What this Chapter Is About

- We study the value of money in OLG models.
- We develop an important model of money (with applications to asset pricing).

The central question of monetary economics:

Why and when is money valued in equilibrium?

By money we mean bits of green paper with pictures of dead presidents.
Rate of return dominance is a problem:

- Why would anyone hold money in the presence of other assets that offer a higher rate of return?

The answer of the OLG model:

- Money is a bubble.
- Its value derives solely from the expectation that money will be valued tomorrow.
- It is only valued if no other asset offers a higher rate of return.

Can money alleviate dynamic inefficiency?

- Previous models lacked a long-lived asset that would facilitate intergenerational trade.
- Money could solve this problem.
1 An OLG Model of Money

Consider a standard two period OLG model without production or bonds.

The population grows at rate $n$.

Money is introduced as follows.

In period 1, the initial old are given $M$ bits of green paper.

In every subsequent period, the government prints additional paper and hands it to the current old in proportion to current paper holdings.

Effectively, money pays (nominal) interest if held from young to old age.

The money growth rate is constant:

$$\frac{M_{t+1}}{M_t} = 1 + \theta$$
1.1 Household

This is a standard two period household problem with endowments.

Preferences are

\[ u(c_t^y, c_{t+1}^o) \]

The household receives endowments \( e_1, e_2 \) of (perishable) goods.

The price of goods in period \( t \) is \( P_t \).

The budget constraints are therefore

\[
\begin{align*}
P_t(e_1 - c_t^y) &= P_t x_t \\
P_{t+1}(c_{t+1}^o - e_2) &= x_t(1 + \theta)P_t
\end{align*}
\]

The lifetime budget constraint is:

\[
e_1 - c_t^y = \frac{c_{t+1}^o - e_2}{(1 + \theta)P_t/P_{t+1}}
\]

Note that money acts exactly like a bond that pays gross interest

\[
R_{t+1} = (1 + \theta)P_t/P_{t+1}
\]
The Lagrangian is the same as in previous models:

$$\Gamma = u(c^y_t, c^o_{t+1}) + \lambda_t \left[ e_1 - c^y_t + \frac{e_2 - c^o_{t+1}}{R_{t+1}} \right]$$

FOCs:

$$u_1(t) = \lambda_t$$
$$u_2(t) = \lambda_t / R_{t+1}$$

Euler equation:

$$u_1(t) = R_{t+1} u_2(t)$$

A solution to the household problem is a triple \((c^y_t, c^o_{t+1}, x_t)\) which satisfies the Euler equation and the two budget constraints.

Optimal behavior can be characterized by a savings function (which is now a money demand function)

$$x_t = s(R_{t+1}, e_1, e_2)$$ (1)
2 Equilibrium

The government is simply described by a money growth rule:

\[ \frac{M_{t+1}}{M_t} = 1 + \theta \]

Market clearing:

- Money market:
  \[ M_t = N_t P_t x_t \]
  or
  \[ m_t = \frac{M_t}{(N_t P_t)} = s(R_{t+1}) \]

- Goods market:
  \[ e_1 + e_2/(1 + n) = c^y_t + c^o_t/(1 + n) \]
Equilibrium: A sequence of prices and quantities

\[(c^y_t, c^o_t, x_t, P_t, M_t)\]

such that

1. \(M_t\) obeys the money growth equation. [1 eqn]
2. Markets clear [1 independent eqn]
3. Households behave optimally [3 eqn]
3 Characterizing Equilibrium

We look for a difference equation in terms of the economy's state variables.

State variables are $m$ and $p$.

But in this model (and typically) only the ratio $m/p$ matters.

Start from the money market clearing condition

$$m_t = s(R_{t+1}) \quad (2)$$

Subsitute out $R$ using

$$R_{t+1} = (1 + \theta)P_t/P_{t+1} \quad (3)$$

We need an expression for inflation. From

$$\frac{M_{t+1}}{M_t} = \frac{m_{t+1}P_{t+1}N_{t+1}}{m_tP_tN_t}$$

we have

$$R_{t+1} = (1 + \theta)P_t/P_{t+1} = (1 + n)m_{t+1}/m_t$$

The law of motion is

$$m_t = s((1 + n)m_{t+1}/m_t) \quad (4)$$
3.1 The Offer Curve

We want to determine the shape of the law of motion.

The key idea is to use the household’s intertemporal consumption allocation to figure out how money evolves over time.
3.2 Household consumption choice

The lifetime budget constraint has slope $-R_{t+1}$.

Plot the tangencies between budget constraint and indifference curves for all interest rates → **offer curve**.

![Diagram showing the offer curve](image)

What do we know about the offer curve?

1. It goes through the endowment point.
2. At low levels of $R$ the household would like to borrow (but cannot).
3. For interest rates where the household saves very little, income effects are small ⇒ savings rise with $R_{t+1}$.
4. The offer curve intersects each budget line only once.
3.3 Money demand

Money demand equals saving of the young:

\[ m_t = s(R_{t+1}) = e_1 - c_t^y \]  \hspace{1cm} (5)

Hence: the horizontal axis shows \( m_t \).

Money demand also equals capital income of the old:

\[ (1 + n)m_{t+1} = R_{t+1}s(R_{t+1}) = c_{t+1}^o - e_2 \]  \hspace{1cm} (6)

Hence: the vertical axis shows \((1 + n) m_{t+1}\).

The offer curve therefore describes the law of motion for \( m \):

\[ (1 + n)m_{t+1} = F(m_t) \]

where \( F \) is the offer curve.
3.4 Law of motion

Using a line of slope \((1 + n)\) we can find the path of \(m_t\) for any start value \(m_0\).
3.5 Steady state

There is a unique monetary **steady state** (intersection of offer curve and ray through origin).

It is **unstable**.

**Properties of the steady state:**

- Per capita real money balances, $m$, are constant over time.
- The gross rate of return on money is

\[
R_{t+1} = (1 + \theta) \frac{P_t}{P_{t+1}} = (1 + n) \frac{m_{t+1}}{m_t}
\]

Therefore in steady state

\[
R = 1 + n
\]

- Steady state inflation is

\[
\frac{P_{t+1}}{P_t} = \frac{1 + \theta}{1 + n}
\]
3.6 Dynamics

Take $m_0$ as given for now.

What if $m_0 > m_{ss}$?

- This cannot happen because $m_t$ would blow up towards $\infty$.
- But then consumption will exceed total output at some point.

If $m_0 < m_{ss}$: $m_t$ collapses towards 0.

- Because $M$ grows at a constant rate, this must happen through inflation.
- Along this path $R$ falls over time $\Rightarrow$ inflation accelerates.

Intuition:

- If $m_0 = m_{ss}$ people save just enough to keep $m$ constant.
- If $m_0$ is a bit lower, then $R$ is a bit lower. People save less.
- That requires a lower $m_1$, hence more inflation.
- That leads people to save less again, etc.
3.7 Initial money stock

Nothing in the model pins down $m_0$. Any value below $m_{ss}$ is acceptable.

There is a continuum of equilibrium paths.

The reason: money is a bubble.

As long as expectations are such that people are willing to hold $m_0$, we have an equilibrium.

$m_0 = 0$ is also an equilibrium.
3.8 Dynamic Efficiency

Does money solve the dynamic inefficiency problem?

- It might because it permits intergenerational trade.

Two cases:

1. Samuelson case: the offer curve at the origin is flatter than $1 + n$;
2. Classical case: it is steeper than $1 + n$. 
3.8.1 Samuelson case

The economy without money is dynamically efficient.

The reason: the slope of the offer curve at the origin equals the non-monetary interest rate.

There is no trade. The interest rate equals the slope of the IC through the endowment point.

The economy is dynamically inefficient, if the interest rate is lower than \( n \).

Then the offer curve is flatter than the \( 1 + n \) line.
3.8.2 Classical Case

The non-monetary economy is dynamically efficient.

The offer curve is too steep to intersect that $1 + n$ line.

A monetary equilibrium does not exist.

The main result is therefore:

Money is valued in equilibrium only in an economy that would be dynamically inefficient without money.
4 Is this a good theory of money?

Good features of the OLG model of money are:

1. The outcome that money is valued in equilibrium is not assumed (e.g. because money yields utility or is simply required for transactions).
2. The value of money depends on expectations and is fragile.

The problem:

1. The model does not generate rate of return dominance.
2. A key feature of money seems to be missing: liquidity.

How to construct a theory of money that resolves the problems without introducing new ones is an open question.