Reader's Appendix to
Strategic Foreign Direct Investment and Exchange Rate Uncertainty
Proofs of Properties P1-P5

P1. Exchange rate variability increases $\overline{\Pi}^*_{h,e}$ because it is convex in *e*. $\overline{\Pi}^*_{f,e}$ is independent of *e*, whereas $\overline{\Pi}^*_{b,e}$ is independent of *e* for $e < e_0$, but decreasing in *e* for $e > e_0$. Since profits for all three cases are the same under certainty, the conclusion is immediate.

P2. From (17), let $e = e_0 + \mu x$, where μ is a positive scalar, and $E(x)=0$; then:

(A1) $$E(\overline{\Pi}^*_{b,e}) = \int_{x \quad 0} \left\{ \frac{(\overline{A} + \overline{c} \quad 2\overline{d})^2}{9\overline{b}} \right\} h(x)dx + \int_{x \quad 0} \left\{ \frac{(\overline{A} + c/[e_0 + \mu x] \quad 2\overline{d})^2}{9\overline{b}} \right\} h(x)dx \quad \overline{G}$$

where *h(x)* is the density function for *x*. Upon differentiating, it is readily verified that increases in μ reduce the local firm's expected profits.

P3. From (15) and (16), the expected value of switching from the home to foreign plant is:

(A2) $$V_{f,e} \quad V_{h,e} = E_e \left\{ \frac{(c \quad e\overline{c})(2A \quad c \quad e\overline{c})}{4b} + \frac{4(c \quad e\overline{c})(\overline{A} + \overline{d} \quad \overline{c} \quad (c/e))^2}{9\overline{b}} \right\} + (F \quad e_0\overline{F}).$$

The last term, by assumption, is negative. The remaining terms cannot be determined analytically, but given $c = E(e)\overline{c}$, we have:

(A3) $$V_{f,e} \quad V_{h,e} + (e_0\overline{F} \quad F) = \frac{\overline{c}^2\sigma_e^2}{4b} \quad E_e \left\{ \frac{4\overline{c}^2(e \quad \overline{e})(\overline{e}/e)^2}{9\overline{b}} \right\} \quad \frac{\overline{c}^2\sigma_e^2}{4b} \quad \frac{4\overline{c}^2\sigma_e^2}{9E(e)\overline{b}}.$$

(this expression is exact if *e* is log normally distributed). In the symmetric case ($b = E(e)\overline{b}$), (A3) is negative, guaranteeing the result.

P4. The option value of opening the foreign plant, given that the home plant is open, also depends upon whether the local firm enters, as follows:

(A4) $$[V_{b,d} \quad V_{h,d}] \quad [V_{b,e} \quad V_{h,e}] = E_{e \quad e_0} \left\{ \frac{(e\overline{c} + c)(2\overline{A} \quad 16\overline{d} + 7\overline{c} + 7(c/e))^2}{36\overline{b}} \right\}$$

Assuming: $\overline{d} = \overline{c}$, then: $(2\overline{A} \quad 16\overline{d} + 7\overline{c} + 7(c/e))^2 = (2\overline{A} \quad 9\overline{c} + 7(c/e))^2 > (2\overline{A} \quad 2\overline{c})^2$ for $e < e_0$, so that the expression in brackets is uniformly positive.

P5. We prove P5 for the case in which the local firm enters; the case when it doesn't enter is similar. The option value of opening the foreign plant, given that the home plant is open (and the local firm enters) is found from equations (15) and (17):

$$\text{(A5)}\quad V_{b,e} = \underset{e\ e_0}{E}\left\{\frac{(b-c)\ (e\bar{c}-(b-2A)\ c\ (e\bar{c}-c)}{4b} + \frac{4e(b-c/e)\ (\bar{c}-(\bar{A}+\bar{d})\ \bar{c}\ (b-c/e)^2}{9\bar{b}}\right\}$$

Let $e = e_0\phi$, where: $\phi\ [1-\varepsilon, 1+\varepsilon]$, with density function: $g(\phi) = (1/2\varepsilon)$. Then:

$$\text{(A6)}\quad V_{b,e} = \underset{\phi\ 1}{E}\left\{\frac{(e_0\bar{c}(1-\phi)^2(\bar{c}-(2A-e_0\bar{c}(1+\phi)^2)}{4b} + \frac{4(e_0\bar{c}(1-\phi)^2(\bar{c}-(\bar{A}+\bar{d})\ \bar{c}\ (b-\bar{c}/\phi)^2}{9\bar{b}}\right\}$$

The first term on the LHS in brackets must be positive, but the second term provides more difficulty. Recall that $y^*_{h,e} > 0 \quad (\bar{A}+\bar{d})\ 2(b-c/e)^2 > 0 \quad \phi\ (2\bar{c}/(\bar{A}+\bar{d}))$. Using the uniform distribution to evaluate (A6) yields:

$$\text{(A7)}\quad V_{b,e} = \frac{e_0\bar{c}}{8b}\left\{(b-A-e_0\bar{c})^2\varepsilon + \frac{e_0\bar{c}\varepsilon^2}{3}\right\} + \frac{2e_0\bar{c}}{9\bar{b}}\left\{\frac{(\bar{A}+\bar{d})\ \bar{c}^2\varepsilon}{2} + \bar{c}^2\left(1+\frac{\ln(1-\varepsilon)}{\varepsilon}\right)\right\}$$

The first term on the RHS of (A5iii) is clearly positive and increasing in the variance $(\sigma_e^2 = (e_0^2\varepsilon^2/3)^2)$. The second term takes more work. Define:

$$\text{(A8)}\quad J(\varepsilon) = \left\{\frac{(\bar{A}+\bar{d})\ \bar{c}^2\varepsilon}{2} + \bar{c}^2\left(1+\frac{\ln(1-\varepsilon)}{\varepsilon}\right)\right\};$$

Then, it can be shown that: $\left.\frac{dJ(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} = \left(\frac{\bar{A}+\bar{d}\ 2\bar{c}}{2}\right) > 0$ and $\frac{d^2J(\varepsilon)}{d\varepsilon^2} < 0, \quad \varepsilon > 0$.

Finally, $\phi\ (2\bar{c}/(\bar{A}+\bar{d}))\ \varepsilon\ \hat{\varepsilon}\ [(\bar{A}+\bar{d})\ 2\bar{c}^2/(\bar{A}+\bar{d})^2]$. Evaluating $\frac{dJ(\varepsilon)}{d\varepsilon}$ at $\hat{\varepsilon}$ yields:

$$\text{(A9)}\quad J(\hat{\varepsilon}) = \left(\frac{r}{4(1-r)^2}\right)\ln(1-r)\ (r(r-3)-2\ln(r)); \quad r\ \left(\frac{2\bar{c}}{\bar{A}+\bar{d}}\right)\ 1,$$

and $J(\hat{\varepsilon}) > 0, \quad r < 1$ which proves the claim.

NOTE

$$E\left[\pi^*_{b,o} \quad \pi^*_{h,o}\right] = \underset{e \quad e_0}{E}\left\{\frac{\left(\left(e\bar{c} + c\right)\left(2A \quad c \quad e\bar{c}\right)\right)}{4b} + \frac{\left(4\left(e\bar{c} + c\right)\left(\bar{A} + \bar{d} \quad \bar{c} \quad \left(c/e\right)\right)\right)}{9\bar{b}}\right\}$$

$$E\left[\pi^*_{b,n} \quad \pi^*_{f,n}\right] = \underset{e \quad e_0}{E}\left\{\frac{\left(\left(e\bar{c} \quad c\right)\left(2A \quad c \quad e\bar{c}\right)\right)}{4b} + \frac{\left(\left(e\bar{c} \quad c\right)\left(2\bar{A} \quad \bar{c} \quad \left(c/e\right)\right)\right)}{4\bar{b}}\right\}$$

$$E\left[\pi^*_{b,n} \quad \pi^*_{h,n}\right] = \underset{e \quad e_0}{E}\left\{\frac{\left(\left(e\bar{c} + c\right)\left(2A \quad c \quad e\bar{c}\right)\right)}{4b} + \frac{\left(\left(e\bar{c} + c\right)\left(2\bar{A} \quad \bar{c} \quad \left(c/e\right)\right)\right)}{4\bar{b}}\right\}$$

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$$E\left[\pi^*_{b,n} \quad \pi^*_{h,n}\right] = \underset{e \quad e_0}{E}\left\{\frac{\left(\left(e\bar{c} + c\right)\left(2\bar{A} \quad 16\bar{d} + 7\bar{c} + 7\left(c/e\right)\right)\right)}{36\bar{b}}\right\}$$