Notes on Wilhite
(Computational Economics, 2001)

Leigh Tesfatsion
Professor of Economics
Courtesy Professor of Mathematics and ECpE
Iowa State University, Ames, IA 50011-1070
https://www2.econ.iastate.edu/tesfatsi/

Main Reference:
https://www2.econ.iastate.edu/tesfatsi/SmallWorldNetworksBilateralTrade.Wilhite.pdf

NOTE: For details about the C++ Program implementation, contact the author Allen Wilhite at wilhitea@email.uah.edu
A number of ACE researchers have begun to consider small-world networks in relation to economic processes.

For example, Wilhite (2001) uses an ACE model of a bilateral exchange economy to explore the consequences of restricting trade to four different types of networks, including a small-world trade network.
As defined by Watts and Strogatz (1998), a Small-World Network is a network that can be represented as a simple connected graph G exhibiting two properties:

- **Global Reach:** The presence of “short cut” connections between vertices results in short characteristic path length $L(G)$.

- **Local Connectivity:** Each vertex of G is linked to a relatively well-connected set of neighboring vertices, resulting in a large value for the clustering coefficient $C(G)$. 
Wilhite’s Basic Approach

Examine the trade-off between market efficiency and transaction costs under four types of trade networks:

* **Completely connected trade network** (every trader can trade with every other trader);

* **Locally disconnected trade network** (disjoint trade groups);

* **Locally connected trade network** consisting of trade groups aligned around a ring with a 1-trader overlap at each meeting point;

* **Small-world trade network** constructed from the locally connected trade networks by permitting from 1 to 5 randomly specified short-cut trade links between members of non-adjacent trade groups.
Four Possible Trade Networks

**Note:** Depicted links are for a *typical* trader in the *global* network

**Panel a:**
Global Network
Trade routes for one trader

**Panel b:**
Local Disconnected Network
six groups, five agents per group

**Panel c:**
Local Connected Network
six groups, six agents per group

**Panel d:**
Small-world Network
two crossover agents

*Figure 1.* Sketches of four networks (population: 30 agents).
Basic Wilhite Model

- Exchange economy with 2 durable (non-perishable) goods: (1) a good $g_1$ that must be traded in whole units; and (2) an “infinitely divisible” good $g_2$.

- 500 traders *initially* endowed with *random positive* amounts of $g_1$ and $g_2$ ($g_2 = $ numeraire good, i.e., the price of $g_1$ is expressed as units of $g_2$ per unit of $g_1$)

- Each trader $i$ has the same form of utility function measuring preferences for the two goods:
  \[ U_i = g_{i1}^i g_{i2}^i , i = 1,...,500 \]

- Traders are rational, non-strategic, and myopic agents who try to improve their utility in each period by voluntary feasible trades of $g_1$ and $g_2$. 
Opportunity for mutually beneficial trade exists for two traders if they have different reservation prices ("marginal rates of substitution" MRS) giving the rates at which they are just willing to exchange $g_2$ for $g_1$.

To see the intuition for this, consider an “Edgeworth Box” pure exchange economy with two goods - oranges $g_1$ and bananas $g_2$ in fixed supplies 40 and 20 – that are currently allocated between Traders 1 and 2 at point A.

- $\text{MRS}^i(A) = \frac{\Delta U^i(A)/\Delta g^i_1}{\Delta U^i(A)/\Delta g^i_2} = \frac{\Delta g^i_2}{\Delta g^i_1} = \frac{g^i_2}{g^i_1}$
- $U^i(A)$ IC = Indifference Curve of Trader $i$ passing through A
- $\text{MRS}^i(A) = -1$ times the slope of Trader $i$’s indifference curve passing through A
Edgeworth Box Illustration

Region of Pareto improving trades for Traders 1 and 2

Orange Holdings of Trader 2

Banana Holdings of Trader 1

Orange Holdings of Trader 1

Banana Holdings of Trader 2

$U^1(A)$ IC

$U^2(A)$ IC

A
Basic Model...Continued

- Each trader is limited to trades within its own particular neighborhood as determined by the trade network.

- By random selection (without replacement), each trader in each neighborhood searches for trade partners within its neighborhood and selects a trade partner offering a “best” mutually beneficial price (if such a trade partner exists).

- Reservation prices are assumed to be truthfully revealed.
Whenever a suitable pair of trade partners $i$ and $j$ is determined, the two traders trade at the following “split the difference” price as long as the resulting trades are feasible and mutually beneficial:

$$p_{i,j} = \frac{g^i_2 + g^j_2}{g^i_1 + g^j_1}, \quad i, j \in \{1, \ldots, 500\}.$$ 

**Note:**

$$p_{i,j} = \text{MRS}^i \cdot [G^i_1] + \text{MRS}^j \cdot [1 - G^i_1],$$

where $G^i_1 = g^i_1/[g^i_1 + g^j_1]$. 
### Experimental Design: Four Distinct Trade Networks

<table>
<thead>
<tr>
<th>Global network:</th>
<th>Local disconnected network</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) number of groups: 1</td>
<td>(i) number of groups: 50</td>
</tr>
<tr>
<td>(ii) agents in each group: 500</td>
<td>(ii) agents in each group: 10</td>
</tr>
<tr>
<td>(iii) end-agents overlap? no</td>
<td>(iii) end-agents overlap? no</td>
</tr>
<tr>
<td>(iv) number of crossovers: 0</td>
<td>(iv) number of crossovers: 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Local connected network:</th>
<th>Small-world network</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) number of groups: 50</td>
<td>(i) number of groups: 50</td>
</tr>
<tr>
<td>(ii) agents in each group: 11</td>
<td>(ii) agents in each group: 11</td>
</tr>
<tr>
<td>(iii) end-agents overlap? yes</td>
<td>(iii) end-agents overlap? yes</td>
</tr>
<tr>
<td>(iv) number of crossovers: 0</td>
<td>(iv) number of crossovers: 5</td>
</tr>
</tbody>
</table>
50 runs with the same 500-trader populations were conducted for each of the four tested trade networks.

Illustrative depiction of the four tested trade networks for a 30-trader economy:

Panel a: Global Network
Trade routes for one trader

Panel b: Local Disconnected Network
six groups, five agents per group

Panel c: Local Connected Network
six groups, six agents per group

Panel d: Small-world Network
two crossover agents

Note: Depicted links are for a typical trader in the global network.

Note: The short-cut $S$ adds one more trader $T^*$ to group $B$, but only trader $T$ in $B$ can directly trade with $T^*$.

Figure 1. Sketches of four networks (population: 30 agents).
Key Questions Examined

$H1$: Price convergence: Is there a significant difference in the dispersion of prices across each trade network?

$H2$: Speed of convergence: Do the different trade networks require a significantly different number of rounds of trading to reach their steady state?

$H3$: Number of trades: Is there a difference in the number of trades it takes for each network to reach its steady state?

$H4$: Search: Is there a difference in the amount of search and negotiation in each trade network?
Key Experimental Findings

Note: *Round* = One pass through all traders as initiators of trades  
*Equilibrium* = No more mutually beneficial trade opportunities

<table>
<thead>
<tr>
<th></th>
<th>Prices (standard deviation)</th>
<th>Rounds</th>
<th>Total trades</th>
<th>Total searches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global network</td>
<td>1.0046 (0.00168)</td>
<td>8.08</td>
<td>1953.38</td>
<td>2,015,960</td>
</tr>
<tr>
<td>Local disconnected</td>
<td>1.0396 (0.2771)</td>
<td>7.02</td>
<td>1727.7</td>
<td>31,590</td>
</tr>
<tr>
<td>network</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local connected</td>
<td>1.0048 (0.0146)</td>
<td>497.14</td>
<td>93,975.72</td>
<td>2,734,270</td>
</tr>
<tr>
<td>network</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small-world network</td>
<td>1.0045 (0.00724)</td>
<td>242.54</td>
<td>45,944.56</td>
<td>1,236,954</td>
</tr>
</tbody>
</table>

*a Averages calculated from 50 simulations of each network configuration.

* 2,015,960 = [500 x 499 x 8.08] = 500 traders seeking best price from 499 other traders for 8.08 rounds
Price Convergence in the Four Networks

Global

Local Disconnected

Local Connected

Small-World with 5 Crossover Traders
Price Convergence for Different Numbers \{0, 1, 3, 5\} of Crossover Traders

Figure 3. Dynamic price adjustment: adding crossover agents.
Relative Wealth of Crossover Traders

Table III. Wealth of crossover agents.\textsuperscript{a}

<table>
<thead>
<tr>
<th>Crossovers</th>
<th>Agent A</th>
<th>Agent B</th>
<th>Agent C</th>
<th>Agent D</th>
<th>Agent E</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 crossovers</td>
<td>1929.61</td>
<td>887.73</td>
<td>1267.45</td>
<td>670.246</td>
<td>2059.95</td>
</tr>
<tr>
<td>1 crossover</td>
<td>\textbf{1997.17}</td>
<td>874.167</td>
<td>1269.23</td>
<td>675.557</td>
<td>2059.83</td>
</tr>
<tr>
<td>2 crossovers</td>
<td>1990.07</td>
<td>\textbf{1385.97}</td>
<td>1267.61</td>
<td>677.555</td>
<td>2058</td>
</tr>
<tr>
<td>3 crossovers</td>
<td>1984.94</td>
<td>1388.68</td>
<td>\textbf{1318.54}</td>
<td>677.591</td>
<td>2062.74</td>
</tr>
<tr>
<td>4 crossovers</td>
<td>1991.28</td>
<td>1389.3</td>
<td>1308.88</td>
<td>\textbf{985.451}</td>
<td>2066.52</td>
</tr>
<tr>
<td>5 crossovers</td>
<td>1981.82</td>
<td>1376.47</td>
<td>1307.04</td>
<td>981.252</td>
<td>\textbf{2105.27}</td>
</tr>
</tbody>
</table>

\textsuperscript{a} The boldfaced number is the point at which that particular agent established a bridge to another trade group. Wealth from population #23, seed number 18847.
Summary of Key Findings

- The small-world trade network with 5 crossover traders provides most of the market-efficiency advantages of the completely connected trade network while retaining most of the transaction cost economies of the locally connected trade network.

- Wilhite’s findings also suggest that there might exist private micro-level incentives for the formation of small-world trade networks.

- Specifically, the traders in the locally connected network who become crossover traders in the small-world trade network tend to amass greater wealth.
Extensions?

* For economic-social networks it is not satisfactory to focus solely on the implications of a fixed or exogenously-varied network structure.

* Feedback mechanisms at work in economic-social networks can result in endogenous changes in the network structure over time.

* **EXAMPLE:** Whom you have traded with in the past, and with what regularity, can affect how you behave in current trade transactions, which in turn can affect whom you choose and refuse to trade with in the future. For illustration, see the link below to the Trade Network Game (TNG) Lab that permits traders to choose and refuse their trade partners in each trading round. This choice and refusal of trade partners has substantial effects on trade network formation over time.

  TNG Lab Homepage:
  [https://www2.econ.iastate.edu/tesfatsi/tnghome.htm](https://www2.econ.iastate.edu/tesfatsi/tnghome.htm)