1. Consider a competitive firm which produces output, $y$, using labor ($L$) as the only variable input. In order to produce any output, the firm must incur a fixed cost, $F$ (for example, rent factory space). The firm’s production function is given by:

$$y = A L^{1/2} \rightarrow L = \left(\frac{y}{A}\right)^2; \quad A > 0$$

In equation (1), $y$ is output, $L$ is labor input, and $A$ is a productivity parameter. Higher values of $A$ correspond to increases in productivity. The firm’s total costs are given by:

$$TC(y) = \begin{cases} 0 & \text{if } y = 0 \\ (F + wL) = F + w\left(\frac{y}{A}\right)^2 & \text{if } y > 0 \end{cases}$$

In (2), $w$ is the wage rate. The total cost curve shows the firm incurs no costs in the long run, if it decides not to produce; but, if it produces any output it must incur the fixed costs, $F$, and the variable costs ($wL$).

a) The firm’s profits, as a function of output and price, are: $\pi(y) = py - TC(y) = \left(\frac{py - F - w\left(\frac{y}{A}\right)^2}{A}\right)$.

i. Find the output level, $y^* (p; w, A)$, that maximizes profits assuming fixed costs must be paid.

ii. Show that this output level (supply curve) is the firm’s marginal cost curve.

iii. Show how an increase in the wage rate shifts the supply curve.

iv. Show how an increase in productivity shifts the supply curve.

For the remainder of question 1, assume $w = 16, \quad F = 16, \quad A = 8$.

v. By substituting these values of $w, A$ into your answer to part (i), write the firm’s supply curve as a function of price: $y^* (p)$.

vi. Using your answer to part (v), calculate the firms maximized profits as a function of price:

$$\pi^* (p) = py^* (p) - F - w\left(\frac{y^* (p)}{A}\right)^2 = py^* (p) - 16 - \frac{\left(\frac{y^* (p)}{A}\right)^2}{4}$$

b) Find the firm’s long run supply curve. (NOTE: In the short run, the firm must pay fixed costs; in the long run it can avoid the fixed costs by exiting the industry if profits would be negative with $y > 0$.)

i. Use the firm’s supply curve to calculate how much production costs increase when output increases from $y = 80$ to $y = 100$. Use the cost curve to verify your answer.

c) The firm currently sells output at a price of 40 but it has found new markets where it can sell all its output at a price of 50. Assuming competitive profit maximization, use the supply curve to show (graphically and numerically) how much profits increase due to this price change. Verify your answer using the profit function calculated in part a(vi).

d) Modify part (c) by assuming the firm must continue to sell its original output of 80 units in the old market at a price of 40, but is allowed to sell additional units in this new (e.g., a foreign) market at a price of 50. If it sold 20 units there at a price of 50 (and continued to sell the 80 units in its home market at a price of 40), how much would its profits increase? Give a numerical answer and show graphically how to calculate this answer.
2. Consider a consumer who has the following (quasi-linear) utility function:

\[ U(x, y) = x + 80y - \left( \frac{y^2}{4} \right) \]

where \((x, y)\) denotes the goods consumed by the individual. Let \(I\) denote the individual’s income, and \((P_x, P_y)\) denote the prices the individual pays for goods \(x\) and \(y\), respectively.

a) Write the individual’s budget constraint, set up the utility maximization problem and derive the individual’s demand functions for both goods (assume a solution where both goods are consumed).

i. Find the individual’s maximized utility by substituting the demand solutions back in to the utility function {this is called the person’s indirect utility function}.

ii. Discuss how an increase in the price of good \(y\) affects maximized utility and interpret. Take the (partial) derivative of the indirect utility function with respect to \(p_y\); what does this equal?

b) Currently a consumer, with income \(I=5000\), can buy goods at the prices \(P_x = 1, P_y = 40\). A new mall brings to town a discount store (Costco) which sells good \(y\) at the price \(P_y = 30\).

i. Find the consumer’s purchases at this price. Will the consumer be better or worse off as a result of the opening of this store?

ii. Suppose the store charges individuals a one-time fee of $\(F\) to shop in the store. What is the maximum amount this individual would be willing to pay to shop in the store (the alternative is to continue to pay the price \(P_y = 40\) charged by other stores in town)? Give a numerical answer.

iii. Using the demand curve, show graphically how to calculate your answer for part ii. What is this area called?

3. {Efficiency of markets}. Use the supply curve from question 1 and the demand curve from question 2 and find the equilibrium market price and output level. {In essence, you are assuming there are a lot of identical producers and consumers, but it is simpler to just work with one of each}.

a) Suppose you were a dictator and you could choose the output level and price that would maximize the sum of producer and consumer surplus. What output level would you choose? How does it compare to the equilibrium output level you calculated above?

b) Next, assume the government gives producers a production subsidy of 20 per unit sold {thus, if consumers pay a price \(p\) for the good, producers receive \((p+20)\) for each unit produced and sold}. Show how this subsidy affects: (i)equilibrium consumer price \(p\) and producer price \((p+20)\); (ii)equilibrium output; (iii)consumer surplus and (iv)producer surplus. Compare the total change in producer and consumer surplus to the cost to the government (and hence taxpayers) of the subsidy, and discuss how the subsidy affects overall efficiency. Show graphically this change in efficiency.

c) Suppose there is no subsidy but the government now permits trade with the rest of the world. The world price is 50, and the country – since it is small – can import or export as much of the good as it wants at this world price. Starting from the original equilibrium (with no subsidy), show how international trade, at the world price of 50, affects consumer surplus, producer surplus, and overall efficiency. Show the net gain – or loss – from this trade graphically and give an economic explanation of why this area represents the overall welfare change from trade.