the price of electricity dispatched and sold through a poolco, using a closed-firm mathematical formula derived from the analytical concept of SFE models. Baldick [1] uses an example from the literature to compare Cournot and SFE models of bid-based electricity markets with and without transmission constraints. He has demonstrated that the parameterization of the supply function model has a significant effect on the calculated results. Day and Hobbs [13] present a conjectured supply function (CSF) model of competition among power generators on a linearized dc network.

The common feature of these models is to find the market equilibrium under certain assumptions. Some models are successful in studying oligopoly behavior in real electricity markets, but they are generally difficult in obtaining close-form solutions when considering the practical issues in electricity market, such as nonconvexity and discontinuity of cost function and inter-temporal scheduling of generators. Even worse, market equilibrium may not be unique in some circumstances [1], [5], [6], [8], [14]. Thus, a new method needs to be explored to analyze market equilibrium in more realistic and complicated conditions.

There exist at least four distinct approaches to answer the problems concerning strategic interaction in an electricity market [11]. The first is empirical analysis of existing markets. The second approach is to model and solve the analytical market equilibria. The other two relatively new approaches are laboratory experimentation [15] and agent-based market simulation [16], [17].

A recent fast-developing area rests on the application of coevolutionary computation (CCEM) [18] to electricity market analysis. Coevolutionary computation is an extension of conventional evolutionary algorithms (EAs). It models an ecosystem consisting of two or more species. Multiple species

Manuscript received August 10, 2004; revised September 30, 2005. This work was supported in part by Research Grants Council, Hong Kong (PolyU 5214/03E), in part by the Hong Kong Polytechnic University, and in part by the National Natural Science Foundation of China (no. 50207007). Paper no. TPWRS-00433-2004.

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Digital Object Identifier 10.1109/TPWRS.2005.862005

in the ecosystem coevolve and interact with each other and result in the continuous evolution of the ecosystem [18]–[20].

Price uses a coevolutionary genetic algorithm (GA) to model several standard industrial organization games, including a simple model of an electricity pool [21]. He demonstrates that coevolutionary computation has a potential role in applied work requiring detailed market simulation through several examples. Cau develops a coevolutionary approach to study the dynamic behavior of participants over many trading intervals [22]. In a recent paper, Son and Baldick proposed a hybrid coevolutionary programming approach for Nash equilibrium search in games with local optima [23]. The transmission-constrained electricity market examples are studied.

The coevolutionary computation model can be regarded as a special form of the agent-based computational economics model. Agent-based computational economics (ACE) is a computational study of economies modeled as dynamic systems of interacting agents [24]. Each participant in the market under investigation is represented by an agent in the model. The ability of ACE to capture the independent decision-making behavior and interactions of individual agents provides a very good platform for the modeling and simulation of electricity markets. The coevolutionary computation model has many advantages compared to other agent-based models in that it uses the well-developed GA (or other EAs) for agent "strategic learning." GAs are found to be effective analogues of economic learning [25].

A "cooperative coevolutionary algorithm" for unit commitment is first presented by the first author, which can be thought as a simulation of perfect competitive electricity market with price-taking generating units [26]. The coevolutionary computation-based approach is systematically extended to the analysis of oligopolistic electricity markets. This paper presents a new unified framework of CCEM for both the one-shot and the repeated games of oligopolistic electricity markets and shows its high computational efficiency and great potentials in practical application. The standard Cournot model and the newly presented Pareto improvement model are combined together to form an integrated framework to analyze both the one-shot and the repeated electricity market games.

The rest of the paper is organized as follows: Market models used for analysis are presented in Section II. The framework of the CCEM is proposed in Section III and then validated by a simple example in Section IV. Finally, Section V concludes the paper.

II. Market Model Formulation

This paper starts from the Cournot oligopoly model. In the standard static Cournot game, the firms make their quantity decisions at the same time, and each firm behaves independently to maximize its own profit [27]. It is well known that the Cournot oligopoly has a Nash equilibrium in which every firm has its maximum profit while assuming that other firms have fixed their outputs. It is apparent that when all firms have reached such a point, none has any incentive to change unilaterally, and so the situation is viewed as the market equilibrium. However, the situation will be quite different in existence of repeated games. Because the Nash equilibrium is generally Pareto-inefficient, all firms can increase their payoff by jointly modifying their decisions. Thus, a set of "Pareto improvement" solutions exist [27], which can be the results of "tacit collusion." Although explicit collusion is illegal, the danger of tacit collusion always exists [28]. In this section, a new model is formulated to consider this situation. For comparison, a "cooperative" collusion model is also formulated, in which all firms simply work together as a pure monopoly.

A. Standard Cournot Model

Suppose there are I producers; each producer has its cost

$$C_i(q_i) = a_i q_i^2 + b_i q_i + c_i \quad i = 1, \ldots, I \tag{1}$$

$$\underline{q}_i \le q_i \le \overline{q}_i \quad i = 1, \ldots, I \tag{2}$$

where q_i is the quantity generated by producer i; a_i, b_i, and c_i are the coefficients of the producer's cost function with $a_i > 0$, $b_i \ge 0$, and $c_i \ge 0$; and $\underline{q}_i$ and $\overline{q}_i$ are the lower and upper limits of the quantity, respectively.

If it is assumed that there is negligible transmission loss, then the aggregate demand Q will be equal to the total output of all the producers in the market, as shown in the following:

$$Q = \sum_{i=1}^{I} q_i. \tag{3}$$

The market price p depends on Q, and their relationship is represented by the inverse linear demand function in (4-1) or the inverse constant elasticity demand function (4-2) below.

Linear demand:

$$p = A - B \cdot Q. \tag{4-1}$$

Constant elasticity demand:

$$p = \lambda Q^{\frac{1}{\varepsilon}} \tag{4-2}$$

where A and B are the positive coefficients of the linear demand function; λ is a positive constant; and ε is the elasticity that typically will be negative.

The Cournot model is criticized for the problem of sensitivity to elasticity of market demand, which is difficult to be specified in electricity market. To overcome this problem, the constant elasticity demand model is also adopted in this paper, in addition to the conventional linear demand model. Different demand elasticity can be simulated by varying the value of ε to obtain the results nearer to the practical situation.

The profit of the producer i is

$$\pi_i(q_1, \ldots, q_I) = p q_i - C_i(q_i). \tag{5}$$

For simplicity of notation, let

$$\vec{q} = (q_1, \ldots, q_I) \tag{6}$$

and

$$\vec{\pi} = (\pi_1, \ldots, \pi_I). \tag{7}$$

So, if a vector of quantity $\vec{q}^* = (q_1^*, \ldots, q_I^*)$ is a Cournot-Nash equilibrium, it must satisfy

$$\begin{aligned}\pi_i(\vec{q}^*) &= \pi_i\left(q_1^*, \ldots, q_I^*\right)\\ &= \max_{q_i} \pi_i\left(q_1^*, \ldots, q_{i-1}^*, q_i, q_{i+1}^*, \ldots q_I^*\right)\\ &\quad i = 1, \ldots, I.\end{aligned} \tag{8}$$

Therefore, the following first-order condition must be satisfied:

$$\left.\frac{\partial \pi_i\left(q_1^*, \ldots, q_{i-1}^*, q_i, q_{i+1}^*, \ldots, q_I^*\right)}{\partial q_i}\right|_{q_i = q_i^*} = 0, \quad i = 1, \ldots, I. \tag{9}$$

For the linear demand function in (4-1), (9) can be explicitly expressed as (10), and the Cournot equilibrium can be easily obtained

$$2(B + a_i)q_i^* + B\left(\sum_{j \neq i} q_j^*\right) = A - b_i. \tag{10}$$

However, for the constant elasticity demand function in (4-2), the first-order condition will be a complicated nonlinear equation set.

B. Pareto Improvement Model

The Cournot–Nash equilibrium is the result of the one-shot noncooperative game. In the real electricity market, the auction is repeated daily in a pool-based electricity market. This raises the question of what the outcomes will be in the repeated games. The results are given by numerous folk theorems in game theory [29]. The most important one concerning the oligopoly game is presented by Friedman [30]. Since the Cournot–Nash equilibrium is generally not Pareto optimal, it is dissatisfactory to be used as a viable outcome in the repeated oligopoly game. Under some assumptions, the Pareto optimal solutions can be the noncooperative equilibria in infinitely repeated games (supergames) [29], [30]. These equilibria can be achieved by "tacit collusion" among the firms. Since there are many Pareto optimal solutions that can be the noncooperative equilibria of an infinitely repeated game, a particular solution is dependent on any additional conditions imposed on the game. This paper will only focus on searching for the general Pareto improvement solutions of the Cournot oligopoly. The definition for Pareto optimality is given in Definition 1 below. The Pareto improvement model is given after Definition 2 below, which defines Pareto dominance.

Definition 1 (Pareto Optimality): A point $\vec{q}^* = (q_1^*, \ldots, q_I^*)$ is **Pareto optimal** if for every possible $\vec{q} = (q_1, \ldots, q_I)$ and $i = 1, \ldots, I$, either

$$\pi_i(\vec{q}) = \pi_i(\vec{q}^*) \quad i = 1, \ldots, I \tag{11}$$

or there is at least one $i(1 \leq i \leq I)$ such that

$$\pi_i(\vec{q}) < \pi_i(\vec{q}^*). \tag{12}$$

Definition 2 (Pareto Dominance): A vector $\vec{\sigma} = (\sigma_1, \ldots, \sigma_I)$ is said to dominate $\vec{\tau} = (\tau_1, \ldots, \tau_I)$ (denoted by $\vec{\sigma} \succ \vec{\tau}$) if and only if

$$\sigma_i \geq \tau_i \; i = 1, \ldots, I \tag{13}$$

and there is at least one $i(1 \leq i \leq I)$ such that

$$\sigma_i > \tau_i. \tag{14}$$

For clearance, we denote the Cournot–Nash equilibrium by $\vec{q}^{*(n)} = (q_1^{*(n)}, \ldots, q_I^{*(n)})$ and Pareto optimal solutions by $\vec{q}^{*(p)} = (q_1^{*(p)}, \ldots, q_I^{*(p)})$ hereafter.

The objective of the Pareto improvement model is to find the Pareto improvement set (denoted by Ω) of Cournot oligopoly model (1)–(5), i.e., the set of Pareto optimal solutions with some producers better off and none of the producers worse off compared to outcome at the Cournot–Nash equilibrium, which can be written as

$$\begin{aligned}\Omega = \Big\{\vec{q}^{*(p)} \,|\, \pi_i\left(\vec{q}^{*(p)}\right) \geq \pi_i\left(\vec{q}^{*(n)}\right) \text{ and}\\ \nexists \vec{q} = (q_1, \ldots, q_I) \pi(\vec{q}) \succ \pi\left(\vec{q}^{*(p)}\right) 1 \leq i \leq I\Big\}\end{aligned} \tag{15}$$

where $\nexists$ means "do not exist."

The Pareto improvement model (1)–(5) and (15) forms a classical multiobjective optimization problem with inequality constraints. Apparently, at a solution of the multiobjective optimization problem, no producer can achieve more profit without decrease of other producers' profits, and all producers' profits are more than those at the Cournot–Nash equilibrium, so we call it "Pareto improvement solution." Based on the theory in [29] and [30], any such solutions can be the result of the repeated oligopoly game.

C. Collusion Model

Different from tacit collusion achieved in repeated games, explicit collusion is generally illegal under antitrust law. The collusion model is formulated in this paper for comparison.

By collusion, the firms work together to extract as much total profit as they can from the market [27]. Therefore, the objective of the collusion model is

$$\max_{(q_1, \ldots, q_I)} \sum_{i=1}^{I} \pi_i(q_1, \ldots, q_I). \tag{16}$$

Equations (1)–(5) and (16) compose the collusion model, which is an ordinary multivariable optimization problem. The solutions of the model can be denoted by $\vec{q}^{*(c)} = (q_1^{*(c)}, \ldots, q_I^{*(c)})$.

III. Electricity Market Analysis Based on Coevolutionary Computation

This section introduces a new framework of electricity market analysis based on CCEM.

A. General Framework of CCEM

The above market models can be illustrated by Fig. 1. Each producer submits its optimal trading strategy to the power exchange (PX). The trading strategy is different for different market model. Here for the standard Cournot and Pareto improvement models with quantity decision participants, the trading strategy is the quantity to be generated. PX then calculates the market price using (3) and (4-1) or (4-2) in Section II-A according to the demand characteristics and market rules. Each producer can calculate its profit with the market price and its trading strategy according to (5). Fig. 1 suggests an agent-based simulation method for electricity market analysis, in which each participant in the market under investigation is represented by an agent in the model. Each agent makes its decisions based on the knowledge of itself and the environment. The agents interact with each other through the system model. Here the system model consists of the market rules and demand function.

This paper presents an agent-based simulation approach of electricity market based on CCEM. CCEM is developed from traditional EAs, which simulates the coevolutionary mechanism in nature and adopts the notion of ecosystem. The agents simulate the species in an ecosystem. Multiple species coevolve and interact with each other and result in the continuous evolution of the ecosystem. The species are genetically isolated—individuals only mate with other members of their species. They interact with one another within a shared domain model. CCEM is a relatively new area in the research of evolutionary computation. Its theory and applications are still rapidly developing [18]–[20].

The basic CCEM model is shown in Fig. 2 [18], which is an analogue of Fig. 1. Each producer in Fig. 1 is represented by a species in the ecosystem. Each species evolves a bundle of individuals, which represent the candidate trading strategies of the corresponding producer. PX is modeled with the domain model. Each species is evolved through the repeated application of a conventional EA. Fig. 2 shows the fitness evaluation phase of the EA from the perspective of species 1. To evaluate an individual (trading strategy) from species 1, collaboration is formed with representatives (representative trading strategies) from each of the other species. The domain model solves for the system variable (market price). Then species 1 can use the system variable to evaluate the fitness of its individual. Here the fitness is the profit of the corresponding producer. There are many possible methods for choosing representatives with which to collaborate. An obvious one is to simply let the current best individual from each species be the representative, and an alternative one is to randomly select an individual from each species to be the representative [19].

The pseudo-code of a coevolutionary genetic algorithm (CGA) is given in Figs. 3 and 4, in which the evolution of each species is handled by a standard GA. Therefore, a species just means a population of GA in this algorithm and the procedures of the simple genetic algorithm (SGA) proposed in [31] are used in CGA.

The key variables, data structures, and procedures of CGA are defined as follows:

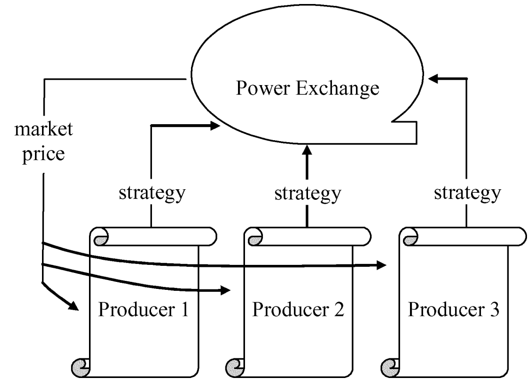

Fig. 1. Illustration of electricity market models.

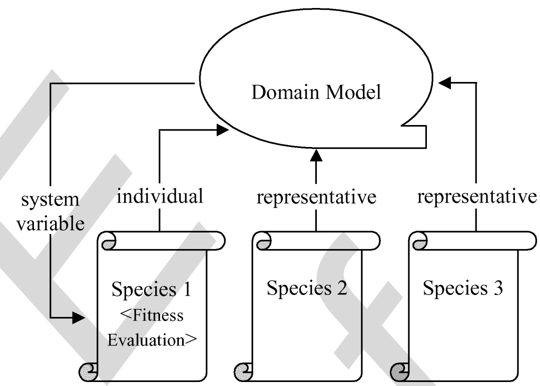

Fig. 2. Framework of CCEM model.

k = 0
for each species i do
  begin
    initialize the species population Pop_0^i
    evaluate fitness of each individual $Ind_0^{i,n}$ *in* Pop_0^i
    choose a representative Rep_0^i *from* Pop_0^i
  end
while termination condition = false do
  begin
    for each species i do
      begin
        reproduction from Pop_k^i *to get* $Mate_k^i$
        crossover and mutation from $Mate_k^i$ *to get* Pop_{k+1}^i
        evaluate fitness of each individual $Ind_{k+1}^{i,n}$ *in* Pop_{k+1}^i
        choose a representative Rep_{k+1}^i *from* Pop_{k+1}^i
      end
    k = *k* + 1
  end

Fig. 3. Pseudo-code of CGA.

k Generation counter of the evolutionary process.

i Sequence number of species, which is corresponding to the producer sequence number. There are totally I species in the ecosystem.

for each individual $Ind_k^{i,n}$ *in* Pop_k^i *do*
begin
form a collaboration with representatives from other species, which is $\left(Rep_k^1, Rep_k^2, \ldots, Ind_k^{i,n}, \ldots, Rep_k^I\right)$
decode the collaboration and form a tentative vector of generating quantities $\left(\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_i, \ldots, \hat{q}_I\right)$ *of all producers*
calculate market price with (3), (4-1) or (4-2)
calculate profit of producer i *with* (5), *which is used as the fitness value of* $Ind_k^{i,n}$
end

Fig. 4. Pseudo-code of fitness evaluation procedure.

n — Sequence number of individuals in a species.

Chromosome — 16-bit binary string (two bytes) is used to encode the quantity q_i generated by the corresponding producer i (namely, the producer's trading strategy) for each species i, which is called the chromosome.

Fitness — Fitness value of a chromosome in species i is the profit of the corresponding producer i when its generating quantity q_i equals the decoded value of the chromosome.

Individual — nth individual $Ind_k^{i,n}$ of species i in the kth generation is composed of a chromosome, its decoded value, and fitness value.

Population — Population Pop_k^i of species i in the kth generation contains a number of individuals $Ind_k^{i,n}$, which is evolved by GA.

Initialization — Chromosomes of each species are randomly initialized to integers between 0 and 65 535 (two bytes). The initial population of species i is denoted by Pop_0^i.

Reproduction — Simple roulette wheel selection or the tournament selection is used for choosing members for the mating pool $Mate_k^i$ from Pop_k^i for species i in the kth generation. Fitter individuals have a higher number of offspring in the succeeding generation.

Crossover — Crossover operator takes two parent chromosomes and generates two offspring chromosomes by swapping some parts of their strings. A two-point crossover operator is used. This means that two crossover positions are selected, and the two parent chromosomes swap the strings between them with certain probability.

Mutation — Bit-wise mutation operator is used, in which each bit of the chromosome is randomly changing from 1 to 0 or vice versa with certain probability. The $k+1$th generation population Pop_{k+1}^i of species i is formed from the kth generation mating pool $Mate_k^i$ through repeated application of crossover and mutation operators.

Choice of representative — Representative Rep_k^i of species i in the kth generation is chosen from Pop_k^i with the aforementioned method.

Fitness evaluation — Process of evaluating fitness of individuals in Pop_k^i is shown in Fig. 4.

Termination condition — Simple termination condition is used that the number of generations reaches its preset upper limit.

GA is used to evolve each species in our model, which is an effective analogue of economic learning. Riechmann shows that economic learning via GA can be described as a specific form of an evolutionary game [25], which is a stochastic process repeatedly turning one population of individuals into another. Each repeated turn of GA consists of several stochastic processes, namely, reproduction, which is interpreted as learning by imitation; crossover, which is interpreted as learning by communication; mutation, which is interpreted as learning by experiment; evaluation, which is interpreted as playing the role of the market as an information revealing device; and selection, which decreases the number of unsuccessful trading strategies [25].

The ability of CCEM model to facilitate the individual participants' strategic learning and the interactions among the participants provides a very good platform for the modeling and simulation of electricity markets.

B. CCEM for Standard Cournot Model

For the Cournot model (1)–(10), the above CGA is used for simulation. In Cournot oligopoly, a producer i only needs to optimize its own profit π_i expressed by (5), with the quantity q_i as the decision variable. The simple roulette wheel selection is used in the reproduction operator of each species. A "greedy" method is used for selecting representatives [19]. In this method, the current best individual from each species is selected as the representative so as to facilitate the fast convergence of the simulation.

From the calculation process above, we can see that the species are coordinated by the market price p. When producer i changes its quantity q_i to gain more profit in (5), it will change the market price according to the inverse demand function in (4-1) or (4-2) and, in turn, changes the profits of other producers. Other producers will behave in the same way. The adjustment process will continue until no one can get more profit by changing its quantity without changes of the quantities of other producers and, in other words, the market reaches Cournot–Nash equilibrium.

C. CCEM for Pareto Improvement Model

The market simulation with Pareto improvement model (1)–(5), (15) is actually a multiobjective optimization problem. The above CGA is used for simulation. A producer i should optimize the Pareto optimal profits (see Definition 1 in Section II-B) for all producers in the market, with its own quantity q_i as the decision variable. Different from CCEM for standard Cournot model, a tournament selection is used in the reproduction operator of each species.

The main techniques used for multiobjective optimization problems are taken from Deb's NSGA-II [32]. The main features of NSGA-II include a fast nondominated sorting procedure, a fast crowded distance computation procedure, and a crowded tournament selection operator. The fast nondominated sorting procedure sorts a population into different nondomination levels. The fast crowded distance computation procedure is to estimate the density of solutions surrounding a particular solution in the population. The crowded tournament selection operator guides the selection process of the algorithm toward a uniformly spread-out Pareto optimal front. The crossover and mutation operators of NSGA-II are the same with standard GA, while the selection operator is different. A binary tournament selection operator is used, and the selection criterion is based on the crowded comparison operator. The details of NSGA-II can be found in [32].

The techniques of NSGA-II are applied to evolution of each species, and thus, the CCA in Figs. 3 and 4 becomes a multiobjective CCEM model.

IV. Case Study

To validate the models and algorithms described in this paper, the test case with three producers from [7] is used. The data of producers' costs in Table I of [7] are reproduced here in Table I. #1, #2, and #3 are the sequence numbers of the producers, respectively.

To illustrate the ability of CCEM to handle the nonlinear models and simulate different market situations, a constant elasticity demand function is used in simulation besides the linear demand function used in [7]. The demand function parameters are listed in Table II. ε is the elasticity of demand, which is typically negative. The demand with the elasticity greater than 1 in absolute value is an elastic demand and the elasticity less than 1 in absolute value for an inelastic demand. The demand curves used for simulation are shown in Fig. 5. The solid line is the linear demand curve; the dashed line is an inelastic demand with the elasticity -0.5; and the dotted line is an elastic demand with the elasticity -1.5. We choose the values of parameter λ so that the three different demand curves intersect at the point with $|\varepsilon| = 1$ on the linear demand curve.

A. CCEM for Standard Cournot Model

We apply the CCEM algorithm described in Section III-B for the standard Cournot model to the test case. The evolution process runs for a maximum 150 generations, with the population size 100. The elitist strategy preserving a single copy of the best individual is adopted. The crossover probability p_c is set to 0.9, and the mutation probability p_m is set to 0.06.

TABLE I
PRODUCERS' COST DATA

Producer's Cost Function	No.	a_i	b_i	c_i
$C_i(q_i) = a_i q_i^2 + b_i q_i + c_i$, $0 \le q_i \le 2000$	#1	0.007859	1.360575	9490.366
	#2	0.010526	-2.07807	11128.95
	#3	0.006478	8.105354	6821.482

TABLE II
DEMAND FUNCTION PARAMETERS

Simulation Cases	Inverse Demand Function	Parameters	
		A or λ	B or ε
Case 1: Linear	$p = A - B \cdot Q$	106.1176	0.0206
Case 2: Inelastic	$p = \lambda Q^{\frac{1}{\varepsilon}}$	3.52×10^8	-0.5
Case 3: Elastic	$p = \lambda Q^{\frac{1}{\varepsilon}}$	9969.7	-1.5

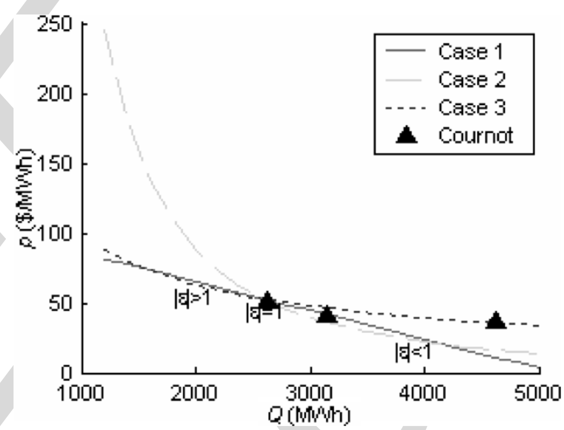

Fig. 5. Market demand and price of CCEM for standard Cournot model.

The Cournot–Nash equilibrium with linear demand function can be easily obtained by solving the linear equation system (10). However, the first-order conditions for Cournot–Nash equilibrium with constant elasticity demand function are generally nonlinear equation systems and difficult to be solved. This paper uses the test cases with constant elasticity demand functions to demonstrate the ability of CCEM for solving nonlinear Cournot models. The simulation results are listed in Table III. The demand curves and market demands and prices at Cournot–Nash equilibrium are also shown in Fig. 5. The symbol solid triangles denote the market aggregate demands and prices at the Cournot–Nash equilibria. They are on different demand curves, which denote the corresponding demand functions used for simulation. To verify the simulation results, an iterative Nash equilibrium search algorithm in [23] has been used to calculate the Cournot–Nash equilibria. The results are the same as those in Table III obtained using CCEM.

From the simulation results, we can observe that CCEM can find the Cournot–Nash equilibrium in all cases. The Cournot–Nash equilibrium with inelastic demand has

TABLE III
COURNOT–NASH EQUILIBRIUM RESULTS

Simulation Cases	p (\$/MWh)	Q (MWh)	Producers' Result		
			No.	q_i (MWh)	π_i (\$)
Case 1: Linear	41.42	3140.4	#1	1103.1	25141
			#2	1044.4	22823
			#3	992.9	19874
Case 2: Inelastic	51.09	2624.9	#1	910.0	29253
			#2	886.4	27726
			#3	828.5	24340
Case 3: Elastic	35.87	4633.2	#1	1652.9	26082
			#2	1447.7	21751
			#3	1532.6	20518

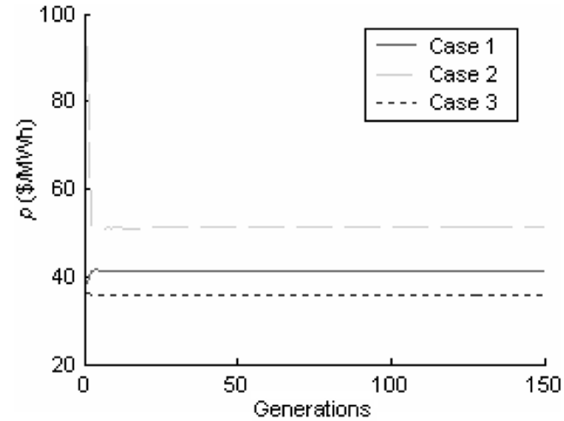

Fig. 6. Variation of market price in coevolution process.

higher market price and less generating output, while the Cournot–Nash equilibrium with elastic demand has lower market price and more generating output. These results are consistent with the experiences from the real markets.

The variations of prices with respect to the generation number of the simulation in the typical runs are shown in Fig. 6. We can see that the market prices converge to the Cournot–Nash equilibrium rapidly in all three cases. CCEM has high efficiency in the market simulations.

Besides, the simulations using different settings of p_c and p_m are also performed to observe the influence of the parameters of the algorithm. The results are listed in Table IV, where d_1, d_2, and d_3 are the Euclidean distances between the simulation results and the actual Cournot–Nash equilibrium quantities for the linear, inelastic, and elastic demand cases, respectively. Since the results are stochastic, the values of d_1, d_2, and d_3 are averaged over ten runs. p_c and p_m vary from 0.0 to 1.0. The other algorithm parameters are the same as those in the above simulations. From the results, it can be observed that the algorithm is more sensitive to p_m than p_c. The simulation results are the best when p_m is between 0.05 and 0.1 and p_c is between 0.5 and 0.9. The simulation results may imply that since the decision space is simple (one variable for each producer), mutation is the main operator to explore new search spaces.

In addition, we use a more realistic five-firm example system in [33] to further validate CCEM. The example system is based on the cost data for the five strategic firm industry in England and Wales subsequent to the 1999 divestiture. The firms' cost data are reproduced here in Table V. The linear demand function in [33] is used in the simulation, where $A = 350$ and $B = 10$. The simulation results are listed in Table VI. The iterative Nash equilibrium search algorithm proposed in [23] is also used to verify the simulation results. This paper shows that CCEM has the potentials in practical electricity market analysis.

TABLE IV
INFLUENCE OF ALGORITHM PARAMETERS ON SIMULATION RESULTS

p_c	p_m	d_1	d_2	d_3	p_c	p_m	d_1	d_2	d_3
0.0	0.000	32.28	21.25	32.57	0.7	0.000	0.39	1.02	0.51
	0.001	6.23	1.34	11.36		0.001	0.20	0.27	0.26
	0.005	2.07	0.66	0.53		0.005	0.14	0.10	0.07
	0.010	1.94	0.23	0.39		0.010	0.05	0.04	0.05
	0.050	0.06	0.05	0.06		0.050	0.06	0.04	0.05
	0.100	0.05	0.05	0.06		0.100	0.05	0.05	0.05
	0.500	0.24	0.14	0.13		0.500	0.25	0.16	0.14
	1.000	26.45	11.84	18.20		1.000	0.05	0.06	1.66
0.2	0.000	1.94	0.29	1.28	0.9	0.000	0.29	0.16	0.15
	0.001	0.69	0.94	0.69		0.001	0.49	0.12	0.12
	0.005	0.30	0.17	0.20		0.005	0.14	0.08	0.06
	0.010	0.06	0.18	0.19		0.010	0.07	0.04	0.05
	0.050	0.05	0.05	0.05		0.050	0.05	0.04	0.05
	0.100	0.05	0.05	0.07		0.100	0.05	0.05	0.05
	0.500	0.21	0.13	0.18		0.500	0.23	0.16	0.14
	1.000	0.07	0.07	4.39		1.000	0.07	0.07	0.63
0.5	0.000	0.18	0.42	0.11	1.0	0.000	0.17	0.15	0.09
	0.001	0.52	0.17	1.05		0.001	1.00	0.13	0.20
	0.005	0.85	0.08	0.09		0.005	0.24	0.08	0.06
	0.010	0.06	0.09	0.06		0.010	4.81	0.09	0.05
	0.050	0.06	0.05	0.05		0.050	0.05	0.05	0.05
	0.100	0.06	0.05	0.05		0.100	0.06	0.04	0.05
	0.500	0.28	0.18	0.17		0.500	0.16	0.19	0.16
	1.000	0.07	0.05	1.13		1.000	0.07	0.06	0.64

TABLE V
PRODUCERS' COST DATA

Producer's Cost Function	No.	a_i	b_i	c_i
$C_i(q_i) = a_i q_i^2 + b_i q_i + c_i$	#1	1.3435	12	0.0000
	#2	2.3075	12	0.0000
	#3	0.8945	8	0.0000
	#4	0.9650	8	0.0000
	#5	2.3075	12	0.0000

B. CCEM for Pareto Improvement Model

We use the algorithm described in Section III-C to perform the simulation. The algorithm parameters are the same as those in the standard Cournot model simulations in the last section.

The basic simulation results are listed in Table VII. Since the Pareto improvement solutions are not unique, the average and standard deviation values of market prices, market aggregate demand (sum of all producers' output), producer quantities, and profits are calculated. The data in Table VII are the average values with the standard deviation values in the brackets. For the cases with linear and elastic demand, the market prices are highly concentrated, with the average of 60.84 and \$53.41/MWh, respectively. These average prices are higher than the prices at the Cournot–Nash equilibria. The producer quantities are distributed, with the average values less than the Cournot–Nash equilibrium quantities. Although the individual producer quantities are distributed, the sums of all producers' output are concentrated, with the average values much less than that of the Cournot–Nash equilibrium. The producer profits are distributed, with the average values more than the Cournot–Nash equilibrium profits. The results show that the producers raise their profits by collectively withholding the quantities supplied and increasing the market price. Although the profits can be allocated among the producers quite differently, the market prices are similar.

For the case with the inelastic demand, the situation is quite different. From Table VII, we can see that the producer quantities are driven to near zero, and the market price gets impossibly high. In turn, the producer profits get impossibly high. Because the demand is inelastic, the producers can raise up the market price to an abnormal level by collectively withholding their outputs. This situation can also be observed in the real market.

Besides, the influence of the algorithm parameters on simulation results is also studied. Since the influence is similar to that of the standard Cournot model in Table IV, the results are not shown for brevity.

For comparison, the results of the collusion model in Section II-C are also listed in Table VIII, where the producers work cooperatively to maximize their total profit. The model can be solved by any ordinary nonlinear optimization method. For the cases with linear and elastic demand, the market prices and demands are near to the average values of the Pareto improvement results in Table VII. For the cases with the inelastic demand, the producers' quantities are driven near zero, and the market price and producers' profit become infinitely large. In fact, for the cases with linear and elastic demand, it is easy to prove that the collusion results are Pareto optimal. However, the collusion results do not necessarily belong to the Pareto improvement solutions. For the case with linear demand, we can see that although the total profit of all producers are the highest in the collusion result, the profit of producer 3 is lower than the Cournot–Nash equilibrium in Table III. This result cannot be achieved without any explicit agreement on division of total profit among the producers.

Using the producers' profits as the coordinates of a point in a three-dimensional space, the Pareto improvement solutions can be visualized as in Fig. 7. Only the Pareto improvement solutions with the linear demand function are shown in the figure. The Pareto front is determined by an iterative Monte Carlo (IMC) search method, and the crosses identify the Pareto optimal profit points. The big circle designates the collusion profits of each producer. The small circles designate the Pareto improvement profits. The symbol solid triangle designates the Cournot–Nash equilibrium profits, which is under the Pareto front. We can see that the collusion result lies on the Pareto front but outside of the Pareto improvement solutions. The Pareto improvement profits are fairly concentrated. The Pareto improvement profits obtained by CCEM coincide with the Pareto front determined by the IMC method, and thus, the correctness can be justified. The same conclusion can be reached in the elastic demand case.

V. Conclusion

This paper presents a CCEM model for electricity market analysis. The coevolutionary computation model can be looked at as a special form of the agent-based computational economics model. Each participant in the market is represented by a species in the model, and the interactions among market participants are embodied in the coevolutionary process of the species.

The standard Cournot model and the newly presented Pareto improvement model are combined together to form an integrated framework to analyze both the one-shot and the repeated electricity market games. The linear and constant elasticity demand functions are used in simulation.

It has been found that CCEM can find the Cournot–Nash equilibrium in standard Cournot model and the Pareto improvement solutions in Pareto improvement model with high efficiency. It can handle the nonlinear market models that are difficult to be handled by conventional methods. Cournot model is often criticized for the problem of sensitivity to specification of market demand, and thus, the framework presented in this paper using different demand functions can greatly help to overcome this difficulty.

TABLE VI
COURNOT–NASH EQUILIBRIUM RESULTS

Simulation Cases	p (£/MWh)	Q (GWh)	Producers' Result		
			No.	q_i (GWh)	π_i ($\times 10^3$ £)
5-firm system	80.40	26.96	#1	5.391	6.869
			#2	4.679	5.615
			#3	6.142	8.560
			#4	6.069	8.413
			#5	4.680	5.615

TABLE VII
PARETO IMPROVEMENT RESULTS

Simulation Cases	p ($/MWh)	Q (MWh)	Producers' Result		
			No	q_i (MWh)	π_i ($)
Case 1: Linear	60.84 (2.53)	2198 (123)	#1	780.0(95.0)	31928(3812)
			#2	741.2(93.2)	29533(3843)
			#3	676.8(87.8)	25766(3460)
Case 2: Inelastic	—	0.104 (0.016)	#1	0.031(0.000)	—
			#2	0.031(0.006)	—
			#3	0.042(0.015)	—
Case 3: Elastic	53.41 (2.34)	2559 (168)	#1	897.3(101.0)	30658(2870)
			#2	842.7(63.7)	28050(2336)
			#3	819.3(111.4)	25695(2968)

TABLE VIII
COLLUSION RESULTS

Simulation Cases	P ($/MWh)	Q (MWh)	Producers' Result		
			No.	q_i (MWh)	π_i ($)
Case 1: Linear	60.56	2211.5	#1	867.9	35968
			#2	811.3	32762
			#3	532.3	19265
Case 2: Inelastic	Infinite	Infinte-simal	#1	Infintesimal	Infinite
			#2	Infintesimal	Infinite
			#3	Infintesimal	Infinite
Case 3: Elastic	52.03	2652.4	#1	1016.8	33907
			#2	922.5	29830
			#3	713.0	21205

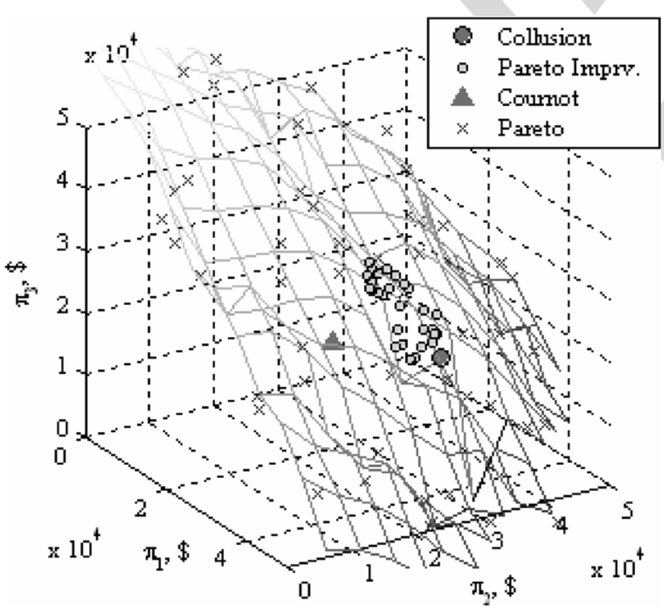

Fig. 7. Pareto improvement solutions.

APPENDIX

The techniques of NSGA-II are applied to the evolution of each species in the Pareto improvement model simulation. The main features of NSGA-II include a fast nondominated sorting procedure, a fast crowded distance computation procedure, and a crowded tournament selection operator. The main process of NSGA-II is described as follows [32].

Initially, a random parent population P_0 is created. The population is sorted into different nondomination levels [32]. Each solution is assigned a fitness (or rank) equal to its nondomination level (1 is the best level, 2 is the next-best level, and so on). Thus, minimization of fitness is assumed. At first, the usual binary tournament selection, recombination, and mutation operators are used to create an offspring population Q_0 of size N.

In the kth $(k \geq 1)$ generation of the evolutionary process, a combined population $R_{k-1} = P_{k-1} \cup Q_{k-1}$ is formed. The population R_{k-1} is of size $2N$. Then, the population R_{k-1} is sorted into different nondomination levels. Since all previous and current population members are included in R_{k-1}, elitism is ensured. The N members of the population P_k are chosen from the nondominated fronts in the order of their ranking. The new population P_k is now used for selection, crossover, and mutation to create a new population Q_k of size N. The NSGA-II procedure is also shown in Fig. 8.

In NSGA-II, a binary tournament selection operator is used, and the selection criterion is based on crowded comparison [32]. The diversity among nondominated solutions is introduced by using the crowded comparison procedure, which is used in the tournament selection and during the formation of population P_k $(k \geq 1)$.

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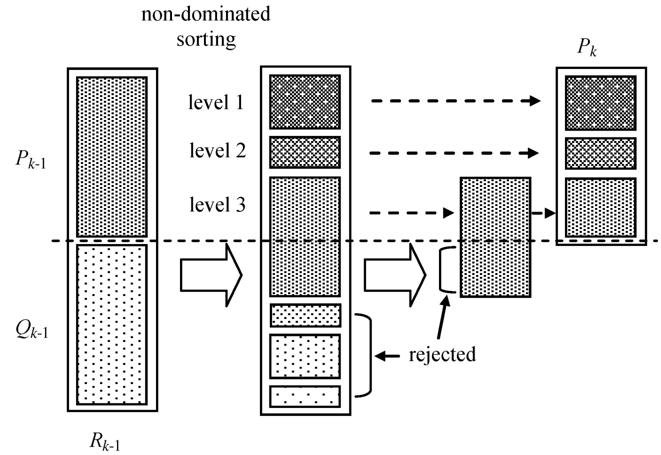

Fig. 8. NSGA-II procedure.

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