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Game Theory: Basic Concepts and Terminology

A **GAME** consists of:

- a collection of decision-makers, called *players*;
- the possible information states of each player at each decision time;
- the collection of possible moves (decisions, actions, plays,...) that each player can choose to make in each of his possible information states;
- a procedure for determining how the move choices of all the players collectively determine the possible outcomes of the game;
- preferences of the individual players over these possible outcomes, typically measured by a *utility* or *payoff* function.

EMPLOYER

		C	D
WORKER	C	(40,40)	(10,60)
	D	(60,10)	(20,20)

Illustrative Modeling of a Work-Site Interaction as a "Prisoner's Dilemma Game"

D = Defect (Shirk)   C = Cooperate (Work Hard),

(P1,P2) = (Worker Payoff, Employer Payoff)

A **PURE STRATEGY** for a player in a particular game is a complete contingency plan, i.e., a plan describing what move that player should take in each of his possible information states.

A **MIXED STRATEGY** for a player i in a particular game is a probability distribution defined over the collection $\mathcal{S}_i$ of player i's possible pure strategy choices. That is, a mixed strategy assigns a nonnegative probability Prob(s) to each pure strategy s in $\mathcal{S}_i$, with

$$\sum_{s \in \mathcal{S}_i} \text{Prob}(s) \ = \ 1 \ . \tag{1}$$

EXPOSITIONAL NOTE:

For simplicity, the remainder of these brief notes will develop definitions in terms of pure strategies; the unqualified use of "strategy" will always refer to pure strategy. Extension to mixed strategies is conceptually straightforward.

ONE-STAGE SIMULTANEOUS-MOVE N-PLAYER GAME:

- The game is played just once among N players.
- Each of the N players *simultaneously* chooses a strategy (move) based on his current information state, where this information state does *not* include knowledge of the strategy choices of any other player.
- A payoff (reward, return, utility outcome,...) for each player is then determined as a function of the N simultaneously-chosen strategies of the N players.

Note: For ONE-stage games, there is only one decision time. Consequently, a choice of a strategy based on a current information state is the same as the choice of a move based on this current information state.

ITERATED SIMULTANEOUS-MOVE N-PLAYER GAME:

- The game is played among N players over successive iterations $T = 1, 2, \ldots TMax$.
- In each iteration T, each of the N players *simultaneously* makes a move (action, play, decision,...) conditional on his current information state, where this information state does *not* include the iteration-T move of any other player.
- An iteration-T payoff (reward, return, utility outcome,...) is then determined for each player as a function of the N simultaneously-made moves of the N players in iteration T.
- If $T < TMax$, the next iteration T+1 then commences.
- The information states of the players at the beginning of iteration T+1 are typically updated to include at least some information regarding the moves, payoffs, and/or outcomes from the previous iteration T.

Note: For ITERATED games there are multiple decision times. Consequently, a choice of a move based on a current information state does not constitute a strategy (complete contingency plan). Rather, a strategy is the choice of a move for the current iteration, given the current information state, together with a designation of what move to choose in each future iteration conditional on every possible future information state.

"PAYOFF MATRIX" FOR A ONE-STAGE SIMULTANEOUS-MOVE 2-PLAYER GAME:

Consider a one-stage simultaneous-move 2-player game in which each player must choose to play one of M feasible strategies $S_1, \ldots, S_M$. The *Payoff Matrix* for this 2-player game then consists of an $M \times M$ table that gives the payoff received by each of the two players under each feasible combination of moves the two players can choose to make.

More precisely, each of the M rows of the table corresponds to a feasible strategy choice by Player 1, and each of the M columns of the table corresponds to a feasible strategy choice by Player 2. The entry in the ith row and jth column of this $M \times M$ table then consists of a pair of values $(P_1(i,j), P_2(i,j))$.

The first value $P_1(i,j)$ denotes the payoff received by Player 1 when Player 1 chooses strategy S_i and Player 2 chooses strategy S_j, and the second value $P_2(i,j)$ denotes the payoff received by Player 2 when Player 1 chooses strategy S_i and Player 2 chooses strategy S_j. **See the 2-player example depicted on the next page**.

This definition is easily generalized to the case in which each player has a different collection of feasible strategies to choose from (different by type and/or number).

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NASH EQUILIBRIUM FOR AN N-PLAYER GAME:

A specific combination $(S_1^*, \ldots, S_N^*)$ of feasible strategy choices for an N-player game, one strategy choice S_i^* for each player i, is called a *(Pure Strategy) Nash equilibrium* if no player i perceives any feasible way of achieving a higher payoff by switching unilaterally to another strategy S_i'.

DOMINANT STRATEGY FOR AN N-PLAYER GAME:

A feasible strategy for a player in an N-player game is said to be a *dominant strategy* for this player if it is this player's *best response* to *any* feasible choice of strategies for the other players.

For example, suppose S_1^* is a dominant strategy for player 1 in an N-player game. This means that, no matter what feasible combination of strategies $(S_2, \ldots, S_N)$ players 2 through N might choose to play, player 1 attains the highest feasible (expected) payoff if he chooses to play strategy S_1^*.

QUESTIONS:

(1) Does the previously depicted worker-employer game have a Nash equilibrium?

(2) Does either player in this game have a dominant strategy?

(3) What is the key distinction between a dominant strategy and a strategy constituting part of a Nash equilibrium?

PARETO EFFICIENCY:

Intuitive Definition:

A feasible combination of decisions for a collection of agents is said to be *Pareto efficient* if there does *not* exist another feasible combination of decisions under which each agent is at least as well off and some agent is strictly better off.

More Rigorous Definition: N-Player Game Context

For each $i = 1, \ldots, N$, let P_i denote the payoff attained by player i under a feasible strategy combination $S = (S_1, \ldots, S_N)$ for the N players. The strategy combination S is said to be *Pareto efficient* if there does *not* exist another feasible strategy combination S' under which each player i achieves at least as high a payoff as P_i and some player j achieves a strictly higher payoff than P_j. The payoff outcome $(P_1, \ldots, P_N)$ is then said to be a *Pareto efficient payoff outcome*.

QUESTION:

Does the previously depicted worker-employer game have a Pareto efficient strategy combination?

PARETO DOMINATION:

Intuitive Definition: A feasible combination of decisions for a collection of agents is said to be *Pareto dominated* if there *does* exist another feasible combination of decisions under which each agent is at least as well off and some agent is strictly better off.

More Rigorous Definition: N-Player Game Context For each $i = 1, \ldots, N$, let P_i denote the payoff attained by player i under a strategy combination $S = (S_1,\ldots,S_N)$ for the N players. The strategy combination S is said to be *Pareto dominated* if there *does* exist another feasible strategy combination S' under which each player i achieves at least as high a payoff as P_i and some player j achieves a strictly higher payoff than P_j.

QUESTION:

Does the previously depicted worker-employer game have strategy combinations that are Pareto dominated?

COORDINATION FAILURE:

Intuitive Definition: A combination of decisions for a collection of agents is said to exhibit *coordination failure* if mutual gains, attainable by a collective switch to a different feasible combination of decisions, are not realized because no individual agent perceives any feasible way to increase their own gain by a unilateral deviation from their current decision.

More Rigorous Definition: N-Player Game Context A strategy combination $S = (S_1, \ldots, S_N)$ is said to exhibit *coordination failure* if it is a Pareto-dominated Nash equilibrium.

QUESTIONS:

Does the previously depicted worker-employer game have a move combination that exhibits coordination failure?

Might the *iterative* play of this worker-employer game help alleviate coordination failure problems?