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HUMAN CAPITAL INVESTMENT AND THE LOCALLY RATIONAL CHILD*

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ABSTRACT

In traditional studies of parent-child transfers, the distortion in educational investment in poorer children is primarily attributed to the absence of a human capital loan market and hence to the unequal access of children to educational opportunity. This paper cautions that the distortion might also be attributed, in part, to disincentive problems that cause children to make inefficient use of their educational opportunities. The latter possibility is demonstrated in the context of an overlapping generations economy with multiple family dynasties in which the ability levels of children are random and unobservable and parents allocate part of their income to educational investment in their children. Children, in turn, decide how much effort to exert in school; but their effort decisions are affected by their family circumstances, in particular by the marginal returns to schooling they perceive their parents to have experienced. In this case the distortionary effects of government tax-transfer policies result in children making suboptimal effort choices. Small income transfers from richer to poorer families do increase social welfare by enabling poorer families to invest more optimally in their children. As the transfer policy becomes increasingly egalitarian, however, effort levels soon begin to diminish and social welfare ultimately falls to a level well below that achieved with no income transfers.

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1. INTRODUCTION

The relationship between human capital and lifetime earnings is well-established in the economics literature. Particularly well-established is the positive link between increased years of education and increased real earnings. This link is apparent across all ethnic and racial groupings in the United States and, indeed, seems similarly well-established in other developed and lesser-developed countries [Schultz (1988, pp. 616-617)].

In the absence of some form of public intervention, however, optimal investment in human capital is unlikely to occur. Applying the usual neoclassical principle, the Pareto optimal investment rule would be for each child to be educated up to the point where the present value of the investment costs is equal to the expected present value of the child's future returns to education [Becker (1975), Rosen (1977)]. The difficulty is that anticipated future returns to education are typically not permitted as collateral by private financial lenders. Investments in a child's human capital will thus generally be tied to the well-being of the child's parents rather than to his own innate skills and earnings capacity. If the costs of human capital investment are substantial, liquidity constraints on the poor will thus result in underinvestment (from society's perspective) in the education of poor children. In addition, the suboptimal stock of human capital embodied in poor children will inflate the returns to human capital for wealthy children, leading to an overinvestment (from society's perspective) in the education of these children.

This issue is discussed by Loury (1981) in his classic article on intergenerational transfers of wealth. Loury sets out a two-period lived overlapping generations model in which the ability levels of children are randomly distributed and unobservable. Each parent, concerned for the utility attained by his child, allocates part of his income to investment in his child's human capital. Children are passive recipients of this human capital investment; they have no say either in its determination or in its consequences. Although average incomes are equal for all families when computed over a sufficiently long time horizon, there is some initial stickiness in relative family income rankings due to the inability of families to borrow against future earnings capacities. The public provision of education through purely redistributive nondistortionary tax policies is then shown to ameliorate this problem and improve social welfare by reducing the linkage between family income and educational investment.

Recent studies have generalized Loury's model in a number of interesting directions. For example, Durlauf (1996a,b) investigates a model with an imperfect human capital loan market and neighborhood technological externalities in the production of human capital. Families endogenously organize themselves into neighborhoods, and the tax base of a neighborhood determines a common level of human capital for each neighborhood child. This human capital, together with a productivity shock partly reflecting neighborhood-specific factors, then determines the future income of the child. In contrast to Loury (1981), Durlauf establishes that initial family income inequalities can persist in this context.

Other human capital studies such as Lucas (1988), Galor and Zeira (1993), Perotti (1993), and Galor and Tsiddon (1996) assume that children have direct control over physical resources and invest in their own human capital in order to maximize their expected lifetime utility, with or without the additional support of parental investments or bequests. As in Loury (19981), borrowing against future earning capacity is not permitted. In addition, however, the economy is subject to local externalities such as home environment or political voting effects and also to global technological externalities whereby the human capital investment of any one agent has a positive externality on the productivity of other agents. These externalities imply that initial disparities in family incomes due to imperfections in the human capital loan market may persist over time unless countered by appropriate income redistributions.

An important issue remains to be addressed, however. Suppose children are not modelled either as passive recipients of human capital investment or as rational investors in human capital on the basis of lifetime utility prospects. Rather, suppose the schooling effort exerted by children is affected by their particular family circumstances. In the absence of technological externalities, will distortions in educational investment in poorer children then be due solely to imperfections in the human capital loan market, and hence to the unequal access of children to educational opportunity? Or will distortions also be attributable in part to incentive problems that cause the children, themselves, to make inefficient use of the educational opportunities they receive?

As will be more carefully discussed in subsequent sections, empirical findings suggest the potential importance of the middle-ground case in which the schooling effort of children is assumed to be affected by family circumstances. This paper explores this case in the context of an overlapping generations model with human capital investment that extends the model by Loury (1981) in two principal ways.

First, each child's schooling effort is assumed to depend in part upon the child's perception of his marginal returns to schooling, which in turn is shaped by the child's perception of the marginal returns to schooling attained by his parent. Thus, family effects have a direct impact on each child's behavior.[1]

Second, the treatment of government tax policy within our model has several novel features. In contrast to Loury (1981), we assume that only distortionary income tax policies are available to government in its attempt to redistribute income from richer to poorer families, where the total number of families is arbitrary. Also, the government is forced to satisfy a hard budget constraint in every period, in the sense that all distributed subsidies must be financed by current tax receipts. In contrast, Loury (1981, p. 853) only requires that government satisfy a budget constraint in expectation, implying that recourse must be made

[1]Similarly, Streufort (1991) assumes that young adults estimate their return to schooling by sampling the return to schooling attained by adult workers in the labor force, where this sampling may be truncated from above due to underclass social isolation. Unlike the model developed here, however, Streufort employs a partial equilibrium framework in which schooling and labor (at each schooling level) are inelastically supplied and all other markets are ignored.

to a strong law argument if this is to be interpreted as a hard constraint. Finally, the tax policy in our model is characterized by a single parameter. As this parameter is increased from zero to an upper bound, the tax policy progresses from a purely libertarian policy with no redistribution of income to a perfectly egalitarian policy under which family incomes are equalized in each period. This permits us to analyze and compare welfare and income outcomes across a broad range of tax policies.

Our model demonstrates why the simple equalization of educational opportunity through income transfers to poor families may not result in Pareto optimal human capital investments, even in the absence of technological externalities. The crucial observation is that a government income tax policy can bias downward childrens' perceived marginal returns to schooling at the same time that it equalizes parental income transfers to children. We find, in particular, that a small degree of income subsidization for low income families does increase human capital production, raising both social welfare and GNP. Yet social welfare and GNP both decline precipitously along with schooling effort as government imposes progressively more egalitarian tax policies. Moreover, increasing the number of families in existence at each point in time only hastens and deepens the decline.

The basic human capital investment framework is set out in Section 2. Section 3 develops a particular model within this framework with functional specifications chosen to match the specifications of Loury (1981) as closely as possible for comparison purposes. Section 4 discusses empirical support for the view taken in Section 3 that family circumstances strongly affect a child's choice of schooling effort. Various dynamic properties of the human capital investment model are analytically derived in Section 5. The results of extensive simulation experiments conducted with this model are reported in Section 6. The final Section 7 gives concluding comments.

2. THE BASIC ECONOMY

This section sets out a human capital investment framework, hereafter referred to as the Basic Economy, that generalizes the model used by Loury (1981) to investigate the relationships among human capital investment, income distribution, and government tax and transfer policies. Loury assumes that the subsequent earnings of a child are determined by two factors: innate ability; and parental investment in the child's education. In the Basic Economy it is assumed that the child's subsequent earnings are also determined in part by the intensity of effort that the child devotes to his schooling.

Specifically, the Basic Economy is an overlapping generations economy that begins in period 1 and extends into the infinite future. The economy consists of a consumer sector and a government policymaker with tax and transfer powers. The rate of population growth is constant and equal to zero. Each agent lives for just two periods, a first period ("childhood") and a second period ("parenthood"). One child is born to each agent at the beginning of his second period of life. The economy has only one consumable resource, Q, assumed to be completely perishable and divisible. Adopting the standard convention that goods are distinguished by date of availability, the amount of Q available during period t represents "good t."

In the initial period 1, the population consists of N parents and N children, divided into N parent-child pairings. The N pairings constitute N distinct "family dynasties," $i = 1, \ldots, N$. Each child born in a subsequent period t is then assigned to one of the N family dynasties on the basis of the dynasty belonged to by his parent.

Consider any one family dynasty, say dynasty i. Dynasty i in the initial period 1 consists of one parent, P_{i1}, together with his child C_{i1}. The parent P_{i1} is assumed to have an exogenously given amount of pre-tax earnings y_{i1} measured in period 1 good, and is also characterized by an exogenously given intensity of effort level n_{i0} representing the effort that he put into his schooling as a child. In each subsequent period $t \geq 2$, dynasty i then consists of one parent P_{it}, born in period $t-1$, together with the child C_{it} of P_{it} born at the beginning of period t.

In each period $t \geq 1$, the child C_{it} is endowed with a random skill level $\alpha_{it} \in [0, 1]$ for producing Q, where α_{it} is assumed to be **unobservable**. In addition, the child receives a certain amount of resources e_{it} from his parent P_{it} in support of his education, where e_{it} is measured in period t good. The child must decide how much effort n_{it} he will devote to his schooling. The level of pre-tax earnings $y_{i,t+1}$ achieved by the child when he reaches adulthood in the subsequent period $t+1$ is assumed to depend positively on all three factors. Formally, the child's income (or human capital production) function takes the form

$$y_{i,t+1} \ = \ h(\alpha_{it}, e_{it}, n_{it}) \ , \quad h_1 > 0, \ h_2 > 0, \ h_3 > 0 \ , \qquad (1)$$

where $y_{i,t+1}$ is measured in good $t+1$.

Suppose the government policymaker at the beginning of period 1 selects a tax policy τ for determining the taxes and subsidies to be imposed on the earnings of each parent in each subsequent period. Let τ_{it} denote the tax or subsidy level imposed on the dynasty i parent P_{it} in period t under tax policy τ, where $\tau_{it} < 1$ denotes a tax and $\tau_{it} > 1$ denotes a subsidy (or "negative tax"). The after-tax earnings of P_{it} then take the form $\tau_{it} y_{it}$. It will be supposed that τ_{it} is a function of the pre-tax income y_{jt} earned by each dynasty j in period t, the tax policy τ, and the total number of dynasties N. That is,

$$\tau_{it} \ = \ G_i(y_{1t}, \ldots, y_{Nt}, \tau, N) \qquad (2)$$

for some function $G_i(\cdot)$, $i = 1, \ldots, N$.

The child C_{it} must decide what effort n_{it} he will devote to his schooling. It is assumed that the child is positively influenced in this decision by two factors: (a) the amount of resources e_{it} that his parent invests in his education; and (b) his expected marginal return to schooling effort, denoted by

$$r^e_{it} \;=\; (\partial \tau_{i,t+1} y_{i,t+1} / \partial n_{it})^e \;. \tag{3}$$

Note, in particular, that the child's choice of effort cannot **directly** depend on his ability level, since ability levels are assumed to be unobservable.

Formally, the choice of effort n_{it} takes the form[2]

$$n_{it} \;=\; n(e_{it}, r^e_{it}) \;, \quad n_1 > 0, \; n_2 > 0 \;. \tag{4}$$

It follows from (4) that a parent can influence his child's schooling effort in two ways. First, a parent can directly transfer resources to his child, as measured by the human capital investment level e_{it}. Second, as will be clarified below, the information the child obtains about his parent's income and schooling effort can influence the child's expected marginal return to schooling effort. Note, also, that the government tax policy τ indirectly affects the child's schooling effort through the investment level e_{it}, since e_{it} is allocated out of the parent's disposable income, and τ may also influence the child's schooling effort by influencing his expected marginal return to schooling effort.

The parent P_{it} faces a more complicated decision problem in period t than his child. Specifically, he must decide how much of his disposable income to devote to his own consumption versus how much to devote to investment in the education of his child. The utility attained by P_{it} in period t is assumed to be a function $U(c_{it}, V_{i,t+1})$ of his own period t consumption, c_{it}, and the utility $V_{i,t+1}$ he anticipates for his child in period $t+1$.

Let s_{it} denote the state vector describing the situation of the parent P_{it} at the beginning of period t in terms of his pre-tax earnings y_{it}, his past schooling effort $n_{i,t-1}$, and his income

[2] The functional form (4) can be viewed as the reduced form solution to the following optimization problem for the child: choose an effort level that maximizes expected adult after-tax income, net of effort costs. The child's effort supply function is assumed to be upward sloping in expected marginal return. Parental investments in education are assumed to make schooling easier, increasing the child's supply of effort at every possible level of expected marginal return. The empirical support for this modelling of schooling effort is assessed in Section 4, below.

tax or subsidy level τ_{it}, as well as the government's tax policy τ and the total number of dynasties N. That is, define

$$s_{it} \;=\; (y_{it}, n_{i,t-1}, \tau_{it}, \tau, N)\ . \tag{5}$$

It is assumed that the state vector (5) represents all of the information that is potentially available to the child C_{it} at the beginning of period t. Consequently, the child's expected marginal return to schooling effort (3) must ultimately be determined as a function of this state vector:[3]

$$r^e_{it} \;=\; r^e(s_{it})\ . \tag{6}$$

It follows from (6) that the child's expectations regarding returns to schooling are potentially affected by the tax policy τ as well as by his family's particular tax or transfer level τ_{it}. Thus, the tax system can influence the child's schooling effort—i.e., his human capital investment decision—by distorting the child's perceived returns to schooling. In contrast, in Loury (1981) the schooling effort of the child is effectively held constant, implying that the tax system cannot influence the human capital investment decision of the child.

Given (1), (4), and (6), the state vector s_{it} satisfies a recurrence relation of the form

$$\begin{aligned} s_{i,t+1} \;&=\; (\ h(\alpha_{it}, e_{it}, n(e_{it}, r^e(s_{it}))),\ \ n(e_{it}, r^e(s_{it})), \tau_{i,t+1}, \tau, N) \\ &\equiv\; F(\alpha_{it}, e_{it}, \tau_{i,t+1}, s_{it})\ . \end{aligned} \tag{7}$$

The budget set facing P_{it} in period t takes the form

$$B(s_{it}) \;=\; \{(c_{it}, e_{it}) \geq 0 \mid c_{it} + e_{it} = \tau_{it} y_{it}\}\ . \tag{8}$$

The value function $V_{it}(s_{it})$ for the parent P_{it} is then defined to be the maximum expected utility attainable by P_{it} in period t, given the state s_{it}. By construction, this value function

[3]The functional form (6) encompasses everything from rational to adaptive expectations. In Section 3 a specific form for relation (6) will be introduced which will imply that children form their expectations myopically on the basis of the particular circumstances they observe for their parents.

satisfies a dynamic programming recurrence relation of the form

$$V_{it}(s_{it}) \;=\; \max_{c_{it}, e_{it} \in B(s_{it})} E_{i,t}[U(c_{it}, V_{i,t+1}(s_{i,t+1})) \mid c_{it}, e_{it}, s_{it}] \;, \tag{9}$$

subject to $s_{i,t+1}$ being given by relation (7). The conditional expectation in (9) is taken with respect to the ability level α_{it} and the tax rate $\tau_{i,t+1}$ appearing in (7). Note that this construction implies that the parent knows the income function $h(\cdot)$ and the function $n(\cdot)$ that determines the effort level of his child.

3. AN ILLUSTRATIVE SPECIAL CASE OF THE BASIC ECONOMY

In this section, the dynamic properties of the Basic Economy are investigated for particular specifications for the income function, the effort function, the utility function, and the tax policy set out in general form in Section 2. These specifications are chosen to match as closely as possible the example studied by Loury (1981, pp. 855–857).

Consider, then, a special case of the Basic Economy in which the income function (1) for each dynasty i in each period t is given by

$$\begin{aligned} y_{i,t+1} &= h(\alpha_{it}, e_{it}, n_{it}) \\ &\equiv \lambda \cdot (\alpha_{it})^m \cdot (e_{it})^u \cdot (n_{it})^v \end{aligned} \tag{10}$$

for arbitrary positive constants λ, m, u, and v satisfying $0 < (u+v) \leq 1$. Suppose, also, that the effort function (4) for each dynasty i in each period t is given by

$$\begin{aligned} n_{it} &= n(e_{it}, r^e(s_{it})) \\ &\equiv (e_{it})^a \cdot (r^e(s_{it}))^b \end{aligned} \tag{11}$$

for arbitrary positive constants a and b satisfying $0 < (a+b) \leq 1$.

The utility attained by each dynasty i parent P_{it} in each period t is assumed to be given by

$$U_{it} \;=\; (c_{it})^{\gamma} \cdot (V_{i,t+1})^{1-\gamma} \;, \tag{12}$$

where c_{it} denotes the period t consumption of P_{it}, $V_{i,t+1}$ denotes the utility which P_{it} anticipates for his child in period $t+1$, and γ is an arbitrary constant assumed to satisfy $0 < \gamma < 1$.[4] Suppose $(1-\gamma)^k \log(V_{i,t+k})$ approaches zero as k becomes arbitrarily large. If anticipations are correct, implying $U_{it} = V_{it}$ for all t, it then follows that

$$\begin{aligned} \log(U_{it}) &= \gamma \log(c_{it}) + (1-\gamma) \log V_{i,t+1} \\ &= \gamma \log(c_{it}) + (1-\gamma)[\gamma \log(c_{i,t+1}) + (1-\gamma)\log(V_{i,t+2})] \\ &\vdots \\ &= \gamma[\sum_{k=0}^{\infty} (1-\gamma)^k \log(c_{i,t+k})] \ . \end{aligned} \tag{13}$$

Consequently, the ex post "true" utility of each dynasty i parent P_{it} in each period t is a function of the consumption levels of all current and future members of dynasty i.

The government must select a tax policy that is budgetarily feasible. Since the resource Q for the Basic Economy is nonstorable, budgetary feasibility in each period t requires that the sum of the after-tax incomes of the N dynasties should not exceed the sum of their before-tax incomes. Letting τ_{it} denote the tax or subsidy imposed on the dynasty i parent P_{it} in period t, the condition expressing this feasibility is

$$\sum_{i=1}^{N} \tau_{it} y_{it} \ \leq \ \sum_{i=1}^{N} y_{it} \ . \tag{14}$$

It will be supposed that the government selects a tax policy from among a family of feasible tax policies parameterized by τ, where τ lies in the interval $[0, 1/N]$. Specifically, given any τ in $[0, 1/N]$, and given the pre-tax income levels $\{y_{it} : i = 1, \ldots, N\}$ of the N dynasties in period t, the tax or subsidy level τ_{it} imposed on each parent P_{it} is determined by the relation

$$\tau_{it} y_{it} \ \equiv \ [1 - (N-1)\tau] \cdot y_{it} \ + \ \tau \cdot (\sum_{j \neq i} y_{jt}), \quad i = 1, \ldots, N \ . \tag{15}$$

[4] If $\gamma = 1$, parents places no weight on their progeny, and hence do not invest in education. Given (10), this in turn dooms all future generations to have zero income.

The tax policies τ in $[0, 1/N]$ all satisfy the budget feasibility condition (14) with exact equality, by construction. Moreover, the tax policies range from "libertarian" to "egalitarian" as τ increases from 0 to $1/N$. Given the tax policy $\tau = 0$, the tax rates τ_{it} are identically equal to 1, implying that no redistribution is undertaken. On the other hand, the tax policy $\tau = 1/N$ is egalitarian in the sense that each of the N dynasties in each period t receives precisely $1/N$ of the total income earnings of the N dynasties. Given any existing configuration of pre-tax incomes, the tax policy becomes progressively more egalitarian as τ ranges from 0 to $1/N$, in the sense that the variance of the after-tax incomes across dynasties declines in each period t.

For later purposes, it is useful to point out the free-riding problem that arises under the egalitarian tax policy, $\tau = 1/N$. In this case the after-tax income of each dynasty is given by mean dynasty income. Consequently, an increased effort level by any one dynasty only changes that dynasty's after-tax income to the extent that it has an effect on mean dynasty income; and, ceteris paribus, the larger is N, the smaller this effect will be. Consequently, an egalitarian tax policy gives dynasties an incentive to free ride on the effort levels of others, and this free-riding problem worsens as the number of dynasties increases.

More can now be said about r^e_{it}, the marginal return to schooling effort anticipated by the dynasty i child C_{it} in period t. Recall from relation (3) that r^e_{it} is assumed to be measured in terms of C_{it}'s after-tax income earnings for period $t+1$. Let r_{it} denote the actual marginal return to schooling effort attained by C_{it} in the subsequent period $t+1$. Using (10) and (15), one has

$$\begin{aligned} r_{it} &= \partial \tau_{i,t+1} y_{i,t+1} / \partial n_{it} \\ &= \partial [1 - (N-1)\tau] y_{i,t+1} / \partial n_{it} \\ &= [1 - (N-1)\tau] \partial y_{i,t+1} / \partial n_{it} \\ &= [1 - (N-1)\tau] v \cdot [y_{i,t+1} / n_{it}] \ . \end{aligned} \tag{16}$$

Thus, the **marginal** return to schooling effort, r_{it}, is proportional to the **average** return to schooling effort, $y_{i,t+1}/n_{it}$. Note that $[1-(N-1)\tau]v$ is strictly positive for all τ in the admissable range $[0, 1/N]$.

As discussed in Section 2, r^e_{it} must ultimately be describable as a function $r^e(s_{it})$ of the state vector s_{it}, where s_{it} consists of all information available to the child C_{it} at the beginning of period t. Consistent with this observation, it will be supposed that the child estimates his own average return to schooling effort, $y_{i,t+1}/n_{it}$, in (16) by considering the average return to schooling effort, $y_{it}/n_{i,t-1}$, attained by his parent.[5] Thus,

$$\begin{aligned} r^e_{it} &= r^e(s_{it}) \\ &\equiv [1-(N-1)\tau]v \cdot [y_{it}/n_{i,t-1}] \ . \end{aligned} \tag{17}$$

It follows from (17) that, for **each** child, the expected marginal return to schooling effort declines ceteris paribus with increases in τ, i.e., with increases in the degree of egalitarianism of the government's tax policy. As will be clarified in Section 6, this negative effect can offset the positive effect of income transfers on human capital investment.

Finally, two important restrictions will be placed on the stochastic properties of the model. First, it will be supposed that the random ability levels of children across the dynasties $i = 1, \cdots, N$ are governed by stationary independent probability distributions $(f_1(\cdot), \ldots, f_N(\cdot))$ in each period t. Second, it will be supposed that each parent P_{it} believes he is unable, by his own actions, to affect the tax or subsidy level to be imposed on his child.

[5]In effect, then, each child acts as a "price taker" by assuming that the price (average return) he gains per unit of schooling effort expended is independent of the total amount of effort he exerts. Moreover, each child estimates this price to be the same as the price obtained by his parent. The latter expectational assumption is consistent with the views of Wilson (1987), Case and Katz (1991), and Streufert (1991), among others, who argue that children can have clouded perceptions of the true returns to schooling because their primary source of information is their own immediate family and neighborhood. Nevertheless, as is discussed more carefully in Section 4, additional empirical work is needed to learn how children actually do infer their returns to schooling.

More precisely, given any function $J(\tau_{i,t+1})$ of $\tau_{i,t+1}$, it will be supposed that

$$E[J(\tau_{i,t+1}) \mid c_{it}, e_{it}, s_{it}] \; = \; E[J(\tau_{i,t+1}) \mid s_{it}] \; . \tag{18}$$

Thus, P_{it} believes that his consumption decision c_{it} and his human capital investment decision e_{it} are not relevant for estimating the tax or subsidy level $\tau_{i,t+1}$ to be imposed on his child. Essentially, this reduces to assuming that each parent views himself as being too small to affect government tax policy.

Before developing various dynamic properties for the model set out in this section, it will be useful to examine the empirical support for the view expressed in relations (11) and (17) that a child's choice of schooling effort is strongly affected by family circumstances.

4. FAMILY EFFECTS ON SCHOOLING EFFORT

Beginning with the publication of the Coleman report (1966), researchers have consistently found that parental education and income have strong effects on the level of human capital investment in children. At the same time, school quality indicators have been found to have only a small impact on school attendance or on achievement test scores. Similarly, holding family and neighborhood attributes fixed, measures of the quality of schooling received as a child have been found to have only negligible effects on subsequent adult earnings.[6]

In part, these findings reflect the fact that local taxes support approximately 50 percent of all public expenditures on elementary and secondary schools, so that school quality strongly reflects family and neighborhood decisions regarding human capital investment [Kozol (1991, pp. 56-57)]. The evidence also suggests, however, that redistributing income towards poorer school districts has had only negligible effects on human capital investment.[7] Consequently,

[6] Hanushek (1986,1996) provides exhaustive literature reviews regarding the effects of school quality on school attendance and achievement test scores. Betts (1996) provides a similarly detailed literature review regarding the effects of school quality on earnings.

[7] Betts (1996, Table 6-11) provides a review of derivative estimates that have been obtained for the change in years of school resulting from a change in the level of expenditures per pupil. The average estimate is that a one percent increase in per pupil expenditures increases years of schooling by just .04 years.

family background appears to have an independent effect on human capital investment apart from its effect on school quality.

Researchers have also consistently found a strong positive correlation between parent and child lifetime incomes.[8] If children are investing in schooling so as to maximize their lifetime earnings, conditional on complete information about returns to schooling, it is not clear why this should be the case.

Taubman (1989) discusses two possible explanations for this empirical linkage. The first explanation emphasizes the role of children's education as a consumption good for parents. If the education of a child is a normal good, then higher-income families will invest more in their children's education than will poorer families. The second explanation treats the education of a child as an investment good but assumes that poorer families are liquidity constrained and so cannot invest optimally in their children's human capital.

However, the impact of both income and liquidity constraints on educational investments should have been reduced over the course of the century as poverty rates have fallen and as transfers to poor school districts have made budget constraints less binding. Thus, current differences in schooling outcomes between poorer and wealthier families should be less a consequence of family income and liquidity constraints than was true in the past, which may explain why additional transfers of resources to poor school districts have had little apparent effect on schooling outcomes in recent times.

In this paper we propose a third possible explanation for the observed linkage between parental and child lifetime incomes: that children condition their expectations of returns to schooling on their parents' returns to schooling. If true, then egalitarian transfers of income to poorer families may bias downward children's expected returns to schooling and reduce children's incentives to invest effort in schooling, even as those transfers remove liquidity

[8] Polachek and Siebert (1993) have extensively reviewed the literature on the economics of earnings. Solon (1992) and Zimmerman (1992) have analyzed the intergenerational correlation of lifetime earnings between fathers and sons.

constraints on families.

The latter explanation is consistent with recent sociological research into urban underclass isolation. For example, Wilson (1987) has argued that, in neighborhoods with large shares of welfare recipients, "children will seldom interact on a sustained basis with people who are employed or with families that have a steady breadwinner. The net effect is that....the relationship between schooling and postschool employment takes on a different meaning. The development of cognitive, linguistic, and other educational and job-related skills necessary for the world of work in the mainstream economy is thereby adversely affected."

Unfortunately, despite the importance of expectations to the welfare dependency hypothesis, very few empirical studies have focused on the role of expectations in determining schooling effort levels [Manski (1993)]. Sandell and Shapiro (1980) and Blau and Ferber (1991) have found that expectations of intermittent employment can cause individuals to select less training-intensive employment. To our knowledge, however, no studies have measured the impact of family or neighborhood transfer payments on the expectation formation of youths. The limited evidence on the role of welfare payments on child schooling effort is not definitive.

For example, Currie (1995) reports that white children of welfare recipients are more likely to repeat a grade, but that there is no significant effect of welfare recipiency on black children. Butler (1990) finds that AFDC payments appear to increase the odds that a child will continue beyond high school, but also increase the odds that a child will stop after the ninth grade. Both studies use extremely small samples, so conclusions must be drawn with caution. Nevertheless, a conclusion that AFDC payments may raise or lower child educational attainment is not inconsistent with the model to be explored here, in which income transfers simultaneously relax liquidity constraints and lower expected returns to human capital investments.

5. DYNAMIC PROPERTIES OF THE BASIC ECONOMY

In this section a number of propositions are established for the Basic Economy, given the particular functional specifications described in Section 3. It is demonstrated that the parent will invest some positive amount in his child's human capital regardless of the tax policy. In the long run, however, the expected income and schooling effort levels of each dynasty are strictly lower under the egalitarian tax policy than under the libertarian tax policy. Furthermore, the gap between the egalitarian and libertarian income and effort levels increases as the number of dynasties increases. The reason for this is that, for sufficiently large t, a period t child's expected marginal return to schooling effort decreases unambiguously as the number of dynasties increases in the egalitarian economy but is unaffected by the number of dynasties in the libertarian economy. Thus, the economy that attempts to redistribute income equally among all of its citizens is plagued by free-rider effects that drive expected income and schooling effort toward zero as the number of dynasties increases.

The formal statements of these and other propositions are presented below, and proofs are given in the Appendix. The first proposition provides a more concrete analytical representation for the recurrence relation satisfied by the value function $V_{it}(s_{it})$.

PROPOSITION 1. Suppose the value function for the child C_{it} upon reaching adulthood in period $t+1$ can be expressed in the form

$$V_{i,t+1}(s_{i,t+1}) \;=\; (y_{i,t+1})^{\delta_{t+1}} \cdot (n_{it})^{-\theta_{t+1}} \cdot K_i(\beta_{t+1}, g_{i,t+1}) \tag{19}$$

for some nonnegative coe±cient vector $\beta_{t+1} \equiv (\delta_{t+1}, \theta_{t+1})'$ and some function $K_i(\beta_{t+1}, g_{i,t+1})$ of β_{t+1} and $g_{i,t+1} \equiv (\tau_{i,t+1}, \tau, N)$ that depends on the probability distribution function $f_i(\cdot)$ governing ability levels for dynasty i. Then the value function for the parent P_{it} of C_{it} in period t takes the form

$$V_{it}(s_{it}) \;=\; (y_{it})^{\delta_t} \cdot (n_{i,t-1})^{-\theta_t} \cdot K_i(\beta_t, g_{it}) \;, \tag{20}$$

where $\beta_t \equiv (\delta_t, \theta_t)'$ is a nonnegative coe±cient vector that satis¯es a matrix recurrence relation of the form

$$\beta_t \;=\; \gamma\pi \;+\; (1-\gamma)M\beta_{t+1} \;, \tag{21}$$

with M given by

$$\begin{pmatrix} u+(a+b)v & -(a+b) \\ bv & -b \end{pmatrix} ,$$

and where $K_i(\beta_t, g_{it})$ satis¯es a recurrence relation of the form

$$K_i(\beta_t, g_{it}) \;=\; H_i(\beta_{t+1}, g_{it}) \cdot E[K_i(\beta_{t+1}, g_{i,t+1})^{1-\gamma} \mid g_{it}] \;. \tag{22}$$

As a corollary to Proposition 1, it can be shown that the period t optimal consumption and investment levels for the dynasty i parent are proportional to his period t after-tax income.

COROLLARY 1. Under the assumptions of Proposition 1, the parent P_{it}'s optimal choices for human capital investment e_{it} and consumption c_{it} take the form

$$e^o_{it} \;=\; \tau_{it}y_{it} \cdot (1-\gamma)w_{t+1}/[\gamma + (1-\gamma)w_{t+1}] \;; \tag{23}$$

$$c^o_{it} \;=\; \tau_{it}y_{it} \cdot \gamma/[\gamma + (1-\gamma)w_{t+1}] \;, \tag{24}$$

where $w_{t+1} \equiv (u+av)\delta_{t+1} - a\theta_{t+1}$, $t \geq 1$.

As Proposition 1 indicates, the assumption that a parent's utility depends on the utility attained by his child results in a family of possible parent value functions parameterized by (β_t). In particular, given any sequence (β_t) satisfying the recurrence relation (21), the corresponding value function is given by (20). The structure of this value function, and hence also the structure of the optimal human capital investment decision (23) and the optimal consumption decision (24), changes over time unless $\beta_t = \beta_{t+1}$ for all periods $t \geq 1$.

Since the seminal article by Barro (1974), most researchers assuming parental altruism for children (e.g., Loury (1981)) have assumed without comment that each parent has a stationary-structured value function, i.e., a value function $V(s)$ that depends only on the current state of the economy, s, and not on the current time t. The next proposition establishes that there indeed exists a unique stationary-structured value function for each parent in the illustrative Basic Economy. For easier comparison with the findings of Loury (1981) and other previous researchers, it will henceforth be assumed that each parent's value function coincides with this unique stationary-structured value function.

PROPOSITION 2. There exists a unique stationary-structured value function for each parent P_{it} in the Illustrative Basic Economy: namely, the value function (20) parameterized by the unique stationary solution $\bar{\beta} = (\bar{\delta}, \bar{\theta})' > 0$ for the recurrence relation (21), where

$$\bar{\delta} \quad = \quad \gamma[1 + (1-\gamma)b]\Delta \ ; \tag{25}$$

$$\bar{\theta} \quad = \quad \gamma[(1-\gamma)bv]/\Delta \ , \tag{26}$$

and $\Delta > 0$ denotes the determinant of the matrix $[I - (1-\gamma)M]$.

Given these stationary-structured value functions, it is straightforward to show that each parent's optimal choices (23) and (24) for human capital investment and consumption remain positive as long as he earns a positive after-tax income $\tau_{it}y_{it}$.

The next two propositions characterize the long-run behavior of dynasty income and effort levels under libertarian and egalitarian tax policies. For ease of notation, let

$$x_{it} \ = \ (\log(y_{it}), -\log(n_{i,t-1}))' \tag{27}$$

denote the indicated logarithmic transformation of the period-t income and effort levels $(y_{it}, n_{i,t-1})'$ for dynasty i. Also, let $E[\cdot \mid s_1]$ denote an expectation conditional on the period 1 social state vector,

$$s_1 \ \equiv \ ((y_{11}, n_{10}), \ldots, (y_{N1}, n_{N0}), \tau, N) \ . \tag{28}$$

The expectation is assumed to be taken with respect to the joint distribution for ability levels across dynasties over time.

For the libertarian tax policy $\tau = 0$, it follows from (15) that the after-tax income $\tau_{it}y_{it}$ for each dynasty i in each period t satisfies $\tau_{it}y_{it} = y_{it}$, implying that no redistribution occurs. The next proposition shows that the expected income and effort levels of each dynasty i, in log form, converge over time to stationary limiting values under a libertarian tax policy.

PROPOSITION 3. Suppose that government in period 1 implements a permanent libertarian tax policy $\tau = 0$, known to all agents in all dynasties. Then for each dynasty i,

$$\lim_{t\to\infty} E[x_{it} \mid s_1] \;=\; [I - M']^{-1}d_i \ , \tag{29}$$

where M' denotes the transpose of the matrix M appearing in relation (21), and the 2×1 vector d_i has the form

$$d_i \;=\; (\log(C) + mE[\log(\alpha_i)], -\log(D))' \tag{30}$$

for constant terms C and D that are independent of the number of dynasties, N. If the stationary probability distributions f_i and f_j that govern ability levels for dynasties i and j in each period t are identical, then $d_i = d_j$.

In the case of an egalitarian tax policy $\tau = 1/N$, it follows from (15) that the after-tax income for each dynasty i in each period t satisfies

$$\tau_{it}y_{it} \;=\; \sum_{j=1}^{N} y_{jt}/N \;\equiv\; \bar{y}_t \ , \tag{31}$$

implying that each dynasty receives an equal share of total social income. The next proposition provides a partial characterization of long-run outcomes for this case. More precisely, given the assumption that the expected values for both mean social income and individual dynasty incomes in log form converge over time to finite stationary values, analytical expressions are derived for long-run expected dynasty incomes and effort levels in log form.

PROPOSITION 4. Suppose that government in period 1 implements a permanent egalitarian tax policy $\tau = 1/N$, known to all agents in all dynasties. Suppose, also, that for each dynasty i,

$$\lim_{t\to\infty} E[\log(\bar{y}_t) \mid s_1] \;=\; K_i \;+\; \lim_{t\to\infty} E[\log(y_{it}) \mid s_1] \tag{32}$$

for some constant K_i, where the limits in (32) are ¯nite valued. Then for each dynasty i,

$$\lim_{t\to\infty} E[x_{it} \mid s_1] \;=\; [I - M']^{-1} h_i \ , \tag{33}$$

where M' denotes the transpose of the matrix M appearing in relation (21), and the components of the 1×2 vector $h_i' = (h_{1i}, h_{2i})'$ take the form

$$h_{1i} \;=\; \log(C) + mE[\log(\alpha_i)] - bv\log(N) + [u + av]K_i \ ; \tag{34}$$

$$h_{2i} \;=\; -\log(D) + b\log(N) - aK_i \ . \tag{35}$$

The constant terms C and D in (34) and (35) are the same as in (30).

COROLLARY 2. Suppose the hypotheses of Proposition 4 hold for constant terms K_i that are independent of the number of dynasties, N. Then the expected long-run marginal return to schooling e®ort in log form for each dynasty i is a strictly decreasing function of N of the form

$$\lim_{t\to\infty} E[\log(r^e_{it}) \mid s_1] \;=\; L_i - R\log(N) \ , \tag{36}$$

where the constants L_i and $R > 0$ are independent of N.

Given the hypotheses of Corollary 2, the following proposition establishes that each dynasty can expect to have a lower long-run income under egalitarianism than under libertarianism.

PROPOSITION 5. Suppose condition (32) holds for constant terms K_i that are independent of the number of dynasties, N. Then the expected long-run income and e®ort levels in log form for each dynasty under an egalitarian tax policy $\tau = 1/N$ are strictly decreasing functions of N. Moreover, these long-run levels are strictly lower than the long-run levels achieved under a libertarian tax policy $\tau = 0$ for each $N > 1$.

Recall from Proposition 3, equation (29), that each dynasty's expected long-run income in log form is independent of the number of dynasties, N, when government implements a libertarian tax policy $\tau = 0$. Proposition 5 therefore implies that, the greater the number of dynasties, the greater is the extent to which the dynasties are worse off in the long run under egalitarianism than under libertarianism.

The simulation results reported below in Section 6 indicate that, averaged across runs in log form, social income levels, dynasty income levels, and dynasty effort levels all do converge to stationary positive limiting values under the egalitarian tax policy. Morever, the expected income, effort, and social welfare levels achieved under the egalitarian tax policy fall well short of the levels achieved under the libertarian tax policy, even for $N = 2$.

The final proposition of this section provides another interesting check on the validity of the simulation results reported below in Section 6. If average dynasty income constitutes a sufficiently good approximation for the true expected dynasty income for any dynasty i in any period t, then the true expected tax/subsidy level τ_{it} for dynasty i in period t is a strictly increasing function of the tax policy τ. That is, the more egalitarian the government's tax policy, the higher the subsidy level (or the lower the tax level) that dynasty i can expect to receive (or to pay).

PROPOSITION 6. Suppose that average dynasty income in some period t converges to the true expected income for some dynasty i in period t, conditional on the period 1 social state vector s_1, as the number of dynasties, N, becomes arbitrarily large. That is, suppose

$$lim_{N\to\infty}[\sum_{j=1}^{N} y_{jt}/N] \;=\; E[y_{it} \mid s_1] \; . \tag{37}$$

Then for all su±ciently large N,

$$\frac{\partial E[\tau_{it} \mid s_1]}{\partial \tau} \; > \; 0 \; . \tag{38}$$

6. SIMULATION RESULTS

The illustrative Basic Economy outlined in Section 3 is too complex to allow a detailed analytical characterization of the economy's responses to changes in the tax policy τ. Simulation experiments were therefore conducted to determine how different tax policies influence economic growth and human capital investment in this economy.

To operationalize the model, specifications are needed for the number of dynasties, the distribution of abilities, and the parameters characterizing the utility and income functions. The number of dynasties N was first set at 2. The ability α of each child in each dynasty was assumed to be distributed uniformally over the range $(0, 1)$, implying a mean ability of .5. The value of γ, the exponent on current consumption in the utility function (12), was set to .5. The parameters of the income function (10) were set at $\lambda = .5$, $m = .5$, $u = .5$, and $v = .5$. Since λ serves as a scaling variable, we can generate proportionally larger or smaller measures of income without altering the relative levels of the variables by rescaling the value of λ. Finally, the parameters of the effort function (11) were set at $a = .5$ and $b = .5$.

Various parameterizations were tried to determine the sensitivity of the qualitative results to changes in the model parameters. The qualitative results proved to be quite robust to modifications in the parameters.

Each simulation was replicated 40 times over a 40 generation horizon. In each run, each dynasty received 40 independent draws from the uniform ability distribution, one draw per generation. For all runs, the starting values for parental income and schooling effort were initialized to be one. First, the purely libertarian tax policy ($\tau = 0$) was run. For each dynasty in each year, the simulation generated values for income, consumption, schooling effort, and human capital investment. Then, holding fixed the sequence of ability draws, the simulation was rerun with progressively more egalitarian tax policies, ending with the purely egalitarian tax policy ($\tau = 1/N = .5$). For each tax policy τ, we also measured after-tax income and the tax/subsidy position of each dynasty. Finally, gross national products were computed by summing across dynasties at a point in time. Statistical summaries of the simulations are reported in Tables 1-4.

Table 1 contains the results from our base run. For each variable, we report the mean and standard deviation taken over 40 replications of 39 generations of generated data. The initial generation was deleted since all runs had the same initial levels of income and effort. For the two-dynasty model, this implies 3120 observations ($39 \times 40 \times 2$). The simple correlation between current and once-lagged variables are also reported to determine the extent to which parent and child outcomes are correlated.

—Table 1 About Here—

Immediately apparent from Table 1 is that tax policies can increase human capital production, as in the model by Loury (1981). With two dynasties, the tax policy τ can vary from 0 to .5. A relatively small tax policy ($\tau = .04$) increases income and parental investment in children. It also reduces the variance in those parental investments across households. By transferring income toward poorer households, poorer parents are able to invest in the education of their high ability children. This reduces the inefficiency in human capital investment caused by the relative overinvestment in the children of high income parents and the relative underinvestment in children of low income parents. As a result, the economy as a whole benefits. Moreover, introduction of the tax policy begins to reduce the variance in after-tax income. The reduction in uncertainty about future income and consumption is a second benefit from the tax policy.

The result that tax policies can increase expected GNP is in marked contrast to the implications of the model of King and Rebelo (1990), who found that increasing taxes uniformally reduced GNP growth. Nevertheless, Table 1 also indicates that a moderate tax policy dominates relatively more egalitarian tax policies. Although transfers allow poorer parents to invest more in their children, the income taxes used to finance the transfers also lower the expected marginal returns to these investments for all children. To see this, note that, ceteris paribus, an increase in τ decreases the child's expected marginal return to schooling effort in equation (17). As children see the expected marginal return from their human capital investments declining, they apply less effort to their schooling. As seen in Table 1, this reduction in effort begins at relatively modest tax policy levels, and effort continues to decline as the tax policy becomes progressively more egalitarian. In the limit, schooling effort under the perfectly egalitarian tax policy $\tau = .5$ is only 55 percent of the level under the libertarian tax policy $\tau = 0$. As a result, the GNP attained under a policy that guarantees income equality averages just over one-half the level of GNP attained under a policy that involves no income redistribution.[9] These findings are consistent with the predictions of Proposition 5.

Table 1 also tells how a parent's income is related to his child's income. As tax policies become more egalitarian, the correlation between the before-tax incomes of a parent and child falls monotonically. The same is not true of after-tax income. The reduction in after-tax income correlation across generations is more moderate. As τ rises from 0 to .15, the correlation coefficient declines by .45 for before-tax income, but by only .19 for after-tax income. Thereafter, while intergenerational income correlations continue to fall for before-tax income, they begin to rise for after-tax income. In the limit, the intergenerational after-tax income correlation under the perfectly egalitarian tax policy $\tau = .5$ is larger than under the libertarian tax policy $\tau = 0$. The depressing effect of the perfectly egalitarian tax policy on human capital investment reduces the rate of growth and the variance in GNP to such an extent that it increases the intergenerational correlation in incomes. Since GNP is shared equally by all households, the intergenerational correlation in dynasty after-tax income is exactly the same as the intergenerational correlation in GNP.

[9] Recall from Section 5 that, even under a perfectly egalitarian tax policy, each parent will still desire to invest a positive amount of resources in his child's education as long as the parent's after-tax income is positive.

Proposition 6 predicts that dynasties will expect to receive a higher subsidy (or lower tax) from the government as the tax policy becomes more egalitarian. This is clearly borne out in the simulations. The expected value of τ_{it} rises from 1.0 to 1.15 as τ increases from 0 to .5.

One last outcome from the simulations in Table 1 is that households allocate about 39 percent of their after-tax income to their children and the remainder to consumption. Olson (1983, p. 40) estimated for the United States that two-parent families with two children born in 1980 would allocate about 37 percent of their income to raising their children to age 22. Thus, the parameterizations in Table 1 yield reasonable estimates of parental resource allocations.

It is important to determine the sensitivity of the conclusions derived thus far to changes in the parameters. Simulations were run assuming different values of the parameters of the utility function, the income function, and the number of dynasties. In Table 2, we lower the utility function weight γ on current consumption to .25, thereby raising the utility function weight $[1 - \gamma]$ on future consumption to .75. As a result, parents increase their investment in their children and the economy grows to almost five times the levels in Table 1. However, none of the qualitative implications change relative to those derived from Table 1. Small tax policies raise expected income and lower the variance in income, but progressively more egalitarian tax policies ultimately lower income, GNP, schooling effort, and human capital investment. Initially, taxes lower the intergenerational correlation in after-tax income, but ultimately the highest intergenerational correlations are found under the perfectly egalitarian tax policy. The share of income devoted to children rises to 59 percent, well above the actual national average in the United States.

—Table 2 About Here—

In Table 3, the utility parameters are the same as in Table 1, but we increase the income elasticity (i.e., the elasticity of a child's income) with respect to parental human capital investment from $u = .5$ to $u = .75$ and lower the income elasticity with respect to schooling effort from $v = .5$ to $v = .25$. This change in parameterization leaves the income elasticity with respect to the scale factor λ unchanged at 1; see equation (10). Comparing Table 3 with Table 1, tax policies involving increased income transfers now initially have a relatively larger positive effect on dynasty incomes and on GNP, and the adverse effects of income transfers on these variables do not outweigh the positive effects until later in the progression toward more egalitarian tax policies. Nonetheless, the general Table 1 pattern of results still holds. Perfectly egalitarian tax policies are still dominated by purely libertarian tax policies, and the intergenerational correlation in after-tax income is still highest under the egalitarian tax policy. The share of household income devoted to children is 45 percent, a bit high relative to the 37 percent reported by Olson (1983).

—Table 3 About Here—

As the number of dynasties increases, GNP growth should be smoothed since extreme individual ability draws will have a smaller effect on national income. This is shown clearly in Table 4. The coefficient of variation in GNP is about two-thirds the level in Table 1. However, the increase in the number of dynasties also reduces expected marginal returns to schooling effort, consistent with Corollary 2. Individuals have a greater incentive to free ride on the economy as tax policies become more egalitarian. In the perfectly egalitarian regime, schooling effort falls to .64, only 43 percent of the schooling effort in the perfectly egalitarian two-dynasty economy in Table 1. The free-rider problem is so great under the perfectly egalitarian tax policy that GNP with five dynasties is not much larger than with two dynasties, and dynasty average income is less than half that in the two-dynasty economy. Increasing the number of dynasties exacerbates the disincentive effects of egalitarian tax policies.

—Table 4 About Here—

All of the statistics reported thus far have looked at the generated data over the entire trajectory of 40 generations. The analytical results reported in Section 5 predict that these data values should converge in expectation to steady state levels under each of the two tax policy extremes, libertarianism and egalitarianism. In Figure 1, the time path of average GNP is graphed for six tax policies spanning these two extremes. In each case the time paths appear to level off after fifteen to twenty generations. The time paths show very similar GNP levels for the three most libertarian tax policies, and then show progressively lower GNP levels for the remaining three most egalitarian tax policies.

—Figure 1 About Here—

Another method for assessing the various tax policies is to measure the expected lifetime utility levels associated with these policies. By relation (13), the log of the true lifetime utility U_{i1} achieved by a dynasty i parent in period 1 can be expressed as an infinite discounted sum in log form of all of the instantaneous utilities achieved by dynasty i parents in periods $t \geq 1$. We constructed a truncated approximation for this infinite discounted sum by summing over the instantaneous log-utilities achieved by dynasty i parents in generations fifteen through thirty-five only. This approximation was exponentiated to reclaim an estimate $\hat{U}_{i1}$ for U_{i1}. The expected lifetime utility of a period 1 parent under a given tax policy τ was then estimated by first averaging the estimates $\hat{U}_{i1}$ across dynasties $i = 1, \ldots, N$ for each of forty replications of the economy under the tax policy τ, and then further averaging these averages over the forty runs. To check the robustness of our expected lifetime utility estimate, we also calculated a second estimate by repeating the same procedure using a truncated approximation which included the instantaneous utilities achieved by dynasty i parents in generations twenty through forty only.

Table 5 reports these estimated expected lifetime utilities for a number of different income, effort, and utility parameter specifications. These results generally mimic the results reported above for the simple GNP measure of welfare.

—Table 5 About Here—

For each reported parameter specification, the perfectly libertarian tax policy ($\tau = 0$) yields higher welfare than the perfectly egalitarian tax policy ($\tau = .5$), but small positive tax policies dominate both of these extreme tax policies. The utility welfare measure does rank modestly egalitarian tax policies (e.g., $\tau = .08$) more highly than does a simple comparison of GNP. The reason is that positive tax policies lower uncertainty about income, even if they may slightly lower expected income. Small tax policies have bigger welfare-improving effects as the utility weight γ on current consumption decreases, and also as the income function parameter u increases, i.e., as parental human capital investments become more productive. Small tax policies become less welfare-enhancing as the number of dynasties increases.

7. CONCLUSION

This paper examines the impact of redistributive income tax policies on human capital investment, per capita GNP, and social welfare in the context of a multi-dynasty overlapping generations economy. Simulation experiments indicate that modest redistributive tax policies raise human capital investment, per capita GNP, and social welfare by increasing the human capital investments of relatively poor families in their children. Further welfare gains are obtained from the reduction in uncertainty regarding after-tax incomes that results from the implementation of the transfer program.

Nevertheless, the income taxes that fund these transfers also lower children's perceptions of their marginal returns to schooling. The resulting adverse effects on schooling effort quickly overtake the beneficial effects of the transfers as the tax policy becomes progressively more egalitarian. In the limit, a purely egalitarian tax policy lowers schooling effort to such a great extent that per capita GNP and social welfare fall well short of the levels achieved under a purely libertarian tax policy. These adverse effects on schooling effort worsen as the number of dynasties increases. Also, human capital investment and per capita GNP growth are more adversely affected in economies in which parents place a relatively large weight on current consumption, and in economies in which parental investments in their children's human capital are relatively less productive.

These findings are meant as a cautionary tale. Questions can certainly be raised concerning the particular model specifications used in their generation. In particular, much more work needs to be done to understand how children actually determine their schooling effort. Moreover, the robustness of our findings to changes in the assumption that the government maintains forever a stationary tax policy τ, known to all agents, should clearly be examined. It would also be useful and interesting to examine what happens when ability levels are correlated intergenerationally within families.

Nevertheless, we believe our findings highlight an important public policy issue. If children do indeed vary their schooling effort in response to their perceived returns to schooling, then underinvestment in the education of poorer children may be due in part to incentive problems caused by distortionary taxes that bias downward these perceived returns and lead children to make inefficient use of their educational opportunities.

REFERENCES

Barro, R., "Are Government Bonds Net Wealth?," Journal of Political Economy 82 (1974), 1095–1117.

Becker, G., Human Capital (Chicago: University of Chicago Press, 1975).

Betts, J. R., "Is There a Link Between School Inputs and Earnings? Fresh Scrutiny of an Old Literature," in Gary Burtless (ed.), Does Money Matter:

The E®ect of School Resources on Student Achievement and Adult Success (Washington, D.C.: Brookings Institution Press, 1996).

Blau, F. D., and Ferber, M. A., "Career Plans and Expectations of Young Women and Men: The Earnings Gap and Labor Force Participation," *Journal of Human Resources* 25 (Fall 1991), 581–607.

Butler, A. C., "The Effect of Welfare Guarantees on Children's Educational Attainment," *Social Science Research* 19 (June, 1990), 175–203.

Case, A., and Katz, L., "The Company You Keep: The Effects of Family and Neighborhood on Disadvantaged Youths," NBER Working Paper No. 3705, 1991.

Coleman, J. S., Campbell, E., et al., *Equality of Educational Opportunity* (Washington, D.C.: U.S. Department of Health, Education, and Welfare, Office of Education, 1966).

Currie, J., *Welfare and the Well-Being of Children* (Chur, Switzerland: Harwood Academic Publishers, 1995).

Durlauf, S., "Neighborhood Feedbacks, Endogenous Stratification, and Income Inequality," in *Disequilibrium Dynamics: Theory and Applications*, Edited by W. Barnett, G. Gandolfo, and C. Hillinger, Cambridge University Press, Cambridge, 1996a.

Durlauf, S., "A theory of Persistent Income Inequality," *Journal of Economic Growth* 1 (1996b), 75–94.

Galor, O., and Zeira, J., "Income Distribution and Macroeconomics," *Review of Economic Studies* 60 (1993), 35–52.

Galor, O., and Tsiddon, D., "The Distribution of Human Capital and Economic

Growth," Working Paper No. 18-96, The Sackler Institute of Economic Studies, Tel-Aviv University, July 1996.

Hanushek, E. A., "The Economics of Schooling: Production and Efficiency in Public Schools," *Journal of Economic Literature* 24 (September, 1986), 1141–1177.

Hanushek, E. A., "School Resources and Student Performance,"in G. Burtless (ed.), *Does Money Matter: The E®ect of School Resources on Student Achievement and Adult Success* (Washington, D.C.: Brookings Institution Press, 1996).

King, R. G., and Rebelo, S., "Public Policy and Economic Growth: Developing Neoclassical Implications," *Journal of Political Economy* 98 (October 1990), S126–S150.

Kozol, J., *Savage Inequalities* (New York: Crown Publishers, Inc., 1991).

Loury, G. C., "Intergenerational Transfers and the Distribution of Earnings," *Econometrica* 49 (July 1981), 843-867.

Lucas, R. E., "On the Mechanics of Human Development," *Journal of Monetary Economics* 22 (1988), 3-42.

Manski, C., "Adolescent Econometricians: How Do Youths Infer the Returns to Schooling," in C. Clotefelter and M. Rothschild (eds.), *Studies of Supply and Demand in Higher Education*, University of Chicago Press, 1993.

Olsen, L., *Cost of Children* (Lexington: Lexington Books, 1983).

Orazem, P., and L. Tesfatsion, "Human Capital Investment and the Locally Rational Child," Economic Report No. 31, Iowa State University, Revised March 1997.

Perotti, R., "Political Equilibrium, Income Distribution, and Growth," *Review of Economic Studies* 60 (1993), 755–776.

Polochek, S. W. and Siebert, W. S., *The Economics of Earnings* (Cambridge:Cambridge University Press, 1993).

Rosen, S., "Human Capital: A Survey of Empirical Research," in Ronald Ehrenberg (ed.) *Research in Labor Economics*, Vol. 1. (Greenwich, CT: JAI Press, 1977).

Schultz, T. P., "Education Investments and Returns," pp. 544-630 in *Handbook of Development Economics* (Amsterdam:North Holland, 1988).

Sandell, S. H. and Shapiro, D., "Work Expectations, Human Capital Accumulation, and the Wages of Young Women," *Journal of Human Resources* 15 (Summer 1980), 335–353.

Solon, G. R., "Intergenerational Income Mobility in the United States," *The American Economic Review* 82 (June, 1992), 393–408.

Streufert, P., "The Effect of Underclass Social Isolation on Schooling Choice," Institute for Research on Poverty, Discussion Paper No. 954-91, University of Wisconsin, August 1991.

Taubman, P., "Role of Parental Income in Educational Attainment," *The American Economic Review* 79 (May, 1989), 57–61.

Wilson, W. J., *The Truly Disadvantaged: The Inner City, the Underclass, and Public Policy* (Chicago: The University of Chicago Press, 1987).

Zimmerman, D. J., "Regression Toward Mediocrity in Economic Stature," *American Economic Review* 82 (June, 1992), 409–429.

APPENDIX: PROOFS OF SECTION 5 PROPOSITIONS

PROOF OF PROPOSITION 1 AND COROLLARY 1. Suppose the special function specifications in Section 3 hold. Suppose also, as the induction hypothesis, that the value function for the child C_{it} upon reaching adulthood in period $t+1$ takes the form (19), where $\beta_{t+1} \equiv (\delta_{t+1}, \theta_{t+1})$. Consider the utility level which is then achieved by the parent P_{it} of C_{it} for any possible realization α_{it} for the ability level of C_{it}:

$$\begin{aligned} c_{it}^{\gamma} V_{i,t+1}(s_{i,t+1})^{1-\gamma} &= [\tau_{it} y_{it} - e_{it}]^{\gamma} \cdot \left[y_{i,t+1}^{\delta_{t+1}} \eta_{it}^{-\theta_{t+1}} K_i(\beta_{t+1}, g_{i,t+1}) \right]^{1-\gamma} \\ &= [\tau_{it} y_{it} - e_{it}]^{\gamma} \cdot e_{it}^{w_{t+1}[1-\gamma]} \cdot y_{it}^{bd_{t+1}[1-\gamma]} \cdot \eta_{i,t-1}^{-bd_{t+1}[1-\gamma]} \\ &\quad \lambda^{\delta_{t+1}(1-\gamma)} \cdot k(\tau, N)^{-b\theta_{t+1}[1-\gamma]} \cdot \alpha_{it}^{m\delta_{t+1}[1-\gamma]} \cdot K_i(\beta_{t+1}, g_{i,t+1})^{1-\gamma} \ , \end{aligned} \tag{39}$$

where

$$w_{t+1} \equiv (u + av)\delta_{t+1} - a\theta_{t+1} \ ; \tag{40}$$

$$d_{t+1} \equiv v\delta_{t+1} - \theta_{t+1} \ ; \tag{41}$$

$$g_{i,t+1} \equiv (\tau_{i,t+1}, \tau, N) \ ; \tag{42}$$

$$k(\tau, N) \equiv [1 - (N-1)\tau]v \ . \tag{43}$$

Recall the assumption formalized in Section 3, equation (18), that each parent views himself as being too small to affect government tax policy through his own actions. In particular, parent P_{it}'s expectation concerning the tax or subsidy $\tau_{i,t+1}$ to be imposed on his child is assumed to be independent of the parent's choice of c_{it} and e_{it}. Consequently, the expected value of (39) conditional on c_{it}, e_{it}, and s_{it} takes the form

$$[\tau_{it} y_{it} - e_{it}]^{\gamma} \cdot e_{it}^{(1-\gamma)w_{t+1}} \cdot [\text{terms independent of } e_{it}] \ . \tag{44}$$

Maximizing (44) with respect to e_{it} then yields the time-t optimal solution e_{it}^o for e_{it}, and hence also the optimal solution c_{it}^o for $c_{it} = [\tau_{it} y_{it} - e_{it}]$:

$$e_{it}^o = \tau_{it} y_{it} \cdot (1-\gamma) w_{t+1} / [\gamma + (1-\gamma) w_{t+1}] \ ; \tag{45}$$

$$c_{it}^{o} \quad = \quad \tau_{it} y_{it} \cdot \gamma / [\gamma + (1-\gamma) w_{t+1}] \ . \tag{46}$$

This establishes Corollary 1.

Substituting (45) back into (39), and taking the expectation of (39) conditional on s_{it}, one obtains an expression for the time-t value function $V_{it}(s_{it})$ in (9) of the form

$$V_{it}(s_{it}) \ = \ y_{it}^{\delta_t} \cdot \eta_{i,t-1}^{-\theta_t} \cdot K_i(\beta_t, g_{it}) \ . \tag{47}$$

In relation (47),

$$\beta_t \quad = \quad (\delta_t, \theta_t) \ ; \tag{48}$$

$$\delta_t \quad = \quad \gamma + (1-\gamma)[u + (a+b)v]\delta_{t+1} - (1-\gamma)(a+b)\theta_{t+1} \ ; \tag{49}$$

$$\theta_t \quad = \quad (1-\gamma) b v \delta_{t+1} - (1-\gamma) b \theta_{t+1} \ . \tag{50}$$

Also, $K_i(\beta_t, g_{it})$ is given by the expression

$$K_i(\beta_t, g_{it}) = H_i(\beta_{t+1}, g_{it}) \cdot E[K_i(\beta_{t+1}, g_{i,t+1})^{1-\gamma} \mid g_{it}] \ , \tag{51}$$

where

$$H_i(\beta_{t+1}, g_{it}) \equiv A(\beta_{t+1}) \cdot \lambda^{\delta_{t+1}(1-\gamma)} \cdot k(\tau, N)^{-b\theta_{t+1}[1-\gamma]} \cdot \tau_{it}^{\gamma + w_{t+1}[1-\gamma]} \cdot E[\alpha_{it}^{(1-\gamma) m \delta_{t+1}}] \tag{52}$$

and

$$A(\beta_{t+1}) \equiv \left[\frac{\gamma}{\gamma + (1-\gamma) w_{t+1}} \right]^{\gamma} \cdot \left[\frac{(1-\gamma) w_{t+1}}{\gamma + (1-\gamma) w_{t+1}} \right]^{(1-\gamma) w_{t+1}} \ . \tag{53}$$

Let π denote the column vector $(1,0)'$, and let M denote the matrix

$$\begin{pmatrix} u + (a+b)v & -(a+b) \\ bv & -b \end{pmatrix} \ .$$

It then follows from equations (49) and (50) that $\beta_t \equiv (\delta_t, \theta_t)'$ satisfies the matrix recurrence relation

$$\beta_t \ = \ \gamma \pi + (1-\gamma) M \beta_{t+1} \ . \tag{54}$$

Since the matrix M is nonsingular, relation (54) can also be inverted to give β_{t+1} as a function of β_t. Thus, together with (54), relation (51) yields a well-defined recurrence relation.

Consequently, Proposition 1 has been established. Q.E.D.

PROOF OF PROPOSITION 2. The determinant Δ of the matrix $[I - (1-\gamma)M]$ is given by

$$\Delta \;=\; (1 - (1-\gamma)[u + (a+b)v]) \cdot (1 + (1-\gamma)b) + (1-\gamma)^2 bv(a+b) \; . \tag{55}$$

Recalling that u, v, a, b, and γ are assumed to be positive constants satisfying $0 < (u+v) \leq 1$, $0 < (a+b) \leq 1$, and $0 < \gamma < 1$, the determinant (55) is positive. Consequently, the inverse of the matrix $[I-(1-\gamma)M]$ exists, and the unique stationary solution for the matrix recurrence relation (54) is given by

$$\bar{\beta} \;=\; [I - (1-\gamma)M]^{-1} \cdot \gamma\pi \; . \tag{56}$$

Finally, by direct calculation, the components of the stationary solution $\bar{\beta} = (\bar{\delta}, \bar{\theta})'$ are given by

$$\bar{\delta} \;=\; \gamma[1 + (1-\gamma)b]/\Delta \; ; \tag{57}$$

$$\bar{\theta} \;=\; \gamma[(1-\gamma)bv]/\Delta \; . \tag{58}$$

Since Δ is positive along with b and v, and $0 < \gamma < 1$, these components are strictly positive. Q.E.D.

PROOF OF PROPOSITION 3. By assumption, the tax policy in effect is $\tau = 0$, implying that the dynasties are not coupled together in any way. Consequently, it suffices to focus on any one dynasty, say dynasty i.

State equations describing the movement over time of the dynasty i state vector $(y_{it}, n_{i,t-1})$ can be determined by making use of the special income and effort function specifications (10) and (11), together with the optimal consumption and investment solution functions (45) and (46). These state equations take the form

$$y_{i,t+1} = C \cdot (\alpha_{it})^m \cdot (y_{it})^{u+(a+b)v} \cdot (n_{i,t-1})^{-bv} \; ; \tag{59}$$

$$n_{it} = D \cdot (y_{it})^{a+b} \cdot (n_{i,t-1})^{-b} \; , \tag{60}$$

where

$$\bar{w} \equiv (u+av)\bar{\delta} - a\bar{\theta} = \gamma[(u+av)+(1-\gamma)ub]/\Delta > 0 \; ; \tag{61}$$

$$B \equiv (1-\gamma)\bar{w}/[\gamma+(1-\gamma)\bar{w}] \; ; \tag{62}$$

$$C \equiv \lambda \cdot (v)^{bv} \cdot (B)^{u+av} \; ; \tag{63}$$

$$D \equiv (B)^a \cdot (v)^b \; . \tag{64}$$

Let $z_{it} \equiv E[x_{it} \mid s_1]$, where $x_{it} \equiv (\log(y_{it}), -\log(n_{i,t-1}))'$. Taking the s_1-conditional expectation of the log of each side of each state equation (59) and (60), one obtains a linear nonhomogeneous difference equation for z_{it} of the form

$$z_{i,t+1} = d_i + M'z_{it} \; , \; t \geq 1 \; , \tag{65}$$

where M' denotes the transpose of the matrix M described in the proof of Proposition 1, and

$$d_i \equiv (\log(C) + mE[\log(\alpha_{it}) \mid s_1], -\log(D))' \; . \tag{66}$$

A straightforward calculation shows that the determinant of $[I - M']$ is given by $[(1 - u - (a+b)v)(1+b) + bv(a+b)]$. Under the sign restrictions imposed on the coefficients a, b, u, and v in Section 3, this expression is positive. Consequently, a particular solution for the difference system (65) is given by the stationary solution

$$z_i \equiv [I - M']^{-1} d_i \; . \tag{67}$$

Using a straightforward proof by contradiction, it can be shown that the particular solution (67) is a stable solution for the nonhomogenous difference system (65) if and only if the origin **0** in Euclidean 2-space is a stable solution for the homogeneous difference system

$$z^*_{i,t+1} \;=\; M' z^*_{it} \;,\; t \geq 1 \;. \tag{68}$$

In turn, it follows from basic results in difference equation theory that **0** is a stable solution for (68) if and only if all of the characteristic roots r of the matrix M' are less than 1 in absolute value.

Consider, then, all possible solutions r for the characteristic equation

$$0 \;=\; |\, M' - rI \,| \;=\; r^2 + r[b-s] - ub \;, \tag{69}$$

where $s \equiv [u + (a+b)v]$. By the quadratic formula, in order to have a solution r for (69) be less than 1 in absolute value, it is necessary and sufficient that

$$|\, (s-b) \pm \sqrt{(b-s)^2 + 4ub} \,| \;<\; 2 \;. \tag{70}$$

By assumption, the coefficients u, v, a, and b have positive values satisfying $(u+v) \leq 1$ and $(a+b) \leq 1$, implying $b < 1$ and $u < s \equiv [u + (a+b)v] \leq 1$. Consequently,

$$0 \;<\; [(b-s)^2 + 4ub] \;<\; [(b-s)^2 + 4sb] \;=\; (b+s)^2 \;, \tag{71}$$

and

$$-2 \;<\; -2b < (s-b) \pm \sqrt{(b-s)^2 + 4ub} \;<\; 2s \;\leq\; 2 \;. \tag{72}$$

Thus, all of the characteristic roots r of the matrix M' are less than 1 in absolute value.

In summary, the origin **0** in Euclidean 2-space has been shown to be a stable solution for the homogeneous difference system (68), implying that the particular solution (67) is a stable solution for the original nonhomogeneous difference system (65). Finally, it is clear from the definition (66) for d_i that this particular solution will be the same for any two dynasties whose ability levels are determined in accordance with the same stationary probability density function. Q.E.D.

PROOF OF PROPOSITION 4. By assumption, the tax policy in effect is $\tau = 1/N$, implying that the period t after-tax income $\tau_{it}y_{it}$ of each dynasty i is given by mean social income $\bar{y}_t$. Thus, recalling the definitions (63) and (64) for the constant terms C and D, the state equations for this case take the form

$$y_{i,t+1} = C(N) \cdot (\alpha_{it})^m \cdot (\bar{y}_t)^{u+av} \cdot (y_{it})^{bv} \cdot (n_{i,t-1})^{-bv} \ ; \tag{73}$$

$$n_{it} = D(N) \cdot (\bar{y}_t)^a \cdot (y_{it})^b \cdot (n_{i,t-1})^{-b} \ , \tag{74}$$

where,

$$C(N) \equiv C \cdot (N)^{-bv}; \tag{75}$$

$$D(N) \equiv D \cdot (N)^{-b} \ . \tag{76}$$

Let p_{it} denote $E[\log(y_{it}) \mid s_1]$ and let q_{it} denote $E[\log(n_{i,t-1}) \mid s_1]$. Then, expressing each state equation in logarithmic form, and taking s_1-conditional expectations of each side of each equation, one obtains

$$p_{i,t+1} = R(N) + [u+av] \cdot E[\log(\bar{y}_t) \mid s_1] + [bv] \cdot p_{it} - [bv] \cdot q_{it} \ ; \tag{77}$$

$$q_{i,t+1} = S(N) - a \cdot E[\log(\bar{y}_t) \mid s_1] - b \cdot p_{it} - b \cdot q_{it} \ , \tag{78}$$

where

$$R(N) = \log(C) + m \cdot E[\log(\alpha_{it})] - [bv] \cdot \log(N) \ ; \tag{79}$$

$$S(N) = -\log(D) + b \cdot \log(N) \ . \tag{80}$$

By assumption, p_{it} and $E[\log(\bar{y}_t) \mid s_1] - K_i$ converge to a common value as t becomes arbitrarily large; let this common value be denoted by $\bar{p}_i$. It can then be shown that the system of state equations (77) and (78) has a stationary solution $(p_{it}, q_{it}) = (\bar{p}_i, \bar{q}_i)$ for some value $\bar{q}_i$ if and only if $\bar{z} = (\bar{p}_i, \bar{q}_i)$ satisfies the matrix equation

$$[I - M']\bar{z} \;=\; h_i \;, \tag{81}$$

where $h_i = (R(N) + [u + av]K_i, S(N) - aK_i)'$ and M' denotes the transpose of the matrix M described in the proof of Proposition 1. As demonstrated in the proof of Proposition 3, the matrix $[I - M']$ is nonsingular, implying that $\bar{z} = [I - M']^{-1}h_i$ is the unique solution for (81). Finally, as is also established in the proof of Proposition 3, all of the characteristic roots r of the matrix M' are less than 1 in absolute value. Consequently, it can also be shown that the stationary solution $\bar{z}$ is stable. Q.E.D.

PROOF OF COROLLARY 2. By the specification (17) for the expected marginal return to schooling effort and the assumption $\tau = 1/N$, one has

$$E[\log(r^e_{it}) \mid s_1] \;=\; \log(v/N) + E[\log(y_{it}/n_{i,t-1}) \mid s_1] \;. \tag{82}$$

Also, it follows from Proposition 4 that

$$\lim_{t\to\infty} E[\log(y_{it}/n_{it}) \mid s_1] \;=\; [Q_i + b[1 - u - v]\log(N)]/\det \;, \tag{83}$$

where $\det \equiv \det[I - M'] = (1 - u - [a + b]v)(1 + b) + bv[a + b] > 0$, and where

$$Q_i \;\equiv\; (1 - a)[\log(C) + mE\log(\alpha_i)] - \log(D)[1 - u - av] + K_i[u + av - a] \;, \tag{84}$$

with C and D as defined in (63) and (64), respectively. Relation (36) is then obtained by combining (82) and (83) and noting that $(\det - b[1 - u - v]) = (1 - u - av) > 0$. Q.E.D.

PROOF OF PROPOSITION 5. As established in the proof of Proposition 3, the determinant of $[I - M']$ is positive, where the matrix M is given in explicit form in the proof of Proposition 1. The proof of Proposition 5 then follows by direct calculation using the form of the long run solutions given in Propositions 3 and 4 and the exogenous parameter restrictions $0 < v$, $0 < b$, and $0 < u < 1$. Q.E.D.

PROOF OF PROPOSITION 6. It follows from definition (15) for τ_{it} that

$$E[\tau_{it} \mid s_1] \;=\; 1 \;+\; (-N + E[(\sum_{j=1}^{N} y_{jt}/y_{it}) \mid s_1]) \cdot \tau \;. \tag{85}$$

Given the regularity condition (37), and noting by Jensen's inequality that $E(1/x) > 1/E(x)$, one obtains

$$\frac{\partial E[\tau_{it} \mid s_1]}{\partial \tau} \quad \approx \quad (-1 + E[y_{it} \mid s_1] \cdot E[(1/y_{it}) \mid s_1]) \cdot N \;>\; 0 \tag{86}$$

for all sufficiently large N. Consequently, the desired result holds. Q.E.D.