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INTRODUCTORY NOTES ON

WALRASIAN GENERAL EQUILIBRIUM MODELING

https://www2.econ.iastate.edu/tesfatsi/WalrasIntro.pdf

https://www2.econ.iastate.edu/tesfatsi/Syl502.htm

Key Questions:

- Basic assumptions underpinning the Walrasian General Equilibrium (WGE) Model

- What is a "Walrasian equilibrium"?

- How should an individual person's "welfare" be measured?

- How should "social welfare" be measured for an economy?

- In what sense is social welfare "optimized" in a Walrasian equilibrium?

- How robust is WGE modeling to changes in its basic assumptions?

1   Introduction

Decentralized market economies are complex dynamic systems. Large numbers of micro agents engage repeatedly in local interactions, giving rise to global regularities such as employment and growth rates, income distributions, market institutions, and social conventions. These global regularities in turn feed back into the determination of local interactions. The result is an intricate system of interdependent feedback loops connecting micro behaviors, interaction patterns, and global regularities.

Macroeconomists have grappled with the modeling of decentralized market economies for hundreds of years. Nevertheless, the Walrasian general equilibrium model devised by the nineteenth-century French economist Leon Walras (1834-1910) still remains the fundamental paradigm that frames the way many macroeconomists think about this issue.

As detailed by Katzner [1] and Takayama [6], Walrasian equilibrium in modern-day form is a precisely formulated set of conditions under which feasible allocations of goods and services can be price-supported in an economic system organized on the basis of decentralized markets with private ownership of productive resources. These conditions postulate the existence of a finite number of price-taking profit-maximizing firms who produce goods and services of known type and quality, a finite number of consumers with exogenously determined preferences who maximize their utility of consumption taking prices and dividend payments as given, and a Walrasian Auctioneer (or equivalent clearinghouse construct) that determines prices to ensure each market clears.[1] Assuming consumer nonsatiation, the First Welfare Theorem guarantees that every Walrasian equilibrium allocation is Pareto efficient.

As preliminary preparation for undertaking a systematic survey of current macroeconomic theory, these notes provide a brief overview of the Walrasian general equilibrium model. In subsequent parts of the course, as each macroeconomic model is surveyed in turn, connections to (and departures from) the basic Walrasian paradigm will be highlighted.

2 Structural and Behavioral Underpinnings of the Basic Walrasian Framework

As detailed in Katzner [1], the *Walrasian general equilibrium (WGE)* model represents a precisely formulated set of conditions under which feasible allocations of goods and services can be supported by price systems in decentralized market economies characterized by price-taking consumers and firms and the private ownership of capital and labor. The standard textbook version of the Walrasian general equilibrium model incorporates nine basic assumptions, as follows:

A1: There is a fixed finite number of distinct consumption good and capital good types. Each good is *private* in the sense that it is both excludable and rival, where *excludable* means that people can be excluded from consuming the good and *rival* means that

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[1]The colorful term "Walrasian Auctioneer" was first introduced by Leijonhufvud [2]. He explains the origins of the term as follows (personal correspondence, May 10, 2004): "I had come across this statement by Norbert Weiner, made in the context of explaining Maxwell's Demon to a lay audience, to the effect that 'in the physics of our grandfathers' information was costless. So I anthropomorphized the tâtonnement process to get a Walras's Demon to match Maxwell's." See also Leijonhufvud [3].

one person's consumption of the good reduces the amount available for consumption by others. *Note:* For capital goods, consumption = depreciation (used up portion) of the capital good.

A2: There is a fixed finite number of agents, called consumers, who have preferences over different bundles of consumption goods and who own nonnegative initial endowments of capital goods and labor.

A3: The preferences of each consumer are exogenously given and can be represented by a utility function.

A4: There is a fixed finite number of agents, called firms, who produce consumption goods for sale to consumers using labor services and capital services purchased from consumers as inputs to production. Consumers are the ultimate owners of firms; profits are distributed back to consumers as dividends in proportion to their ownership shares.

A5: The income of consumers comes from dividends and from the sale of capital services and labor services.

A6: Markets for services and consumption goods are *complete*. This means that, for each valued service and consumption good, there is a market price at which it can be bought or sold.

A7: Consumers, taking expected goods prices, wages, rental rates, and dividends as given, choose demands for consumption goods and supplies of capital and labor services to maximize their utility subject to a budget constraint (expenditure less than or equal to expected income) and physical feasibility conditions (nonnegativity and endowment constraints).

A8: Firms, taking expected goods prices, wages, and rental rates as given, choose supplies of goods and demands for capital and labor services to maximize expected profits subject to technological feasibility conditions (nonnegativity constraints, and production relations associating inputs with possible outputs).

A9: All purchase and sale agreements are costlessly arranged and enforced.

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Under assumptions A1-A9, then, the world is viewed as highly orderly. Property rights are well established and costlessly enforced. Potentially disruptive behavior such as incorrect expectations, the breaking of contracts, theft, power struggles, and status competition is not permitted. Within this orderly world, researchers have established additional restrictions on utility functions and production relations guaranteeing the existence of a unique *Walrasian equilibrium*, that is, a unique set of relative prices and corresponding demand and supply quantities at which, assuming fulfilled expectations, all consumers are maximizing their utility, all firms are maximizing their profits, and all markets clear.

A simple example of a Walrasian general equilibrium model is outlined in the next section. A detailed presentation of the equations underlying this example can be found in the course packet for this course.

3   A Simple WGE Illustration

Consider, for concreteness, the following illustrative Walrasian general equilibrium model over a time period $T = [t^0, t^1]$. The economy at time t^0 consists of n utility-maximizing consumers, $i = 1, \ldots, n$, and two profit-maximizing firms, X and Y. Each consumer is endowed with labor, capital, an ownership share in firm X, and an ownership share in firm Y. During period T the firms X and Y produce distinct consumption goods x and y, respectively, using labor services and capital services purchased from consumers at the beginning of period T. The consumption goods are sold to the consumers at the end of period T, and all profits are distributed back to the consumers as dividends in proportion to their ownership shares. The structure of this economy is schematically depicted in Figure 1.

Each consumer i at time t^0 has expectations regarding the dividends (i.e., profit distributions) he will receive from firms X and Y, as well as expectations with regard to prices (i.e., price of good x, price of good y, wage rate for labor services, and rental rate for capital services). Conditional on these expectations, the consumer chooses his demands for goods x and y and his supplies of capital and labor services to maximize his period T utility subject to physical feasibility constraints and a budget constraint (expenditures less than or equal to income). The solution to this utility maximization problem give the demands and supplies for consumer i as a function of his dividend expectations, his price expectations, his tastes

(utility function), and his labor and capital endowments.

Each firm X and Y maximizes its period T expected profits subject to nonnegativity and technology constraints, conditional on expected prices. The solutions to these profit-maximizing problems give the supplies and demands for each firm as a function of its price expectations and its technology (production and cost functions).

DEFINITION: A specific vector e^* consisting of numerical values for consumer supplies and demands for services and consumption goods, firm demands and supplies for services and consumption goods, nonnegative prices, expected prices, and expected dividends is said to be a *Walrasian equilibrium* if the following four conditions hold:

(a)Individual Optimality: At e^*, each consumer i is maximizing his utility, conditional on expected prices and expected dividends, and each firm X and Y is maximizing its profits conditional on expected prices.

(b) Fulfilled Expectations: At e^*, expected prices coincide with actual prices and expected dividends coincide with actual dividends calculated as consumer shares of firm profits.

(c) Market Clearing: At e^*, excess supply (i.e., supply minus demand) is greater than or equal to zero in each market for each service and consumption good.

(d) Walras' Law: At e^*, the total value of excess supply is zero.

Remark on Walras' Law: Suppose consumers are *nonsatiated* at e^*, meaning they would demand more goods and services if they had more income. Then conditions (a) and (b) necessarily imply condition (d). Consequently, given consumer nonsatiation, condition (d) can be omitted from the definition of a Walrasian equilibrium. See the appendix for a more detailed discussion of the important economic implications of condition (d).

4 Pareto Efficiency and the First Welfare Theorem

How should a person's welfare be measured? Under what circumstances is it appropriate to say that one person is better off than another?

Political philosophers addressing this question generally focus on political rights (including property rights) as well as on economic standards of living. For example, the famous

political philosopher John Rawls [4] has attempted to construct welfare measurements in terms of a "primary good" index, where by primary goods he means goods which are generalized means for the attainment of a decent quality of life. Thus, primary goods include political goods (liberties such as freedom of speech, freedom of religion, etc.) as well as basic economic goods (shelter, food, clothing, etc.).

Economists, on the other hand, have primarily devised welfare criteria that focus on non-wastage of resources, where resources are interpreted in two different but related ways: non-wastage of *physical resources* ("productive efficiency," or simply "efficiency") and non-wastage of *utility* ("Pareto-efficiency"). Political rights are considered to be part of the given background context. In particular, property ownership is generally taken as given; and the discussion of welfare focuses primarily on the possible benefits of a redistribution of income (i.e., of the payments received by the owners of human and physical capital) rather than on a redistribution of property ownership per se.

Consider first the concept of (productive) efficiency, as defined in standard economic texts:

DEFINITION: An economy is said to exhibit *(productive) efficiency* if, within the limitations of technology and resources, there is no feasible way to increase the amount of produced output, holding fixed the current amount of inputs to production, or to decrease the amount of inputs to production, holding fixed the current amount of produced output.

In contrast, the concept of Pareto efficiency considers whether utility is being wasted. The general definition of Pareto efficiency is as follows:

DEFINITION: A feasible allocation of goods and services for an economy is said to be *Pareto efficient* if there does not exist any other feasible allocation of goods and services for the economy under which the utility of each person in the economy is at least as great and the utility of at least one person is strictly greater.

Assuming consumer nonsatiation (more preferred to less consumption) and positive marginal productivity for inputs to production, efficiency in the sense of nonwastage of physical resources is a necessary but not sufficient condition for Pareto efficiency. It is now possible to state what is widely perceived to be the most important implication of Walrasian equilib-

rium, the First Welfare Theorem. For a proof of this theorem, see for example Takayama [6, Section C, pp. 185–204], especially Theorem 2.C.1 on page 192.

THE FIRST WELFARE THEOREM: Let a Walrasian general equilibrium model be given, as outlined in sections 1 and 2. Suppose the utility function of each consumer is strictly increasing with respect to each of its arguments, implying that no consumer is ever satiated with respect to consumption and hence each consumer entirely exhausts his budget in equilibrium. Then, any Walrasian equilibrium for this model is Pareto efficient, in the sense that the allocation of goods and services resulting from this equilibrium is a Pareto efficient allocation.

5   Robustness of WGE Modeling

The Walrasian general equilibrium model outlined in Sections 2 and 3 can be enhanced with more explicit analytical details and more sophisticated features. However, its defining *structural* characteristic is that no direct agent-agent interactions take place. Rather, all agent-agent interactions are mediated by an implicit clearinghouse referred to as the *Walrasian Auctioneer*.

More precisely, *strategic interaction* is said to occur between two agents if the choice of a decision for at least one of the agents depends upon what he perceives or expects the decision of the other agent to be. The Walrasian general equilibrium model reflects the view that, in decentralized market economies, price systems reduce or even eliminate the need for economic agents to interact strategically.

As discussed and illustrated more carefully in Tesfatsion [7], a companion slide-set to these notes, the two key observations here are as follows: (i) Expected (or presumed known) values for prices and dividend payments constitute the only links among consumers and firms in the Walrasian general equilibrium model prior to actual trades; and (ii) no trades take place except at equilibrium values. Since expected (or presumed known) prices and dividend payments are treated as parameters by these agents in their decision problems, these decision problems reduce to "control" problems. That is, the decision problem for each agent only includes decision variables fully under the agent's own control; the decision variables for

other agents do not appear, implying there is no strategic interaction.

In systems science parlance, the global allocation problem has been decomposed into a collection of individual agent allocation problems by the introduction of linking variables (prices and dividends). The equilibrium values for the linking variables are determined by calculations performed by the fictitious Walrasian Auctioneer; they do not arise from any actions of the consumers or firms within the model.

For example, the Walrasian Auctioneer might be presumed to use the following tatonnement (iterative solution) process to discover equilibrium prices and dividend payments: The Auctioneer starts by "calling out" arbitrary positive prices for goods and services to firms and consumers as well as nonnegative dividend payments for consumers, requiring that the consumers and firms in turn report back to the Auctioneer their optimal demand and supply quantity responses. If these quantity responses imply, in aggregate, that a good or service with a positive price is in excess demand (supply), the Auctioneer then calls out a slightly higher (lower) price for this good or service as well as revised dividend payments for consumers. The Walrasian Auctioneer stops calling out new prices and dividend payments only when all markets for goods and services are in equilibrium in the following sense: (a) There is no excess demand in any market; (b) any good or service in excess supply has had a zero price called out for it; and (c) the dividend payments called out by the Auctioneer equal the actual dividends that would be paid out by the firms, given the final called-out prices.

The robustness of the Walrasian general equilibrium model to changes in its pricing structure can be tested by considering three basic questions.

Question 1: How might strategic interaction become important if firms set prices for their inputs and outputs?

Firms' actions become strategically linked together if they understand and exploit the fact that the demand for their outputs and the distribution of labor and capital rental services across firms depend on the prices they set as well as on the prices set by other firms. For example, firm X might be able to bid away labor services from firm Y by offering a higher wage than the wage set by firm Y; and similarly with regard to attracting an increased supply of capital services and an increased output demand. Consequently, the attraction

and retention of service suppliers and customers now involves a careful consideration of the pricing strategies of other firms.

Question 2: How might expectations and learning rules become important if firms set prices for their inputs and outputs?

Realistically, firms would not have costless access to complete and correct information regarding the supply and demand functions they face for inputs and outputs, information that is critical for the price-setting process. In this case, firms would face a *dual control problem* at each point in time in the sense that the manager of each firm would have two potentially conflicting objectives:

> *Information exploitation*: Set prices to maximize expected total profits conditional on my *current* information regarding the supply and demand functions faced by my firm.

> *Information exploration*: Set prices in an attempt to learn more about the supply and demand functions faced by my firm, so that *future* profits might be increased even if these learning efforts lead to lower *current* profits.

The situation is further complicated by the fact that the supplies and demands for a firm's inputs and outputs depend not only on its own prices but also on the prices set by other firms. Indeed, past prices will also affect the firm's current supplies and demands to the extent that these past prices affect current consumer budget constraints and search behavior.

Also, firms can now offer different wage rates to observationally equivalent workers and different goods prices to observationally distinct consumers for units of the same type of good. Consequently, it is highly unrealistic to assume that consumers can costlessly acquire complete and correct information regarding the wage rates and goods prices they face. Rather, consumers would presumably have to undertake some form of sequential search to learn about the current distribution of wage rates and goods prices. Presumably, however, this search would involve opportunity costs for consumers in terms of delayed consumption and foregone wages. Consequently, consumers might decide to sample only a small fraction of the available wage offers and goods prices and then to accept the highest wage offer and lowest goods prices found to date instead of carrying out a complete global search of all possibilities.

The rule by which a consumer (or any agent) decides to stop sequential sampling is called a *sequential stopping rule* in the statistical decision theory literature. When consumers use stopping rules, a nondegenerate distribution of wage rates can exist in "equilibrium" for a single type of labor, and a nondegenerate distribution of prices can exist in "equilibrium" for a single type of good. Moreover, consumers deciding to supply labor to or buy goods from a firm in some time period may simply decide to stick with this firm in future time periods without engaging in more search (habit, brand effects,...). These considerations can further complicate the strategic price-setting rivalry among firms.

Question 3: How might bankruptcy rules, rationing rules, and inventory management become important if firms set prices for their inputs and outputs?

Markets are no longer guaranteed to clear at the levels of *planned* supplies and demands, since wage rates and goods prices might be "wrong."

If the demand for a good happens to exceed its actual supply, some type of formally or informally established rationing rule is needed to determine who gets the scarce supply of goods. If this situation arises frequently, a firm might want to institute an inventory plan so that excess demand can be satisfied out of inventory.

In the reverse case in which the actual supply of a good exceeds its demand, unintended inventories arise, and the firm would presumably want to take this possibility into consideration when making its price and quantity decisions. Even with an inventory plan, the possibility of insolvency (negative net worth) arises for a firm if it cannot sell all of its supply of goods since the firm might then be unable to fulfill its obligations with respect to wage and capital rental payments. If insolvency occurs, some type of formally or informally established rule is needed to determine how any assets still in the firm's possession are to be divided among its various creditors.

The robustness of the WGE model to modifications of its assumptions, and more generally its suitability as a benchmark for macroeconomic modeling, will be considered at greater length in later parts of this course. In particular, we will discuss a study [5] that examines what happens when a standard macroeconomic model based on the Walrasian paradigm is modified by the removal of all externally imposed global coordination conditions, leaving decision-making consumers and firms to fend for themselves.

References

[1] Katzner, Donald, *The Walrasian Vision of the Microeconomy: An Elementary Exposition of the Structure of Modern General Equilibrium Theory*, University of Michigan Press, Ann Arbor, 1989.

[2] Leijonhufud, Axel, "Keynes and the Keynesians: A suggested interpretation," *American Economic Review* 57(2), 1967, 401-410.

[3] Leijonhufvud, Axel, *On Keynesian Economics and the Economics of Keynes*, Oxford University Press, Oxford, 1968.

[4] Rawls, John, *A Theory of Justice*, Harvard University Press, 1971.

[5] Sinitskaya, E., and L. Tesfatsion, "Macroeconomies as Constructively Rational Games," *J. of Economic Dynamics and Control* 61(2015), 152-182. Preprint: https://www2.econ.iastate.edu/tesfatsi/MacroConstructiveRationalityWP.SinitskayaTesfatsion.pdf

[6] Takayama, A., *Mathematical Economics* Cambridge University Press, l985.

[7] Tesfatsion, L., *Robustness of Walrasian General Equilibrium Modeling* (Slide-Set) https://www2.econ.iastate.edu/tesfatsi/WalrasianGERobustness.LT.pdf

Econ 502 Lecture Packet for Walrasian General Equilibrium

L. Tesfatsion, "Introduction to Walrasian General Equilibrium Modeling"

https://www2.econ.iastate.edu/tesfatsi/WalrasIntro.pdf

L. Tesfatsion, "Walrasian General Equilibrium: Benchmark of Coordination Success?"

https://www2.econ.iastate.edu/tesfatsi/Walbench.pdf

L. Tesfatsion, "Robustness of Walrasian General Equilibrium Modeling" (Slide-Set)

https://www2.econ.iastate.edu/tesfatsi/WalrasianGERobustness.LT.pdf

L. Tesfatsion, "Game Theory: Basic Concepts and Terminology (Notes & Slide-Set)"

https://www2.econ.iastate.edu/tesfatsi/gamedef.pdf

L. Tesfatsion, "Non-Walrasian Equilibrium: Illustrative Examples"

https://www2.econ.iastate.edu/tesfatsi/nonwalra.pdf