Overview of the Santa Fe Artificial Stock Market Model

Leigh Tesfatsion
Professor of Economics
Courtesy Professor of Mathematics &
Electrical and Computer Engineering (ECpE)
Iowa State University
Ames, IA 50011-1070
15 April 2009
Basic References (See Econ 308 Syllabus for Links)

https://www2.econ.iastate.edu/classes/econ308/tesfatsion/syl308.htm

Ref.[1]  ** L. Tesfatsion, "Stock Market Basics”
Ref.[2]  ** L. Tesfatsion, "Rational Expectations, the Efficient Market Hypothesis, and the Santa Fe Artificial Stock Market Model”
Ref.[3]  * L. Tesfatsion, "Detailed Notes on the Santa Fe Artificial Stock Market Model” (NOTE: Ref.[3] contains a detailed glossary of terms. Also, the equation numbers appearing below in this slide-set are the same as in Ref.[3].)
  https://www2.econ.iastate.edu/tesfatsi/BuildingTheSFASM.BLeBaron.pdf
Introduction: The Santa Fe Artificial Stock Market (SF-ASM) Model

- Originated in work at the Santa Fe Institute in late 1980s and early 1990s.

- **Authors:**
  - Blake LeBaron (economics);
  - W. Brian Arthur (economics);
  - John Holland (psychology/EE/CS, and father of GAs);
  - Richard Palmer (physics);
  - Paul Taylor (computer science).
The SM-ASM Model...Continued

- **Seminal Research:** One of the earliest attempts to develop and implement a *computational financial market model with heterogeneously learning traders*.

- Relatively simple model that attempts to address several important and controversial questions in financial economics.

- Many modeling issues not satisfactorily resolved by the SF-ASM model have been taken up in *later research* (see Ref.[4]).
Basic Objectives of Authors

- Provide a test-bed for exploring the rational expectations hypothesis (REH, Ref.[2])
- Consider a traditional stock market model with traders assumed to satisfy the REH
- Replace traditional REH traders with traders who learn to forecast stock prices over time
- Study dynamics around a well-studied REH equilibrium (fundamental pricing, Ref.[1])
Basic Objectives...Continued

• Examine whether the introduction of trader learning helps to explain empirical observations.

• In particular, does it help to explain well documented anomalies = deviations from fundamental stock pricing?

• Compare statistical characteristics of price and trading volume outcomes (model outcomes vs. actual empirical outcomes).
Basic Model Features (cf. Ref.[3])

- Discrete-time model: $t = 0,1,2,...$
- Market participants consist of $N$ stock market traders plus an “auctioneer”

**KEY:** Traders are identical except each trader individually forms expectations over time through inductive learning.

- Each trader has same initial wealth $W_0$ in the initial time period.
Basic Model Features… Continued

• Financial assets available for purchase at beginning of each period $t = [t, t+1)$:
  
  ★ **Risk-free asset** $F$ (\(\infty\) supply) paying a **constant** known 1-period net return rate $r$

  ★ **N shares of a risky stock** $A$. Each share
    • pays an **uncertain** dividend $d_{t+1}$ at the end of each period $t$ (beginning of each period $t+1$);
    • has an **uncertain** one-period net return rate $R_t$ over each holding period $t$. 
• Let $p_t$ denote the price of a share of the risky stock A at time $t$

• The **expected net return rate** $R_t$ **on this share over period** $t$ (i.e. from time $t$ to time $t+1$) is defined as

$$R_t = \frac{p_{t+1}^e - p_t + d_{t+1}^e}{p_t}$$

• This definition implies that

$$p_t = \frac{d_{t+1}^e + p_{t+1}^e}{1 + R_t}$$
Basic Model Features… Continued

• The expected net return rate $R_t$ on a share of the risky stock A over period $t$ satisfies:

$$p_t = \frac{[p_{e_{t+1}} + d_{e_{t+1}}]}{[1 + R_t]}$$

• **Basic rule of thumb for an investor in period $t$:** Given $r = \text{net return rate on the risk-free asset}$, **SELL** shares of A in period $t$ if $R_t < r$ because this implies $p_t$ is **GREATER THAN** the current fundamental value of these shares:

$$p_f^t = \frac{[p_{e_{t+1}} + d_{e_{t+1}}]}{[1 + r]}$$
Basic Model Features... Continued

- **Stock Dividend** $d_t$ paid at beginning of each period $t = [t,t+1)$ is generated by a random process unknown to the traders (see equ.(1) in Ref.[3])

- Wealth-seeking traders have identical **utility of wealth function** $U(W)$ exhibiting constant absolute risk aversion.
Basic Model Features… Continued

- In beginning of each period $t$, each trader chooses a portfolio $(X,Y)$, where $X =$ holdings of risky stock A and $Y =$ holdings of risk-free asset F.

- Each trader’s objective in period $t$ is to maximize his expected utility of wealth $E U(W_{t+1})$ subject to the constraint

$$W_{t+1} = \text{value in period } t+1 \text{ of the asset portfolio } (X,Y) \text{ purchased in period } t$$
Basic Model Features… Continued

• In beginning of each period $t$, each trader has a set of $K$ if-then forecasting rules.

• Each forecasting rule forecasts the expected sum $[p_{t+1} + d_{t+1}]$ and generates an update of the rule’s “forecast variance.”

**Forecast variance** = a weighted average of a rule’s past squared forecast errors (deviations between actual and forecasted price-plus-dividend sums).
• Form of an **if-then forecasting rule**:

Let

$$VAR = \text{Updated forecast variance};$$

$$E[p_{t+1} + d_{t+1}] = a[p_t + d_t] + b.$$
Basic Model Features… Continued

- The **specificity** of a forecasting rule = number of specific conditions incorporated into its “if” condition statement C.
- A forecasting rule is **activated** if its “if” condition statement C matches the trader’s current market state information.
- The **fitness** of a forecasting rule depends *inversely* on the rule’s forecast variance (error rate) and *inversely* on its specificity (thus encouraging parsimonious info use).
Time Line of Activities in Period $t$

- Period-$t$ dividend $d_t^*$ is publicly posted.
- Each trader $i=1,...,N$ determines a forecast
  \[
  E[p_{t+1} + d_{t+1}] = a'[p_t + d_t] + b'
  \]
  as a function of the *yet-to-be determined* period-$t$ market price $p_t$.
- He then generates a demand function giving his expected-utility-maximizing share holdings $X_i$ as a function of $p_t$:
  \[
  X_i = X_i(p_t) \]
Each trader $i = 1, \ldots, N$ submits his demand function to the Auctioneer, who determines the **period-$t$ market clearing price** $p_t^*$.

\[
\sum X_i = \sum X_i(p_t)
\]
• The Auctioneer publicly posts $p_t^*$.

• Each trader $i$ purchases $X_i(p_t^*)$.

• Each trader $i$ uses $(p_t^*,d_t^*)$ to update the fitness of the forecasting rule he used in period $t-1$ to generate a forecast $E[p_t + d_t]$.

• Each trader $i$ with probability $p_u$ then updates his entire forecasting rule set via a genetic algorithm involving recombination, elitism, and mutation operations.
Each trader $i = 1, \ldots, N$ updates his set of forecasting rules with probability $p_u$ in each period $t$ using a genetic algorithm (GA).

Thus, updating of forecasting rule sets happens in different time periods for different traders.

$p_u$ is an important parameter determining the speed of learning.
GA Classifier Learning…Continued

- Current market state $\Rightarrow$ 12-bit array
- Each bit position $\Rightarrow$ Distinct possible feature of the current market state

-- Bit in kth position takes on value 1 if kth feature is true

-- Bit in kth position takes on value 0 if kth feature is false
GA Classifier Learning…Continued

- **12-bit array** used to describe market state

- **First six bit positions**

  ➔ *Fundamental Features*

  Is the current market price above or below the fundamental price level in the previous time period? *(six different discrepancy values)*
Next four bit positions

- **Technical Features**

Is the current market price above an n-period moving average of past prices? (four different values of n)

Last two bit positions

- **Fixed Bit Values** (no information)
12-Bit Array for GA Classifier Learning

<table>
<thead>
<tr>
<th>Bit</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Price*interest/dividend &gt; 1/4</td>
</tr>
<tr>
<td>2</td>
<td>Price*interest/dividend &gt; 1/2</td>
</tr>
<tr>
<td>3</td>
<td>Price*interest/dividend &gt; 3/4</td>
</tr>
<tr>
<td>4</td>
<td>Price*interest/dividend &gt; 7/8</td>
</tr>
<tr>
<td>5</td>
<td>Price*interest/dividend &gt; 1</td>
</tr>
<tr>
<td>6</td>
<td>Price*interest/dividend &gt; 9/8</td>
</tr>
<tr>
<td>7</td>
<td>Price &gt; 5-period MA</td>
</tr>
<tr>
<td>8</td>
<td>Price &gt; 10-period MA</td>
</tr>
<tr>
<td>9</td>
<td>Price &gt; 100-period MA</td>
</tr>
<tr>
<td>10</td>
<td>Price &gt; 500-period MA</td>
</tr>
<tr>
<td>11</td>
<td>On: 1</td>
</tr>
<tr>
<td>12</td>
<td>Off: 0</td>
</tr>
</tbody>
</table>

Note on Rules 7-10: MA = Moving Average = Weighted average of past observed prices

Note on Rules 1-6: 
\[ \text{pr/d} > 1 \] if and only if \[ p > \frac{[p+d]}{(1+r)}, \text{ i.e., iff the current price } p \text{ for a share of the risky stock A exceeds the “fundamental” value of this share realized in the previous time period. (Refer back to slide 10.)} \]
Why this market state description?

* Permits testing for the possible emergence of *fundamental trading* (heavy reliance on first six bit positions) versus *technical trading* (heavy reliance on next four bit positions) versus *uninformed trading* (heavy reliance on the last two bit positions).
GA Classifier Learning...Continued

- Each forecast rule if[C]-then[forecast this] is conditioned on a 12-bit market state C.
- Each bit in C has one of 3 possible values: 1 (true), 0 (false), or # (I don’t care).

**Specificity of C =** number of 1 and 0 bits

- C matches actual 12-bit market state if:
  - (a) C has a 1 or # symbol in every position the actual market state has a 1;
  - (b) C has a 0 or # symbol in every position the actual market state has a 0.
Experimental Design

- **Key Treatment Factor: Probability** $p_u$
  Controls when each trader updates their forecasting rule set in any given time period

- **Slow-Learning Regime:** $p_u = 1/1000$
  (GA learning invoked every 1000 trading periods on average for each trader)

- **Medium-Learning Regime:** $p_u = 1/250$
  (GA learning invoked every 250 trading periods on average for each trader)
Experimental Findings

- **Slow-Learning Regime**: $p_u = 1/1000$
  Simulated data resemble data generated for a rational expectations equilibrium (REE) benchmark for which 100% market efficiency holds by assumption.

- **Medium-Learning Regime**: $p_u = 1/250$
  Complex outcomes - market does not settle down to a recognizable equilibrium. Simulated data in accordance with many empirical “anomalies” (deviations from REH) seen in actual stock markets.
Frequency of Use of “Technical Trading” Bits 7-10 in REE vs. Complex Regimes

Figure 3. Number of technical-trading bits that become set as the market evolves, (median over 25 experiments in the two regimes).
Final Remarks

☐ For a balanced detailed critique of the Santa Fe Artificial Stock Market (SF-ASM), see the working paper by Blake LeBaron at the pointer below.

☐ In this paper, LeBaron discusses the advantages and disadvantages of various design aspects of the SF-ASM, including the use of “classifier systems” for the representation and evolution of forecasting rules.

https://www2.econ.iastate.edu/tesfatsi/BuildingTheSFASM.BLeBaron.pdf