INTERNATIONAL TRADE IN THE PRESENCE OF PRODUCT DIFFERENTIATION, ECONOMIES OF SCALE AND MONOPOLISTIC COMPETITION

A Chamberlin–Heckscher–Ohlin approach

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The paper presents a generalization of the Heckscher–Ohlin theory by admitting the existence of sectors in which there is monopolistic competition. The structure of preferences is based on Lancaster's work. It is shown without requiring homotheticity in the production of differentiated products that the intersectoral pattern of trade can be predicted from factor endowments but not from pre-trade commodity prices or factor rewards, except under special circumstances. It is also shown how the share of intra-industry trade is related to differences in income per capita and how the volume of trade depends on differences in income per capita and relative country-size. Other empirical implications are also discussed.

1. Introduction

The interest in the effects of product differentiation, economies of scale, and monopolistic competition on international trade has existed for many years. Nevertheless, traditional theories of international trade have not been extended to incorporate these elements. With the growth in recent years of formal models of industrial organization, there now seem to exist more than ever before the conditions necessary for an integration of theories of industrial organization with theories of international trade.

Two recent studies — Krugman (1979) and Lancaster (1979, ch. 10) — which used a one-sector model began the new literature on the effects of product differentiation, monopolistic competition, and economies of scale, on

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problems of international trade. Despite the fact that each one of them used a different approach to the specification of preferences, they reached the same broad conclusions regarding the nature of intra-industry trade and gains from specialization that are secured by taking advantage of economies of scale.

It is my purpose to provide in this paper an integration of the Heckscher-Ohlin approach to international trade with a Chamberlin-type approach to product differentiation, economies of scale, and monopolistic competition. In the present framework, Heckscher-Ohlin are used to explain intersectoral trade while Chamberlin is used to explain intra-industry trade [see also Dixit and Norman (1980, ch. 9), Lancaster (1980), and Krugman (1981)]. The theory that emerges from this study is a proper generalization of the Heckscher-Ohlin theory and it yields interesting as well as useful results. For example, I provide a factor price equalization theorem without requiring homotheticity of the production function in the differentiated product industry. Thus, it is shown that in the presence of monopolistic competition, we can predict the pattern of intersectoral trade from factor endowments even when the production function of the differentiated product, which exhibits economies of scale, is not homothetic. A capital rich country will be a net exporter of the capital intensive good while a labor rich country will be a net exporter of the labor intensive good. Differentiated products will be imported and exported by every country.

Secondly, it is shown that it is generally impossible to predict the pattern of trade from information about pre-trade relative commodity prices or relative factor rewards. However, when differentiated products are produced with a homothetic production function and consumers spend fixed budget shares on each good (Cobb-Douglas utility functions), relative factor rewards can be used to predict the intersectoral pattern of trade. The country with the lower wage-rental ratio will be a net exporter of the labor intensive good while the other country will be a net exporter of the capital intensive good. But, even in this case relative commodity prices cannot be used to predict the intersectoral pattern of trade, because country size matters. Other things being equal, the larger country has a lower relative price of the good produced with economies of scale. Lancaster (1980) has termed this feature 'false comparative advantage'. In order to overcome the size bias, I develop an index which I call the scale-adjusted price, which can be used to predict the pattern of trade. Thus, the country with the lower scale-adjusted relative price of a good will be a net exporter of this good.

Third, I prove a general theorem on the composition of trade in terms of intersectoral versus intra-industry trade. The theorem says that within some well defined limits a redistribution of factor endowments which enlarges the difference in capital–labor ratios available in each country, reduces the share of intra-industry trade in the total volume of trade. The volume of trade is
not related monotonically to the difference in capital labor ratios, unless
country sizes are kept constant. The volume of trade is declining as pure size
differentials increase.

Fourth, based on this theorem, as well as on additional considerations, I
propose two hypotheses concerning the relationship between the share of
intra-industry trade and incomes per capita. The first hypothesis, which deals
with a cross-section-comparison, is that the bilateral share of intra-industry
trade is negatively correlated with the absolute difference in bilateral incomes
per capita. The second hypothesis, which deals with a time series comparison,
is that the share of intra-industry trade in world trade is negatively
correlated with the dispersion of the countries' incomes per capita. The first
hypothesis finds support in a recent study by Loetscher and Wolter (1980):
I am not aware of any studies which shed light on the second hypothesis.

Since the theory of consumer behavior that is used in this study is new
and probably not widely known, sections 2 and 3 provide a detailed
description of consumer behavior and the behavior of monopolists that use
true demand functions for profit maximization. These sections build on
Lancaster's recent important study [see Lancaster (1979)]. Here I wish only
to point out that I have chosen to work with Lancaster's demand theory
because it enables me to discuss monopolistic competition in terms that are
commonly used in industrial organization, and I consider this to be an
advantage. For example, in this model, a firm that produces a variety of a
certain product chooses a specification and a price, knowing the demand
curve that it faces. Its demand curve depends on its product's price, its
specification, and also on the prices and product specifications of its closest
competitors, i.e. those whose varieties are closest to the firm's variety. If the
firm increases its price, it loses customers and those who remain buy less, i.e.
its market share declines.

An equilibrium of a two-by-two closed economy is described in section 4.
Then, in section 5, I discuss trading equilibria of a two-country world. There,
I interpret well known theorems from the Heckscher-Ohlin theory and prove
the theorem on the relationship between the share of intra-industry trade and
the difference in factor endowments. Then, in section 6, I consider the
predictive power of pre-trade commodity prices and factor rewards regarding
the pattern of intersectoral trade. Finally, in section 7, I discuss some
empirical implications of the theory.

2. Consumers

Assume that a typical consumer consumes two goods — a manufactured
commodity and food. Food is a homogeneous product with a single
specification; there is no more than one type of food. Manufactured goods,
on the other hand, have many potential specifications so that there are many
types (varieties) of the manufactured product. I assume that there is a continuum of types of the manufactured product that can be produced, and that there is a one-to-one correspondence between these types and points on a circumference of a circle. Thus point $b_i$ in fig. 1 represents a product of a particular type, and so do points $b_{i-1}$ and $b_{i+1}$. Moreover, each product type has a corresponding point on the circumference of the circle in fig. 1.\(^1\)

![Fig. 1](image)

It is now assumed that among all varieties of the manufactured good that can possibly be produced, each consumer has a most preferred type. The meaning of the most preferred type assumption is as follows: If the consumer is faced with the bundle of $x$ units of manufactured goods and $y$ units of food and he is free to choose the specification of the manufactured good of which he will receive $x$ units, then, independent of the quantities $x$ and $y$, he will always prefer a particular type, referred to as his 'ideal' type. Observe that this assumption implies something about the units in which quantities of manufactured goods are measured. At this stage it is perhaps appropriate to mention an assumption which will be made about production in the next section, i.e. that units in which quantities of manufactured goods are measured and technologies are such that the production function of manufactured goods is independent of specification. This means that if a given combination of factors of production produces, say, $X$ units of goods

\(^1\)Lancaster (1979) prefers to work with product specifications which can be represented by a line instead of a circle. For many purposes the line specification is more appropriate but it is more convenient to work with the circle specification. Both specifications yield similar results if in the case of a line specification one is willing to make special assumptions about the behavior of the 'edges' [see Lancaster (1979, ch. 6)]. The circle (loop) specification appears in Vickrey (1964, ch. 8). See also Salop (1979).
of type \( b_i \) in fig. 1, then the same combination of factors of production will produce \( X \) units of every other possible type of manufactured goods. This assumption is needed in order to assure the possibility of a symmetrical equilibrium. In cases in which one wants to give up symmetry, this assumption can be relaxed.

Now let \( u(x, y) \) be a consumer's utility function which represents his preference ordering over food and quantities of his ideal manufactured good. For a complete representation of preferences we also have to specify preferences over food and other types of manufactured goods which are not the consumer's ideal type. This is done by assuming the existence of a function \( h(v) \), defined for \( 0 \leq v \leq \pi l = 1 \), where \( l = 1/\pi \) is the radius of the circle in fig. 1, such that the consumer is indifferent between \( x \) units of the ideal manufactured product and \( h(v)x \) units of a good whose location on the circumference of the circle is at distance \( v \) (shortest arc distance) from the consumer's ideal manufactured product. The function \( h(v) \) is called the compensation function and it is assumed to have the following properties:

\[
\begin{align*}
    h(0) &= 1 \quad \text{and} \quad h(v) > 1 \quad \text{for} \quad v > 0, \\
    h'(0) &= 0 \quad \text{and} \quad h'(v) > 0 \quad \text{for} \quad v > 0, \\
    h''(v) &> 0 \quad \text{for} \quad v \geq 0.
\end{align*}
\]

(1a)  
(1b)  
(1c)

Thus, the further away a product is located from the ideal product, the more of it is required to make the consumer indifferent between it and one unit of the ideal product. Also, due to (1c), the further away a product is located from the ideal product, the larger the required marginal compensation. A typical compensation function is presented in fig. 2.
Let \( x(v) \) denote the quantity of the manufactured product located at distance \( v \) from the consumer's ideal product that is being consumed by the consumer. Then, if he also consumes \( y \) units of food, his utility level is

\[
u = u\left[\frac{x(v)}{h(v)}, y\right]. \tag{2}\]

It is assumed that a consumer consumes only one type of manufactured goods. In this case eq. (2) provides a complete specification of his preferences. I assume that \( u(\cdot) \) is increasing in each argument, strictly quasi-concave and homothetic.

In the present framework a consumer makes two decisions. The first decision concerns the variety of the manufactured good that he will consume. Among all the varieties available in the market, and taking account of the relative prices of the available varieties, the consumer chooses the product that fits him best. Then he makes the second decision, which is the allocation of his budget between food and the manufactured product that he has chosen to consume. The second decision is a standard decision in consumer theory. If he chooses to consume a variety which is located at a distance \( r \) from his ideal product, his demand functions will be (due to homotheticity of preferences):\(^2\)

\[
x(v)/h(v) = x_v[p_xh(v), p_y], \tag{3a} \]
\[
y = x_y[p_xh(v), p_y], \tag{3b} \]

where \( p_x \) is the price of the manufactured good located at distance \( v \) from the consumer's ideal type, \( p_y \) is the price of food, \( I \) is the consumer's income and \( x_v(\cdot) \) and \( x_y(\cdot) \) are functions homogeneous of degree \((-1)\), with the property

\[
p_xh(v)x_v[p_xh(v), p_y] - p_yx_y[p_xh(v), p_y] = 1. \]

The interpretation of the demand functions in (3) is as follows. Suppose the consumer decides to purchase a variety of the manufactured product which is located at distance \( v \) from his ideal type, and whose price is \( p_x \). Then, his effective price per unit of the ideal type is \( p_xh(v) \), because in his welfare calculations \( h(v) \) units of the available good are equivalent to one unit of the ideal type. Hence, the right hand side of (3a) represents the demand for units of the ideal type and (3b) represents the demand for food, with \( p_xh(v) \) standing for the price of a unit of the ideal type and \( p_y \) standing for the price of food. Since \( x_v(\cdot) \) represents demand for units of the ideal

\(^2\)Formally the consumer's problem is max \( u[x(v)/h(v), y] \) subject to \( p_xx(v) + p_yy = I \). Define a new variable \( x' = x(v)/h(v) \). Then, transforming variables, the original problem can be rewritten as max \( u(x', y) \) subject to \( p_xx' + p_yy = I \). This yields, due to the homotheticity of \( u(\cdot) \), the demand functions \( x' = x_v[p_xh()], p_y], \( y = x_y[p_xh(v), p_y]I \), which imply (3). The function \( x_v(\cdot) \) is declining in its first argument and increasing in the second.
type, we have — on the left hand side of (3a) — \( x(r) h(r) \), which represents
the quantity of the ideal type that is equivalent from the point of view of the
consumer to \( x(r) \) units of the product located at distance \( r \) from the ideal
type. It is now clear which variety the consumer will choose to purchase —
he will choose the variety which provides him with the lowest effective price
of the ideal product.

So far I have discussed the properties of an isolated consumer. In order to
analyze market behavior I also need to specify properties of the population
of consumers. This will be done in the remaining part of this section.

It is assumed that there is a continuum of consumers and that all
consumers have the same income and the same utility function. Since income
is derived from ownership of factors of production, the assumption is really
that all consumers have the same fraction of ownership of all factors of
production. However, not all consumers have the same ideal type of the
manufactured product. Preferences for the ideal type are assumed to be
uniformly distributed on the circumference of the circle in fig. 1. This means
that when \( L \) is the population size in a country, the density of consumers
whose ideal type is \( h_i \) (see fig. 1) is \( \xi = L/(2\pi L) = L/2 \) in this country, with the
same density applying to every point on the circumference of the circle. Thus,
although eq. (3) represents every consumer's demand function, the
consumers' reference points in the form of ideal products are evenly
distributed on the circumference of the circle. These assumptions assure
symmetry in aggregate demand for varieties of manufactured products. The
role of this symmetry will become clearer in later discussions.

3. Producers

I assume that food is produced by means of labor and capital with a usual
twice differentiable, increasing, linear homogeneous, strictly quasi-concave
production function \( Y = F_Y(L_Y, K_Y) \). This production function has associated
with it the cost function:

\[
C_Y(w, r, Y) = c_Y(w, r) Y. \tag{4}
\]

where \( w \) is the wage rate, \( r \) is the capital rental rate, and \( c_Y(w, r) \) is an
increasing, concave, linear homogeneous function which represents marginal
(= average) costs. This cost function yields demand functions:

\[
L_Y = a_{L_Y}(w, r) Y, \tag{5a}
\]

\[
K_Y = a_{K_Y}(w, r) Y, \tag{5b}
\]

where \( a_{L_Y}(w, r) = \tilde{c}c_Y(w, r)/\tilde{w} \) is the labor–output ratio and \( a_{K_Y}(w, r) = \tilde{c}c_Y(w, r)/\tilde{r} \) is the capital–output ratio.
As usual, the food sector will produce a positive and finite output level only if the price of food equals marginal costs of production, and it will produce no food if the price of food falls short of its marginal costs of production. Thus, if the industry is to produce at a finite level, we require:

\[ p_f = c_f(w, r). \]  

(6)

Independently of whether the food industry produces or not, its profits are always zero.

Now consider the manufacturing sector. It is assumed that the production function of a single variety is \( X = F_X(L_X, K_X) \), where \( L_X \) and \( K_X \) are labor and capital employed in the production of this variety. The production function applies to every variety represented by a point on the circumference of the circle in fig. 1. It is assumed that at positive output levels \( F_X(\cdot) \) is twice differentiable, increasing, strictly quasi-concave, and that it exhibits economies of scale at least for a range of output from zero to some upper limit \( \bar{X} \), and that the economies of scale are declining as output increases. Let \( C_X(w, r, X) \) be the cost function associated with the production function \( F_X(L_X, K_X) \). The inverse of the elasticity of the cost function with respect to output represents the usual measure of economies of scale [see Hanoch (1975)]. Thus, if \( \theta(w, r, X) \) is our measure of economies of scale, we have

\[ \theta(w, r, X) = \frac{c_X(w, r, X)}{C_X(w, r, X)X}, \]  

(7)

where \( C_{XX}(\cdot) \) represents marginal costs of production. There are local economies of scale at the output level \( X \) if \( \theta(w, r, X) > 1 \), there are local diseconomies of scale if \( \theta(w, r, X) < 1 \), and there are local constant returns to scale if \( \theta(w, r, X) = 1 \). My assumptions on the structure of economies of scale imply

\[ \theta(w, r, X) > 1 \quad \text{for} \quad 0 < X < \bar{X} \leq +\infty, \]  

(8a)

\[ \frac{\partial \theta(w, r, X)}{\partial X} X < 0. \]  

(8b)

\(^1\)I do not assume homotheticity of \( F_X(\cdot) \). Ohlin, for example, thought that in the presence of economies of scale the efficient techniques of production do depend on the level of output [see Ohlin (1933, p. 107)].

\(^4\)An example of a cost function that satisfies these assumptions is

\[ C_X(w, r, X) = \begin{cases} r + w & \text{for} \quad X > 0, \\ 0 & \text{for} \quad X = 0. \end{cases} \]
The demand functions for factors of production are derivable from the cost function:

\[ L_x(w, r, X) = \frac{\partial C_x(w, r, X)}{\partial w} \quad (9a) \]
\[ L_x(w, r, X) = \frac{\partial C_x(w, r, X)}{\partial r} \quad (9b) \]

It remains to explain how a firm chooses its output level, the variety it produces and the price it charges. Due to the economies of scale in production of manufactured goods, generally not all possible types of products will be produced. My interest lies in equilibria in which only a finite number of varieties is produced. Consequently, the following discussion is limited to such situations.\(^5\)

Suppose that the products identified by points \(b_{i-1}\) and \(b_{i+1}\) in fig. 1 are produced and that they are offered for sale at prices \(p_{xi-1}\) and \(p_{xi+1}\), respectively. Suppose also that no other variety on the segment connecting \(b_{i-1}\) and \(b_{i+1}\) is produced. Now take a firm that considers producing a variety that is located between \(b_{i-1}\) and \(b_{i+1}\) at, say, \(b_i\). If it attracts any customers at all, the first to come will be those for whom \(b_i\) is the ideal product. This means that the price it charges cannot exceed the effective price for consumers whose ideal product is \(b_i\) when they have to buy either \(b_{i-1}\) or \(b_{i+1}\). Hence, if the firm is to operate at a positive output level its price has to satisfy

\[ p_{xi} \leq \min \{p_{xi-1} h(v_{i-1}), p_{xi+1} h(v_{i+1})\} \]

where \(v_{i-1}\) is the arc distance between \(b_{i-1}\) and \(b_i\) and \(v_{i+1}\) is the distance between \(b_i\) and \(b_{i+1}\). We have thus identified a price at which the demand facing the firm is zero. Clearly, demand is also zero for every higher price.\(^6\)

Now suppose that the price chosen by firm \(i\) is sufficiently low so that it attracts customers. Then, clearly, consumers whose ideal product lies within the vicinity of \(b_i\) are part of its customers. But we can determine precisely the market width of this firm. Among all consumers whose ideal product lies between \(b_i\) and \(b_{i+1}\), consider the subset of those whose ideal product lies at arc distance smaller or equal to \(d_i\) from \(b_i\), where \(d_i\) is implicitly defined by

\[ p_{xi} h(d_i) = p_{xi+1} h(v_{i+1} - d_i). \quad (10) \]

Let \(d_i\) be at arc distance \(d_i\) from \(b_i\). Then eq. (11) says that a consumer whose ideal product is \(b_i\) is just indifferent between purchases of \(b_i\) and \(b_{i+1}\), since his effective price is the same in both cases.\(^7\) It is clear that all the

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\(^5\)If the cost function is linear in output, as in the example of footnote 4, then in every equilibrium only a finite number of varieties is produced.

\(^6\)This discussion, as well as what follows, is based on the assumption that firms cannot price-discriminate.

\(^7\)Eq. (10) applies for \(p_{xi}\) such that \(p_{xi} \leq p_{xi+1} h(v_{i+1})\) and \(p_{xi} h(v_{i+1}) \geq p_{xi+1}\).
consumers whose ideal product is between \( b_i \) and \( b_i \) will be attracted to the firm that produces product \( b_i \).

Similarly, if \( b_i \) is at arc distance \( d_i \) from \( b_i \) and \( d_i \) is implicitly defined by

\[
p_{x_i} h(d_i) = p_{x_i-1} h(v_{i-1} - d_i),
\]

then all consumers whose ideal product is between \( b_i \) and \( b_i \) will also be the firm's customers. The logic behind (11) is similar to the logic behind (10).\(^8\)

We have thus determined the market width of our firm. All consumers whose ideal product is between \( b_i \) and \( b_i \) become the firm's customers. These customers make up a share \((d_i + \tilde{d}_i)/2\) of potential buyers of manufactured products.

From (10) and (11), we can solve \( d_i \) and \( \tilde{d}_i \) as functions of \( p_{x_i-1}, p_{x_i+1}, p_{x_i}, v_{i-1} \) and \( v_{i+1} \). Of particular interest are the variables \( p_{x_i}, v_{i-1} \) and \( v_{i+1} \), which are decision variables of the firm; \( p_{x_i} \) is its price while \( (v_{i-1}, v_{i+1}) \) determine the type of product produced by it. The firm can choose \( v_{i-1} \) and \( v_{i+1} \) subject to \( v_{i-1} + v_{i+1} = 2D_i \), where \( 2D_i \) is the arc distance between \( b_i \) and \( b_{i+1} \). Using this constraint, the solution to \( d_i \) and \( \tilde{d}_i \) can be represented as follows:

\[
\tilde{d}_i = \delta(p_{x_i}, v_{i-1}; p_{x-1}, p_{x+1}, D_i), \tag{12a}
\]

\[
d_i = \delta(p_{x_i}, v_{i-1}; p_{x-1}, p_{x+1}, D_i). \tag{12b}
\]

Implicit differentiation of (10) and (11) enables us to calculate the partial derivatives of the functions \( \delta(\cdot) \) and \( \tilde{\delta}(\cdot) \); these derivatives are calculated in the appendix. It is shown there that both \( \delta(\cdot) \) and \( \tilde{\delta}(\cdot) \) are declining with \( p_{x_i} \), and that \( \tilde{\delta}(\cdot) \) is decreasing with \( v_{i-1} \) while \( \delta(\cdot) \) is increasing with \( v_{i-1} \).

Now, using (3) and (12), we can calculate the demand function facing the producer of variety \( b_i \):

\[
Q(p_{x_i}, v_{i-1}; p_{x}, p_{x-i-1}, p_{x+1}, D_i; \xi I) = \xi I \left[ \int_0^{\delta_i} \alpha x[p_{x_i} h(v), p_{x-i-1} h(v)] dv + \int_0^{\tilde{\delta}_i} \alpha x[p_{x+1} h(v), p_{x-i-1} h(v)] dv \right]. \tag{13}
\]

This demand function depends on the price and variety chosen by the producer as well as on prices of competing goods — food and the closest varieties of manufactured products. An increase in the price of variety \( b_i \) affects demand through two channels. First, it reduces the number of

\(^8\)Eq. (11) applies for \( p_{x_i} \) such that \( p_{x_i} \leq p_{x-i-1} h(v_{i-1}) \) and \( p_{x_i} h(v_{i-1}) \geq p_{x-i-1} \).
customers who purchase the product through a reduction in \( \delta(\cdot) \) and \( \delta'(\cdot) \) (see (A.1) and (A.4) in the appendix). Second, it reduces the quantity purchased by every remaining customer (since \( \pi(\cdot) \) is declining in its first argument). Hence, the demand function \( Q(\cdot) \) is downward sloping. Unlike a price change, a change in specification affects demand in two opposing directions. An increase in \( \nu_{i-1} \) changes only the number of customers who purchase the firm's product; there is no change in the quantity purchased by a customer who does not leave the firm. However, an increase in \( \nu_{i-1} \) causes a net loss of customers on one side of the market and a net gain of customers on the other side of the market (see (A.2) and (A.4) in the appendix). For example, if the utility function is Cobb–Douglas, \( u(x,y) = x^ay^{1-a}, 0 < a < 1 \), then (13) implies \( Q(\cdot) = \frac{s_x[I(\delta(\cdot) + \delta'(\cdot))/p_{xi}]}{\nu_i} \).

Now we have all the relevant information concerning the demand function faced by a typical producer of the manufactured product. The demand function specified in (13) is a true demand function which is assumed to be known to the producer. The producer of \( b_i \) is assumed to take as given the price of food and the actions taken by other producers of manufactured goods, and he is assumed to maximize profits. His profits are

\[
\Pi_i = p_{xi}Q(p_{xi}, \nu_{i-1}; \ldots) - C_x[w, r, Q(p_{xi}, \nu_{i-1}; \ldots)].
\]

Hence, the first order conditions for profit maximization are:

\[
p_{xi}\left[ 1 + \frac{1}{E(p_{xi}, \nu_{i-1}; \ldots)} \right] = C_{xx}[w, r, Q(p_{xi}, \nu_{i-1}; \ldots)].
\]

\[
[p_{xi} C_{xx}[w, r, Q(p_{xi}, \nu_{i-1}; \ldots)]]^{-1} \frac{\partial Q(\cdot)}{\partial \nu_{i-1}} = 0,
\]

where \( E(\cdot) \) is the elasticity of \( Q(\cdot) \) with respect to \( p_{xi} \). Condition (15) is the standard condition for maximum profits of a monopolist, i.e. marginal revenue equals marginal costs. Condition (16) is the condition for profit maximizing product differentiation. As long as the product's price exceeds marginal costs of production, it pays to slightly change the variety produced so as to increase demand. However, since \( E(\cdot) < 0 \), (15) implies that the price does indeed exceed marginal costs. Therefore the profit maximizing variety is such that slight changes in specification of the product do not change the

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9This effect is the main theme in Novshek and Sonnenschein (1979).
10The demand function \( Q(\cdot) \) is well defined for the set of parameters that satisfy the inequalities of footnotes 7 and 8. By a proper extension of the domain of \( \delta(\cdot) \) and \( \delta'(\cdot) \), the domain of \( Q(\cdot) \) can be extended. In any case, my discussion is limited to the domain specified in footnotes 7 and 8.
quantity demanded. Formally, due to (15), eq. (16) reads

$$\frac{\partial Q(\cdot)}{\partial v_{i-1}} = 0.$$  \hspace{1cm} (17)

This completes the specification of the behavior of firms. We shall now discuss equilibria.\(^{11}\)

4. Equilibrium in a closed economy

My discussion is limited to symmetrical equilibria. It is assumed that the varieties that are available in equilibrium are equally spaced on the circumference of the circle. This means that if, say, \(N\) varieties are consumed, the arc distance between any two adjacent varieties that are available is \(D = 2/N\). And if \(n\) varieties are produced, then in a closed economy equilibrium requires

\[ N = n. \]  \hspace{1cm} (18)

Due to the symmetrical position of each producer of a manufactured product, each type of manufactured product sells in equilibrium for the same price \(p_x\). This is seen from (10) and (11) by observing that the variety \(b_i\) is located at equal distance from its closest competitors if and only if \(p_{xi-1} = p_{xi} = p_{xi+1} = p_x\). This implies that the equilibrium values of \(\delta(\cdot)\) and \(\bar{\delta}(\cdot)\) are both \(D/2 = 1/N\), while \(v_{i-1} = D = 2/N\). Using this information to evaluate (13) at equilibrium yields

\[ Q(\cdot) = 2\xi \int_0^{1/N} \alpha_x[p_x h(v), p_y] h(v) \, dv. \]  \hspace{1cm} (19)

It is also shown in the appendix that at such an equilibrium \(\delta(\cdot)/\partial v_{i-1} = -\frac{1}{2}\) and \(\bar{\delta}(\cdot)/\partial v_{i-1} = \frac{1}{2}\), and that this implies \(\frac{\partial Q(\cdot)}{\partial v_{i-1}} = 0\). This means that the profit maximizing product differentiation condition (17) is satisfied. It stems from the fact that in our symmetrical equilibrium a firm that tries to slightly alter the specification of its product loses (gains) at the ‘upper’ end of its market the same number of customers that it gains (loses) at the ‘lower’ end of its market, while at the same time the marginal consumers that are gained consume the same quantity of the product as the marginal consumers that are lost. Hence, slight changes in specification do not alter the quantity demanded.

\(^{11}\)I have not discussed second order conditions, but assume that they are satisfied.
Now, using (19) and (A.11) in the appendix we can calculate the elasticity of demand at an equilibrium point:

\[
\bar{E}(p_x, p_y, N) \equiv \left[ \frac{1}{N} \int_0^1 \alpha_x[p_x h(v), p_y] h(v) dv \right]^{-1} \\
\times \left[ p_x \int_0^1 \alpha \left[ p_x h(v), p_y \right] [h(v)]^2 dv \\
- \frac{1}{2h'(1/N)} \alpha \left[ p_x h(1/N), p_y \right] [h(1/N)]^2 \right].
\]

Since \( \alpha_x(\cdot) \) is homogeneous of degree \((-1)\), \( \alpha_{x1} < 0 \) and \( \alpha_{x2} > 0 \), it can be shown that \( \bar{E}(\cdot) < -1 \), as required for the fulfillment of (15).

It is useful to define at this stage a new function which describes the degree of monopoly power faced by a producer of a manufactured product. The function \( R(p_x, p_y, N) \) is defined as the equilibrium ratio of price \( p_x \) to marginal revenue. This is a standard measure of monopoly power: the larger this ratio, the larger the monopoly power:

\[
R(p_x, p_y, N) \equiv \left[ 1 + \frac{1}{\bar{E}(p_x, p_y, N)} \right]^{-1}. \tag{21}
\]

The function \( R(\cdot) \) obtains values larger or equal to one and it approaches one (no monopoly power) when the elasticity of demand approaches infinity. The elasticity of demand, on the other hand, approaches infinity when the compensation function \( h(\cdot) \) flattens out, which means that all possible manufactured varieties become perfect substitutes in consumption. This is seen from (20) by evaluating it at \( h'(1/N) \to 0 \).

It is also interesting to note that the degree of monopoly power depends on the available number of differentiated products. One would expect the degree of monopoly power to decline as the number of products increases, because an increase in the number of products indicates a sense ‘more’ competition. This is indeed the case if the utility function is of the Cobb–Douglas type and the elasticity of \( h(v) \) is increasing with \( v \), but it cannot be shown in the general case, except when \( N \) is very large. Observe that due to (1b) \( \bar{E}(\cdot) \) approaches infinity and \( R(\cdot) \) approaches one as \( N \) approaches infinity, which means that all monopoly power is lost when there are infinitely many varieties. This means that for sufficiently large \( N \), the degree of monopoly power is declining with \( N \). In what follows I assume that \( R(\cdot) \) is declining with \( N \).

It is assumed that in the long run there is free entry into industries and
that labor and capital are mobile between both firms and sectors. As a result, every firm faces the same factor prices and free entry drives profits down to zero. Hence, in a long-run equilibrium the following zero-profit conditions have to be satisfied:

\[ p_x = c_Y(w, r), \quad (22) \]
\[ p_x X = C_X(w, r, X), \quad (23) \]

where \( X \) is the output level of a typical firm in the industry producing differentiated products.

A firm that produces \( X \) equates marginal revenue to marginal costs. Evaluating (15) at an equilibrium, this condition reads:

\[ p_x \left[ 1 + \frac{1}{E(p_x, p_y, \ldots)} \right] = C_X(w, r, x). \]

Now combining it with (23), (21) and (7), we obtain

\[ R(p_x, p_y, N) = \theta(v, r, X), \quad (24) \]

i.e. the degree of monopoly power equals the degree of economies of scale.

Conditions (22)-(24) provide a complete specification of the long run equilibrium conditions for firms. Given factor prices and the number of varieties available to consumers, they provide solutions for equilibrium commodity prices and the equilibrium output level of a firm in the manufacturing sector. The equilibrium output level of a firm in the food sector cannot be determined due to the existence of constant returns to scale in the production of food. It remains to specify the equilibrium conditions in the markets for goods and factors of production.

The demand functions for factors of production are described by eqs. (5) and (9). If we let \( Y \) represent total output of food, and we remember that there are \( n \) firms in the manufacturing sector, each one producing \( X \) units of the variety in which it specializes, we can write down the equilibrium conditions in factor markets as:

\[ a_{LY}(w, r) Y + L_X(w, r, X)n = L, \quad (25) \]
\[ a_{KY}(w, r) Y + K_X(w, r, X)n = K, \quad (26) \]

where \( L \) is the country's total labor force (which equals its population \( 25 \)) and \( K \) is the country's capital stock. The left hand side of (25) represents aggregate demand for labor. It is composed of labor demanded by the food
sector and of labor demanded by the \( n \) firms in the manufacturing sector. The right hand side of (25) represents labor supply. A similar interpretation applies to eq. (26), which is the equilibrium condition in the capital market.

It remains to specify equilibrium conditions in commodity markets. Using (19), the equilibrium condition in the market for manufactured products can be written as

\[
2\zeta I \int_0^{1/N} \alpha_x[p_x h(v), p_y] h(v) dv = X, \tag{27}
\]

i.e. the demand for a particular variety is equal to its supply. Now, the set of consumers who consume a particular variety of manufactured products also consumes \( Y/N \) units of food. Therefore, using (3b), the equilibrium condition in the market for food can be written as

\[
2\zeta I \int_0^{1/N} \alpha_y[p_x h(v), p_y] dv = Y/N. \tag{28}
\]

Finally, dividing (27) by (28), we obtain

\[
\frac{1}{1/N} \int_0^{1/N} \alpha_x[p_x h(v), p_y] h(v) dv = \frac{X}{Y/N}. \tag{29}
\]

The system of equilibrium conditions for the closed economy is represented by eqs. (18), (22)–(26) and (29). This system is homogeneous of degree zero in \((p_x, p_y, w, r)\). It provides, therefore, a solution to \( X, Y, N, n \) and three relative prices, say \( p_x/p_y, w/p_y, \) and \( r/p_y \).\(^{12}\) I assume that in an equilibrium \( n \) is relatively large. This is needed for two reasons. First, there is the so-called integer problem. Strictly speaking \( n \) should be an integer, but there is nothing in the equilibrium conditions which ensures that \( n \) is an integer. However, if \( n \) is large enough, our equilibrium will be a good approximation to the true equilibrium in which \( n \) is an integer. Second, if \( n \) is small, say 2, the concept of equilibrium that we have employed may not be appropriate. For these reasons I require \( n \) to be large. This outcome can be assured by appropriate assumptions on the degree of economies of scale in manufacturing. In particular, if the economies of scale are small enough, \( n \) will be large in equilibrium.

This completes the discussion of equilibrium in a closed economy. It is

\(^{12}\) will always assume the existence of an equilibrium.
required as background to the main concern of this paper, i.e. international trade, which is taken up in the next section.

5. International trade in a two-country world

Consider a world which consists of two countries — a home country and a foreign country — both being of the type discussed in the previous sections. Assume that the technologies are identical across countries, which implies that cost functions are the same in every country. All consumers are assumed to have the same utility function $u(x, y)$ and the same compensation function $h(v)$. In every country the consumers are uniformly distributed on the circumference of the circle in fig. 1 according to their ideal product. Countries may, however, differ in population size and stocks of capital. Hence, as in the familiar Heckscher-Ohlin model, countries differ only in factor endowments (population size equals labor endowment).

The foreign country’s variables are denoted with asterisks while the home country’s variables are denoted without asterisks. For example, $L^*$ and $K^*$ are used to denote the foreign country’s labor force (population) and capital stock, while $\xi^* = L^*/2$ is used to denote the foreign country’s population density. The letter $N$, on the other hand, is still used to denote the number of varieties of manufactured products that are available to consumers. In the presence of frictionless international trade, as is assumed here, all varieties that are produced in the world economy are available to consumers in every country; the location of production does not affect the availability of products to consumers.

Now consider an equilibrium in the world economy that has the following two properties:

(a) No sectoral specialization, i.e. every country produces food and manufactured products. Clearly, no single type of manufactured product will be produced in both countries for the same reason that no two firms in a country will produce the same variety; there is always specialization in the production of varieties. What is required here is that no country produces only food or only manufactured products;

(b) Symmetry in the market for differentiated products, in the sense that every two adjacent varieties are equally spaced on the circumference of the circle. This implies that all varieties are sold at the same price.

If these two conditions are met, a subset of the equilibrium conditions for the world economy consists of country specific variants of (22)–(26), which can be written as:

$$p_y = c_y(w, r).$$

(30)
\[ p_x X = C_X(\omega, r, X), \]  
\[ R(p_x, p, N) = \theta(\omega, r, X), \]  
\[ a_{LY}(\omega, r) Y + L_Y(\omega, r, X)n = L, \]  
\[ a_{KY}(\omega, r) Y + K_Y(\omega, r, X)n = K, \]  
\[ p_y = c_Y(\omega^*, r^*), \]  
\[ p_x X^* = C_X(\omega^*, r^*, X^*), \]  
\[ R(p_x, p, N) = \theta(\omega^*, r^*, X^*), \]  
\[ a_{LY}(\omega^*, r^*) Y^* + L_Y(\omega^*, r^*, X^*)n^* = L^*, \]  
\[ a_{KY}(\omega^*, r^*) Y^* + K_Y(\omega^*, r^*, X^*)n^* = K^*. \]

Eqs. (30)--(34) constitute the home country’s block of equilibrium conditions, while eqs. (35)--(39) constitute the foreign country’s block of equilibrium conditions. A typical country-specific block describes, basically, the country’s supply sector. Given the world price of food \( p_y \), and the prices of competing manufactured products \( p_x \), a country’s producers equate marginal revenue to marginal costs and, due to entry competition, firms find themselves charging prices which equal average costs (in the food industry marginal revenue equals the price of food and marginal costs equal average costs). These conditions are described by the first three equations of a country specific block. Given commodity prices, these three equations determine the country’s factor prices and the output level of a typical firm in the manufacturing sector. If the mapping from \((\omega, r, X)\) to \((p_x, p, N)\), which is described by (30)--(32) and (35)--(37), is univalent, then both countries will have the same factor prices and the same level of output of a firm in the manufacturing sector. For example, if manufactured goods are produced with a homothetic production function, then univalence of the mapping is assured by the usual assumption of no factor intensity reversal.\(^{13}\) I have thus proved the following:

\(^{13}\)If \( F_\ell(\cdot) \) is homothetic, then \( C_X(\cdot) \) obtains the separable form \( C_X(\omega, r, X) = c_X(\omega, r)e(X) \), with \( e'(X) > 0 \). In this case \( \theta(\cdot) \) is only a function of output and it does not depend on factor prices. Then (32) and (37) imply \( X = X^* \). Eq. (31) can now be written as \( p_xX/e(X) = c_X(\omega, r) \), while (37) can be written as \( p_xX/e(X) = c_X(\omega^*, r^*) \). Since \( [\tilde{c}_X(\cdot)/\tilde{r}] / [\tilde{c}_X(\cdot)/\tilde{w}] \) represents the capital-labor ratio employed by the firm, the absence of factor intensity reversals assures univalence of the mapping from \((\omega, r)\) to \([p_xX/e(X), p_y]\) given by (30) and \( p_xX/e(X) = c_X(\omega, r) \).
Proposition 1. If both countries have the same technologies as specified above, the univalence of mapping conditions is satisfied, in the trading equilibrium every country produces both food and manufactured products (i.e. no sectoral specialization) and all varieties of manufactured products are equally spaced (i.e. symmetrical equilibrium), then in the trading equilibrium factor prices are the same in both countries and the output level of a firm in the manufacturing sector is the same in both countries.

I have thus provided a generalization of the factor price equalization theorem. Observe that in the context of differentiated products the factor price equalization theorem is widened to include equalization of output levels of manufacturing firms and it does not require homotheticity in the production of differentiated products.

For the sake of concreteness, let us now assume that in equilibrium manufactured goods are relatively capital intensive. Then, comparing (33)–(34) with (38)–(39) and taking account of proposition 1, we have immediately:

**Proposition 2.** Suppose that the conditions of proposition 1 are satisfied and that manufactured products are relatively capital intensive. Then the country with the higher capital–labor ratio produces less food per capita and more varieties per capita than the country with the lower capital–labor ratio. If both countries have the same capital–labor ratio, they produce the same quantity of food per capita and the same number of manufactured products per capita.

Since in the present context manufacturing firms produce the same output level, independent of country association, the country that produces more varieties per capita produces also more manufactured goods per capita. It is, therefore, clear that proposition 2 is a version of the Rybczynski theorem. The particular feature introduced by the existence of differentiated products is that changes in factor endowments change the number of varieties produced, and therefore the number of firms, rather than output per firm.

Since in the equilibrium I have been discussing every manufacturing firm produces the same output level, every manufacturing firm receives also the same share of customers. Hence, the remaining equilibrium conditions — which are worldwide conditions — can be expressed as (compare to (18) and (29)):

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14 If homotheticity in the production of manufactured products is not assumed, relative factor intensities may depend on the output level of manufacturing firms. This is why I consider only relative factor intensities in equilibrium (this is a cross section comparison). Alternatively, one may employ a strong notion of no factor intensity reversal, requiring, say, the manufacturing sector to employ more capital per unit labor for all output levels.

15 This is similar to the result derived by Mayer (1976) in a model with price uncertainty.
\begin{align*}
N &= n + n^*, \quad (40) \\
\frac{1}{N} \int_0^{1/N} x \left[ p_x h(v), p_y \right] h(v) \, dv = \frac{X}{(Y + Y^*)/N} = \frac{X^*}{(Y + Y^*)/N}, \quad (41)
\end{align*}

Condition (40) just says that the number of varieties available to consumers equals the number of varieties produced in the home country plus the number of varieties produced in the foreign country. The left hand side of (41) represents the ratio of manufactured goods to food consumed by that part of the world population that is being served by a single manufacturing firm (remember that all consumers have the same functions \( u(\cdot) \) and \( h(\cdot) \)). This has to equal the firm's output divided by the output of food that is being allocated to this population. But since this population is a typical segment of \( N \) identical segments, it gets \( 1/N \) of the world's output of food. In such a typical segment the ratio of home country residents to foreign country residents is \( \zeta/\zeta^* = L/L^* \).

Eqs. (30)-(41) provide a complete representation of a symmetrical equilibrium. This system is homogeneous of degree zero in \((p_x, p_y, w, r, w^*, r^*)\). It provides a solution (whenever such a solution exists) to \( X, X^*, Y, Y^*, n, n^*, N \) and the relative prices \( p_x/p_y, w/p_y, r/p_y, w^*/p_y, r^*/p_y \).

Now what can we say about the pattern of trade? Clearly, as long as no country specializes in the production of food, there will always be intra-industry trade in manufactured products. This is so since the home country will import the \( n^* \) varieties produced in the foreign country while the foreign country will import the \( n \) varieties produced in the home country. Hence, the existence of intra-industry trade depends on differences in factor proportions only to the extent that differences in factor proportions lead to an equilibrium in which one country specializes in the production of food. However, the share of intra-industry trade in total trade does depend on differences in factor proportions.

It is clear that in a symmetrical equilibrium the ratio of manufactured goods to food in consumption is the same in every country. This is, of course, part of condition (41). But this means that the country that produces more food per capita in a trading equilibrium has to export food while the country that produces more varieties per capita has to be a net exporter of manufactured products. Combining this observation with proposition 2, we obtain:

**Proposition 3.** Suppose that the conditions of proposition 1 are satisfied and that independent of the scale of operation manufactured products are relatively capital intensive. Then, although both countries are exporters and importers of
manufactured products, the country with the higher capital–labor ratio is a net exporter of manufactured goods and an importer of food, while the country with the lower capital–labor ratio is an exporter of food and a net importer of manufactured products. If both countries have the same capital–labor ratio, all trade is intra-industry trade and there is no intersectoral trade, i.e. no food is exported or imported and net exports (imports) of manufactured products are zero in every country.

This proposition is, of course, a generalization of the Heckscher–Ohlin theorem. In the present context we use Heckscher–Ohlin to explain intersectoral trade while intra-industry trade is explained by the existence of economies of scale and differentiated products. We have seen that in the absence of a divergence in the capital–labor ratios with which countries are endowed, there will be only intra-industry trade while in the presence of such a divergence there will be both intra-industry and inter-industry trade. Now I want to show that the larger the divergence in the capital–labor ratios with which countries are endowed, the smaller the share of intra-industry trade in world trade.

In order to discuss the relationship between intra-industry trade and factor proportion divergences, we need an index of the share of intra-industry trade in world trade. I will use the standard index [see Grubel and Lloyd (1975, p. 22)]. Assuming that the home country’s capital labor ratio is smaller or equal to the foreign country’s capital–labor ratio, i.e. the home country exports food, this index translates in the present case into

\[
Intra = 1 - \frac{p_y(Y - A_x) + p_x(n^*A_x - nA_x^*)}{p_y(Y - A_x) + p_x(nA_x^* + n^*A_x)},
\]

where \(A_x\) = aggregate consumption of food in the home country, \(A_x\) = aggregate consumption of a variety of manufactured goods in the home country, and \(A_x^*\) = aggregate consumption of a variety of manufactured goods in the foreign country. Remember that, due to the symmetry in our system, a country consumes the same quantity of each variety.

Due to the balance of trade (income) constraint, i.e. exports equal imports, which translates in our case into

\[p_y(Y - A_x) + p_xnA_x^* = p_xn^*A_x,\]

we can write (42) as

\[
Intra = \frac{n/A_x}{n^*/A_x^*}.
\]
Observe that $0 \leq Intra \leq 1$, $n = 0$ implies $Intra = 0$, which is the case when there is only intersectoral trade, and $n^*A_x = nA_x^*$ implies $Intra = 1$, which is the case when all trade is intra-industry trade. Hence, $Intra$ is a true share measure.

Now I want to argue that for a particular experiment $Intra$ declines as the divergence between the foreign and the home country’s capital labor ratio increases. In this experiment we reallocate the world’s labor and capital stock so that the capital–labor ratio employed in the foreign country increases while the capital–labor ratio employed in the home country declines, but only within the limits in which commodity prices and factor rewards do not change. These limits are imposed in order to preserve the economic size of the world. This leads to an increase in $(n^*/A^*_x)/(n/A_x)$, which can be seen as follows. Each country spends the same proportion of its income on manufactured products. Hence, $(n^*/A^*_x)/(n/A_x) = (n^*/A^*_I)/(n/A_I)$, where $A_I$ and $A^*_I$ represent aggregate income in the home and foreign country, respectively. However,

$$\frac{n^*}{A^*_I} = \frac{n^*}{p_yY^* + p_xX^*n^*} = \frac{1}{p_yY^*/n^* + p_xX^*},$$

where $X^*$ is output per firm in the manufacturing sector, and it is not affected by the reallocation of labor and capital because prices remain constant and therefore $n + n^* = N$ remains constant. Now, since the capital–labor ratio increases in the foreign country, $Y^*/n^*$ declines (the Rybczynski effect), which implies an increase in $n^*/A^*_I$. A similar argument shows that $n/A_I$ decreases. Hence, the ratio $(n^*/A^*_I)/(n/A_I)$ increases and $Intra$ declines. We have thus proved:16

**Proposition 4.** Assume that the world economy is in an equilibrium in which factor prices are equalized and the home country has a lower (or equal) capital–labor ratio than the foreign country. Then if we reallocate the world’s labor and capital stock in a way which increases the foreign country’s capital–labor ratio and reduces the home country’s capital–labor ratio without disturbing commodity prices and factor rewards, then the share of intra-industry trade as measured by $Intra$ will decline.

This proposition suggests that there exists a relationship between the composition of trade (in terms of intra- versus inter-industry trade) and the dispersion of relative factor endowments. It suggests that endowment ratio similarity breeds intra-industry trade while dissimilarity breeds intersectoral trade.

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16Krugman (1981) provides a specific example of this proposition.
trade. The insights of this proposition are used in section 7 to formulate two testable hypotheses.

We have thus seen that the share of intra-industry trade is a declining function of the absolute difference in the capital-labor ratios. Does there exist also a monotonic relationship between the difference in capital-labor ratios and the volume of trade? The answer is 'yes' if the larger gap in relative factor endowments is obtained by a re-distribution which does not change the relative size of countries, while the answer is 'no' if the relative size changes.

In order to see these points, consider two trading countries with equalized factor prices, and with the home country being the labor rich country (i.e. the home country exports food as food is the labor intensive good). The volume of trade is $p_x(Y-A_x) + p_x(n A_x^* + n^* A_x)$. Let $\mu$ be the share of the home country in world income. Then $A_y = \mu(Y + Y^*)$, $A_x = \mu X$ and $A_x^* = (1 - \mu)X$. Using these relationships as well as (33), (34), (38) and (39) to solve for $Y$, $Y^*$, $n$, $n^*$, the volume of trade can be written as

$$V = A^{-1} \left[ p_x (1 - \mu) \mu (L K Y - K L Y) - p_x \mu (A_y K - a_{KY} L) X + p_x \mu (a_{LY} K - a_{KY} L) X \right],$$

where $a_{LY}$, $a_{KY}$, $L$, $K$ are the same in both countries and $A = a_{LY} K - a_{KY} L > 0$.

Now suppose that we transfer labor from the foreign country to the home country and capital from the home country to the foreign so as to preserve each country's size in terms of its national income (a marginal reallocation does not change factor and commodity prices). Hence, $dL = -dL^*$, $dK = -dK^*$, $w dL + r dK = 0$ and $d\mu = 0$. Using these conditions, a direct calculation shows $dV = 2A^{-1} p_x \mu X dL/r > 0$, which proves:

**Proposition 5.** Assume that the world economy is in an equilibrium in which factor prices are equalized. Then if we reallocate the world's labor and capital stock in a way which increases the capital–labor ratio in the capital rich country and reduces the capital–labor ratio in the capital poor country, but preserves the relative size of each country and does not disturb commodity prices and factor rewards, the volume of trade will increase.

In order to see the relative size effect on the volume of trade, consider the case in which both countries have the same capital–labor ratio, i.e. $L = \mu \bar{L}$, $K = \mu \bar{K}$, $L^* = (1 - \mu) \bar{L}$ and $K^* = (1 - \mu) \bar{K}$, where $\bar{L}$ and $\bar{K}$ are the world's labor force and capital stock, respectively. In this case all trade is intra-industry trade and

$$V = A^{-1} 2 (a_{LY} \bar{K} - a_{KY} \bar{L}) p_x \mu (1 - \mu).$$

Since food is labor intensive,
(a_k\gamma \hat{K} - a_{kY} \hat{L}) > 0$, and we see that $V$ is increasing in $\mu$ when $\mu < \frac{1}{2}$ and it is decreasing in $\mu$ when $\mu > \frac{1}{2}$. Hence, we have proved:

**Proposition 6.** Assume that both countries have the same capital–labor ratio. Then a redistribution of resources which preserves each country’s initial capital–labor ratio increases the volume of trade if it reduces the inequality in country size, and it reduces the volume of trade if it increases the inequality in country size. The volume of trade is largest when both countries are of equal size.

Let me conclude this section with a comparison of the present model with the familiar two-sector Heckscher–Ohlin model. I wish to argue that eqs. (30)-(39) provide a natural generalization of the production structure of the Heckscher–Ohlin model. In order to see this point, consider what happens when the manufacturing sector produces with constant returns to scale. In this case $C_x(w, r, X) = c_x(w, r)X$ and it is immediately obvious that eqs. (30)-(31) and (35)-(36) reduce to the Heckscher–Ohlin pricing equations. Since with constant returns to scale $L_x(w, r, X) = a_{lx}(w, r)X$ and $K_x(w, r, X) = a_{kx}(w, r)X$, eqs. (33)-(34) and (38)-(39) represent in this case the Heckscher–Ohlin equilibrium conditions in factor markets. However, with constant returns to scale in the production of $X$, the factor market equilibrium conditions depend on aggregate output of manufactured products, $nX$, and they do not depend on the number of firms in the industry. Thus, industry output is well determined, but not its composition in terms of the number of firms and output per firm.

It remains to consider the implication of constant returns to scale for (32) and (37). From (7), constant returns to scale imply $\theta(\cdot) \equiv 1$. Hence, in the present case (32) reads $R(p_x, p_y, N) = 1$. This can be satisfied in either one of two cases. First, when $h(v)$ is flat, which means that different types of manufactured products are perfect substitutes in consumption. This interpretation I believe to be closest in spirit to the common view of the Heckscher–Ohlin model. In this case any number of varieties can be produced; it is simply not relevant. Second, when $h(v)$ satisfies (1). In this case $R(p_x, p_y, N) = 1$ implies that $N$ goes to infinity. This means that every consumer can purchase his own ideal product — due to the constant returns to scale in production, manufactured products are custom made! This interpretation is also consistent with the Heckscher–Ohlin model. Since every consumer is of measure zero, aggregate output of manufactured products is still well determined, even though there is a continuum varieties produced. This interpretation seems to provide wider applicability to the standard model than had been recognized in the past.
6. Predicting the intersectoral pattern of trade

In the traditional Heckscher–Ohlin model there are three predictors of the pattern of trade: (1) relative commodity prices, (2) relative factor rewards, and (3) relative factor endowments. It is well known that under the standard assumptions, which include the assumptions of no factor intensity reversal and identical homothetic preferences, all three predictors provide the same valid prediction of the pattern of trade. Thus, if the home country is relatively labor abundant, then in the pre-trade equilibrium its relative price of the labor-intensive good and its wage-rental ratio are lower than in the capital rich country, and all three predictors suggest that in the presence of international trade the home country will be an exporter of the labor intensive good. In this section I investigate the extent to which relative commodity prices and relative factor rewards can provide valid predictions of the pattern of trade in the Chamberlin–Heckscher–Ohlin model.

The fact that relative factor endowments provide a valid prediction of the intersectoral pattern of trade was established in the previous section (see proposition 3). It was shown that the country with relatively more capital will be a net exporter of the capital intensive good while the country with relatively more labor will be a net exporter of the labor intensive good. This prediction is based on the assumption that one sector is more capital intensive than the other for all factor prices and all output levels of a single firm. The new element that appears here is the requirement that factor intensity reversal should not take place as a result of changes in the output level of a firm in the differentiated product industry. This requirement is relevant only when the production function of manufactured products is not homothetic, and I have allowed for non-homothetic production functions. If manufactured products are produced with a homothetic production function, then employed factor proportions are scale-independent and the absence of factor intensity reversals obtains the usual meaning.

What about relative commodity prices and relative factor rewards? Relative commodity prices cannot provide a valid prediction of the pattern of trade because, due to the economies of scale, a country's size affects its pre-trade relative commodity prices. Thus, the larger a country is, the better advantage it can take of the economies of scale, an advantage which is expected to translate into relatively lower prices of manufactured products. Hence, if we observe two countries in autarky which are identical except for size, we may expect the larger country to have a relatively lower price of manufactured goods. However, if trade opens, there will be no intersectoral trade between these countries — all trade will be intra-industry trade. It is therefore clear that in this case relative commodity prices cannot be used in the usual way to predict the pattern of trade.\(^\text{17}\) This raises the following

\(^{17}\text{Lancaster (1980) calls this situation 'false comparative advantage'.}\)
question: Is there a way to adjust prices for the scale effect so as to obtain \textit{scale-adjusted} prices which provide a valid prediction of the pattern of trade? There is an interesting case in which this can be done: when (1) the utility function is Cobb–Douglas, and (2) manufactured goods are produced with a homothetic production function. In this case relative factor rewards also provide a valid prediction of the intersectoral pattern of trade. In what follows I discuss this case, including a description of the scale-adjusted prices in a form which is empirically measurable. Then I discuss the difficulties that exist in other cases.

Let the production function $F_X(L_X,K_X)$ be homothetic. Then its cost function can be written in the form

$$C_X(w, r, X) = c_X(w, r)e(X).$$  \hfill (44)

In this case (7) implies

$$\vartheta(X) = \frac{e(X)}{e'(X)X},$$  \hfill (45)

i.e. the elasticity of scale is independent of factor prices. In addition, (9) implies:

$$L_X(w, r, X) = a_{LZ}(w, r)e(X), \hfill (45a)$$

$$K_X(w, r, X) = a_{KZ}(w, r)e(X), \hfill (46b)$$

where $a_{LZ}(\cdot) \equiv \frac{\partial c_X(\cdot)}{\partial w}$ and $a_{KZ}(\cdot) \equiv \frac{\partial c_X(\cdot)}{\partial r}$. Observe that $a_{LZ}(\cdot)$ is not the labor–output ratio and $a_{KZ}(\cdot)$ is not the capital–output ratio. The labor–output ratio is $a_{LZ}(\cdot)e(X)/X$ while the capital–output ratio is $a_{KZ}(\cdot)e(X)/X$, and they depend not only on factor prices but also on the level of output. However, the capital–labor ratio does not depend on the output level.

Let the utility function be

$$u(x, y) = x^s y^{1-s}, \quad 0 < s < 1.$$  \hfill (47)

Then:

$$\alpha_x[p_x h(v), p_y] \equiv s/[p_x h(v)], \hfill (48a)$$

$$\alpha_y[p_x h(v), p_y] \equiv (1 - s)/p_y, \hfill (48b)$$
\[ R(N) = 1 + 2e_h(1/N), \quad (49) \]

where \( e_h(\cdot) \) is the elasticity of the function \( h(\cdot) \).

Consider now the equilibrium of an isolated country in which \( F_x(L_x, K_x) \) is homothetic and in which the utility function is Cobb–Douglas. Using (44)–(49), the equilibrium conditions (18), (22)–(26), (29) can be written as:

\[ N = n, \quad (50) \]
\[ p_y = c_y(w, r), \quad (51) \]
\[ p_x X/e(X) = c_Z(w, r), \quad (52) \]
\[ R(n) = \theta(X), \quad (53) \]
\[ a_{LY}(w, r)Y + a_{LZ}(w, r)e(X)n = L, \quad (54) \]
\[ a_{KY}(w, r)Y + a_{KZ}(w, r)e(X)n = K, \quad (55) \]
\[ \frac{nX}{Y} = \frac{s}{1 - s} \frac{p_y}{p_x}. \quad (56) \]

Now define two auxiliary variables:

\[ p_z = p_x X/e(X), \quad (57) \]
\[ Z = e(X)n. \quad (58) \]

Using (57)–(58) and food as numeraire (i.e. \( p_y = 1 \)), the equilibrium conditions (50)–(56) can be written in the form:

\[ N = n, \quad (50') \]
\[ l = c_y(w, r), \quad (51') \]
\[ p_z = c_Z(w, r), \quad (52') \]
\[ R(n) = \theta(X), \quad (53') \]
\[ a_{LY}(w, r)Y + a_{LZ}(w, r)Z = L, \quad (54') \]
\[ a_{KY}(w, r)Y + a_{KZ}(w, r)Z = K, \]  
\[ \frac{Z}{Y} = \frac{s}{1-s} \frac{1}{p_z}. \]

Now observe that (51')-(52') and (54')-(55') represent a standard Heckscher-Ohlin production structure, with \( p_z \) being the relative price of \( Z \). There exist, therefore, functions \( \eta_w(p_z), \eta_r(p_z), \phi_Y(p_z, L, K), \phi_Z(p_z, L, K) \), such that in the absence of sectoral specialization:

\[ w = \eta_w(p_z), \]  
\[ r = \eta_r(p_z), \]  
\[ Y = \phi_Y(p_z, L, K), \]  
\[ Z = \phi_Z(p_z, L, K). \]

The functions \( \phi_Y(\cdot) \) and \( \phi_Z(\cdot) \) are homogeneous of degree one in \((L, K)\). \( \phi_Y(\cdot) \) is decreasing in \( p_z \), and \( \phi_Z(\cdot) \) is increasing in \( p_z \). Now define

\[ \phi(p_z, K/L) = \frac{\phi_Z(p_z, 1, K/L)}{\phi_Y(p_z, 1, K/L)}. \]

The function \( \phi(\cdot) \) is increasing in \( p_z \), and due to (60),

\[ \frac{Z}{Y} = \phi(p_z, K/L). \]

Assuming that manufactured products are capital intensive implies that \( \phi(\cdot) \) is increasing in its second argument (Rybczynski) and that \( \eta_w(\cdot) \) is decreasing and \( \eta_r(\cdot) \) is increasing in \( p_z \) (Stolper–Samuelson).

For a given capital–labor ratio, \( K/L \), I plot (56') and (62) in fig. 3 [this is similar to fig. 7.2 in Caves and Jones (1977)]. The intersection between these two curves determines the equilibrium values of \( p_z \) and \( \rho = Z/Y, \tilde{p}_z \) and \( \tilde{\rho} \). Then, substituting \( \tilde{p}_z \) into (59) and (69), we obtain equilibrium factor prices and output levels. Having done this, we use (58) and (53') to calculate equilibrium values of \( X \) and \( n \), and then, using (57), we calculate the equilibrium value of \( p_x \). I will come back to these calculations later.

Now suppose that there are two identical countries, a home country and a foreign country, except that \( L^* = \lambda L, K^* = \lambda K, \lambda \geq 1 \). Hence, both countries have the same factor proportions but the foreign country is possibly larger. I want to compare these countries' pre-trade equilibria. Since they have the same factor proportions, the equilibrium values of \( p_z \) and \( \rho \) are the same in
both countries (see fig. 3). But this implies, via (59)–(60) and the homogeneity of degree one of the functions $\phi_x(\cdot)$ and $\phi_z(\cdot)$, that:

$$w^* = \tilde{w},$$  \hspace{1cm} \text{(63a)}

$$\tilde{r}^* = \tilde{r},$$  \hspace{1cm} \text{(63b)}

$$\tilde{Y}^* = \lambda \tilde{Y},$$  \hspace{1cm} \text{(64a)}

$$\tilde{Z}^* = \lambda \tilde{Z},$$  \hspace{1cm} \text{(64b)}

where ‘tildes’ indicate equilibrium values. I have thus shown that factor rewards in terms of food are the same in both countries and that the foreign country produces $\lambda$ times the amount of food and $Z$ produced in the home country.

Now let us compare $(\tilde{n}, \tilde{X})$ with $(n^*, X^*)$. Eq. (53') implies that $(\tilde{n}, \tilde{X})$ and $(n^*, X^*)$ both lie on the upward sloping curve in fig. 4 that satisfies $R(n) = \theta(X)$, while (58) implies that $(\tilde{n}, \tilde{X})$ lies on the downward sloping curve $e(X)n = \tilde{Z}$ and that $(n^*, X^*)$ lies on the downward sloping curve $e(X)n = \tilde{Z}^*$. The curve $e(X)n = \tilde{Z}^*$ is vertically $\lambda$ times higher than the curve $e(X)n = \tilde{Z}$, and it is drawn on the assumption $\lambda > 1$. Hence $Q$ is the home country’s equilibrium point and $Q^*$ is the foreign country’s equilibrium point.

Finally, observe that due to the fact that $\tilde{p}_x = \tilde{p}_z$, $\tilde{X} > \tilde{X}$, and the elasticity of $e(\cdot)$ is smaller than one, (57) implies

$$\tilde{p}_x < \tilde{p}_x,$$  \hspace{1cm} \text{(65)}
which means that the relative price of manufactured products is lower in the foreign country. We have thus seen that:

1. factor prices in terms of food are the same in both countries,
2. the larger country produces more varieties, with a higher output per firm, and
3. the relative price of manufactured products is lower in the larger country.

In the pre-trade equilibrium both countries have the same relative factor rewards and use therefore the same techniques of production. Since they also have the same factor proportions, the larger country employs in each sector more labor and capital in direct proportion to its relative size. This results in a proportionately higher output of food, and due to the economies of scale, a more than proportionately higher output of manufactured products, with the higher output of manufactured products being composed of more varieties and a higher output per firm. The relatively larger supply of manufactured products makes their relative price lower in the larger country.\(^{18}\)

If we were to use pre-trade relative commodity prices to predict the pattern of trade, we would predict that the large country will be a net exporter of manufactured products and an importer of food. However, it is

\(^{18}\)It is interesting to note that average welfare is higher in the larger country for two reasons: (1) income per capita in terms of food is the same, but income per capita in terms of manufactured products is higher, so that every consumer can afford to buy more manufactured products; and (2) there are more varieties produced, which means that more consumers find manufactured products which are closer to their ideal products. If the location of the equally spaced varieties that are produced is drawn from a uniform distribution, the expected utility of a consumer in the larger country is higher than the expected utility of a consumer in the smaller country.
clear from proposition 2 (as well as directly from the present context) that as a result of equal factor proportions all international trade will be intra-industry trade within the manufacturing sector, and that every country will produce its own consumption of food.\(^{19}\) It is therefore clear that in the world I have been discussing, pre-trade relative commodity prices do not serve as reliable predictors of the pattern of trade. Moreover, the very same economic factors that lead prior to trade to a relatively lower price of manufactured products in the larger country (when both countries have the same factor proportion), make the post-trade relative price of manufactured products lower than the relative price of manufactured products in either one of the countries prior to trade. This stems from the fact that the introduction of international trade combines both countries into an integrated economy. Hence, pre-trade relative commodity prices do not provide the usual bounds on the post-trade relative commodity prices, for we have just seen that the post-trade relative price of manufactured goods will be outside (and to the left of) the bounds determined by pre-trade relative prices of these goods. If countries have different factor proportions, the location of relative commodity prices in a trading equilibrium relative to the location of pre-trade relative commodity prices depends on the above-mentioned scale effect and the usual Heckscher-Ohlin effect. Clearly, if factor proportions do not differ by much, the scale effect dominates.

How useful then are pre-trade commodity prices in predicting the pattern of trade? Observe that the pre-trade values of \(p_z\) do predict the intersectoral pattern of trade. Suppose the foreign country has a higher capital-labor ratio. Then its equilibrium point in fig. 3 is on a downward sloping curve to the left of \(A\), say at \(A'\). This is so because due to the fact that manufactured products are relatively capital intensive, the foreign country's upward sloping curve will be above the upward sloping curve of the home country (the Rybczynski effect). In this case the foreign country will have a lower \(p_z\), which via (59) means a higher wage rate and a lower rental rate in terms of food. Hence, the country with the lower \(p_z\) will be a net exporter of manufactured products. But \(p_z\) can be considered to be the scale-adjusted relative price of manufactured products, which is the index we have been looking for.

The implication of all this is quite simple. Since \(p_z = p_x X/e(X)\), then in order to predict the pattern of trade we need to know both pre-trade relative commodity prices and the scale of operation of manufacturing firms in each country. In addition, we need to have an estimate of \(e(X)\), which is a component of the cost function. This information can be used to calculate the scale-adjusted relative price of manufactured products in each country in order to predict the intersectoral pattern of trade.

\(^{19}\)For this result there is no need to assume the absence of factor intensity reversals.
Now let us consider the predictive power of factor rewards. We have seen that countries with identical factor proportions have the same factor rewards in terms of food. Hence, they have the same wage–rental ratio. In such cases the Heckscher–Ohlin prediction is that the opening of international trade will lead to no active trade. In the present context the opening of international trade leads to no active intersectoral trade, but it does lead to intra-industry trade. Since the Heckscher–Ohlin theory is concerned with intersectoral trade, it is fair to argue that its factor rewards oriented prediction of the pattern of trade remains valid if we can also show that in the presence of differences in factor proportions the capital rich country has a higher wage–rental ratio in the pre-trade equilibrium. In the case considered so far this relationship holds, as I have shown above. Hence, in the present case relative scale-adjusted commodity prices and relative factor rewards provide a valid prediction of the intersectoral pattern of trade.

What happens when preferences cannot be represented by a Cobb–Douglas utility function? In this case — even with homothetic production functions — factor rewards, as well as the simple index of scale-adjusted relative prices developed above, cannot be used to predict the pattern of trade. The reason for this difficulty stems from the fact that whenever the elasticity of substitution in consumption does not equal one (as it does in the Cobb–Douglas case), aggregate relative demand depends not only on relative commodity prices but also on the number of varieties available to consumers. For example, if the utility function is CES and the elasticity of substitution is larger than one, it can be shown that the relative demand for manufactured goods increases as the number of varieties increases, or alternatively that the share of income spent on food declines as the number of varieties increases. In addition, a non-unitary elasticity of substitution implies that $R(\cdot)$ depends on relative commodity prices in a rather complex way. As a result, the simple links that I have presented above break down. In this case size differences can lead to differences in relative factor rewards and scale-adjusted relative commodity prices as a result of differences in relative demands. The result is that the pattern of trade cannot be predicted from price information but only from information about relative factor endowments.

In order to see this point in a clear way, consider two countries, one of which has $\lambda$ times more labor and capital than the other, with $\lambda > 1$. Suppose that in the pre-trade equilibrium both countries have the same wage rate and rental rate in terms of food, which due to (52') also means that they have the same scale adjusted price $p_z$. In this case the large country produces $\lambda$ times the food and $Z$ produced by the small country. Preferences are assumed to differ from Cobb–Douglas. I will use superscript $\lambda$ to denote equilibrium values of the large country, while variables without a superscript denote equilibrium values for the small country.
Comparing the equilibrium conditions of the two countries under the assumption of equal factor prices which result in an equal scale-adjusted price \(p_z\), taking \(p_x^* = p_y^*\) (since food is used as numeraire), the following relations have to hold:

\[ R(p_y, p_x^*, n^*) = 0(X^\lambda), \]  

\[ e(X^\lambda)n^\lambda = \lambda Z, \]  

\[ p_x^* X^\lambda/e(x^\lambda) = p_z. \]  

Eq. (66) is the same as condition (53'), except that now the degree of monopoly power depends also on prices. Eq. (67) says that \(Z^\lambda = \lambda Z\), which has to be satisfied if factor prices are equal, while (68) says that \(p_x^* = p_z\), which also has to be satisfied if factor prices are equal. Finally, consider the general demand equilibrium condition (29) which replaces (56) and (56'). Let the left hand side of (29) be represented by the function \(\rho(p_y, p_x, N)\), which is the relative demand for manufactured products. Then, using \(N = n\), (29) reads

\[ \rho(p_y, p_x, n) = \frac{nX}{Y} = \frac{Zp_z}{Y} \frac{1}{p_x}. \]

With equal factor prices, \(p_z Z/Y\) is the same in both countries. Hence, demand equilibrium implies

\[ p_x^* \rho(p_y, p_x^*, n^*) = p_z Z/Y = p_x \rho(p_y, p_x, n). \]  

Equality of factor prices is consistent with equilibrium if and only if there exist \(p_x^*, n^\lambda\), and \(X^\lambda\) which satisfy eqs. (66)–(69). We have four equations with three unknowns. Therefore a necessary condition for a solution to exist is either that at least two equations are dependent or that at least one equation is redundant. In the Cobb–Douglas case (69) is redundant, because \(p_x \rho(\cdot) = s/(1-s)\), and so I was able to use (66)–(68) to solve for \(p_x^*, n^\lambda, X^\lambda\). However, if the utility function is not Cobb–Douglas, eq. (69) is not redundant, and there will, generally, not exist a solution to the above four-equation system. This means that under these circumstances relative factor prices and relative scale-adjusted commodity prices will not be the same in both countries. Since they have the same relative endowments, there will be no intersectoral trade when trade is opened, but by relying on relative factor rewards or relative scale-adjusted commodity prices one would wrongly predict the existence of intersectoral trade.
7. Some empirical implications

Now suppose that there are many countries and many commodity groups, but maintain the assumption that labor and capital are the only factors of production. A commodity group consists of different varieties of the same good: cars, T.V. sets, bicycles, etc. Some of these groups may consist of homogeneous products.

Using the results from Jones (1974), it is clear that with homothetic production functions in a trading equilibrium, countries with higher capital labor ratios produce on average more capital intensive goods. In particular, if we consider two countries whose factor prices are not equalized, then the capital rich country will produce varieties from at least one commodity group such that the capital intensity of this group is higher than the capital intensity of all commodity groups being produced in the capital poor country, or the capital poor country produces varieties whose capital intensity is lower than the capital intensity of all commodity groups being produced in the capital rich country.

If countries are far apart in their capital-labor ratios, they may happen to produce entirely different groups of commodities, making all bilateral trade inter-industry trade. Intra-industry trade, on the other hand, will take place between countries with close factor proportions. Hence, intra-industry trade tends to be larger between countries with close factor proportions than between countries with far apart factor proportions. Since the higher the capital-labor ratio the higher is income per capita (in a cross country comparison), this raises the hypothesis that a country's share of bilateral intra-industry trade is negatively correlated with the absolute difference in bilateral incomes per capita. Note that this has the flavor of the Linder hypothesis [see Burestam Linder (1961)], but it is restricted to intra-industry trade and it stems from supply considerations, while the Linder hypothesis concerns total volumes of trade, and it is based on the assumption that relative demands change with income per capita.

A negative correlation between the absolute difference in income per capita and the share of intra-industry trade in the bilateral volume of trade has been recently reported for the OECD countries in the early seventies. In a study of the determinants of intra-industry trade, Loertscher and Wolter (1980) explain an index of the share of intra-industry trade which is a variant of Intra that I employed in section 5. In their regression analysis they use absolute differences in incomes per capita as one of the explanatory variables, and they find a negative coefficient which differs from zero at the one percent significance level (see their table 2).

A second hypothesis, which is related to the previous one, can be formulated about the time series of the share of intra-industry trade in the world's volume of trade. The hypothesis is that this share is negatively correlated with a measure of dispersion of incomes per capita. For example,
one could use the variance or the standard deviation of the distribution of incomes per capita across countries to test whether over time such a negative correlation exists, but I have not been able to find a study which provides a test of this hypothesis.

The present theory does not predict strong relationships between volumes of trade and similarity of incomes per capita. We have seen that a link of this type exists only for an appropriate experiment in which relative country size does not change (proposition 5), but I have also shown that changes in relative size have strong effects on the volume of trade. This explains, perhaps, the weak results that emerge from tests of the Linder hypothesis.

The present theory is consistent with the observed differential in the rates of growth of GNP and the volume of trade. In the post World War II period, the volume of trade of the industrial countries grew at an average rate which was almost double the average growth rate of their GNP. The present model is indeed capable of producing a growth rate of the volume of trade which exceeds the rate of growth of GNP. Take, for example, the case of a homothetic production function in the manufacturing sector and Cobb-Douglas preferences (see section 6). Then, if labor and capital grow at a uniform rate which is the same in every country, GNP of each country also grows at this rate. The volume of trade in food grows at the general growth rate, but output and the volume of trade in manufactured products grows faster. Hence, the total volume of trade grows faster than GNP.

Appendix

The purpose of this appendix is to provide explicit calculations of terms that are used in the main body of the paper. Implicit differentiation of (10) and (11) yields the following partial derivatives of the functions \( \delta'(\cdot) \) and \( \ddot{\delta}(\cdot) \) that are defined in (12):

\[
\frac{\partial \delta(\cdot)}{\partial p_{xi}} = \frac{-h[\ddot{\delta}(\cdot)]}{p_{xi}h'[\ddot{\delta}(\cdot)] + p_{xi+1}h'[2D_i - v_{i-1} - \ddot{\delta}(\cdot)]} < 0, \tag{A.1}
\]

\[
\frac{\partial \ddot{\delta}(\cdot)}{\partial v_{i-1}} = \frac{-p_{xi+1}h'[2D_i - v_{i-1} - \ddot{\delta}(\cdot)]}{p_{xi}h'[\ddot{\delta}(\cdot)] + p_{xi+1}h'[2D_i - v_{i-1} - \ddot{\delta}(\cdot)]} < 0. \tag{A.2}
\]

\[
\frac{\partial \dot{\delta}(\cdot)}{p_{xi}} = \frac{-h[\ddot{\delta}(\cdot)]}{p_{xi}h'[\ddot{\delta}(\cdot)] + p_{xi+1}h'[v_{i-1} - \ddot{\delta}(\cdot)]} < 0, \tag{A.3}
\]

\[
\frac{\partial \ddot{\delta}(\cdot)}{\partial v_{i-1}} = \frac{p_{xi-1}h'[v_{i-1} - \ddot{\delta}(\cdot)]}{p_{xi}h'[\ddot{\delta}(\cdot)] + p_{xi-1}h'[v_{i-1} - \ddot{\delta}(\cdot)]} > 0. \tag{A.4}
\]
while differentiation of (13) yields:

\[
\frac{\partial Q(\cdot)}{\partial p_{xi}} = \xi I \left\{ \int_0^{h(v)} \alpha_1(p_x, h(v), p_y)[h(v)]^2 \, dv \right. \\
+ \int_0^{h(v)} \alpha_1(p_x, h(v), p_y)[h(v)]^2 \, dv \left. \right\} \\
+ \xi I \left\{ \alpha_3(p_x, h(\delta(\cdot)), p_y)[h(\delta(\cdot))] \frac{\partial \delta(\cdot)}{ \partial p_{xi} } \right. \\
+ \alpha_3(p_x, h(\delta(\cdot)), p_y)[h(\delta(\cdot))] \frac{\partial \delta(\cdot)}{ \partial p_{xi} } \left. \right\}, \tag{A.5}
\]

\[
\frac{\partial Q(\cdot)}{\partial v_{i-1}} = \xi I \left\{ \alpha_3(p_x, h(\delta(\cdot)), p_y)[h(\delta(\cdot))] \frac{\partial \delta(\cdot)}{ \partial v_{i-1} } \right. \\
+ \alpha_3(p_x, h(\delta(\cdot)), p_y)[h(\delta(\cdot))] \frac{\partial \delta(\cdot)}{ \partial v_{i-1} } \left. \right\}. \tag{A.6}
\]

It is explained in section 4 that in equilibrium \( p_{xi} = p_x \), \( \delta(\cdot) = \delta(\cdot) = 1/N \) and \( v_{i-1} = 2/N \). Using this information to evaluate (A.1)–(A.6) yields:

\[
\frac{\partial \delta(\cdot)}{ \partial p_{xi} } = -\frac{h(1/N)}{2p_x h'(1/N)}, \tag{A.7}
\]

\[
\frac{\partial \delta(\cdot)}{ \partial v_{i-1} } = -\frac{1}{2}, \tag{A.8}
\]

\[
\frac{\partial \delta(\cdot)}{ \partial v_{i-1} } = -\frac{h(1/N)}{2p_x h(1/N)}, \tag{A.9}
\]

\[
\frac{\partial \delta(\cdot)}{ \partial v_{i-1} } = \frac{1}{2}, \tag{A.10}
\]

\[
\frac{\partial Q(\cdot)}{ \partial p_{xi} } = 2\xi I \left\{ \int_0^{1/N} \alpha_3(p_x, h(v), p_y)[h(v)]^2 \, dv \right. \\
- \xi I \alpha_3(p_x, h(1/N), p_y)[h(1/N)]^2/[p_x h'(1/N)]. \tag{A.11}
\]
\[ \frac{\partial Q(t)}{\partial t_i} = 0, \quad (A.12) \]

where, in the calculations of (A.11) and (A.12), use has been made of (A.7)–(A.10).

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