

# Transactive Energy Design for Integrated Transmission and Distribution Systems

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**Abstract**—The increasing deployment of distributed energy resources (DERs) is disrupting every aspect of power system operations, from retail distribution to wholesale production and transmission. This paper reports on the development of an agent-based test system enabling the study of new transactive energy system (TES) designs to ensure the reliable efficient operation of integrated transmission and distribution (ITD) systems with growing DER penetration. This ITD test system is used to explore the ability of a non-profit Distribution System Operator (DSO), participating within an ITD system, to use an innovative TES design to manage the power usage of DER devices in accordance with the local goals and constraints of DER owners, and to extract flexible ancillary services from DER devices in return for appropriate market-based compensation.

**Index Terms**—Transactive Energy System, transmission, distribution, distributed energy resources, flexibility, ancillary services

## NOMENCLATURE

### Parameters and Descriptors:

$b$	Generic symbol for a distribution grid bus
$B$	Generic symbol for a transmission grid bus
$\beta$	Household comfort parameter (Utils/hr( $^{\circ}$ F) $^2$ )
$D$	Generic symbol for a day
DAM	Day-Ahead Market (wholesale)
H	Generic symbol for an hour
$h$	Generic symbol for a household
NH	Total number of households
RTM	Real-Time Market (wholesale)
TB	Household bid parameter indicating bliss inside air temperature ( $^{\circ}$ F)
TMax	Household bid parameter indicating maximum acceptable inside air temperature ( $^{\circ}$ F)
TMin	Household bid parameter indicating minimum acceptable inside air temperature ( $^{\circ}$ F)
$\theta^s$	Scale factor (cents/kWh) for a household's ancillary service supply offer
$\theta^u$	Scale factor (cents/kWh) for a household's power usage demand bid
$U(TB)$	Maximum hourly comfort attainable by a household (Utils/hr)

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### Variables and Functions:

$A^{AvD}$	Average household DSO-allocation (\$/day)
$CM^{AvH}$	Average household comfort (Utils/hr)
LMP	Locational marginal price (\$/MWh)
$NEP^{AvH}$	Average household net energy payment (\$/hr)
$p^D$	Power dispatch (MW) determined in the day-ahead market
$p^{RT}$	Power dispatch (MW) determined in the real-time market
$p^{RET}$	Retail power usage (kW)
$\pi^{DA}$	Locational marginal price (\$/MWh) determined in the day-ahead market
$\pi^{RT}$	Locational marginal price (\$/MWh) determined in the real-time market
$\pi^{RET}$	Retail power price (cents/kWh)
$\pi^s(T_a)$	A household's minimum acceptable payment (cents/kWh) for ancillary service provision
$\pi^u(T_a)$	A household's maximum willingness to pay (cents/kWh) for power usage
$T_a(t)$	Household inside air temperature ( $^{\circ}$ F) at $t$
$T_m(t)$	Household inside mass temperature ( $^{\circ}$ F) at $t$
$t$	Time point (at granularity of seconds)
$V_{b,p}^{lo}$	Lower voltage magnitude limit violation (pu)
$V_{b,p}^{hi}$	Upper voltage magnitude limit violation (pu)
$\%VIB_b$	Voltage imbalance (%)

## I. INTRODUCTION

NET load (i.e., load minus non-controllable generation) is becoming increasingly volatile due to two recent developments. First, variable energy resources such as wind and solar power are increasingly being substituted for thermal generators in response to growing concerns regarding environmental pollution [1]. Second, technological developments such as advanced metering and intelligent devices are permitting retail customers to become more active participants in power system transactions [2].

This increased volatility makes it difficult for system operators to maintain a continual real-time balance between net load and power supply, an essential physical requirement for the reliable operation of power grids. Since large thermal generators (especially nuclear and coal-fired) tend to have slow ramping capabilities, they are not an effective means for countering real-time fluctuations in net load. Although large-scale energy storage devices could in principle be deployed to offset these real-time fluctuations, to date this deployment has not been cost effective.

Responding to this concern, the GridWise Architecture Council [3] has advocated the adoption of *Transactive Energy System (TES)* designs for power systems. TES designs are collections of economic and control mechanisms permitting the dynamic balancing of power demands and supplies across an entire electrical infrastructure, using buyer and seller reservation values<sup>1</sup> as key operational parameters. The ultimate TES objective is to achieve “a loosely coupled set of controls with just enough information exchange to allow for stability and global optimization through local action” [3, p. 10].

To date, TES researchers have primarily focused on the implementation of TES designs for distribution systems. The goal of these designs is to facilitate the reliable and efficient management of prosumer-owned *distributed energy resources (DERs)*. Prosumers are decision-making entities that function either as pure power producers, or as pure power consumers, or as hybrids that switch between production and consumption of power depending on local conditions. DERs include small-scale generators (e.g., rooftop solar panels), small-scale energy storage systems, plug-in electric vehicles, and household appliances with energy storage capabilities and flexible power requirements that permit them to function as virtual batteries.

For example, PowerMatcher [4] is a TES design originally developed by Koen Kok [5] in collaboration with industry partners. PowerMatcher relies on a bid-based market mechanism to balance power supplies and demands within a self-sufficient distribution system that includes both distributed generation and loads arising from appliances and other physical devices.

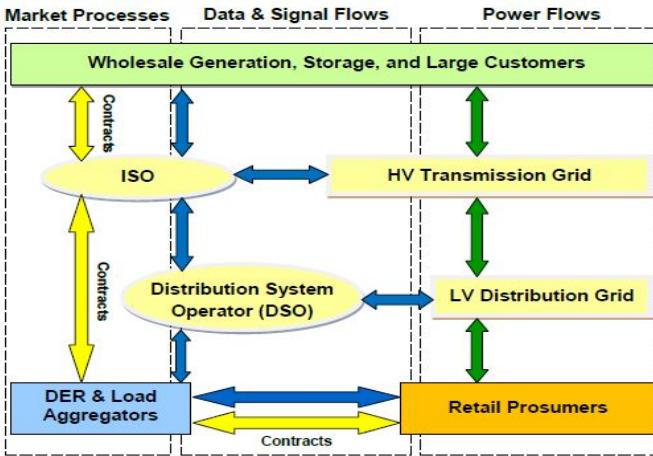


Fig. 1. TES designs can induce tight linkages between transmission and distribution level operations.

However, as depicted in Fig. 1, TES designs implemented within *integrated transmission and distribution (ITD)* systems can induce tight linkages between transmission and distribution level operations through market processes, two-way data and signal flows, and two-way power flows. Full performance evaluation of such designs requires careful consideration of

<sup>1</sup>A *buyer's reservation value* for a good or service at a particular point in time is defined to be the buyer's maximum willingness to pay for the purchase of an additional unit of this good or service at that time. A *seller's reservation value* for a good or service at a particular point in time is defined to be the minimum payment that the seller is willing to receive for the sale of an additional unit of this good or service at that time.

these linkages, including the manner in which these linkages lead to ITD feedback effects over successive days of operation.

Thus, researchers exploring TES designs for ITD systems face five critical challenges:

- The validation of TES designs for ITD systems prior to real-world implementation requires an ITD test system permitting the high-fidelity modeling and simulation of physical attributes, institutional arrangements, and decision-maker behaviors and methods.
- This ITD test system should permit the modeling of ITD systems as open-ended dynamic systems operating over successive days in order to support full dynamic performance evaluation for proposed TES designs.
- This ITD test system should permit careful modeling of linkages between transmission and distribution systems.
- This ITD test system should permit careful evaluation of the physical viability of grid operations and the economic viability of all participants taking their local objectives and constraints into account.
- This ITD test system should easily scale to permit consideration of TES designs for the procurement of power and ancillary services from DERs as the number and diversity of these DERs continues to increase.

Fortunately, *agent-based modeling (ABM)* is well suited for addressing these five challenges. As detailed in [6], [7], TES researchers are increasingly turning to ABM tools in an attempt to bridge the gap between conceptual TES design proposals and validated real-world TES implementations.

This paper reports on the development of *ITD Test System (v3.0)*, an agent-based computational platform permitting careful performance evaluation of TES designs for ITD systems with massively distributed DERs. ITD Test System (v3.0) permits the empirically-based modeling of a centrally-managed wholesale power market operating over a transmission grid linked to one or more distribution systems, each consisting of a collection of DERs operating over a distribution grid.

Section II summarizes existing demand-response research. The basic features of ITD Test System (v3.0) are explained in Section III and illustrated in Section IV for a particular TES design called the Six-Step PowerMatcher Design. ITD Test System (v3.0) is used in Sections V and VI to develop and implement more elaborate ITD Test Cases focusing on the reliability and welfare performance of the Six-Step PowerMatcher Design within an ITD power system. Concluding remarks are given in Section VII. Technical details regarding performance metric construction and household thermal dynamics are relegated to Appendix A and Appendix B, respectively.

## II. RELATIONSHIP TO PREVIOUS RESEARCH

As discussed more fully in [7]–[10], recent breakthroughs in metering technology, referred to as *Advanced Metering Infrastructure (AMI)*, have radically improved the potential for more active customer participation in power system operations. In particular, AMI enables the implementation of *demand-response (DR)* designs intended to encourage fuller demand-side participation in power system operations. Looking ahead to increased AMI penetration, power system researchers are exploring three basic types of DR designs:

(i) *Incentive-Based Load Control*: Down/up adjustments in the power usage of household and business devices are undertaken, either in response to direct requests from designated parties,<sup>2</sup> or via device switches under the remote control of designated parties, with compensation at administratively set rates.

(ii) *Dynamic Pricing*: Down/up power usage adjustments are undertaken by households and/or businesses in response to changes in power prices communicated to them by designated parties.

(iii) *Transactive Energy System (TES)*: Demands and supplies for power and ancillary services by households and businesses are determined by decentralized bid/offer-based transactions.

A key goal of type-(i) DR designs is to permit ancillary services to be extracted from demand-side resources in support of system *reliability*. A key goal of type-(ii) DR designs is to enhance system *efficiency* by permitting household and business customers to purchase power at its true marginal cost. A key goal of type-(iii) DR designs is to enhance the reliability and efficiency of system operations by enabling a balancing of demands and supplies for power and ancillary services across an entire electrical infrastructure on the basis of household and business reservation values.

Another critical distinction among these DR designs is their communication structure. The first two types of DR designs are based on one-way communication of price/control signals to devices. In contrast, TES designs are based on two-way communication.

More precisely, TES researchers are currently exploring two general types of TES designs, each based on two-way communication [11]–[13]. Under *peer-to-peer* TES designs, retail customers repeatedly communicate their current bids and offers directly to each other. Under *aggregator* TES designs, retail customers repeatedly communicate their current bids and offers to one or more designated “*aggregators*” that then use aggregated forms of these bids and offers to communicate price signals back to the residential customers.

Two serious problems have been observed for type-(i) and type-(ii) DR designs based on automated one-way communication of prices/controls to devices. First, as illustrated in Fig. 2, these types of designs can result in substantial power spikes arising from massive synchronized ON-OFF switching of these devices.<sup>3</sup> Second, as established in [15] using both analysis and simulations, one-way DR designs implemented within ITD systems can induce unstable “braided cobweb” dynamics in which prices and power levels display increasingly volatile behaviors over time.

These adverse findings for DR designs based on automated one-way communication suggest that TES designs should be carefully considered as a possible alternative. The two-way communication structures underlying TES designs embody the

<sup>2</sup>These designated parties can be system operators or utilities. They can also be intermediaries who manage collections of customer-owned energy resources in accordance with the operational requirements of system operators or utilities.

<sup>3</sup>The one-way DR design used to generate the findings in Fig. 2 was adapted from [14].

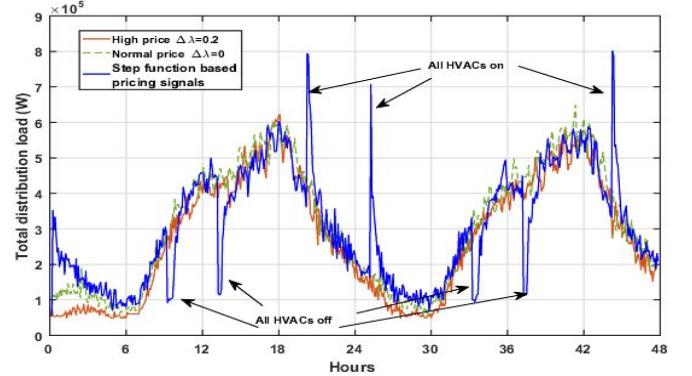


Fig. 2. Demand-response design with automated one-way communication of prices-to-devices. Households have HVAC systems managed by price-responsive controllers. Power spikes arise when the DSO’s price signals result in synchronized ON/OFF controller responses.

premise that distribution system operations should be based more fully on demand bids and supply offers initiated by retail customers in *advance* of actual power usage. A key open issue, however, is whether this two-way communication structure indeed results in increased opportunities to ensure reliable and efficient ITD power system operations over time.

### III. THE ITD TEST SYSTEM

#### A. Key Software Components

ITD Test System (v3.0) is an agent-based computational platform permitting the modeling of transmission and distribution systems linked by market processes, two-way data and signal flows, and two-way power flows; cf. Fig. 1. A partial agent taxonomy for this test system is provided in Fig. 3.

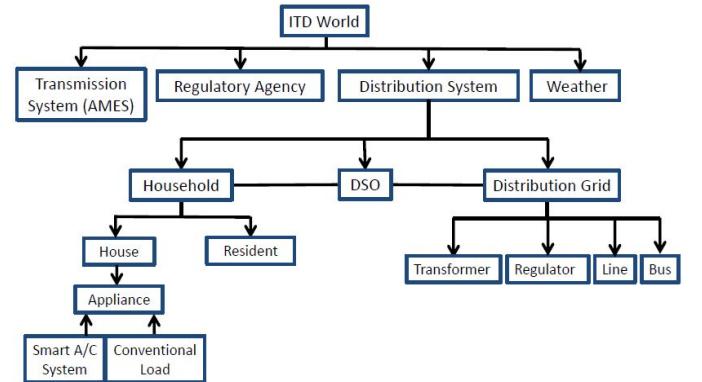


Fig. 3. Partial agent hierarchy for ITD Test System (v3.0). Down-pointing arrows indicate “has a” relationships, and up-pointing arrows indicate “is a” relationships.

As depicted in Fig. 4, the principal software components comprising ITD Test System (v3.0) are as follows:

- C1: A retail power sector, modeled using the IEEE 13-Bus Test System [16];
- C2: A non-profit Distribution System Operator (DSO), implemented in Python;
- C3: Conventional (non-price sensitive) loads, from GridLAB-D [17];

- *C4*: Structural house and A/C system attributes, from GridLAB-D [18];
  - *C5*: A household resident with comfort-cost trade-off preferences, implemented in Python;
  - *C6*: An Equivalent Thermal Parameter (ETP) model for household thermal dynamics, from GridLAB-D [18];
  - *C7*: A smart price-responsive controller for each household's A/C system, implemented in Python;
  - *C8*: A wholesale power sector, implemented via the AMES Wholesale Power Market Test Bed [19];
  - *C9*: A high-level architecture to enable communication (synchronized message passing) among C1-C8, implemented via PNNL's Framework For Network Co-Simulation (FNCS) [20].

send price signals back to households based on the resulting aggregate household bid functions. The control methods allow the DSO to monitor bus voltages and line currents to check for limit violations, and to adjust tap settings and/or to exert direct load control if violations are either observed or anticipated.

With regard to C3 and C4, GridLAB-D [17], [21] is used to model conventional loads, house size and thermal integrity attributes, and A/C system attributes. GridLAB-D provides detailed structural and thermal integrity characteristics for various types of residential and business buildings. It also provides physics-based models for a wide variety of appliances and equipment.

With regard to C5, each household has a resident; cf. Fig. 3. As explained more carefully in Appendix A, the welfare of each resident is measured by three criteria: (i) attained comfort; (ii) net energy payments; and (iii) periodic lump-sum allocations of net revenues (or net costs) from the non-profit DSO, undertaken by the DSO in order to ensure retention of its non-profit status.

With regard to C6, a physics-based ETP model [22] is used to represent the thermal dynamics of each household. As detailed in Appendix B, the ETP model assumes that the thermal state of a household at any given time is described by its inside air and mass temperatures whose movement over time can be expressed by a parameterized system of linear differential equations.

With regard to C7, the A/C system controller for each household consists of the household's latest refreshed state-conditioned bid function. This bid function determines ON/OFF power settings for the household's A/C system in response to DSO-communicated price signals, conditional on the household's current state (including inside air temperature).

With regard to C8, AMES (V3.0) [19] is used to implement a Day-Ahead Market (DAM) and Real-Time Market (RTM) operating over a high-voltage transmission grid, managed by an Independent System Operator (ISO), with congestion managed by locational marginal prices (LMPs). AMES (*Agent-based Modeling of Electricity Systems*) is an open-source agent-based computational platform that captures key structural and institutional aspects of actual U.S. ISO-managed wholesale power systems [23].<sup>4</sup> For later use, AMES wholesale power market operations are described in greater detail in Section III-B.

With regard to C9, the high-level architecture implemented via PNNL's FNCS respects the encapsulation of data, attributes, and methods for each of the agents comprising ITD Test System (v3.0); see Fig. 3.

Components C1-C9 are used to implement an end-to-end power system encompassing both transmission and distribution level operations. The distribution system is linked to the AMES transmission grid at a particular transmission bus. The non-profit DSO participates in the AMES DAM as a Load-Serving Entity (LSE) at this linked bus, purchasing power at wholesale prices that is then resold to households at retail prices. The DSO's power purchases in the DAM are based on

<sup>4</sup>Another AMES feature, important for comprehensive testing of TES designs, is that market participants can have learning capabilities.

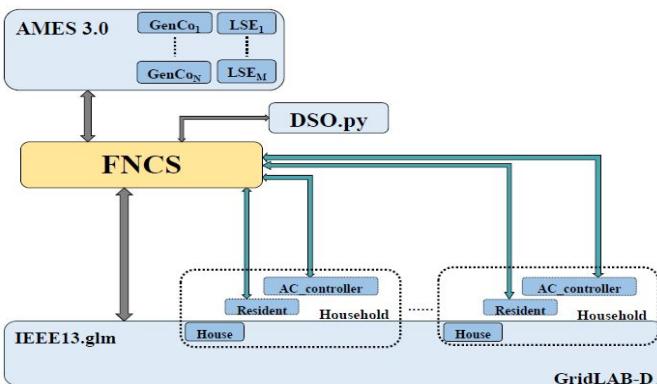


Fig. 4. Key software components for ITD Test System (v3.0).

Brief descriptions will next be provided for components C1-C9. Subsequent sections reporting the outcomes of test cases conducted by means of ITD Test System (v3.0) will help to explain more carefully their functioning and interactions.

With regard to C1, the IEEE 13-Bus System is a distribution system with a 13-bus grid populated by households dispersed across fifteen bus loads; see Fig. 5.

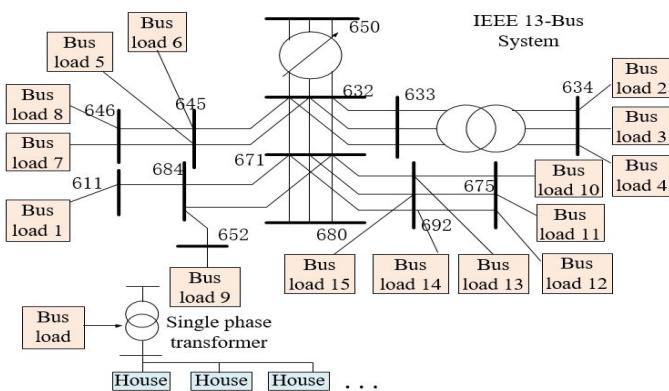


Fig. 5. The 13-bus distribution grid for ITD Test System (v3.0).

With regard to C2, the distribution system is managed by a non-profit DSO tasked with ensuring reliable efficient operations by appropriate use of economic and control methods. The economic methods allow the DSO to receive household bid functions, form aggregate household bid functions, and

its forecast for next-day household power needs, conditional on observed past household power usage levels.

### B. AMES Wholesale Power Market Operations

As depicted in Fig. 6, AMES (V3.0) models the operations of an ISO-managed DAM and RTM. The DAM and RTM operate in tandem over a high-voltage AC transmission grid during successive days. Congestion on the grid is handled by LMP, i.e., by the pricing of power in accordance with the location and timing of its injection into, or withdrawal from, the transmission grid.

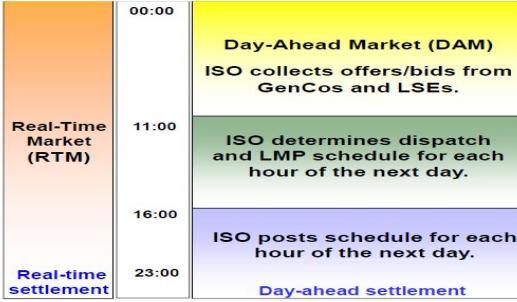


Fig. 6. Daily operation of day-ahead and real-time markets in ITD Test System (v3.0), implemented via AMES (v3.0).

The daily operations of the DAM and RTM in ITD Test System (v3.0) proceed roughly as follows. During the morning of each day D a collection of user-specified *Generation Companies* (*GenCos*) and *Load-Serving Entities* (*LSEs*) submit into the DAM a collection of supply offers and demand bids, respectively, for all 24 hours H of day D+1.

For each hour H, these offers and bids take the following general form:

$$\text{GenCo Price-Responsive Supply: } \pi = a + 2bp ; \quad (1)$$

$$\text{LSE Price-Responsive Demand: } \pi = c - 2dp ; \quad (2)$$

$$\text{LSE Fixed Demand: } p = FD . \quad (3)$$

where:  $\pi$  (\$/MWh) denotes price,  $p$  (MW) denotes power,  $FD$  (MW) denotes a fixed (non-price-responsive) demand for power, and  $a$  (\$/MWh),  $b$  (\$/(MW)<sup>2</sup>h),  $c$  (\$/MWh), and  $d$  (\$/(MW)<sup>2</sup>h) are positive coefficients.<sup>5</sup> The power levels in eqns. (1) through (3) represent constant power levels to be maintained during the entire hour H, either as injections into the grid (power supplies) or as withdrawals from the grid (power demands).

Given these offers and bids, the ISO solves a *Security-Constrained Economic Dispatch (SCED)* optimization subject to standard transmission line, generation capacity, and power balance constraints to determine scheduled GenCo dispatch levels and an LMP  $\pi^{\text{DA}}(B,H,D+1)$  (\$/MWh) at each transmission bus B for each hour H of day D+1. A GenCo is

<sup>5</sup>Although LSEs participating in U.S. DAMs are permitted to submit hourly demand bids for the next-day power needs of their customers in two parts – a price-responsive demand schedule and a fixed power amount – most LSE hourly demand bids take the fixed form (3). For example, the percentage of cleared price-sensitive demand in ISO-New England's DAM has remained nearly constant at 27% from 2012-2016; see [24, Fig. 3-20].

paid  $\pi^{\text{DA}}(B,H,D+1)$  for each MW it is scheduled to inject at B during hour H of day D+1, and an LSE must pay  $\pi^{\text{DA}}(B,H,D+1)$  for each MW its customers are scheduled to withdraw at B during hour H of day D+1.

The RTM runs every five minutes of each day.<sup>6</sup> At the start of each RTM, the ISO determines a load forecast for each transmission bus B for the next five minutes. The ISO then conducts a SCED optimization, conditional on its load forecasts, to resolve any discrepancies between DAM scheduled generation and RTM-forecasted loads. All such discrepancies are settled at RTM LMPs.

For example, suppose the generation scheduled in the day-D DAM for some bus B for period 5Min during day D+1 exceeds the ISO's forecasted load at bus B for period 5Min during day D+1. Then the scheduled generators must "buy back" their excess generation at price  $\pi^{\text{RT}}(B,5\text{Min},D+1)$ , the LMP (\$/MWh) determined in the RTM at bus B for period 5Min of day D+1. Conversely, if the scheduled generation is less than the ISO's forecasted load, the generators are paid for the additional needed generation at price  $\pi^{\text{RT}}(B,5\text{Min},D+1)$ .

### C. Two-Way ITD Feedback in the ITD Test System

A high-level flow diagram for ITD Test System (v3.0) is given in Fig. 7. The strengths of the depicted two-way feedback links between transmission and distribution levels depend on the relative size of the two systems and the extent to which their operations are coupled through market processes, data and signal flows, and power flows.

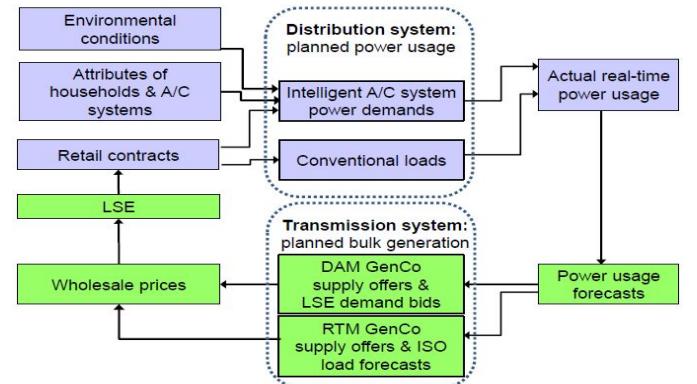


Fig. 7. Flow diagram for ITD Test System (v3.0) depicting two-way feedback between transmission and distribution levels.

More precisely, transmission system operations substantially affect distribution system operations through wholesale to retail power flows. In addition, wholesale prices can potentially affect the determination of retail prices.

Conversely, distribution system operations can potentially affect transmission system operations through three channels. First, household power usage requires wholesale power generation. Second, past household power usage can influence the DAM demand bids of the LSEs that procure wholesale power to service next-day household power needs, which in turn could affect DAM LMPs. Third, past household power

<sup>6</sup>In actual U.S. centrally-managed wholesale power markets, an RTM is conducted at least once every five minutes.

usage can affect the ISO's RTM power usage forecasts, hence RTM LMPs.

#### D. Performance Metrics for the ITD Test System

ITD Test System (v3.0) can record simulation outputs such as voltages (phase-to-ground or phase-to-phase), currents, complex power, reactive power, and active power. These outputs can be in complex or real-number form as appropriate. The sampling time step can also be flexibly chosen. Consequently, ITD Test System (v3.0) can compute ex-post reliability metrics as part of the performance evaluation of a TES design.

In addition, ITD Test System (v3.0) can record welfare (net benefit) outcomes for each decision-making participant, including the non-profit ISO, GenCos, and LSEs at the transmission level and the non-profit DSO and households at the distribution level. Thus, ITD Test System (v3.0) can be used to compute ex-post social/private welfare outcomes as part of the performance evaluation of a TES design.

### IV. TES DESIGN: A POWERMATCHER ILLUSTRATION

#### A. Design Overview

To facilitate understanding of the decentralized layered architecture of TES designs, it is useful to consider a concrete example. This section reports specifications and outcomes for a relatively simple TES distribution-system design based on PowerMatcher [4].

The distribution grid for this illustration is the 13-bus grid depicted in Fig. 5. This grid is populated with 180 households dispersed in groups of twelve across the 15 bus loads. Each of these 180 households has two types of load: (i) conventional (non-price-responsive) load; and (ii) price-responsive load arising from an electric air-conditioning (A/C) system locally managed by an A/C controller with bang-bang (ON/OFF) control settings.

The state of each household is measured by its inside air and mass temperatures,  $T_a$  and  $T_m$ , determined by weather, house structural attributes, and past A/C control settings. Each household strives to ensure  $T_a$  is maintained between a lower level  $T_{\text{Min}}$  and an upper level  $T_{\text{Max}}$ . Within this interval, each household balances comfort against energy cost (or ancillary service compensation), where comfort is measured by nearness of  $T_a$  to a bliss (most desired) inside air temperature  $T_B$ .

The distribution system is managed by a non-profit DSO tasked with ensuring the reliability and efficiency of distribution system operations. The specific goal of the DSO is to ensure that daily aggregate household power usage closely tracks a targetted daily aggregate load profile.<sup>7</sup> In pursuit of this goal, the DSO uses a “Six-Step PowerMatcher Design,” explained in the following subsection.

<sup>7</sup>One possible interpretation is that the DSO seeks to match a previous DAM demand bid in order to avoid RTM imbalance adjustment payments.

#### B. The Six-Step PowerMatcherDesign

The Six-Step PowerMatcher Design, characterized by five rate settings, consists of repeated iterations of the following six steps:

- **Step 1:** The A/C controller for each household collects data on the state of the household at a *data check rate*.
- **Step 2:** The A/C controller for each household sends a state-conditioned A/C power bid to the DSO at a *bid refresh rate*. As clarified below, this bid takes one of four forms depending on the household’s current state: “Must be ON” regardless of price; “Must be OFF” regardless of price; “May Run for Usage” depending on the price charged for power usage; or “May Run as Service” depending on the price paid for ancillary service.
- **Step 3:** The DSO aggregates all household A/C power bids into an aggregate power bid at a specified *aggregate bid refresh rate*.
- **Step 4:** The DSO uses this aggregate power bid to determine a price signal in accordance with its goal(s). A positive price signal denotes a usage price charged and a negative price signal denotes a service price paid.
- **Step 5:** The DSO communicates this price signal to the A/C controller for each household at a specified *price signal rate*.
- **Step 6:** The A/C controller for each household inputs this price signal into the household’s latest refreshed state-conditioned bid at a specified *power control rate*, which triggers an ON/OFF power response from the household’s A/C system.

The bid reported to the DSO by each household at each bid refresh point takes one of four possible forms, depending on the relationship of the household’s actual inside air temperature  $T_a$  to three possible temperatures  $T_{\text{Min}} < T_B < T_{\text{Max}}$ :<sup>8</sup>

- F1: **Must Be OFF ( $T_a \leq T_{\text{Min}}$ )** The house is too cold. The A/C system must stay (or be switched) OFF, regardless of price; hence, the A/C system has no power usage flexibility.
- F2: **May Run as Service ( $T_{\text{Min}} < T_a \leq T_B$ )** The internal air temperature is at or somewhat below the household’s bliss temperature  $T_B$ . The A/C system stays (or is switched) ON if and only if the price  $\pi^s$  paid to the household for ancillary service (power absorption) is at least as great as the minimum acceptable service price  $\pi^s(T_a)$  that the household is willing to receive as compensation for running its A/C system at its ON power usage level  $P^*$ . The function  $\pi^s(T_a)$  is a non-negative decreasing function of  $T_a$ .
- F3: **May Run for Usage ( $T_B < T_a < T_{\text{Max}}$ )** The internal air temperature  $T_a$  is somewhat hotter than the household’s bliss temperature  $T_B$ . The A/C system stays (or is switched) ON if and only if the price  $\pi^u$  charged to the household for power usage does not exceed the maximum price  $\pi^u(T_a)$  that the household is willing to pay for its

<sup>8</sup>This four-part bid is a generalization of the three-part bid proposed by Koen Kok [5, Section 8.1.2] for the power usage of a freezer. As clarified below, given the four-part bid formulation, a household can offer ancillary services (load absorption) as well as express demands for power usage.

ON power usage level  $P^*$ . The function  $\pi^u(T_a)$  is a non-negative increasing function of  $T_a$ .

- F4: **Must Be ON ( $T_{\text{Max}} \leq T_a$ )** The house is too hot. The A/C system must stay (or be switched) ON, regardless of price; hence, the A/C system has no power usage flexibility.

The specific forms used for each household's  $\pi^s(T_a)$  and  $\pi^u(T_a)$  functions are as follows:

$$\pi^s(T_a) = \theta^s \left[ \frac{TB - T_a}{TB - T_{\text{Min}}} \right] \quad \text{for } T_{\text{Min}} < T_a \leq TB ; \quad (4)$$

$$\pi^u(T_a) = \theta^u \left[ \frac{T_a - TB}{T_{\text{Max}} - TB} \right] \quad \text{for } TB < T_a < T_{\text{Max}} , \quad (5)$$

where  $\theta^s$  and  $\theta^u$  are positively valued. Illustrative graphical depictions of a household's May-Run bid functions in service and usage states are provided in Fig. 8.

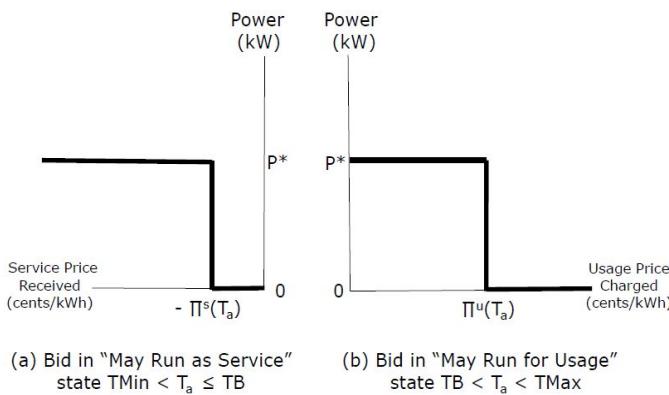


Fig. 8. A household's state-dependent "May Run" bid forms for (a) ancillary service provision and (b) power usage. A negative price denotes a price paid to the household for ancillary service (load absorption). A positive price denotes a price charged to the household for power usage.

The procedure used by the DSO to aggregate household bids at any given time is illustrated in Fig. 9. For simplicity, only two households are depicted, each in a "May Run for Usage" state ( $TB < T_a < T_{\text{Max}}$ ). Household one has a lower inside air temperature  $T_a$  than household two. Hence, the value of (5) for household one (labelled A) is smaller than the value of (5) for household two (labelled B).

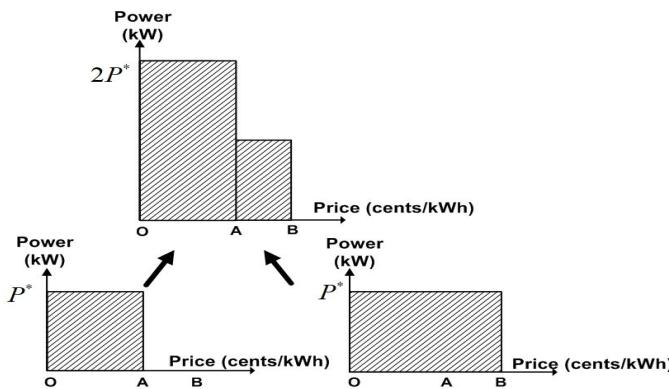


Fig. 9. The DSO's procedure for aggregation of household bids, illustrated for two different households in a "May Run for Usage" state with different inside air temperatures.

### C. Illustrative Findings

As explained in Section IV-A, the goal of the DSO is to ensure that the aggregate power usage of the 180 households during each day D closely tracks a target aggregate load profile for day D. The ability of the DSO to achieve this goal depends on the degree to which aggregate household A/C power usage responds flexibly to changes in the DSO's price signals. This flexibility depends, in turn, on structural house attributes.

This section reports findings from a collection of test cases undertaken to explore the ability of the DSO to achieve a load-tracking goal for a particular hot summer day D under varied structural house attributes. As carefully explained in Appendix B, size and thermal integrity (insulation) parameters are separately categorized into labeled collections with correlated parameter settings adopted from [18]. The collections of *size* parameter settings are labeled from "small" to "large," and the collections of *thermal integrity* parameter settings are labeled from "poor" to "good." Each pairing of labeled sets for size and thermal integrity determines the parameter settings for an ETP model characterizing the thermal dynamics of a house.

More precisely, three different types of houses are considered, distinguished by their *quality*, i.e., by their size and thermal integrity. As seen in Tables VI-VIII in Appendix B, a house is said to be *low quality* if it has a "small" size and "poor" thermal integrity. A house is said to be *medium quality* if it has a "normal" size and "normal" thermal integrity. Finally, a house is said to be *high quality* if it has a "large" size and "good" thermal integrity.

Four test cases are then constructed with systematically varied house quality types. For the first three test cases, all 180 houses have the same quality type (low, medium, or high). For the fourth test case, the 180 houses consist of a (1/3,1/3,1/3) mix of quality types.<sup>9</sup>

For each of the four test cases the five rates for the Six-Step PowerMatcher Design are commonly set as follows: data check rate (1/1s); bid refresh rate (1/300s); aggregate bid refresh rate (1/300s); price signal rate (1/300s); and power control rate (1/300s). Also, parameter values for household bid functions are commonly set as follows:  $T_{\text{Min}} = 68^\circ\text{F}$ ;  $TB = 72^\circ\text{F}$ ;  $T_{\text{Max}} = 76^\circ\text{F}$ ; and  $\theta^s = \theta^u = 100$  (cents/kWh).

The outside air temperature during day D, the same for each household, is for a 24-hour hot summer day (July 1, 2003) in Des Moines, Iowa [25]. To ensure diversity across households, even within quality types, the initial inside air temperature for each household is randomly drawn from the interval  $[68^\circ\text{F}, 76^\circ\text{F}]$ .

As reported in Figs. 10-13, for each of the four test cases the DSO is able to use a suitably selected succession of positive and negative price signals to ensure that actual aggregate household power usage closely matches the DSO's target 24-hour aggregate load profile.

<sup>9</sup>Specifically, the twelve houses located at each of the fifteen bus loads for the distribution grid depicted in Fig. 5 consist of four low-quality houses, four medium-quality houses, and four high-quality houses.

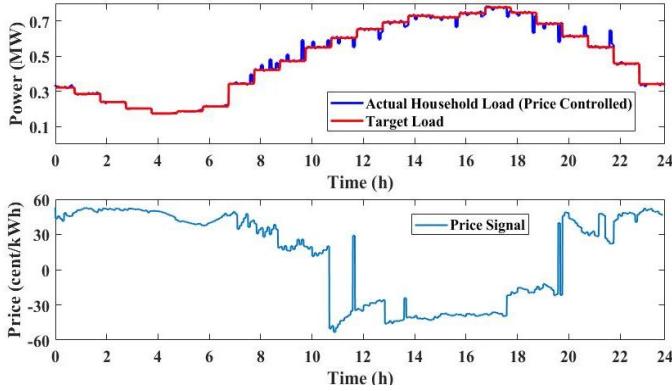


Fig. 10. DSO load-tracking with 100% low quality houses.

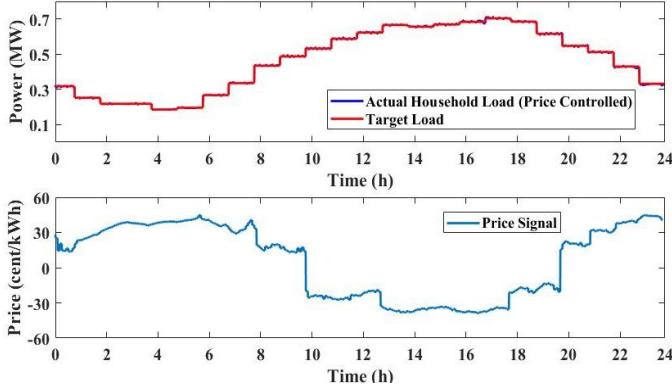


Fig. 11. DSO load-tracking with 100% medium quality houses.

## V. TESTING THE SIX-STEP POWERMATCHER DESIGN WITHIN AN ITD SYSTEM

### A. ITD Test Case Overview

In Section IV, the non-profit DSO tasked with managing the distribution system is assumed to have one simple goal: namely, to maintain aggregate household power usage close to a target aggregate load profile. In this part of our study the non-profit DSO is instead assumed to have three broader goals, as follows:

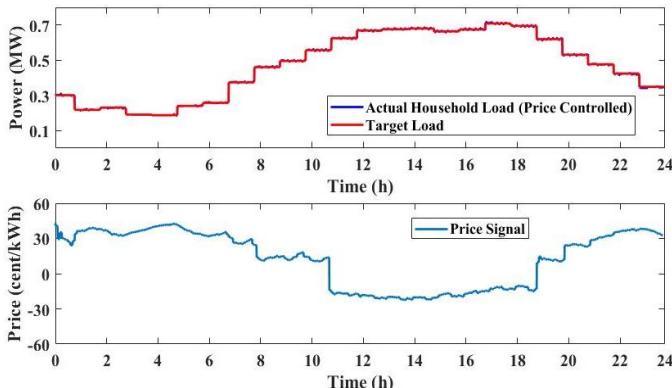


Fig. 12. DSO load-tracking with 100% high quality houses.

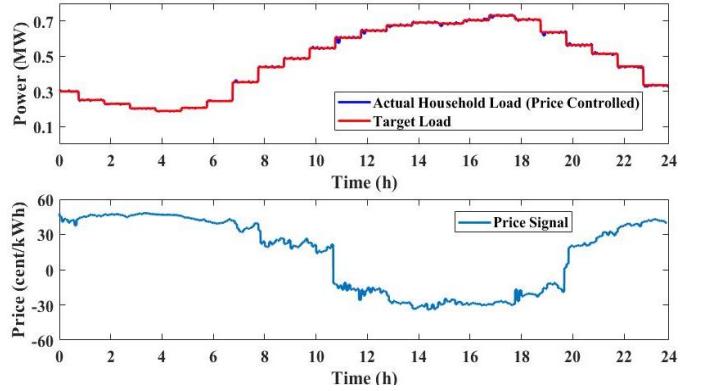


Fig. 13. DSO load-tracking with a (1/3,1/3,1/3) mix of house quality types.

- Goal 1: Maintain the short-run efficiency<sup>10</sup> of distribution system operations, which requires that the retail prices charged to households be accurate reflections of the true marginal costs of power production.
- Goal 2: Maintain the reliability of distribution system operations (e.g., ensure voltage limits are not violated).
- Goal 3: Maintain my non-profit status by “breaking even” over time, i.e., ensure my incoming revenues cover all incurred costs, and that any revenues in excess of incurred costs are returned to households.

This section explains the construction of ITD Test Cases, implemented by means of ITD Test System (v3.0), whose purpose is to explore the extent to which the Six-Step PowerMatcher Design facilitates the achievement of Goals 1-3. Illustrative findings for these ITD Test Cases are reported in Section VI.

### B. Treatment Factors and Maintained Settings

For each ITD Test Case, the distribution grid is configured as in Section IV-A; see Fig. 5. However, this distribution grid is now linked to a 5-bus transmission system<sup>11</sup> implemented by means of AMES (v3.0) [19].

As depicted in Fig. 14, this 5-bus transmission system is populated by five GenCos and four LSEs. The distribution grid is linked to this system at transmission bus 3, and the non-profit DSO functions as LSE 4 at transmission bus 3.

More precisely, the non-profit DSO services household loads at transmission bus 3 as follows. On each day D the DSO submits a power demand bid into the day-D DAM that represents the DSO’s forecast for next-day household power usage. This bid consists of 24 hourly fixed demands (MW) that represent the DSO’s best forecast for household power

<sup>10</sup>In economics, efficiency is generally defined as “non-wastage of resources.” This is interpreted in two basic senses. First, there should be no wastage of valued physical resources. Second, there should be no wastage of “utility” (satisfaction) for people; that is, an outcome should be *Pareto efficient* in the sense that there is no way to make every person at least as well off and at least one person better off by means of some feasible deviation from this outcome.

<sup>11</sup>The basic 5-bus transmission system (without the DSO) is a default test case distributed as part of the AMES v3.0 package [19]. This default test case is a version of the well-known 5-bus test case developed by John Lally [26] at ISO New England that is still widely used for ISO training purposes.

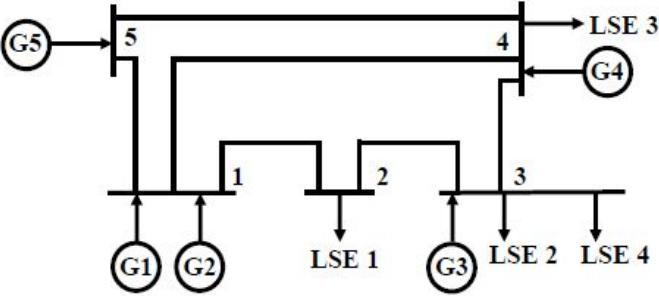


Fig. 14. The five-bus transmission system for the ITD Test Cases. Participants include five Generation Companies (G1-G5) and four Load-Serving Entities (LSEs). The DSO participates as LSE 4 at transmission bus 3.

usage during each hour of day D+1.<sup>12</sup> Each hourly forecast is given by the actual household load observed by the DSO for hour H on day D-1.<sup>13</sup>

The non-profit DSO must pay for its hourly forecasted loads at the hourly LMPs determined for transmission bus 3 in the day-D DAM. In addition, the DSO will subsequently pay (or be paid) additional settlements at RTM LMPs for any deviations between its day-D DAM load forecasts for day D+1 and actual real-time household power usage during day D+1.<sup>14</sup>

The non-profit DSO communicates retail prices to households at a specified price signal rate, in accordance with the Six-Step PowerMatcher Design described in Section IV-A. For simplicity, it is assumed that the DSO sets the retail price for each hour H of day D+1 equal to the LMP the DSO prepaid for forecasted household power usage for H of day D+1 in the day-D DAM, after first converting this LMP (\$/MWh) into retail price units (cents/kWh).

To preserve its non-profit status, the DSO allocates any net revenues (i.e., revenues minus costs) incurred over the course of a day back to the households at the end of this day. This allocation is either a lump-sum payment (if revenues exceed costs) or a lump-sum charge (if costs exceed revenues). The share allocated to each household on each day D is set equal to the household's relative power usage during day D.

In contrast to Section IV, the non-profit DSO is now concerned about household comfort as part of its efficiency goal. The comfort level attained by any household  $h$  during any hour H of any day D+1 is assumed to be a non-increasing function of the deviation of its inside air temperature  $T_a$  from its bliss temperature TB. Specifically, this comfort level (Utils/hr) is modeled as follows:

$$\text{Comfort}^h(\text{H}, \text{D+1}) = U(\text{TB}) - \beta [T_a^h(\text{H}, \text{D+1}) - \text{TB}]^2, \quad (6)$$

where  $\beta$  is a non-negative factor that determines the sensitivity of  $h$  to deviations of  $T_a$  from TB.

The ITD Test Cases are distinguished by their settings for the following two treatment factors:

- The total number of households (NH);

<sup>12</sup>As discussed in footnote 5, LSE demand bids in actual U.S. DAMs largely take a fixed form.

<sup>13</sup>Note that, at the opening of the day-D DAM, the DSO has not yet observed actual household load for day D.

<sup>14</sup>See Section III-B for a more extended discussion of this DAM/RTM two-settlement process.

- The value set for the parameter  $\theta^u$  appearing in the “May Run for Usage” household bid function; see eqn. (5).

All houses for the ITD Test Cases are configured as medium quality types. The value for the parameter  $\theta^s$  appearing in the “May Run as Service” household bid function in eqn. (4) is set at 4 (cents/kWh). The values for  $U(\text{TB})$  and  $\beta$  in the household comfort function (6) are set to 1.5 and 0.02, respectively. The values for all other parameters are set as in Section IV-C.

Each ITD Test Case runs for two successive days. The outside air temperature for all households is for two hot summer days (July 3-4, 2003) in Des Moines, Iowa [25]. To ensure diversity across households, the initial inside air temperature for each household is randomly drawn from the interval [68°F, 76°F].

## VI. ILLUSTRATIVE RESULTS FOR THE ITD TEST CASES: PERFORMANCE OUTCOMES

### A. Overview

This section reports illustrative ITD Test Case performance outcomes for the Six-Step PowerMatcher Design presented in Section IV, making use of the following reliability and welfare metrics:

#### Reliability Metrics:

- Upper voltage magnitude limit violation ( $V_{b,p}^{\text{hi}}$ )
- Lower voltage magnitude limit violation ( $V_{b,p}^{\text{lo}}$ )
- Voltage imbalance (%VIB<sub>b</sub>)

#### Household Welfare and DSO Break-Even Metrics:

- Average hourly household comfort level ( $\text{CM}^{\text{AvH}}$ )
- Average hourly household net energy payment ( $\text{NEP}^{\text{AvH}}$ ) incorporating both payments for power usage and compensation for ancillary service provision
- DSO's average daily lump-sum net-revenue allocation to each household ( $\text{A}^{\text{AvD}}$ )

The construction of each of these metrics is carefully explained in Appendix A.

One important caution is in order. In this section all power and price outcomes are reported in (MW,\$/MWh) units. Any parameter or variable  $v$  originally defined in the Nomenclature table in kW or cents/kWh units that has been converted to MW or \$/MWh units will be denoted as  $\widehat{v}$ .

### B. Reliability Outcomes

The treatment factors for the ITD Test Case outcomes reported in this section are specified as follows. The total number of households, NH, is 180. Also, the household bid parameter  $\widehat{\theta^u}$  is set equal to 1 (\$/MWh).

The 13-bus distribution system for these ITD Test Cases, depicted in Fig. 5, is implemented by means of GridLAB-D (specifically IEEE13.glm). In GridLAB-D, the voltage regulator at the substation of the distribution grid is responsible for guaranteeing that voltage magnitudes at all buses remain within a specified limits [ $V_b^{\min}, V_b^{\max}$ ]. The default setting for these limits in GridLAB-D is [0.90pu,1.10pu].

Figs. 15-16 report the phase voltage magnitudes recorded at 1-minute time-steps at distribution bus 634 under two different

settings for these voltage magnitude limits: (i) GridLAB-D's default setting [0.90pu,1.10pu]; and (ii) a commonly used tighter setting [0.95pu,1.05pu]; see, e.g., [28]. As can be seen, the resulting phase voltage magnitudes are very sensitive to the voltage limit setting.

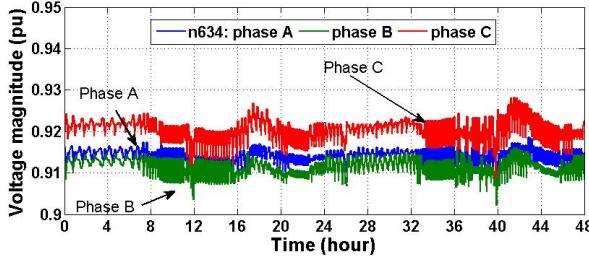


Fig. 15. Phase voltage magnitudes at distribution bus 634, given GridLAB-D's default voltage magnitude limits [0.90pu,1.10pu].

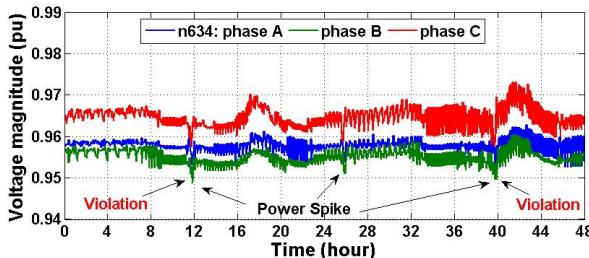


Fig. 16. Phase voltage magnitudes at distribution bus 634, given the tighter voltage magnitude limits [0.95pu,1.05pu].

Table I reports two different types of reliability outcomes at distribution buses 634 and 675. First, outcomes are reported for the maximum deviations above ( $V_{b,p}^{\text{hi}}$ ) and below ( $V_{b,p}^{\text{lo}}$ ) the voltage magnitude limits  $[V_b^{\min}, V_b^{\max}]$  set at each bus  $b$  for each phase  $p \in \{A, B, C\}$ . Second, outcomes are reported for the three-phase voltage imbalance metric  $\%VIB_b$ . Each type of reliability outcome is reported for two different settings for the voltage magnitude limits: GridLAB-D's default limits [0.90pu,1.10pu]; and the tighter limits [0.95pu,1.05pu].

TABLE I  
RELIABILITY METRIC OUTCOMES FOR BUSES 634 AND 675

(a) Voltage Limits:  $0.90\text{pu} \leq V \leq 1.10\text{pu}$

Bus	Metrics	Phase A	Phase B	Phase C
634	$V_{b,p}^{\text{hi}}/V_{b,p}^{\text{lo}}$	0/0	0/0	0/0
	$\%VIB_b$	<b>0.86%</b>		
675	$V_{b,p}^{\text{hi}}/V_{b,p}^{\text{lo}}$	0/0	0/0	0/0
	$\%VIB_b$	<b>1.27%</b>		

(b) Voltage Limits:  $0.95\text{pu} \leq V \leq 1.05\text{pu}$

Bus	Metrics	Phase A	Phase B	Phase C
634	$V_{b,p}^{\text{hi}}/V_{b,p}^{\text{lo}}$	0/0	<b>0/0.0012</b>	0/0
	$\%VIB_b$	<b>0.14%</b>		
675	$V_{b,p}^{\text{hi}}/V_{b,p}^{\text{lo}}$	0/0	0/0	0/0
	$\%VIB_b$	<b>2.01%</b>		

As seen in Table I, the omission of reliability considerations in the Six-Step PowerMatcher design can result in both voltage limit violations and voltage imbalance problems at the distribution level. These findings suggest it would be prudent

to couple this design with reliability constraints. To preserve the decentralized architecture of the design, this coupling could take the form of load controls triggered automatically at the level of buses or even households. Alternatively, it could take the form of DSO-implemented price-signal adjustments.

Another potential problem is that the Six-Step PowerMatcher Design in its current form does not protect against the kind of longer-run reliability problems reported in [15] for a linked retail-wholesale power system. Specifically, given certain structural conditions, price and power outcomes at both the retail and wholesale levels can exhibit increasingly volatile "cobweb dynamics" over successive days of operation.

Interestingly, necessary and sufficient conditions for unstable cobweb dynamics to arise within the system studied in [15] are expressed in terms of the relative slopes of the aggregate supply curve at wholesale and the aggregate demand curve at retail induced by the price-responsive power demands of 500 households. Since the Six-Step PowerMatcher Design permits the DSO to calculate aggregate household demand curves in advance of actual household power usage, it is possible that the DSO could adjust its retail price signals in a manner that reduces or eliminates this longer-run reliability risk.

For the ITD Test Cases reported in this study, the power requirements of the distribution system (Fig. 5) are small in relation to the total power requirements serviced by the transmission system (Fig. 14). Consequently, distribution system load does not have a substantial effect on transmission system price and power outcomes.

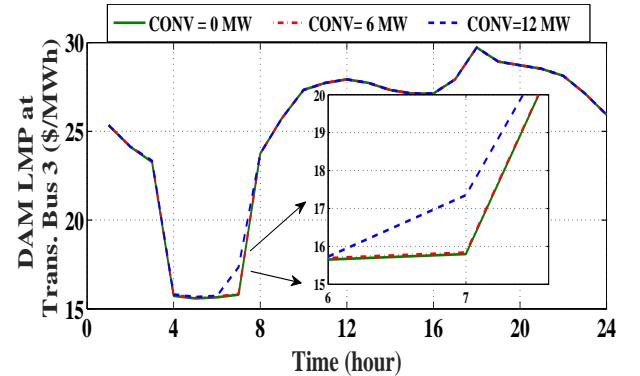


Fig. 17. DAM LMP outcomes at transmission bus 3, given three different distribution load specifications.

This point is illustrated in Fig. 17, which reports DAM LMP outcomes at transmission bus 3 for three distribution load cases. For Case 1, the distribution system and transmission system are not connected. For Case 2, the distribution system is connected at transmission bus 3, and 6MW of conventional load is added to household power usage at distribution bus 362. Finally, for Case 3, the distribution system is connected at transmission bus 3, and 12 MW of conventional load is added to household power usage at distribution bus 362. Comparing these three cases, it is seen that DAM LMPs only begin to be noticeably affected by distribution system operations for sufficiently large distribution loads.

A proper examination of unstable cobweb dynamics and other longer-run reliability issues will require an enhanced

version of ITD Test System (v3.0) that incorporates more realistically sized distribution systems linked to one or more transmission buses.

### C. Welfare Outcomes

In this section welfare outcomes are generated under variously specified settings for the two treatment factors: namely, the total number of households, NH, and the household bid parameter  $\hat{\theta}^u$ .

Recall that the DSO sets the retail price  $\hat{\pi}^{\text{RET}}(H, D+1)$  for hour H on any day D+1 equal to the LMP  $\pi^{\text{DA}}(3, H, D+1)$  determined in the day-D DAM for transmission bus 3 during hour H of day D+1. For the ITD Test Cases, the DAM LMPs at transmission bus 3 typically vary between 15 (\$/MWh) and 30 (\$/MWh) each day.

The parameter  $\hat{\theta}^u$  reflects the maximum willingness of a household to turn off its A/C power consumption when in a “May Run for Usage” state. Aggregate household power usage is sensitive to changes in the value of  $\hat{\theta}^u$ . When  $\hat{\theta}^u$  is small (e.g., equal to 1),  $\pi^{\text{DA}}$  will tend to exceed  $\hat{\pi}^u(T_a)$ ; hence, the A/C system of each household will tend to be turned OFF, even if the household is in a May Run for Usage state ( $T_B < T_a < T_{\text{Max}}$ ). A/C power will be turned ON only when the inside air temperature  $T_a$  attains or exceeds  $T_{\text{Max}}$ .

Power spikes occur when the inside air temperature  $T_a$  reaches or exceeds  $T_{\text{Max}}$  at the same time for a large number of households. Similar power-spike effects occur in reverse when  $\hat{\theta}^u$  is large (e.g., equal to 1000).

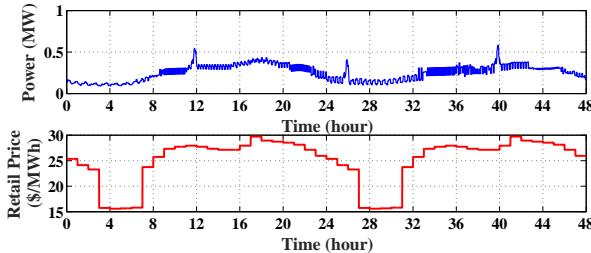


Fig. 18. Price-responsiveness of aggregate household power usage over two successive days, given  $NH = 180$  and  $\hat{\theta}^u = 1$  (\$/MWh).

For example, Fig. 18 reports the aggregate household power usage for  $NH=180$  houses when  $\hat{\theta}^u = 1$  (\$/MWh). The power-spikes observed in Fig. 18 are induced by the form of the household bid function. With greater diversity of bid functions (e.g., more diverse settings for  $T_{\text{Min}}$  and  $T_{\text{Max}}$ ), these power-spikes would be smoothed out. However, as currently implemented, the Six-Step PowerMatcher design could also induce price-synchronized power spiking, as observed in Fig. 2.

Thus, two issues arise. Do power spikes negatively impact either welfare or system reliability to an extent that counter-actions are desirable? Second, could the PowerMatcherDesign be modified to permit such counter-actions?

Tables II and III report household welfare outcomes for two successive simulated days, given systematically varied settings for the total number of households (NH) and the household bid parameter  $\hat{\theta}^u$ . Specifically, ex-post welfare metrics are reported for average hourly household comfort  $CM^{\text{AvH}}$ , average hourly

TABLE II  
WELFARE METRICS FOR DAY 1 (3 JULY 2003)

Day	Case	NH	$\hat{\theta}^u$ (\$/MWh)	$CM^{\text{AvH}}$ (Utils/hr)	$NEP^{\text{AvH}}$ (\$/hr)	$A^{\text{AvD}}$ (\$/day)
1	1	180	1	1.16	0.0062	-0.0283
1	2	180	40	1.33	0.0064	-0.0287
1	3	180	80	1.43	0.0066	-0.0292
1	4	180	10,000	1.47	0.0067	-0.0295
1	5	360	1	1.16	0.0062	-0.0315
1	6	360	40	1.33	0.0064	-0.0323
1	7	360	80	1.43	0.0066	-0.0329
1	8	360	10,000	1.47	0.0067	-0.0339

TABLE III  
WELFARE METRICS FOR DAY 2 (4 JULY 2003)

Day	Case	NH	$\hat{\theta}^u$ (\$/MWh)	$CM^{\text{AvH}}$ (Utils/hr)	$NEP^{\text{AvH}}$ (\$/hr)	$A^{\text{AvD}}$ (\$/day)
2	1	180	1	1.18	0.0059	-0.0273
2	2	180	40	1.35	0.0060	-0.0272
2	3	180	80	1.44	0.0061	-0.0275
2	4	180	10,000	1.47	0.0064	-0.0280
2	5	360	1	1.18	0.0059	-0.0285
2	6	360	40	1.35	0.0060	-0.0288
2	7	360	80	1.44	0.0061	-0.0294
2	8	360	10,000	1.47	0.0063	-0.0308

household net energy payments  $NEP^{\text{AvH}}$ , and average daily household lump-sum allocations  $A^{\text{AvD}}$ .

Table IV then reports hourly comfort levels, hourly net energy payments, and daily allocations averaged over these two successive days. Several interesting regularities are revealed in these reported outcomes that suggest further systematic investigation would be desirable. For example, comfort  $CM^{\text{AvH}}$  and net energy payments  $NEP^{\text{AvH}}$  each systematically increase with increases in  $\hat{\theta}^u$ , all else equal, and are each essentially invariant to changes in NH, all else equal. On the other hand, the household lump-sum allocations  $A^{\text{AvD}}$  systematically become more negative with increases either in NH or in  $\hat{\theta}^u$ , all else equal.

The consistently negative values for  $A^{\text{AvD}}$  in Table IV show that, on average, the DSO is making net payments in the RTM that are not covered by its setting of retail prices. These RTM net payments are additional costs to the DSO that must be allocated to households as lump-sum charges in order for the DSO to retain its non-profit status.

The persistent need for these lump-sum charges indicates that the DSO is persistently underestimating household power

TABLE IV  
WELFARE METRICS AVERAGED OVER DAY 1 AND DAY 2

Case	NH	$\hat{\theta}^u$ (\$/MWh)	$CM^{\text{AvH}}$ (Utils/hr)	$NEP^{\text{AvH}}$ (\$/hr)	$A^{\text{AvD}}$ (\$/day)
1	180	1	1.17	0.0060	-0.0278
2	180	40	1.34	0.0062	-0.0280
3	180	80	1.44	0.0064	-0.0283
4	180	10,000	1.47	0.0065	-0.0287
5	360	1	1.17	0.0060	-0.0300
6	360	40	1.34	0.0062	-0.0305
7	360	80	1.44	0.0063	-0.0311
8	360	10,000	1.47	0.0065	-0.0323

usage in its DAM demand bids. The determination of appropriate DAM demand bids for the DSO is thus another critical design aspect for DSO-managed TES designs implemented for ITD power systems.

## VII. CONCLUSION

This study presents an Integrated Transmission and Distribution (ITD) Test System (v3.0) capable of modeling the ITD operations of a power system over successive days. Such test systems permit comprehensive performance evaluations for Transactive Energy System (TES) designs implemented within end-to-end power systems encompassing both transmission and distribution level operations.

In support of this claim, ITD Test System (v3.0) is used to test the reliability and welfare performance characteristics of a particular DSO-managed TES design, referred to as the Six-Step PowerMatcher Design. This bid-based design permits a non-profit DSO to service the power needs of DER owners, and to compensate DER owners for extracted ancillary services, in a manner consistent with their local goals and constraints.

By construction, the Six-Step PowerMatcher design respects household privacy; and it easily scales to handle increasing DER penetration. In addition, it achieves short-run efficiency to the extent that the wholesale prices passed through to retail prices reflect true marginal costs. However, the results presented in this study indicate it would be prudent to couple this design with additional operational constraints in order to ensure the short-run and longer-run *reliability* of ITD power system operations.

The results reported in this study raise a number of important issues for TES designs managed by non-profit DSOs within ITD power systems. For example, if current U.S. power systems were to implement such a design, what precise form would the DSO's objective(s) and constraints have to take in order to ensure that reliability and efficiency are both enhanced under this design? What precise form would the bid functions of distribution-system participants have to take to ensure their welfare and solvency? And could the DSO manage this design in such a manner that its revenues approximately balance its costs, without any need to rely on out-of-market net revenue (or net cost) allocations back to design participants?

Another critical issue is the form of contract used by a DSO to function as a distribution system intermediary in transmission system transactions. For example, a *swing contract* permits a DER aggregator to offer into a wholesale power market a collection of power paths available for extraction from DERs that have some degree of flexibility in the timing and/or extent of their power operations [27]. The availability of such power-path collections could enable flexible provision of a wide range of service attributes in support of ITD system operations, such as start-time, power level, ramp rate, duration, and volt/VAr support. Could the use of swing contracts facilitate the participation of DSOs as DER aggregators in wholesale power markets?

These issues will be systematically explored in subsequent research, making use of appropriately enhanced versions of ITD Test System (v3.0).

## APPENDIX A

### RELIABILITY AND WELFARE PERFORMANCE METRICS

#### A.1 Performance Metric Construction: Overview

ITD Test System (v3.0) permits the performance of a TES design to be evaluated ex-post by means of a wide variety of reliability and welfare metrics, including the following:

##### *DSO Net Revenue and Reliability Metrics:*

- DSO's average hourly net revenue ( $NR^{DSO}$ )
- Upper voltage magnitude limit violation ( $V_{b,p}^{hi}$ )
- Lower voltage magnitude limit violation ( $V_{b,p}^{lo}$ )
- Current line-limit violation  $I_{\ell,p}^{over}$
- Voltage imbalance (%VIB<sub>b</sub>)

##### *Household Welfare and DSO Break-Even Metrics:*

- Average hourly household comfort level ( $CM^{AvH}$ )
- Average hourly household net energy payment ( $NEP^{AvH}$ ) incorporating both payments for power usage and compensation for ancillary service provision
- DSO's average daily lump-sum net-revenue allocation to each household ( $A^{AvD}$ )

This section explains the construction of each of the above metrics. Explanations for the notation used in these constructions can be found in the Nomenclature table.

One important caution is in order. The construction of the revenue and payment metrics requires careful attention be paid to conversions between retail power levels in kW and wholesale power levels in MW, and between retail power prices in cents/kWh and wholesale power prices in \$/MWh. In particular,  $1\text{MW} = 1000\text{kW}$ , and  $$/\text{MWh} = [100 \text{ cents}/1000\text{kWh}] = [1/10] \times [\text{cents}/\text{kWh}]$ .

In all of the revenue and payment metric constructions detailed below, expressions of the form  $\hat{p}^{RET}$  are used to denote retail power levels expressed in MW units, and expressions of the form  $\hat{\pi}^{RET}$  are used to denote retail price levels expressed in \$/MWh units.

#### A.2 DSO Net Revenue Metric

The DSO is assumed to be a non-profit entity. This means that the DSO is not permitted to keep any excess of revenues over costs, and is not allowed to pay for any excess of costs over revenues. Rather, the DSO's net revenues (i.e., its revenues minus costs) must be allocated out to households either as a lump-sum payment (if revenues exceed costs) or as a lump-sum charge (if costs exceed revenues).

An economic entity is said to *break even* over a specified time interval if its incoming revenues over this time interval are equal to its incurred costs over this time interval. In this section we construct a net-revenue metric  $NR^{DSO}$  for the DSO that measures the extent to which the DSO fails to break even over time, conditional on any given TES design implementation.

Let H denote a particular hour of a 24-hour day, and let D denote any particular day. Recall that the non-profit DSO participates as an LSE in the day-D DAM, managing household power usage at transmission bus 3. The dependence of all DSO price and power outcomes on transmission bus 3 is hereafter suppressed in the notation for clarity of exposition.

Recall that, by assumption, the demand bid (load forecast) submitted by the DSO into the day-D DAM for hour H of day D+1 is given by the actual total household power usage  $\hat{p}^{\text{RET}}(H, \text{D-1})$  observed for hour H of day D-1. The DSO pays  $\pi^{\text{DA}}(H, \text{D+1})\hat{p}^{\text{RET}}(H, \text{D-1})$  in the day-D DAM for this forecasted demand  $\hat{p}^{\text{RET}}(H, \text{D-1})$ .

Let  $\hat{\pi}^{\text{RET}}(H, \text{D+1})$  denote the retail price the DSO charges to households for their actual power usage  $\hat{p}^{\text{RET}}(H, \text{D+1})$  during hour H of day D+1. The amount the DSO receives in payment from households for this actual power usage is then given by  $\hat{\pi}^{\text{RET}}(H, \text{D+1})\hat{p}^{\text{RET}}(H, \text{D+1})$ .

If  $\hat{p}^{\text{RET}}(H, \text{D+1})$  is less than  $\hat{p}^{\text{RET}}(H, \text{D-1})$ , the DSO receives compensation at rate  $\pi^{\text{RT}}(H, \text{D+1})$  in the RTM for hour H of day D+1 for its over-payment. Conversely, if  $\hat{p}^{\text{RET}}(H, \text{D+1})$  is greater than  $\hat{p}^{\text{RET}}(H, \text{D-1})$ , the DSO must pay for the unanticipated additional power usage at rate  $\pi^{\text{RT}}(H, \text{D+1})$ . In either case, *DSO Net Revenue* (\$/hr) for hour H of day D+1, i.e., the DSO's revenue net of costs, is given by

$$\begin{aligned} \text{NR}^{\text{DSO}}(H, \text{D+1}) &= \hat{\pi}^{\text{RET}}(H, \text{D+1})\hat{p}^{\text{RET}}(H, \text{D+1}) \\ &\quad + \pi^{\text{RT}}(H, \text{D+1})[\hat{p}^{\text{RET}}(H, \text{D-1}) - \hat{p}^{\text{RET}}(H, \text{D+1})] \\ &\quad - \pi^{\text{DA}}(H, \text{D+1})\hat{p}^{\text{RET}}(H, \text{D-1}) . \end{aligned} \quad (7)$$

The DSO's average hourly net revenue (\$/hr) during day D+1 is then given by

$$\text{NR}^{\text{DSO}}(\text{D+1}) = \frac{\sum_{i=1}^{24} \text{NR}^{\text{DSO}}(H_i, \text{D+1})}{24} . \quad (8)$$

Consequently, the DSO's average hourly net revenue (\$/hr) over simulated days  $D_1, \dots, D_M$  is given by

$$\text{NR}^{\text{DSO}} = \frac{\sum_{m=1}^M \text{NR}^{\text{DSO}}(D_m)}{M} . \quad (9)$$

### A.3 DSO Reliability Metrics

ITD Test System (v3.0) can record simulation outputs such as voltages (phase-to-ground or phase-to-phase), currents, complex power, reactive power, and active power. These outputs can be in complex or real-number form as appropriate. Consequently, ITD Test System (v3.0) can compute ex-post reliability metrics as part of the performance evaluation of a TES design.

Two reliability goals that are commonly imposed for distribution systems are: (i) maintain the voltage magnitude  $V_{b,p}$  (pu) at each phase  $p \in \{A, B, C\}$  of each distribution bus  $b$  within specified lower and upper limits about a nominal voltage magnitude  $V^o$  (pu); and (ii) maintain the directional (plus or minus) current on each distribution line  $\ell$  within specified lower and upper line capacity limits [28].

In this study we construct metrics to check whether the voltage magnitudes corresponding to a particular bus  $b$  and phase  $p$  are larger than a specified upper limit ( $V_b^{\max}$ ) by reporting the maximum deviation ( $V_{b,p}^{\text{hi}}$ ) above this upper limit across NK recorded data samples, as follows:

$$V_{b,p}^{\text{hi}} = \max_{1 \leq k \leq \text{NK}, V_{b,p,k} \geq V_b^{\max}} (V_{b,p,k} - V_b^{\max}) . \quad (10)$$

Similarly, we construct metrics to check whether the voltage magnitudes corresponding to a particular bus  $b$  and phase  $p$

are smaller than a specified lower limit ( $V_b^{\min}$ ) by reporting the maximum deviation ( $V_{b,p}^{\text{lo}}$ ) below this lower limit across NK recorded data samples, as follows:

$$V_{b,p}^{\text{lo}} = \max_{1 \leq k \leq \text{NK}, V_{b,p,k} \leq V_b^{\min}} (V_b^{\min} - V_{b,p,k}) . \quad (11)$$

Finally, since our test-case distribution grid is radial and has no distributed generation, current is unidirectional (from substation to bus load only) and non-negatively valued. We construct metrics to check whether the line currents corresponding to a particular line  $\ell$  and phase  $p$  are larger than a specified line limit ( $I_{\ell,p}^{\max}$ ) by reporting the maximum deviation ( $I_{\ell,p}^{\text{over}}$ ) above this line limit across NK recorded data samples, as follows:

$$I_{\ell,p}^{\text{over}} = \max_{1 \leq k \leq \text{NK}, I_{\ell,p,k} \geq I_{\ell,p}^{\max}} (I_{\ell,p,k} - I_{\ell,p}^{\max}) . \quad (12)$$

Voltages in three-phase distribution grids can be unbalanced. One commonly-used voltage imbalance metric is the percentage voltage imbalance (%VIB) [29]. This metric can be calculated for any bus  $b$  that consists of three phases, such as the buses numbered 634, 675, and 692 in the 13-bus distribution grid used for all test cases in the current study; see Fig. 5.

Roughly defined, the voltage imbalance metric %VIB <sub>$b$</sub>  for bus  $b$  is calculated by making use of  $b$ 's recorded voltage profile as follows:

$$\% \text{VIB}_b = \frac{\text{Max deviation from average}}{\text{Avg. of phase-to-phase voltages}} \times 100\% . \quad (13)$$

More precisely, let the recorded phase-to-phase voltage magnitudes at each time step  $k$  at a particular bus  $b$  with phases  $(A, B, C)$  be denoted by  $V_{b,AB,k}$ ,  $V_{b,BC,k}$ , and  $V_{b,CA,k}$ . Then %VIB <sub>$b$</sub>  in (13) is calculated as follows:

$$\text{Av}_b = \frac{\sum_{k=1}^{\text{NK}} [V_{b,AB,k} + V_{b,BC,k} + V_{b,CA,k}]}{3 \times \text{NK}} ; \quad (14)$$

$$\text{Dev}_{b,k} = \max\{|V_{b,AB,k} - \text{Av}_b|, |V_{b,BC,k} - \text{Av}_b|, |V_{b,CA,k} - \text{Av}_b|\} ; \quad (15)$$

$$\% \text{VIB}_b = \frac{\max_{k=1, \dots, \text{NK}} \text{Dev}_{b,k}}{\text{Av}_b} \times 100\% . \quad (16)$$

### A.4 Household Comfort Metric

The household's state-conditioned bid function under the Six-Step PowerMatcher Design embodies in simple form the household's attempts to achieve maximum comfort for any given net energy payment level, or equivalently, minimum net energy payments for any attained comfort level.

The maximum comfort that household  $h$  can attain during any hour H of day D+1 is the utility (satisfaction) attained by  $h$  when its inside air temperature  $T_a$  equals its bliss temperature TB. This utility level is denoted by  $U(\text{TB})$  (Utils/hr).

The actual comfort level attained by  $h$  during any hour H of any day D+1 is assumed to be a non-increasing function of the deviation of  $T_a$  from TB. Specifically, this actual comfort level (Utils/hr) is modeled as follows:

$$\text{Comfort}^h(H, \text{D+1}) = U(\text{TB}) - \beta [T_a^h(H, \text{D+1}) - \text{TB}]^2 , \quad (17)$$

where  $\beta \geq 0$  determines the sensitivity of  $h$  to deviations of  $T_a$  from TB. The average hourly comfort level (Utils/hr) attained by household  $h$  during any day D+1 is then given by

$$\text{Comfort}^h(D+1) = \frac{\sum_{i=1}^{24} [\text{Comfort}^h(H_i, D+1)]}{24}. \quad (18)$$

Finally, given NH households in total, the average hourly comfort level (Utils/hr) attained by a household over simulated days  $D_1, \dots, D_M$  is given by

$$\text{CM}^{\text{AvH}} = \frac{\sum_{h=1}^{\text{NH}} \sum_{m=1}^M [\text{Comfort}^h(D_m)]}{\text{NH} \times M}. \quad (19)$$

#### A.5 Household Net Energy Payment Metric

Recall that the retail price  $\hat{\pi}^{\text{RET}}(H, D+1)$  set by the DSO for any hour H of any day D+1 is positively valued if it is a charge for power usage and negatively valued if it is a payment for ancillary service (load absorption). Given this sign convention, the net energy payment (\$/hr) of household  $h$  for hour H of day D+1 is determined as follows:

$$\text{NEP}^h(H, D+1) = \hat{\pi}^{\text{RET}}(H, D+1) \hat{p}^{\text{RET}, h}(H, D+1), \quad (20)$$

where  $\hat{p}^{\text{RET}, h}(H, D+1)$  denotes  $h$ 's power usage or ancillary service during hour H of day D+1. Thus,  $h$ 's average hourly net energy payment (\$/hr) during day D+1 is given by

$$\text{NEP}^h(D+1) = \frac{\sum_{i=1}^{24} \text{NEP}^h(H_i, D+1)}{24}. \quad (21)$$

Finally, given NH households in total, the average hourly net energy payment (\$/hr) for a household over simulated days  $D_1, \dots, D_M$  is given by

$$\text{NEP}^{\text{AvH}} = \frac{\sum_{h=1}^{\text{NH}} \sum_{m=1}^M \text{NEP}^h(D_m)}{\text{NH} \times M}. \quad (22)$$

#### A.6 Household Lump-Sum Allocations from the DSO

To ensure its non-profit status, the DSO must allocate back to households any non-zero net revenue it attains, whether positive or negative. For concreteness, it is assumed the DSO uses the following lump-sum allocation method:

**DSO Allocation Method:** At the end of each day D+1, each household  $h$  receives (or is charged) a lump-sum amount of the DSO's net revenue during day D+1 in proportion to  $h$ 's relative power usage during day D+1.

More precisely, let  $\hat{p}^{\text{RET}, h}(H, D+1)$  denote the power usage of household  $h$  during any hour H of any day D+1. Suppose the total number of households is NH. Then the total household power usage during hour H of day D+1 is

$$\hat{p}^{\text{RET}}(H, D+1) = \sum_{h=1}^{\text{NH}} \hat{p}^{\text{RET}, h}(H, D+1). \quad (23)$$

Let household  $h$ 's share of total household power usage during day D+1 be denoted by

$$\gamma^h(D+1) = \frac{\sum_{i=1}^{24} \hat{p}^{\text{RET}, h}(H_i, D+1)}{\sum_{i=1}^{24} \hat{p}^{\text{RET}}(H_i, D+1)} \quad (24)$$

Then the lump-sum allocation  $A^h(D+1)$  (\$/day) received by (or charged to) household  $h$  by the DSO at the end of day D+1 is given by

$$A^h(D+1) = \gamma^h(D+1) [24 \times \text{NR}^{\text{DSO}}(D+1)]. \quad (25)$$

where  $\text{NR}^{\text{DSO}}(D+1)$  is given in (8).

Finally, the average daily allocation (\$/day) received by (or charged to) a household over simulated days  $D_1, \dots, D_M$  is given by

$$A^{\text{AvD}} = \frac{\sum_{h=1}^{\text{NH}} \sum_{m=1}^M A^h(D_m)}{\text{NH} \times M}. \quad (26)$$

## APPENDIX B

### HOUSEHOLD THERMAL DYNAMICS: TECHNICAL DETAILS

In this study the following Equivalent Thermal Parameter (ETP) model, implemented in GridLAB-D [18], is used to represent the thermal dynamics of a household over  $t \geq t^o$  for some initial time  $t^o$ :<sup>15</sup>

$$\begin{aligned} \dot{T}_a(t) &= \frac{1}{C_a} \left( U_a[T_o(t) - T_a(t)] \right. \\ &\quad \left. + H_m[T_m(t) - T_a(t)] + Q_a(t) \right); \end{aligned} \quad (27)$$

$$\dot{T}_m(t) = \frac{1}{C_m} \left( H_m[T_a(t) - T_m(t)] + Q_m(t) \right), \quad (28)$$

where  $T_a(t)$  denotes inside air temperature at time  $t$ ,  $T_m(t)$  denotes inside mass temperature at time  $t$ , and  $T_o(t)$  denotes outside air temperature at time  $t$ .

The differential system (27)-(28) depends at each time  $t$  on time-dependent terms  $Q_a(t)$  and  $Q_m(t)$  representing heat flow rates to the household's inside air mass and inside solid mass, respectively. The total heat flow rate  $Q_a(t)$  is assumed to be determined by specified fractions of (i) the internal heat gain from equipment and occupants ( $Q_i(t)$ ); (ii) solar radiation ( $Q_s(t)$ ); and (iii) A/C system operations ( $\text{COPP}(t)$ ), as follows:

$$\begin{aligned} Q_a(t) &= [1 - f_i]Q_i(t) + [1 - f_s]Q_s(t) \\ &\quad - [1 - f_{ac}]\text{COPP}(t). \end{aligned} \quad (29)$$

The heat flow rate  $Q_m(t)$  is then assumed to be determined by the remaining heat gain fractions, as follows:<sup>16</sup>

$$Q_m(t) = f_iQ_i(t) + f_sQ_s(t) - f_{ac}\text{COPP}(t). \quad (30)$$

Formulas giving the precise determination of  $Q_i(t)$  and  $Q_s(t)$  can be found in [31].<sup>17</sup>

<sup>15</sup>A careful detailed discussion of GridLAB-D's implementation of the ETP model for a residential household is provided in [18].

<sup>16</sup>This assumption is reasonable for the wood frame construction predominant in U.S. homes; cf. the Flex House model in [30].

<sup>17</sup>Roughly stated, the internal heat gain  $Q_i(t)$  is calculated as the sum of sensible heat from house occupants plus the current real power for each end-use load multiplied by the fraction of the end-use load that is internal to the house. The solar heat gain  $Q_s(t)$  is computed assuming each house's windows are equally oriented in each of eight cardinal directions, on average.

The continuous-time differential system (27)-(28) can equivalently be expressed in the following standard state-space form:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bw(t) ; \quad (31) \\ A &= \begin{bmatrix} -\frac{U_a+H_m}{C_a} & \frac{H_m}{C_a} \\ \frac{H_m}{C_m} & -\frac{H_m}{C_m} \end{bmatrix} ; \\ B &= \begin{bmatrix} \frac{U_a}{C_a} & \frac{1}{C_a} & 0 \\ 0 & 0 & \frac{1}{C_m} \end{bmatrix} ; \\ x(t) &= \begin{bmatrix} T_a(t) \\ T_m(t) \end{bmatrix} ; \\ w(t) &= \begin{bmatrix} T_o(t) \\ Q_a(t) \\ Q_m(t) \end{bmatrix} .\end{aligned}$$

For computational tractability, system (31) is discretized using time periods of length  $\Delta t$ , as follows. Replace the time-derivative  $\dot{x}(t)$  by a finite-difference approximation:

$$\dot{x}(t) \approx \frac{x(t + \Delta t) - x(t)}{\Delta t} . \quad (32)$$

Let time periods be indexed by  $k = 0, 1, \dots$ , where time period  $k$  corresponds to the time interval  $[t^o + k\Delta t, t^o + [k+1]\Delta t]$ . Then the discrete-time approximation is given by the following linear difference-equation system:

$$x(k+1) = x(k) + A_D x(k) + B_D w(k), \quad k \geq 0 , \quad (33)$$

where the matrices  $A_D$  and  $B_D$ , the state vector  $x(k)$ , and the forcing-term vector  $w(k)$  are given by:

$$A_D = A\Delta t; \quad (34)$$

$$B_D = B\Delta t; \quad (35)$$

$$x(k) = \begin{bmatrix} T_a(t^o + k\Delta t) \\ T_m(t^o + k\Delta t) \end{bmatrix}; \quad (36)$$

$$w(k) = \begin{bmatrix} T_o(t^o + k\Delta t) \\ Q_a(t^o + k\Delta t) \\ Q_m(t^o + k\Delta t) \end{bmatrix}. \quad (37)$$

Note that the time-period length  $\Delta t$  is assumed to be sufficiently small to permit forcing terms to be approximated as constant values over each successive time period  $k$ .

Letting  $V(k) \equiv V(t^o + k\Delta t)$  for any time-dependent variable  $V$ , and using equations (29) and (30) to substitute out for  $Q_a(k)$  and  $Q_m(k)$ , the explicit equation used to calculate the inside air temperature of a house for each time period  $k+1$  is then as follows:

$$\begin{aligned}T_a(k+1) &= \left(1 - \frac{\Delta t}{C_a} [U_a + H_m]\right) T_a(k) \\ &\quad + \left(\frac{\Delta t H_m}{C_a}\right) T_m(k) \\ &\quad - \left(\frac{\Delta t [1 - f_{ac}]}{C_a}\right) COPP(k) \\ &\quad + \frac{\Delta t}{C_a} ([1 - f_i] Q_i(k) + [1 - f_s] Q_s(k) + U_a T_o(k)) \\ &\equiv a_1 T_a(k) + a_2 T_m(k) - a_3 P(k) + \omega_a(k) . \quad (38)\end{aligned}$$

Similarly, the explicit equation used to calculate the inside mass temperature of a house for each time period  $k+1$  is as follows:

$$\begin{aligned}T_m(k+1) &= \left(\frac{\Delta t H_m}{C_m}\right) T_a(k) \\ &\quad + \left(1 - \frac{\Delta t H_m}{C_m}\right) T_m(k) \\ &\quad - \left(\frac{\Delta t f_{ac}}{C_m} COP\right) P(k) \\ &\quad + \frac{\Delta t}{C_m} [f_i Q_i(k) + f_s Q_s(k)] \\ &\equiv m_1 T_a(k) + m_2 T_m(k) - m_3 P(k) + \omega_m(k) . \quad (39)\end{aligned}$$

A complete specification for the parameter values  $COP$ ,  $U_a$ ,  $H_m$ ,  $C_a$ , and  $C_m$  appearing in the above ETP model can be determined for a particular House Quality Type = (House Size Type, House Thermal Integrity Type) from the settings for the size and thermal integrity attributes of this house given in Tables VI-VII. Specifically,  $COP$  is set in Table VII, and settings for the remaining parameters  $U_a$ ,  $H_m$ ,  $C_a$ , and  $C_m$  are calculated as follows:<sup>18</sup>

$$U_a = \frac{A_c}{R_c} + \frac{A_d}{R_d} + \frac{A_f}{R_f} + \frac{A_g}{R_g} + \frac{A_w}{R_w} + VH_a FhI; \quad (40)$$

$$H_m = h_s \left[ (A_{wt} - A_g - A_d) + A_{wt} IWR + \frac{A_c n}{EWR} \right]; \quad (41)$$

$$C_a = 3VH_a Fh; \quad (42)$$

$$C_m = Fm_f - 2VH_a Fh . \quad (43)$$

The values for the auxiliary parameters  $A_{wt}$  (gross exterior wall area),  $A_g$  (gross window area),  $A_d$  (total door area),  $A_w$  (net exterior wall area),  $A_c$  (net exterior ceiling area), and  $A_f$  (net exterior floor area), all in  $\text{ft}^2$  units, can be calculated as functions of house size attributes as follows:

$$A_{wt} = 2nh[1 + R]\sqrt{\frac{F}{nR}}; \quad (44)$$

$$A_g = WWR \times A_{wt} \times EWR; \quad (45)$$

$$A_d = n_d \times A_{1d}; \quad (46)$$

$$A_w = (EWR \times A_{wt}) - (A_d + A_g); \quad (47)$$

$$A_c = \frac{F}{n} \times ECR; \quad (48)$$

$$A_f = \frac{F}{n} \times EFR . \quad (49)$$

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<sup>18</sup>The expressions (41) and (47) below for  $H_m$  and  $A_w$  are consistent with the GLD documentation [31] and the GLD source code [32]. The  $H_m$  and  $A_w$  expressions in the GLD documentation [18] appear to be incorrect.

TABLE V  
HOUSEHOLD THERMAL DYNAMICS: SYMBOLS AND DESCRIPTIONS FOR PARAMETERS, VARIABLES, AND FORCING TERMS.

Parameters	Explanations
$A$	Matrix of ETP model coefficients
$A_{1d}$	Area of one door ( $\text{ft}^2$ )
$B$	Matrix of ETP model coefficients
$C_a$	Heat capacity ( $\text{BTU}/^\circ\text{F}$ , or $\text{kW}/^\circ\text{C}$ ) of the inside air mass
$C_m$	Heat capacity ( $\text{BTU}/^\circ\text{F}$ , or $\text{kW}/^\circ\text{C}$ ) of the inside solid mass
COP	Coefficient-of-performance (unit free) for household A/C systems
$\Delta t$	Time-period length (300sec, or 1hr/12)
ECR	Exterior ceiling, fraction of total (decimal %)
EFR	Exterior floor, fraction of total (decimal %)
EWR	Exterior wall, fraction of total (decimal %)
$F$	Floor area ( $\text{ft}^2$ ) = $x \times y \times n$
$f_i, f_s, f_{ac}$	Fractions (decimal %) for heat gain delivery from $(Q_i(t), Q_s(t), -\text{COP}(t))$ to $Q_m(t)$
$H_m$	Thermal conductance ( $\text{Btu}/\text{hr} \cdot {}^\circ\text{F}$ ) between inside air & solid masses
$h_s$	Interior surface heat transfer coefficient ( $\text{Btu}/\text{hr} \cdot {}^\circ\text{F} \cdot \text{ft}^2$ )
$I$	Infiltration air exchange (times per hr)
IWR	Interior/exterior wall surface ratio (unit free)
$k$	Time-period index, $k = 0, 1, \dots$
$m_f$	Total thermal mass, per unit floor area ( $\text{Btu}/^\circ\text{F} \cdot \text{ft}^2$ )
$n$	Number of stories (integer)
$n_d$	Number of doors (integer)
$R$	Floor aspect ratio $y/x$ (unit free)
$R_c$	R-value for ceilings ( $\text{hr} \cdot {}^\circ\text{F} \cdot \text{ft}^2/\text{Btu}$ )
$R_d$	R-value for doors ( $\text{hr} \cdot {}^\circ\text{F} \cdot \text{ft}^2/\text{Btu}$ )
$R_f$	R-value for floors ( $\text{hr} \cdot {}^\circ\text{F} \cdot \text{ft}^2/\text{Btu}$ )
$R_g$	R-value for windows ( $\text{hr} \cdot {}^\circ\text{F} \cdot \text{ft}^2/\text{Btu}$ )
$R_w$	R-value for walls ( $\text{hr} \cdot {}^\circ\text{F} \cdot \text{ft}^2/\text{Btu}$ )
$U_a$	Thermal conductance ( $\text{Btu}/\text{hr} \cdot {}^\circ\text{F}$ ) between internal and external air masses defining the thermal envelope of a house
$VH_a$	Volumetric heat capacity of air at standard conditions ( $\text{Btu}/^\circ\text{F} \cdot \text{ft}^3$ )
WWR	Window-to-exterior-wall ratio (decimal %)
$x, y, h$	Width, depth, and height (ft)
Variables and Forcing Terms	Explanations
$P(t)$	A/C power usage ( $\text{Btu}/\text{hr}$ , or $\text{kW}$ ) at $t$
$Q_a(t)$	Heat flow rate ( $\text{Btu}/\text{hr}$ , or $\text{kW}$ ) to inside air mass at $t$
$Q_i(t)$	Heat flow rate ( $\text{Btu}/\text{hr}$ , or $\text{kW}$ ) from internal appliances and occupants at $t$
$Q_m(t)$	Heat flow rate ( $\text{Btu}/\text{hr}$ , or $\text{kW}$ ) to inside solid mass at $t$
$Q_s(t)$	Heat flow rate ( $\text{Btu}/\text{hr}$ , or $\text{kW}$ ) from solar radiation at $t$
$t$	Time point indicator (sec)
$T_a(t)$	Inside air temperature ( ${}^\circ\text{C}$ or ${}^\circ\text{F}$ ) at $t$
$T_m(t)$	Inside mass temperature ( ${}^\circ\text{C}$ or ${}^\circ\text{F}$ ) at $t$
$T_o(t)$	Outside air temperature ( ${}^\circ\text{C}$ or ${}^\circ\text{F}$ ) at $t$
$w(t)$	Vector of forcing terms at $t$
$x(t)$	Vector of state variables at $t$

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TABLE VI  
HOUSE SIZE TYPES

Parameter	Small	Normal	Large
$A_{1d}$ ( $\text{ft}^2$ )	19.5	19.5	19.5
ECR (decimal %)	1.0	1.0	1.0
EFR (decimal %)	1.0	1.0	1.0
EWR (decimal %)	1.0	1.0	1.0
$F$ ( $\text{ft}^2$ )	900	1400	2250
$h$ (ft)	8	8	8
$h_s$ (Btu/hr- ${}^\circ\text{F}$ - $\text{ft}^2$ )	1.46	1.46	1.46
IWR (unit free)	1.5	1.5	1.5
$m_f$ (Btu/ ${}^\circ\text{F}$ - $\text{ft}^2$ )	2.0	2.0	2.0
$n$ (integer)	1	1	1
$n_d$ (integer)	4	4	4
$R$ (unit free)	1.5	1.5	1.5
WWR (decimal %)	0.07	0.07	0.07

TABLE VII  
HOUSE THERMAL INTEGRITY TYPES.

Parameter	Poor	Normal	Good
COP (unit-free)	3.9	3.9	3.9
$I$ (times per hr)	1.5	1.0	0.5
$f_i$ (decimal %)	0.5	0.5	0.5
$f_s$ (decimal %)	0.5	0.5	0.5
$f_{ac}$ (decimal %)	0.0	0.0	0.0
$R_c$ ( $\text{hr} \cdot {}^\circ\text{F} \cdot \text{ft}^2/\text{Btu}$ )	11	30	48
$R_d$ ( $\text{hr} \cdot {}^\circ\text{F} \cdot \text{ft}^2/\text{Btu}$ )	3	3	11
$R_f$ ( $\text{hr} \cdot {}^\circ\text{F} \cdot \text{ft}^2/\text{Btu}$ )	4	19	30
$R_g$ ( $\text{hr} \cdot {}^\circ\text{F} \cdot \text{ft}^2/\text{Btu}$ )	0.79	1.23	3.26
$R_w$ ( $\text{hr} \cdot {}^\circ\text{F} \cdot \text{ft}^2/\text{Btu}$ )	4	11	22
$VH_a$ ( $\text{Btu}/^\circ\text{F} \cdot \text{ft}^3$ )	0.018	0.018	0.018

TABLE VIII  
DEFINITIONS FOR HOUSE QUALITY TYPES.

House Quality	Low	Medium	High
House Size	Small	Normal	Large
House Thermal Integrity	Poor	Normal	Good

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