1. Simplified version of Heckscher-Ohlin model. Consider a country which can produce two goods: cloth (C) and food (F) using two inputs: labor (L) and land (T). Production of each good requires inputs to be used in fixed proportions as follows (these are called Leontief technologies):

To produce cloth: 4 units of labor and 2 units of land are required for each unit of cloth.
To produce food: 2 units of labor and 4 units of land are required for each unit of food.

Let \( L, T \) represent the total amount of labor and land available in the economy, let \( P_c, P_f \) denote the prices of output and let \( W, R \) denote the prices of labor and land respectively.

a) Find production costs and hence output price (price=marginal cost) for each good in terms of factor prices \( (W, R) \). (Hint: To produce \( Q_c \) units of cloth requires \( 4Q_c \) workers and \( 2Q_c \) units of land and total costs are \( WL_c + RK_c \).

i. Use your answers to solve for inputs prices in terms of output prices. How will an increase in the price of food affect factor prices?

b) Find the production possibility frontier (ppf) for this economy and sketch it. (Hint: it is defined by two linear inequalities that state labor (land) used cannot exceed supply. Use the hint from part (a) and apply the same reasoning for factor use in the food sector).

i. Show how an increase in the supply of land shifts the ppf and the production point where inputs are fully used (in this simple version, there is a unique production point that represents full employment of both inputs).

c) Assume two countries (the US and Mexico) have identical tastes and technology, but the U.S. has more land and Mexico has more labor. Assuming the relative demand (ratio of demand for cloth to demand for food) is independent of income, discuss how autarky goods prices and factor prices differ between the two countries, then discuss how trade affect factor prices in each country. Will factor prices be equalized between the two countries?

d) Modify the above model by assuming US productivity in both sectors doubles. Thus, in the US:

To produce cloth: 2 units of labor and 1 unit of land are required for each unit of cloth.
To produce food: 1 units of labor and 2 units of land are required for each unit of food.

Mexican technology remains unchanged. Viewed in a Ricardian context, the US has an absolute advantage in both goods but a comparative advantage in neither good (due to technology).

i. Find how this US productivity change affects its autarky output prices and factor prices.

ii. Assuming free trade between the US and Mexico, will free trade equalize factor prices? Will trade eliminate the pressure for factor migration?
2. (Factor movement problem). Consider a very simple model with two countries (US, Mexico) and only one good. Output of this good, in each country, depends upon technology, the amount of land, and the amount of labor in each country. Assume the following production technology:

US: \[ Q^{us} = 25 \left( T^{us} \right)^{2/3} \left( L^{us} \right)^{1/3} \]; \( T^{us} = 216 \); \( L^{us} = 125 \)

Mexico: \[ Q^{mex} = 16 \left( T^{mex} \right)^{2/3} \left( L^{mex} \right)^{1/3} \]; \( T^{mex} = 125 \); \( L^{mex} = 125 \)

In the above equations, \( T^{us} \) and \( T^{mex} \) represent the (usable) land area in each country, and \( L^{us} \) and \( L^{mex} \) represent the population (= number of workers) initially in each country.

(a) For each country, find and sketch the labor demand curve. Also, calculate the equilibrium wage, return on land and per capita income in each country (all measured in terms of output).

(b) Suppose labor is free to migrate from Mexico to the US. Let \( I \) stand for the number of “guest workers” in the US (from Mexico), so that the total US labor force is \( L^{us} + I \) and the total Mexican labor force is \( L^{mex} - I \).

i. Assuming there are no restrictions on labor movements, and that labor moves until wages are equalized, find the equilibrium number of guest workers.

ii. How does the “guest worker program” affect: (i) US wages, (ii) the return on land in the US; (iii) Mexican wages; and (iv) the return on land in Mexico? How does it affect total world output? Qualitative answers (e.g., increase or decrease) suffice.

iii. Does the “US” gain from this guest worker program? To answer, consider both the income distribution affects and what happens to per capita US income. In answering, note that:

\[ Y^{us} = Q^{us} - W^{us} I = 25 \left( T^{us} \right)^{2/3} \left( L^{us} + I \right)^{1/3} - W^{us} I \]; \( T^{us} = 216 \); \( L^{us} = 125 \)

That is, US income \( Y^{us} \) is total US output minus the total wages paid to guest workers, and thus per capita US income is: \( Y^{us} / L^{us} \).

iv. Suppose the “guest workers” were allowed to become US citizens so that now US per capita income is \( Q^{us} / (L^{us} + I) \) \{that is, there are no labor payments to guest workers since the immigrants become part of US population, which affects both the numerator and denominator in calculating US income\}. How does the immigration affect US per capita income?

v. Discuss the difficulties of deciding whether immigration “benefits the US” in this latter case.

(c) Suppose that each worker in the US, besides receiving his wages, also receives a fixed payment of “F” from the government (for example, free schooling for children or free medical care for all workers). The expense for this program is financed by a tax on landowners. If a guest worker program allows workers to freely choose where to work (and if Mexico has no such plan for workers), will allowing unrestricted guest workers into the US necessarily raise US per capita income (as measured in terms of the original population)? Discuss.
(Chapter 8, Trade Policy) Consider a small country (e.g., Peru) with the following demand and supply curves for sugar:

Supply = 2P_s; \ Demand = 180 − P_s

Assume Peru can import sugar at a given world price of: \( P_s = 30 \). Further, assume that Peru imposes an import tariff of \( t \) per unit of import.

(a) Show how: domestic price, consumption and production change as \( t \) increases. Also, calculate how consumer surplus, producer surplus, and government tariff revenue change as \( t \) increases. What happens to overall welfare in Peru as \( t \) increases?

i. If \( t > 30 \), what happens to the level of trade?

(b) Compare the domestic equilibrium when \( t = 20 \) to the case in which there is an import quota of 30 units (but no tariff). How do imports, domestic price, production and consumption compare under the two plans? What happens to the tariff revenue? Which policy is better for the country?

(c) Suppose, instead of an import tax (tariff), the Peruvian government subsidized imports at a rate of \( s \) per unit of import. Thus, for each unit of sugar imported, the importer receives pays 30 to the foreign exporter, but receives \( s \) from the government, so the net cost to the importer is \( (30-s) \). Show how this import subsidy affects: (i) domestic price; (ii) consumer surplus; (iii) producer surplus; (iv) government expenditures on the subsidy; and (iv) overall welfare.

i. Is there any import quota that would have the same effect as the import subsidy?

(d) Suppose the government’s goal is to reduce domestic consumption of sugar, in order to cut back on obesity and diabetes. It can accomplish this goal with an import tariff or a consumption tax. Which is the better goal? Demonstrate your answer by comparing the welfare implications of an import tariff of 20 and a consumption tax of 20.
4. (8 points extra credit) (More sophisticated version of H-O model). Similar to question #1, suppose there are two goods (C and F) and two inputs, K and L. {I use K, for capital, instead of land, because that is more traditional}. However, the technology for producing each good now allows smooth substitutability between the inputs. The production functions are:

\[ Q_c = K_c^{1/3} L_c^{2/3}; \quad Q_f = K_f^{2/3} L_f^{1/3} \]

where \( \{K_c, L_c\} \) are the inputs (capital, labor) used in sector C and \( \{K_f, L_f\} \) are the inputs used in sector F. Let \( W \) denote the wage rate (price of L) and \( R \) the rental rate (cost of using K, capital). Finally, let \( P_c, P_f \) denote the output prices of goods C and F, respectively.

(a) Derive the cost function for each good by minimizing cost for a given output level. Which good is capital intensive? Why? \{Possible procedure for good C: Minimize \( (WL_c + RK_c) \) subject to the constraint \( Q_c = K_c^{1/3} L_c^{2/3} \), which implies \( K_c = (Q_c)^3 (L_c)^{-2} \); thus, minimize:

\[ (WL_c + R(Q_c)^3 (L_c)^{-2}) \]

with respect to \( L_c \). Use a similar procedure for good F.\}

(b) Given output prices, show how an increase in the supply of labor changes output of each good. Use your result to predict which good the labor-abundant country will export.

i. Given output prices, does the increase in input (labor) supply change factor prices? Explain.

(c) Assuming both goods are produced (so that Price=Marginal Cost for each good), show how an increase in \( P_f \) will affect factor prices \( (W, R) \). Does either factor price rise proportionally more than \( P_f \)? If so, explain which one and why.

(d) Use your answer to parts (b) and (c) to explain how trade will affect the distribution of income in a labor-abundant country (perhaps like China) and how it will affect the distribution of income in a capital-abundant country (like the U.S.). Will everybody gain from trade? Explain.

(e) Assume the U.S. is capital-abundant. Which group in the U.S. is likely to favor import tariffs and which group is likely to impose trade restrictions? Explain.