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**WALRASIAN GENERAL EQUILIBRIUM:**
**BENCHMARK OF COORDINATION SUCCESS?**[1]

**Key Questions:**

- Is Walrasian equilibrium an appropriate benchmark of coordination success for decentralized market economies?
- What does coordination success mean? What should it mean?
- How robust is the concept of Walrasian equilibrium to various weakenings of its assumptions?

# 1 Introduction

As detailed in an earlier packet reading, the *Walrasian general equilibrium (WGE)* model represents a precisely formulated set of conditions under which feasible allocations of goods and services can be supported by price systems in decentralized market economies characterized by price-taking consumers and firms and the private ownership of capital and labor.

The most salient structural characteristic of the WGE model is its strong dependence on the *Walrasian Auctioneer*, a fictitious central clearing house. Firms and consumers interact only with the Walrasian Auctioneer; face-to-face interactions among firms and consumers are not permitted.

In particular, prices and dividend payments mediated by the Walrasian Auctioneer constitute the only links among consumers and firms prior to actual trades. Since consumers take prices and dividend payments as given aspects of their decision problems, outside of their control, their decision problems reduce to simple optimization problems with no perceived dependence on the actions of other agents. A similar observation holds for the decision problems faced by the price-taking firms. The equilibrium values for the linking price and

[1]This packet reading relies heavily on Tesfatsion [17].

dividend variables are determined by market clearing conditions imposed through the Walrasian Auctioneer pricing mechanism; they are not determined by the actions of consumers, firms, or any other agency supposed to actually reside within the economy.

The WGE model is an elegant affirmative answer to a logically posed issue: can efficient allocations be supported through decentralized market prices? It does not address, and was not meant to address, how production, pricing, and trade actually take place in real-world economies through various forms of procurement processes.

What, specifically, is standardly meant by "procurement processes" in the business world? Customers and suppliers must identify what goods and services they wish to buy and sell, in what volume, and at what prices. Potential trade partners must be identified, offers to buy and sell must be prepared and transmitted, and received offers must be compared and evaluated. Specific trade partners must be selected, possibly with further negotiation to determine contract provisions, and transactions and payment processing must be carried out. Finally, customer and supplier relationships involving longer-term commitments must be managed.

Theories always simplify, and substituting equilibrium assumptions for procurement processes is one way to achieve an immensely simplified representation of an economic system. For economic systems known to have a globally stable equilibrium, this simplification might be considered reasonable since procurement processes do not affect the system's long-run behavior. Even in this case, however, the path of adjustment could be of considerable practical concern as a determinant of the speed of convergence. For economic systems without a globally stable equilibrium, procurement processes determine how the dynamics of the system play out over time from any initial starting point.

As carefully detailed by Fisher [1] and Takayama [15, Chapters 2-3], economists have not been able to find empirically compelling sufficient conditions guaranteeing existence of Walrasian equilibria, let alone uniqueness, stability, and rapid speed of convergence, even for relatively simple modelings of market economies. For extensions of the Walrasian framework to dynamic open-ended economies, such as overlapping generations economies, multiple equilibria commonly occur and the Pareto efficiency of these equilibria is no longer guaranteed.

The explicit consideration of procurement processes would therefore appear to be critically important for understanding how numerous market economies have managed in practice to exhibit reasonably coordinated behavior over time. As eloquently expressed by Fisher [1, p. 16]:

> "The theory of value is not satisfactory without a description of the adjustment processes that are applicable to the economy and of the way in which individual agents adjust to disequilibrium. In this sense, stability analysis is of far more than merely technical interest. It is the first step in the reformulation of the theory of value."

Nevertheless, as seen in previous packet readings, current macroeconomic models still heavily rely on an underlying WGE framework with market clearing externally imposed through a Walrasian Auctioneer construct. In this manner they sidestep the need for any explicit modeling of procurement processes as essential aspects of the circular flow among firms and consumers.

To highlight the extraordinary strength of the Walrasian Auctioneer assumption, a natural way to proceed is to examine what happens in a standard WGE model if the Walrasian Auctioneer pricing mechanism is removed and if prices and quantities are instead required to be set entirely through the procurement actions of the firms and consumers themselves. This removal is most definitely *not* a small perturbation of the model.

Two basic types of problems immediately arise: (1) *Pricing Problems:* the price/wage bids and offers of price-setting buyers and sellers might not be at the "right" (Walrasian equilibrium) levels either because these buyers and/or sellers are behaving strategically, or because they are uncertain what the "right" price bids and offers might be; and (2) *Matching Problems:* Even if prices/wages are "right", searching buyers and sellers might fail to locate and match with each other in a manner supporting a Walrasian equilibrium allocation due to information problems and transactions costs.

## 2 A Simple Example with Price-Setting Firms

In this section a simple example is presented that illustrates how strategic interaction necessarily arises among profit-seeking[2] firms competing for the scarce dollars of budget-

[2] The assumption that firms are profit-*seeking* is considerably weaker and less controversial than the assumption that firms are profit-*maximizing*.

constrained consumers once the fictitious construct of a Walrasian Auctioneer is removed and the firms must set their own prices. More detailed presentations of such "non-Walrasian" models are developed in a subsequent packet reading.

Consider an economy that consists of two price-setting profit-seeking firms, each producing a distinct consumption good at constant marginal cost, and a utility-seeking consumer who obtains utility from the consumption of these two goods. Suppose that the profit obtained by Firm i from the sale of good $i$, $i = 1, 2$, is given by

$$p_i q_i - c_i q_i \quad , \tag{1}$$

where $p_i$ denotes the price of good $i$, $q_i$ denotes the amount sold of good $i$, and the constant marginal cost $c_i$ is positive.

Suppose the consumer has a utility function of the form

$$U(q_1, q_2) \quad = \quad \ln(q_1 - b_1) + \ln(q_2 - b_2) \quad , \tag{2}$$

where $b_1$ and $b_2$ are given nonnegative constants and "ln" denotes the natural logarithm (i.e., the logarithm to the base e). Suppose, also, that the income $I$ of the consumer is a positive exogenously-given constant. The utility maximization problem faced by the consumer then takes the following form: Given goods prices $p_1$ and $p_2$, solve

$$\max_{q_1, q_2} U(q_1, q_2) \tag{3}$$

subject to the budget and feasibility constraints

$$p_1 q_1 \; + \; p_2 q_2 \quad = \quad I \quad ; \tag{4}$$

$$q_1, q_2 \geq 0 \quad . \tag{5}$$

The solution to the utility maximization problem (3) through (5) yields the following demand functions for $q_1$ and $q_2$:

$$q_1^* \quad = \quad b_1/2 + [I - b_2 p_2]/2p_1 \quad = \quad D_1(p_1, p_2) \quad ; \tag{6}$$

$$q_2^* \quad = \quad b_2/2 + [I - b_1 p_1]/2p_2 \quad = \quad D_2(p_1, p_2) \quad , \tag{7}$$

where dependence on the exogenous variables $b_1$, $b_2$, and $I$ has been notationally supressed for expositional simplicity. Note that $q_1^*$ is independent of $p_2$ in (6) if and only if $b_2 = 0$, and that $q_2^*$ is independent of $p_1$ in (7) if and only if $b_1 = 0$. This illustrates the general rule of thumb that the optimal consumer demand for any *one* good will depend on the prices of *all* goods. This dependence arises because the demands of a consumer for goods in any one period are simultaneously and jointly constrained by a single budget constraint that requires total expenditures for all goods to equal total income (possibly modified by borrowing or lending). An exception to this rule arises for the special case of an additive and purely logarithmic utility function, e.g., the utility function in (2) with $b_1 = 0$ and $b_2 = 0$; in this case, as seen in (6) and (7), the demand for each good reduces to being a fixed proportion of income with a proportionality factor that depends only on own price.[3]

Suppose the market protocol governing market behavior in this economy is as follows: Firm 1 and Firm 2 simultaneously announce prices $p_1$ and $p_2$, promising to meet any forthcoming demands for their goods from the consumer as long as this can be done with non-negative profits. A strategic interaction problem then arises for each profit-seeking firm, because the profits of each firm depend on the price set by the other firm. Specifically, the profit function of Firm 1 takes the form

$$\pi_1(p_1, p_2) \quad = \quad [p_1 - c_1]D_1(p_1, p_2) \quad , \tag{8}$$

and the profit function of Firm 2 takes the form

$$\pi_2(p_1, p_2) \quad = \quad [p_2 - c_2]D_2(p_1, p_2) \quad . \tag{9}$$

Suppose the values taken on by the parameters appearing in the above-described model economy are as follows:

$$b_1 = 1/2;\ \ b_2 = 1;\ \ I = 1;\ \ c_1 = 0;\ \ c_2 = 0 \quad . \tag{10}$$

In this case, the consumer demand functions (6) and (7) reduce to the following particular forms:

$$q_1^* \quad = \quad 1/4 + [1 - p_2]/2p_1 \quad = \quad D_1(p_1, p_2) \ \ ; \tag{11}$$

$$q_2^* \quad = \quad 1/2 + [2 - p_1]/4p_2 \quad = \quad D_2(p_1, p_2) \ \ . \tag{12}$$

[3] As this argument demonstrates, the logarithmic utility function (2) with $b_1 = b_2 = 0$ is extremely special and imposes extremely strong and empirically non-supported separability restrictions on consumer demand functions. Having positive "subsistence needs" $b_1$ and $b_2$ is a step towards realism that retains analytical tractability.

Suppose, also, that each firm can set its goods price at only one of two possible values, low $L$ or high $H$, where

$$L = 1/2 \text{ and } H = 3/4 . \tag{13}$$

In this case, the demands $D_1(p_1, p_2)$ faced by Firm 1 for all possible settings of the prices $p_1$ and $p_2$ are as follows:

$$\begin{aligned} D_1(L,L) &= 3/4 \; ; \\ D_1(L,H) &= 1/2 \; ; \\ D_1(H,L) &= 7/12 \; ; \\ D_1(H,H) &= 5/12 \; . \end{aligned}$$

Similarly, the demands $D_2(p_1, p_2)$ faced by Firm 2 for all possible settings of the prices $p_1$ and $p_2$ are as follows:

$$\begin{aligned} D_2(L,L) &= 5/4 \; ; \\ D_2(L,H) &= 1 \; ; \\ D_2(H,L) &= 9/8 \; ; \\ D_2(H,H) &= 11/12 \; . \end{aligned}$$

Using (8) and (9), the profit payoff matrix faced by the two firms then takes the form depicted in Table 1. (For clarity, each profit level has been multiplied by 48 so that profits are represented as whole number rather than as fractions.) The first number in each reported payoff pairing denotes the profit for Firm 1 and the second number denotes the profit for Firm 2.

A *Nash equilibrium* for Firm 1 and Firm 2 is any pair ($p_1^*$,$p_2^*$) of pricing strategies such that, given $p_1^*$, Firm 2 has no incentive to deviate from $p_2^*$, and given $p_2^*$, Firm 1 has no incentive to deviate from $p_1^*$. As seen in Table 1, there exists a unique Nash equilibrium for the model economy at hand: namely, the pricing strategy pair ($H$,$H$). In fact, an even stronger property holds for ($H$,$H$). The pricing strategy $H$ constitutes a *dominant* pricing strategy for each firm in the sense that $H$ is the best price for each firm to set in response to *any* price set by its rival.

| | | Firm 2 L | Firm 2 H |
|---|---|---|---|
| Firm 1 | L | (18,30) | (12,36) |
| | H | (21,27) | (15,33) |

Table 1: Profit Payoff Matrix

Morever, $(H,H)$ is also Pareto efficient *for the two firms (not necessarily for the consumer!)*, meaning there is no other feasible pricing strategy pair under which both firms would be at least as well off and at least one would be better off. In fact, *every one* of the four possible strategy combinations for Firm 1 and Firm 2 depicted in Table 1 is Pareto efficient, which illustrates the inherent weakness of the Pareto efficiency concept.

Note that Firm 1 would actually prefer the profit payoff it obtains under the pricing strategy pair $(L,L)$ to the profit payoff it obtains under the pricing strategy pair $(H,H)$, but it has no power to enforce this outcome. Even if Firm 1 were to announce that it plans to choose price $L$, it would still be best for Firm 2 to choose price $H$; and the profit payoff for Firm 1 under $(L,H)$ is worse than the profit payoff for Firm 1 under $(H,H)$.

Various open questions remain. For example, is $(H,H)$ *always* a dominant (hence Nash) pricing strategy pair for the game at hand, regardless of the particular parameter values? Is there always a Nash equilibrium (even if not a dominant pricing strategy pair), regardless of parameter values? If so, is this Nash equilibrium always Pareto efficient for the two firms? Could there be a Nash equilibrium which is Pareto dominated, for example, in the sense that there is another feasible pricing strategy pair that yields at least as much profit for both firms and strictly more profit for at least one? And what about the welfare of the consumer in all of this? And the sensitivity of the conclusions to the form of the utility function?

# 3 Concluding Remarks

In conclusion, the WGE model demonstrates how price systems are capable, in principle, of coordinating the supplies and demands of private agents in models of decentralized market economies that assume all agents are price *takers*. However, the Walrasian general equilibrium model is incomplete as a model of real-world decentralized market economies, since many private agents in such markets are price *setters*. When price setting by private agents is introduced into the standard WGE model, it simply breaks down.

Although a definitive *theory* regarding the working of decentralized market economies with price-setting agents is currently lacking, we do have some analytical, experimental, and natural data evidence that bears on this question.

For example, some researchers have attempted to introduce price setting into appropriately modified general equilibrium models; see, for example, Fisher [1], Pingle and Tesfatsion [6, 7], Shubik [10, Chapter 14], and Smets-Wouters [11, 12]. The relationship between these attempts and the now rather extensive partial-equilibrium literature in industrial organization on sequential bargaining games with price-setting agents nevertheless remains unsettled; see, for example, Fudenberg and Tirole [2, Chapter 10].

In the human-subject market experiments conducted by Charles Plott, Vernon Smith and others in which the market structure is kept fairly simple - e.g., a single market with a unique market clearing point - various auction mechanisms in which buyers and sellers sequentially make bids and offers for units of good have been regularly observed to result in convergence to a market clearing outcome. See, for example, Kagel and Roth [3] and Smith [13, 14]. Thus, the Walrasian general equilibrium model does have predictive content in certain experimental settings even when its behavioral assumptions (in particular, price taking) are known to be false.

Also, various researchers have constructed relatively simple computational models of decentralized market economies with imperfectly informed but price-*taking* agents – see, for example, Sargent [9, Chapter 5]. The experiments conducted for these economies suggest that convergence to a market clearing equilibrium can sometimes be ensured if the learning algorithms used by agents to update their price expectations are suitably restricted by the modeler.

On the other hand, in "bottom up" agent-based computational economics (ACE) studies of decentralized market economies in which consumers and firms are modeled as autonomous interacting price-*setting* agents who evolve their behavior over time, experiments indicate that multiple persistent configurations ("attractors") can exist for each specification of the initial structural conditions if interaction effects are sufficiently strong, as in labor markets. See, for example, Tesfatsion [16] and Tesfatsion and Judd [18].

In the real world, decentralized market economies with price-setting agents have evolved an enormous variety of institutions (legal systems, credit systems, social norms, ...) that help to prevent market breakdowns. Given these supporting institutions, it does appear that many decentralized market economies are able to achieve, on average, a rough balancing of supplies and demands for goods and services. More generally, these economies are able to stay approximately coordinated over time, in the sense that inflation and unemployment do not get wildly out of control and a positive GDP growth rate is sustained on average. Consequently, despite our current inability to explain, theoretically, the process by which real-world decentralized market economies accomplish such feats, the fact that such economies do accomplish such feats is incontrovertible evidence that coordination issues remain a key unresolved aspect of macroeconomic theory.

What still needs to be done, then, is to combine the use of theoretical model analysis, market lab experiments, and computational experiments in an attempt to explain actual empirical data on the workings of real-world decentralized market economies with price setting agents. In particular, do such economies in any sense tend to evolve naturally, or are they the result of deliberate top-down planning? To what extent, if any, is their success or failure the result of top-down interventions?

As detailed above, a serious investigation of these issues will require the consideration of various topics that have traditionally been identified with other disciplines:

1. Strategic interaction (Game Theory)
2. Dual control (Systems Science)
3. Optimal search/sampling rules (Statistical Decision Theory)
4. Inventory planning (Management Science and Operations Research)
5. Descriptive behavioral characterizations, e.g., of the way in which individuals and groups resolve or account for uncertainty in their decision making (Sociology and Psychology)

6. Descriptive institutional characterizations, e.g., of the customs, rules, and/or laws regarding bankruptcy procedures, rationing, wage contracting, credit contracting, etc. (Business and Law)

The resulting models can be quite complex. Consequently, the analysis, experimental lab study, or computational study of such models will require sophisticated mathematical tools from complex systems theory, a good knowledge of statistical experimental design, and/or a good working knowledge of a powerful computer programming language such as Java, C++, or C# to succeed.

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