Chapter 4:  
Technology and Cost

1 Introduction

• Firms should transform efficiently inputs into outputs.
• What is a firm?
  – What happens inside a firm?
  – How are firms structured? What determine size?
  – How are individuals organized/ motivated?
2 Technology and cost for a single product firm

• Profit-maximizing firm must solve a related problem
  – minimize the cost of producing a given level of output
  – combine two features of the firm
    * production function
    * cost function

2.1 The production function

• Production function: how inputs are transformed into output
• 2 inputs: labor ($L$) and capital ($K$)
• $Q = f(L, K)$ is twice continuously differentiable.
• Marginal product: amount of output increase associated with a small increase in the amount of input.
– Marginal product of labor

\[ MP_L(L, K) = \frac{\partial f}{\partial L} \]

– Marginal product of capital

\[ MP_K(L, K) = \frac{\partial f}{\partial K} \]

**Example:** \( Q = L^\alpha K^\beta \)

**Returns to scale (r-t-s).** Let \( \lambda > 1 \). A technology \( Q \) exhibits

– increasing r-t-s if \( f(\lambda L, \lambda K) > \lambda f(L, K) \);
– decreasing r-t-s if \( f(\lambda L, \lambda K) < \lambda f(L, K) \);
– constant r-t-s if \( f(\lambda L, \lambda K) = \lambda f(L, K) \).

**Example:** Does the technology \( Q = L^\alpha K^\beta \) exhibits increasing, decreasing or constant r-t-s?
2.2 The cost function

- *Cost function*: relationship between output choice and production costs.

- Wage rate \((w)\), capital price \((r)\)

- Cost function:

\[
C = wL + rK
\]

- Firm’s objective is

\[
\begin{cases}
\min_{L,K} wL + rK \\
\text{s.t. } Q = f(L, K)
\end{cases}
\]
• Constraint becomes: $L = \tilde{f}(Q, K)$, and the minimization program

$$\min_{K} w \tilde{f}(Q, K) + rK$$

• F.O.C. gives $K^*(Q; w, r)$ and thus $L^*(Q; w, r)$.

• The cost function becomes

$$C = wL^*(Q; w, r) + rK^*(Q; w, r)$$

$$\rightarrow C'(Q)$$

• $C'(Q)$: total cost of producing $Q$ units of outputs.

• Example: if the technology is $Q = L^\alpha K^\beta$ for $\alpha = 0.5$, $\beta = 0.5$, $w = 1$ and $r = 9$, what is the cost function?
• In general:

\[ C(Q) = F + VC(Q) \]

• Average cost: cost per unit of output produced

\[ ATC(Q) = AFC(Q) + AVC(Q) \]

\[ ATC(Q) = \frac{F}{Q} + \frac{VC(Q)}{Q} \]

• Marginal cost: extra cost from producing one more unit of output

\[ MC(Q) = MVC(Q) \]

\[ MC(Q) = \frac{dVC(Q)}{dQ} \]

• Example: \( C(Q) = F + 2Q^2 \). Graph.
2.3 Cost and output decisions

- Maximization program is
  \[ \max_q \{ \Pi(q) = TR(q) - TC(q) \} \]

- Firms maximizes profit where
  \[ MR = MC \]
  provided
  - output should be greater than zero
  - implies that price is greater than average variable cost
  - shut-down decision

- Enter if price is greater than average total cost
  - must expect to cover sunk costs of entry
2.4 Economies of scale

- Definition: average costs fall with an increase in output
- Represented by the scale economy index
  \[ s = \frac{AC(q)}{MC(q)} \]
- \( s > 1 \): economies of scale
- \( s < 1 \): diseconomies of scale

- Sources of economies of scale
  - “the 60% rule”: capacity related to volume while cost is related to surface area
  - product specialization and the division of labor
  - “economies of mass reserves”: economize on inventory, maintenance, repair
  - indivisibilities
2.5 Indivisibilities, sunk costs and entry

- Indivisibilities make scale of entry an important strategic decision:
  - enter large with large-scale indivisibilities: heavy overhead
  - enter small with smaller-scale cheaper equipment: low overhead
- Some indivisible inputs can be redeployed
  - aircraft
- Other indivisibilities are highly specialized with little value in other uses
  - market research expenditures
  - rail track between two destinations
- The latter are sunk costs: nonrecoverable if production stops
- Sunk costs affect market structure by affecting entry
3 Multi-product firms

- Many firms make multiple products
  - Ford, General Motors, 3M etc.
- What do we mean by costs and output in these cases?
- How do we define average costs for these firms?
  - total cost for a two-product firm is

\[ C(Q_1, Q_2) \]

- marginal cost for product 1 is

\[ MC_1 = \frac{\partial C'(Q_1, Q_2)}{\partial Q_1} \]

- but average cost cannot be defined fully generally
- need a more restricted definition: ray average cost
3.1 Ray average cost

- Assume that a firm makes two products, 1 and 2 with the quantities $Q_1$ and $Q_2$ produced in a constant ratio of 2:1.
- Then total output $Q$ can be defined implicitly from the equations $Q_1 = 2Q/3$ and $Q_2 = Q/3$
- More generally: assume that the two products are produced in the ratio $\lambda_1/\lambda_2$ (with $\lambda_1 + \lambda_2 = 1$).
- Then total output is defined implicitly from the equations $Q_1 = \lambda_1 Q$ and $Q_2 = \lambda_2 Q$
- **Ray average cost** is then defined as:

$$RAC(Q) = \frac{C(\lambda_1 Q, \lambda_2 Q)}{Q}$$
3.2 An example of ray average costs

• Assume that the cost function is

\[ C(Q_1, Q_2) = 10 + 25Q_1 + 30Q_2 - \frac{3}{2}Q_1Q_2 \]

• Marginal costs for each product are:

\[ MC_1 = \frac{\partial C(Q_1, Q_2)}{\partial Q_1} = 25 - \frac{3}{2}Q_2 \]

\[ MC_2 = \frac{\partial C(Q_1, Q_2)}{\partial Q_2} = 30 - \frac{3}{2}Q_1 \]

• Ray average costs:
  – assume \( \lambda_1 = \lambda_2 = 0.5 \)

\[ Q_1 = 0.5Q \]

\[ Q_2 = 0.5Q \]
\[ RAC'(Q) = \frac{C(0.5Q, 0.5Q)}{Q} \]
\[ = \frac{10 + 25\frac{Q}{2} + 30\frac{Q}{2} - \frac{3QQ}{2}}{Q} \]
\[ = \frac{10}{Q} + \frac{55}{2} - \frac{3Q}{8} \]

• assume \( \lambda_1 = 0.75, \lambda_2 = 0.25 \)

\[ RAC'(Q) = \frac{C(0.75Q, 0.25Q)}{Q} \]
\[ = \frac{10 + 75\frac{Q}{4} + 30\frac{Q}{4} - \frac{9}{32}Q^2}{Q} \]
\[ = \frac{10}{Q} + \frac{105}{4} - \frac{9Q}{32} \]
3.3 Economies of scale and multiple products

• Definition of economies of scale with a single product

\[ s = \frac{AC(Q)}{MC(Q)} = \frac{C(Q)}{Q \cdot MC(Q)} \]

• Definition of economies of scale with multi-products

\[ s = \frac{C(Q_1, Q_2, ..., Q_n)}{MC_1 Q_1 + MC_2 Q_2 + ... + MC_n Q_n} \]

• This is by analogy to the single product case
  – relies on the implicit assumption that output proportions are fixed
  – so we are looking at ray average costs in using this definition