Economics 102: Problem Set 3

Due date: October 12 (Monday), 2009.
Note: you can either submit your homework right after class or put it in the box outside my office (Heady Hall 469) before 5:00pm on the due date.

Problem 1 Suppose an industry has the following production function:

\[ Y = F(K, L) = K\sqrt{L} \]

(i) Holding capital constant at \( K = 1 \). Compute the marginal product of labor \( MPL(K, L) \) for \( L = 0, 1, 2, 3, 4, 5 \).

(ii) Does this production function exhibit diminishing marginal product of labor? Prove your conclusion.

(iii) Does this production function have constant returns to scale? Prove your conclusion.

(iv) Holding capital constant at \( K = 1 \). Suppose the industry hires 4 workers, can you infer from the equation \( MPL(K, L) = W/P \) the workers’ real wage?

(v) Suppose the industry is experiencing a major technological innovation, and the new production function is

\[ F(K, L) = \alpha K\sqrt{L}, \]

where \( \alpha \) is a constant and \( \alpha > 1 \). Suppose the market real wage for the workers remains the same (as determined in (iv)). Solve for the optimal employment \( L^* \).

Does the industry want to hire more workers after the technological innovation?

Problem 2 We have two countries. Country 1 has the following production function:

\[ Y = F(K, L), \]

and country 2 has the following production function:

\[ Y = 2F(K, L). \]

Suppose the production function \( F(K, L) \) has diminishing marginal product of labor.

(i) For given \( K \) and \( L \), which country’s \( MPL(K, L) \) is higher?

(ii) Suppose the two countries have the same capital stock \( K = 1 \). Suppose country 1’s labor force is \( L = 1,000 \) and country 2’s labor force is \( L = 1,000 \). Suppose there’s full employment in both countries.

(a) Suppose labor cannot move freely between the two countries. Which country has a higher real wage?

(b) What happens if labor can move freely between the two countries?