A Bad Peace or a Good War:
A Structural Estimation Model of Spousal Conflict and Divorce

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Abstract

The optimal balance between keeping marriages intact, despite sustained conflict, or allowing for divorce continues to be a subject of policy debate, even after years of changes to divorce laws. To understand the tradeoffs, I construct a structural game theoretic model of spousal interaction with information asymmetries which may generate Pareto inferior outcomes, including inefficient conflict or divorce. Models with conflict as an equilibrium outcome have not been analyzed before in the household bargaining literature. The structural parameters are estimated using data from the National Survey of Families and Households, a nationally representative panel of 13,000 households with unique questions on conflict and spousal beliefs about the outside options. The model fits the data well and has good out-of-sample predictive properties. The estimation results imply that a large majority of spouses are deeply hurt by conflict and think that their own post-divorce opportunities are poor. Exaggerating a partner's disutility from conflict and underestimating the outside prospects generate inefficient negotiation outcomes in the model. The estimated parameters are mostly in line with intuition. For instance, marital heterogamy indicators, such as the difference in spousal ages, tend to have a negative impact on the value of marriage relative to divorce. Mandatory separation periods before a divorce decree can be granted adversely affect the outside options. Stronger child support enforcement generally makes divorce more attractive to wives and less attractive to husbands, but the effect varies with educational attainment. I simulate several policy changes and find that the elimination of the separation requirements increases the fraction of divorced couples by 8.4 percent and is a weak deterrent to conflict. Perfect child support enforcement reduces the incidence of divorce and conflict by as much as 9.2 and 18.4 percent, respectively.

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Chapter 1

Introduction

Marriage is vital to the production of many economic, social, and health benefits for adults and children. However, available empirical evidence suggests that spousal conflict is sometimes more detrimental to a harmonious personal life than divorce. Thus, it is not surprising that the optimal balance between keeping marriages intact, despite persistent spousal disputes, or allowing for separation continues to be a subject of policy debate, even after years of changes to divorce laws.[1]

In this dissertation, I try to explain why some married couples have frequent disputes but keep living together, while other couples cooperate, and the rest separate. I construct and estimate a game theoretic model of family negotiations with three potential equilibrium outcomes: cooperation, conflict, and divorce. The structural approach allows me to quantify the welfare effects of persistent spousal disagreements, weigh the tradeoffs between conflict and divorce, and evaluate the impact on household bargaining of public policies targeting divorce.

The phenomenon of conflict in an intact marriage is empirically relevant. For

[1]My primary focus in this dissertation is on conflicts between spouses in an intact marriage that do not escalate to physical violence, which is dealt with by the law in a separate fashion. The term "spousal conflict" is synonymous here to "marital conflict," "parental conflict," and "family conflict," while the term "divorce" is equivalent to "separation" and "marital break-up."

instance, in the National Survey of Families and Households (NSFH), which covers a probability sample of 13,000 households, 23% of married couples report that they have disputes at least several times a week. Remarkably, four-fifths of these couples are still intact when they are reinterviewed five and a half years later. Moreover, a considerable proportion do not deal with conflict in a constructive way. 27% of couples admit that they seldom calmly discuss serious disagreements and 10% often heatedly argue or shout at each other.

The NSFH also reports what spouses typically argue about. Married couples often clash over household tasks, money, and spending time together. From an economics perspective, these types of disagreements are particularly interesting, since they pertain to the allocation of valuable household resources. Chronic unresolved disputes attest to a failure in the process of family decision-making and may indicate an inefficient resource allocation.

The discussion of marital clashes in the economics literature has been limited to extreme forms of disputes and their consequences, such as spousal homicide or suicide (e.g., Dee, 2003; Stevenson and Wolfers, 2006) and physical abuse (e.g., Tauchen et al., 1991; Stevenson and Wolfers, 2006; Bowlus and Seitz, 2006). Persistent disagreements that may involve heated arguments and shouting but do not escalate to severe violence have been largely neglected.

Comprehensive analysis of consequences of spousal conflict comes from the psychology literature as summarized by Booth et al. (2001) and Grych and Fincham (2001). Although many effects analyzed in this literature are associative and only a few have been plausibly shown to be causal, there is little doubt that even nonviolent clashes indicate profound negative outcomes for the well-being of the family. For adults, marital disagreements are associated with depression. Disputes are also tied to alcoholism, various illnesses, and a deterioration of the parent-child relationship.

Children who live in homes with conflict demonstrate high anxiety levels, low self-esteem, depression, and bad health. Parental clashes are often linked to children's conduct problems, such as frequent loss of temper, aggressive behaviors, and trouble with the police, in school, and with peers. Emery (1982), Amato et al. (1995), and Jekielek (1998) suggest that the offspring may, in fact, sometimes benefit from divorce, especially when the conflict between their parents is very intense. However, Hanson (1999) finds that for many indicators of well-being, divorce has a deleterious impact on children regardless of the level of conflict at home.

In comparison to the existing literature, this research has several novel features. First, marital conflict is treated as a third distinct equilibrium outcome of spousal bargaining, whereas most papers consider only the polar cases of cooperation and divorce and ignore any alternatives in between. Second, I model marital negotiations as a noncooperative game. Despite the clear advantages of this approach long noted by Lundberg and Pollak (1994), it still remains a far less popular methodological choice among family economists than the cooperative bargaining and collective representations of spousal interactions. Unlike these two relatively simple but restrictive modeling techniques, the noncooperative framework allows me to endogenize Pareto inferior outcomes and incorporate two sources of asymmetric information (on the differential impacts of conflict and divorce). Third, the rich data in the NSFH are crucial for identification and enable the construction of indicators of conflict, optimism or pessimism about one's own divorce prospects, and beliefs about the spouse's divorce prospects. Moreover, these unique data in conjunction with the modeling approach allow me to directly address the issue of the welfare implications of conflict rather than to focus on its symptoms (e.g., fighting, depression, and various types of bad behavior). Lastly, I parameterize the outside options in terms of separation period requirements and the strength of child support enforcement, compute the effects of these variables separately on husbands and wives, and conduct policy experiments of great public significance.

To endogenize spousal conflict along with cooperation and divorce, I construct a fairly simple game, in which a husband and a wife bargain over the marital surplus. As a result of the negotiations, the couple ends up in one of three distinct marital states: cooperation, conflict, or divorce. The state of cooperation occurs if the spouses reach an agreement about the transfer. The state of conflict happens if the negotiations fail, but the couple prefers to remain intact. The state of divorce is the result of the decision to divorce by either spouse. Divorce is assumed to be unilateral, since it is allowed in nearly all U.S. states.

A crucial element of the model is the presence of asymmetric information along two dimensions. First, husbands and wives may be differentially affected by marital conflict. For simplicity, a spouse may be either a "soft bargainer" who is deeply hurt by persistent disagreements, or a "hard bargainer" who is not particularly hurt. Second, all married individuals may have differential post-divorce opportunities. An "optimist" thinks that the divorce prospects are good, while a "pessimist" believes that the prospects are poor. Knowledge of the true individual type is private information of a spouse, and the other spouse has beliefs that may or may not be accurate. Information about the second dimension of the asymmetry is directly observed to a researcher in the NSFH, while information about the first dimension is primarily inferred from conflict and divorce outcomes.

Under relatively unrestrictive assumptions, I formally establish several dominance results for the game and prove the existence of an equilibrium. Next, I analytically derive the likelihood contribution of a couple, which is the probability of observing a particular marital state as an equilibrium outcome. The structural parameters are obtained by the maximum simulated likelihood method. In short, the empirical strategy is to predict the marital state – cooperation, conflict, or divorce – of a couple observed as of the second NSFH wave (1992-94) given data from the initial interview in 1987-88. In addition to basic demographic characteristics of spouses, the first wave includes information on their outside options and beliefs about the partners' outside options. In the second wave, conflict is defined conservatively based on how often couples report open disagreements and how belligerently those disagreements are handled.

I parameterize spousal divorce utilities using policy and other variables collected from secondary sources. I control for local marriage market conditions by constructing availability ratios using the 1990 Decennial Census (see Goldman et al., 1984; Fossett and Kiecolt, 1991). I also include indicators for the length of mandatory separation periods before a divorce decree can be granted by courts, which varies across U.S. states (Friedberg, 1998). Moreover, I incorporate the strength of child support enforcement by computing state-specific collection rates for child support cases on file (Nixon, 1997). The detailed specification of the divorce utilities is essential for policy experiments, in which I evaluate the effect of altering separation requirements or the strength of child support enforcement.

The model fits the data reasonably well, as χ^2 goodness-of-fit statistics exceed 5% critical levels for very few subsets of the estimation sample. Moreover, it has a good out-of-sample predictive ability, since the extent of overprediction of the divorce rate in the prediction sample is small. The main estimation results are as follows. The model implies that a predominant majority of spouses are deeply hurt by marital conflict and think that their own divorce opportunities are poor. Still, overestimating a partner's disutility from conflict and underestimating the outside prospects lead to inefficient negotiation outcomes. I also consider the impact of demographic characteristics on utility in each state. Husband's age and Roman Catholic religious affiliation tend to have a positive impact on spousal utilities when the marriage is intact.[2] Marital heterogamy indicators, such as the difference in spousal ages, have a negative impact. There is a positive effect from marital duration and from common children under 6 years of age and a negative effect from stepchildren. Yet, in the state of conflict, the presence of a stepchild turns out to partially mitigate the disutility impact of persistent disputes. In the state of divorce, more favorable conditions in the local marriage market increase the payoff, but the effect is several times larger for women than men. Mandatory separation requirements, especially in the case of longer periods, reduce the divorce utilities. Higher child support collection rates reduce the divorce payoffs of high school and college educated husbands (who presumably have to transfer more income to ex-wives) and improve the post-marital prospects of wives without a high school degree or with a college diploma.

Given the structural parameters, I simulate the impact of less stringent separation requirements and better enforcement of child support payments on the incidence of each bargaining outcome. I find that the elimination of separation periods increases the fraction of divorced couples by 0.9 percentage points (or 8.4% of the observed divorce rate), but is a weak deterrent to persistent disputes. In contrast, strong child support enforcement has a potential to simultaneously reduce the incidence of divorce and conflict by as much as 1 and 1.9 percentage points (or 9.2 and 18.4% of the respective rates).

The rest of this dissertation is organized as follows. In Chapter 2, I review the existing economics literature on family decision-making. In Chapter 3, I describe the economic model, study its properties, and prove the existence of a bargaining equilibrium. Details on the data and nonstructural analysis are provided in Chapter 4.

[2] The effect of a demographic variable on spousal utilities in an intact marriage (i.e., in the states of cooperation and conflict) is identified relative to the impact of the variable on the divorce payoffs.

In Chapter 5, I parameterize the model and outline the estimation strategy. The discussion of the estimation results, policy simulations, goodness of fit and out-of-sample predictive power of the model is given in Chapter 6. I conclude in Chapter 7 and relegate technical details, as well as additional results to appendices.

Chapter 2

Literature Review

In spite of many decades of active research, the modeling of spousal interactions still remains a methodologically challenging task. The purpose of this chapter is to show that the noncooperative game theoretic framework offers substantial advantages in simultaneously modeling spousal cooperation, conflict, and divorce over other popular methodological approaches.

2.1 Traditional Framework and Its Limitations

Traditional, or "unitary," models of the family, such as the renowned altruist model of Becker (1974b), suppose that either all family members have identical preferences or that the preferences of only one member matter in the household decision making process. Given that this approach effectively assumes away a fundamental source of disagreements (i.e., different preferences), these models cannot provide an adequate account of observable cooperation, conflict, and divorce.

Moreover, traditional models presume the pooling of resources in a household, which is persuasively rejected by data from diverse societies all over the world (Bourguignon et al., 2006). For example, the pooling hypothesis is tested and rejected by Schultz (1990), who uses household data on hours worked and fertility in Thailand, by Thomas (1990, data on nutrition, child survival and health in Brazil), Haddad and Hoddinott (1994, child anthropometric status in Côte d'Ivoire), Browning et al. (1994, clothing spending in Canada), Hoddinott and Haddad (1995, consumption of food, alcohol, and cigarettes in Côte d'Ivoire), Lundberg et al. (1997, clothing expenditures in the U.K.), Browning and Chiappori (1998, spending on household services, food, transportation, clothing, recreation, tobacco, and alcohol in Canada), Phipps and Burton (1998, food, clothing, childcare, and transportation expenditures in Canada), Thomas et al. (2002, child health in Indonesia), Duflo (2003, nutrition of grandchildren in South Africa), Ward-Batts (2003, expenditures on housing, food, tobacco, cosmetics, clothing, and toys in the U.K.), and Duflo and Udry (2004, consumption of adult and prestige goods, education, staples, and vegetables in Côte d'Ivoire).

2.2 Cooperative Bargaining and Collective Models

Cooperative bargaining models, e.g., Manser and Brown (1980) and McElroy and Horney (1981), emerged in response to the limitations of the traditional framework. These papers acknowledge the importance of interaction between spouses, but treat the negotiation process as a "black box." Specifically, the idea is to impose suitable assumptions (perfect information, Pareto efficiency, symmetry, and invariance of the negotiation outcome, etc.), so that the standard cooperative bargaining solutions of Nash (1953) or Kalai and Smorodinsky (1975) can be applied.

Collective models of the family, e.g., Chiappori (1988), dispense with almost all assumptions of the cooperative bargaining approach, except perfect information and

Pareto efficiency of observable outcomes. Chiappori (1992) shows that these remaining assumptions imply the existence of a "sharing rule." In that case, the family decision-making process can be thought of as consisting of two stages. Total family income is first divided between household public goods and the private spending of each family member. Then, family members independently decide how to allocate their shares across private goods. Browning and Chiappori (1998) elaborate on the idea of the "sharing rule" by demonstrating that the symmetry of the Slutsky substitution matrix is lost in the collective setting.

2.3 Criticism of Cooperative Bargaining and Collective Models

Cooperative bargaining and collective models have little to say about spousal conflict in an intact marriage. Most existing applications consider only the polar cases of cooperation and divorce and ignore any possibilities in between. The "separate spheres" model of Lundberg and Pollak (1993) is an exception. In this model, the alternative to cooperation is not divorce, but an inefficient noncooperative equilibrium within marriage.[1] According to the authors, the dominated outcome may result from a game of voluntary contributions in which spouses separately consume their incomes and independently supply household public goods. However, the "separate spheres" model cannot generate this noncooperative equilibrium as an observable outcome without additional ad hoc restrictions.

More importantly, an inherent weakness of the cooperative bargaining and collective approaches lies in the dubious validity of the Pareto efficiency and perfect information assumptions. For instance, Udry (1996) shows that farming households

[1] The "separate spheres" model does not allow for divorce as an outcome of spousal interaction.

in Burkina Faso do not achieve an efficient allocation of resources across the production activities of family members. Likewise, Duflo and Udry (2004) demonstrate that household expenditure patterns in Côte d'Ivoire deviate from optimality and spouses do not fully insure each other against short term variation in individual incomes. Therefore, as pointed out by Lundberg and Pollak (1996), Pareto efficiency needs to be carefully investigated and not simply assumed.

The assumption of perfect information needs to be relaxed, as well. As discussed by Becker et al. (1977), married individuals may face uncertainty about their own and their mate's needs, their capacity to get along with each other, and so on. Moreover, Becker (1991) notes that some information asymmetries may persist for many years of marriage.

Empirically, the perfect information assumption has been proven false. Friedberg and Stern (2006) find a substantial discrepancy in the NSFH data that I use between the opinions of spouses about their own well-being after hypothetical divorce and their partners' beliefs about it. For example, in my estimation sample, 16% of wives say that their overall happiness would be the same or better after divorce when the husbands, in fact, believe it would be worse. Likewise, 18% of husbands report that their happiness would stay the same or improve when the wives, on the contrary, believe it would worsen.

2.4 Noncooperative Game Theoretic Approach

The clear advantages of the noncooperative game theoretic approach to modeling spousal interactions have long been noted by Lundberg and Pollak (1994). Unlike the cooperative bargaining and collective frameworks, noncooperative games impose few restrictions on the nature of equilibrium outcomes and can readily handle asymmetric information. However, they require a precise specification of the bargaining protocol (Kreps, 1990) and, therefore, remain a relatively less popular choice among family economists when analyzing marriage.[2]

Methodologically, my approach to modeling family negotiations is similar to the noncooperative bargaining approach of Friedberg and Stern (2006). In that paper, the interaction between the spouses is assumed to be a one-stage game. The husband offers a side payment to the wife that allocates marital surplus. The wife can either accept the offer, in which case the marriage stays intact, or reject the offer, in which case divorce ensues. Each spouse knows his or her own outside option, but may have an incorrect belief about the outside option of the partner. This information imperfection results in a bargaining inefficiency. Some divorces could be avoided and the total expected value of the marital match would be higher if there were no asymmetric information.

In contrast to Friedberg and Stern (2006), I additionally allow for marital conflict as an outcome of bargaining and consider an extra source of information asymmetry related to the impact of persistent disputes on individuals.[3]

[2] Noncooperative game theoretic models are very popular in other areas of family economics. For instance, Hiedemann and Stern (1999) and Engers and Stern (2002) specify and estimate structural game theoretic models of family bargaining regarding the provision of long-term care to elderly parents. Another example is Hao, Hotz, and Jin (2005), who consider a game in which parents can threaten to withhold transfers in order to prevent adolescent children from engaging in risky behaviors.

[3] Recent structural models of Tartari (2005) and Bowlus and Seitz (2006) allow for marital conflict and domestic violence, respectively. However, they do not do so in the context of family bargaining over marital surplus and ignore the possibility of information asymmetry. Moreover, the focus of these papers is different from mine. Tartari (2005) analyzes the relationship between marital status and a child's cognitive achievement, while Bowlus and Seitz (2006) look at the response of wives to physical abuse via employment and divorce decisions.

Chapter 3

Economic Model

3.1 Bargaining Protocol and Marital States

Consider an intact marriage with two decision-makers, a husband and a wife. The spousal interaction is assumed to comprise a two-stage bargaining game over the allocation of marital surplus. A schematic structure of the game is given in Figure 3.1.

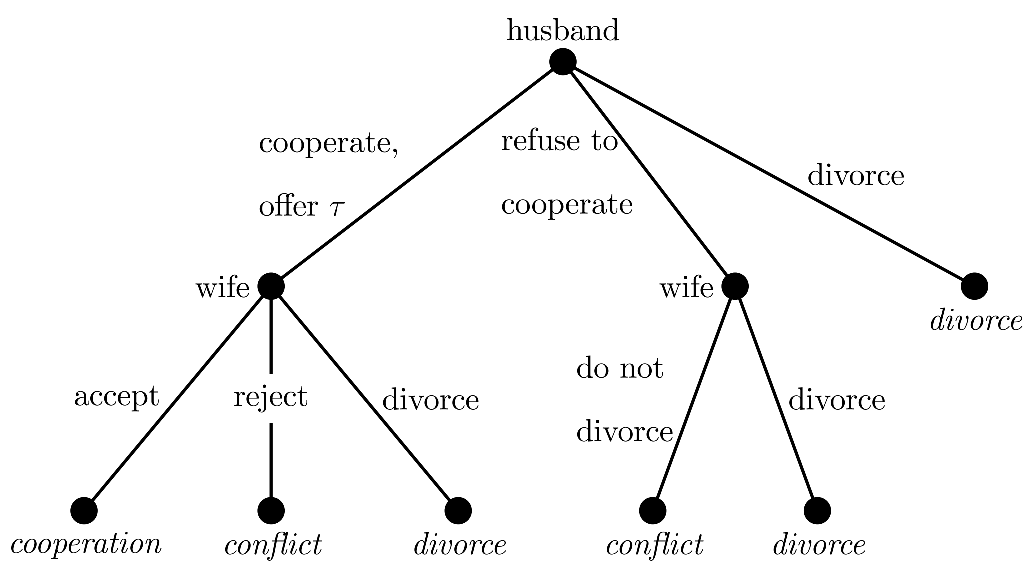

Figure 3.1: Structure of the Game

The husband moves first and (1) proposes cooperation and offers some transfer τ, (2) refuses to cooperate but abstains from separating, or (3) announces divorce. The strategies are respectively denoted by $(\tau; \mathcal{C})$, $\mathcal{R}$, and $\mathcal{D}$, and the transfer may be negative.

The wife observes her husband's action and makes her move. If he chose $(\tau; \mathcal{C})$, she (1) accepts the offer, (2) rejects it without separating, or (3) announces divorce. If he picked action $\mathcal{R}$, she either (1) abstains from separating, or (2) divorces. If the husband chose $\mathcal{D}$, the bargaining game is over before the wife gets to move.

The assumption about the order of the player moves in the game is relatively strong, but I must make it here, since the NSFH contains no information on the specifics of the actual bargaining protocol. In fact, this premise may not be as restrictive as it appears, and the model is straightforward to estimate assuming that the wife is the first to act.[1]

The game ends with one of three mutually exclusive and exhaustive outcomes, which are referred to as "marital states." The marital state of cooperation occurs if the spouses reach an agreement about the transfer, i.e., the husband makes an offer and the wife accepts it. The state of conflict happens if the negotiations fail, but the couple remains intact, i.e., either the husband offers a transfer and the wife rejects it, or he refuses to cooperate and she abstains from separating. The state of divorce is the result of a unilateral decision by either spouse to terminate the marriage. Divorce is assumed to be unilateral, since it is allowed in nearly all U.S. states.[2]

[1] Friedberg and Stern (2006) find that the order of the moves in their model has practically no impact on the results.

[2] A very different bargaining protocol would be needed to model divorce that requires consent of both spouses ("mutual divorce"). I do not address the issue of mutual divorce in this dissertation.

3.2 Spousal Types and Beliefs

Spouses differ with respect to individual characteristics, many of which are observable to their partners, but others of which may be unobservable. Here, I introduce two dimensions of unobservable individual heterogeneity, which induces asymmetric information in the marital negotiation process.

First, spouses may be differentially affected by conflict. This individual trait is referred to as "bargaining strength," and, for simplicity, it is restricted to two possible levels. A spouse may be either (1) deeply hurt by conflict (in which case the spouse is said to be a "soft bargainer"), or (2) not particularly hurt ("hard bargainer").

Second, spouses may have differential divorce opportunities. This trait is called "optimism," and, again, it is limited to two levels. A spouse may think that the divorce prospects are either (1) good (the spouse is an "optimist"), or (2) poor ("pessimist").

A spousal type is a combination of the trait levels. In total, there are four possible types: "hard bargainer – optimist," "hard bargainer – pessimist," "soft bargainer – optimist," and "soft bargainer – pessimist," respectively denoted as HO, HP, SO, and SP. The husband's types are indexed by k and the wife's by l.

Knowledge of one's true type is private information, and a spouse has beliefs about the partner's type. Let δ^l stand for the probability the husband assigns to the event that his wife's type is l. Husband's beliefs satisfy the usual restrictions: $0 \leq \delta^l \leq 1$ for any l and $\sum_l \delta^l = 1$.

Given the structure of the game and the specification of the spousal payoffs in the following section, beliefs of the wife do not affect the outcome of the negotiations.

3.3 Payoffs

Spouses receive payoffs in the form of utilities that are specific to marital state, sex, and possibly type.

In the state of cooperation, the husband obtains utility $u_h(-\tau)$ and the wife receives $u_w(\tau)$. These payoffs are invariant with respect to spousal types, but depend on the transfer, τ. The symbolic minus before τ in $u_h(\cdot)$ indicates that the husband is worse off when a higher transfer is made to the wife. Functions $u_h(\cdot)$ and $u_w(\cdot)$ are continuous and monotone in τ:

$$u_h\left(-\tau^1\right) < u_h\left(-\tau^2\right) \text{ and } u_w\left(\tau^1\right) > u_w\left(\tau^2\right) \text{ if } \tau^1 > \tau^2.$$

The domain of the utility functions is restricted to transfer set $[\tau_{\min}, \tau_{\max}]$, where $\tau_{\min}$ is a very large negative number and $\tau_{\max}$ is a very large positive number. This technical assumption allows me to rigorously establish several properties of the game and plays no practical role in the empirical application.

In the state of conflict, no transfer is made, but the payoff of the husband, denoted as v_h^k, and the payoff of the wife, v_w^l, are specific to the trait of "bargaining strength." All else equal, a husband who is a "hard bargainer – optimist" obtains the same utility v_h^H as a husband who is a "hard bargainer – pessimist," with $v_h^{HO} = v_h^{HP} = v_h^H$, and "soft bargainers" receive v_h^S, with $v_h^{SO} = v_h^{SP} = v_h^S$. Analogous restrictions are imposed on the payoffs of the wife. According to the definition of the trait of "bargaining strength," a "soft bargainer" is relatively worse off in the case of conflict:

$$v_h^S < v_h^H \text{ and } v_w^S < v_w^H.$$

In line with Lundberg and Pollak (1993), the spousal payoffs in the state of conflict are inside the utility possibility frontier that results from cooperation. Technically, this assumption implies the existence of an admissible transfer $\tau^0 \in [\tau_{\min}, \tau_{\max}]$ such that each spouse would be better off if they cooperated:

$$u_h\left(-\tau^0\right) > v_h^H \text{ and } u_w\left(\tau^0\right) > v_w^H.$$

I also specify that transfers $\tau_{\min}$ and $\tau_{\max}$ correspond to unbearably large sacrifices of utility on part of the wife and husband, so that conflict would be preferred:

$$v_w^S > u_w\left(\tau_{\min}\right) \text{ and } v_h^S > u_h\left(-\tau_{\max}\right).$$

In the state of divorce, the husband's utility, denoted as y_h^k, and the wife's utility, y_w^l, are specific to the trait of "optimism." A husband who is a "hard bargainer – optimist" obtains the same utility y_h^O as a husband who is a "soft bargainer – optimist," with $y_h^{HO} = y_h^{SO} = y_h^O$, and "pessimists" receive y_h^P, with $y_h^{HP} = y_h^{SP} = y_h^P$. Similar conditions are imposed on the payoffs of the wife.

Given the definition of the trait of "optimism," a "pessimist" is relatively worse off in the case of divorce:

$$y_h^P < y_h^O \text{ and } y_w^P < y_w^O.$$

3.4 Solution Approach

The bargaining game can be solved by backward recursion. When the wife responds to the husband, she chooses the strategy that maximizes her utility given her type l.

Suppose that the husband proposes cooperation and offers transfer τ. The wife accepts the offer if $u_w(\tau) \geq y_w^l$ and $u_w(\tau) \geq v_w^l$.[3] She rejects the offer, but abstains from separating if $v_w^l \geq y_w^l$ and $v_w^l > u_w(\tau)$. Lastly, she announces divorce if $y_w^l > v_w^l$ and $y_w^l > u_w(\tau)$. Now, suppose the husband refuses to cooperate. If $v_w^l \geq y_w^l$, the

[3]I specify the actions when inequalities hold as strict equalities and focus on equilibria in pure strategies.

wife optimally chooses not to separate. Otherwise, she is better off by announcing divorce.

In turn, the husband anticipates what the response of each type of wife would be to any strategy he chooses (a "hard bargainer – optimist" may respond differently than, say, a "soft bargainer – pessimist"). Since the husband does not know the wife's true type but has beliefs about the types, then, for any strategy choice, he knows the probability distribution of the outcome and the payoff. Therefore, it is natural to treat the husband of a given type k as choosing a strategy by maximizing his expected utility.

Let the expected utilities of the husband of type k corresponding to strategies $(\tau; \mathcal{C})$, $\mathcal{R}$, and $\mathcal{D}$ be respectively denoted as $\hat{E}\mathcal{V}_h^k(\tau; \mathcal{C})$, $\hat{E}\mathcal{V}_h^k(\mathcal{R})$, and $\hat{E}\mathcal{V}_h^k(\mathcal{D})$. To write these expected utilities in closed form, I use indicator functions:

$$1\begin{pmatrix} \text{condition } 1 \\ \vdots \\ \text{condition } m \end{pmatrix} = \begin{cases} 1, & \text{if conditions 1 through } m \text{ are true,} \\ 0, & \text{otherwise.} \end{cases}$$

If the husband proposes cooperation and offers transfer τ, his expected utility is:

$$\begin{aligned} \hat{E}\mathcal{V}_h^k(\tau; \mathcal{C}) &= \sum_l \delta^l \left[u_h(-\tau) \cdot 1 \begin{pmatrix} u_w(\tau) \geq y_w^l \\ u_w(\tau) \geq v_w^l \end{pmatrix} + v_h^k \cdot 1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix} + \right. \\ &\quad \left. + \, y_h^k \cdot 1 \begin{pmatrix} y_w^l > v_w^l \\ y_w^l > u_w(\tau) \end{pmatrix} \right]. \end{aligned}$$

If the husband refuses to cooperate, his expected utility is:

$$\hat{E}\mathcal{V}_h^k(\mathcal{R}) = \sum_l \delta^l \left[v_h^k \cdot 1\left(v_w^l \geq y_w^l\right) + y_h^k \cdot 1\left(y_w^l > v_w^l\right) \right].$$

If the husband announces divorce, his expected utility is:

$$\hat{E}\mathcal{V}_h^k(\mathcal{D}) = y_h^k.$$

The optimization problem of the husband of type k can be formally stated as:

$$\max_{\{\mathcal{C},\ \mathcal{R},\ \mathcal{D}\}} \left\{ \max_{\tau} \hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}),\ \hat{E}\mathcal{V}_h^k(\mathcal{R}),\ \hat{E}\mathcal{V}_h^k(\mathcal{D}) \right\}. \tag{3.1}$$

3.5 Game Properties and Equilibrium Existence

The properties of the game require close scrutiny due to two technical issues. First, the husband has uncountably many strategies of type $(\tau;\mathcal{C})$, which implies that the game is infinite. Second, the expected utility of proposing cooperation is discontinuous in τ. Therefore, I cannot apply standard theorems to prove the existence of an equilibrium.

Fortunately, it is possible to establish several important properties of the husband's expected utility functions and considerably simplify his optimization problem.

First, I show that there is a set of a priori dominated transfers.

Theorem 1 For all transfers in set $\left\{\tau : u_h(-\tau) < y_h^k\right\}$:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) \le \max\left\{\hat{E}\mathcal{V}_h^k(\mathcal{R}), \hat{E}\mathcal{V}_h^k(\mathcal{D})\right\}.$$

Proof. To start with, note that for any l:

$$1\begin{pmatrix} y_w^l > v_w^l \\ y_w^l > u_w(\tau) \end{pmatrix} + 1\begin{pmatrix} v_w^l \ge y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix} + 1\begin{pmatrix} u_w(\tau) \ge y_w^l \\ u_w(\tau) \ge v_w^l \end{pmatrix} = 1,$$

$$1\left(y_w^l > v_w^l\right) + 1\left(v_w^l \ge y_w^l\right) = 1,$$

and also:

$$\sum_l \delta^l = 1.$$

Then, for convenience, rewrite $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ and $\hat{E}\mathcal{V}_h^k(\mathcal{R})$ as:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) = y_h^k \sum_l \delta^l 1\begin{pmatrix} y_w^l > v_w^l \\ y_w^l > u_w(\tau) \end{pmatrix} +$$

$$+v_h^k \sum_l \delta^l 1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix} + u_h(-\tau) \sum_l \delta^l 1 \begin{pmatrix} u_w(\tau) \geq y_w^l \\ u_w(\tau) \geq v_w^l \end{pmatrix},$$

$$\hat{E}\mathcal{V}_h^k(\mathcal{R}) = y_h^k + \left(v_h^k - y_h^k\right) \sum_l \delta^l 1 \left(v_w^l \geq y_w^l\right).$$

Note that irrespective of τ, there are two mutually exclusive and exhaustive possibilities: either $v_h^k < y_h^k$, or $v_h^k \geq y_h^k$.

Consider arbitrary τ such that $u_h(-\tau) < y_h^k$. Suppose it is the case that $v_h^k < y_h^k$. It follows that:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) \leq y_h^k \sum_l \delta^l 1 \begin{pmatrix} y_w^l > v_w^l \\ y_w^l > u_w(\tau) \end{pmatrix} +$$

$$+y_h^k \sum_l \delta^l 1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix} + y_h^k \sum_l \delta^l 1 \begin{pmatrix} u_w(\tau) \geq y_w^l \\ u_w(\tau) \geq v_w^l \end{pmatrix} =$$

$$= y_h^k \equiv \hat{E}\mathcal{V}_h^k(\mathcal{D}) \leq \max\left\{\hat{E}\mathcal{V}_h^k(\mathcal{R}), \hat{E}\mathcal{V}_h^k(\mathcal{D})\right\}.$$

Next, suppose it is the case that $v_h^k \geq y_h^k$. Then:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) \leq y_h^k \sum_l \delta^l \left[1 \begin{pmatrix} y_w^l > v_w^l \\ y_w^l > u_w(\tau) \end{pmatrix} + 1 \begin{pmatrix} u_w(\tau) \geq y_w^l \\ u_w(\tau) \geq v_w^l \end{pmatrix}\right] +$$

$$+v_h^k \sum_l \delta^l 1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix} = y_h^k + \left[v_h^k - y_h^k\right] \sum_l \delta^l 1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix}.$$

By properties of indicator functions, $1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix} \leq 1\left(v_w^l \geq y_w^l\right)$. Also, $\delta^l \geq 0$ for every l. Then, given $v_h^k - y_h^k \geq 0$, it follows that:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) \leq y_h^k + \left[v_h^k - y_h^k\right] \sum_l \delta^l 1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix} \leq$$

$$\leq y_h^k + \left(v_h^k - y_h^k\right) \sum_l \delta^l 1 \left(v_w^l \geq y_w^l\right) = \hat{E}\mathcal{V}_h^k(\mathcal{R}) \leq \max\left\{\hat{E}\mathcal{V}_h^k(\mathcal{R}), \hat{E}\mathcal{V}_h^k(\mathcal{D})\right\}.$$

Since $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) \leq \max\left\{\hat{E}\mathcal{V}_h^k(\mathcal{R}), \hat{E}\mathcal{V}_h^k(\mathcal{D})\right\}$ for any $\tau : u_h(-\tau) < y_h^k$, irrespective of the values of v_h^k and y_h^k, the inequality is true for all $\tau : u_h(-\tau) < y_h^k$. ∎

The intuition behind the result is that large transfers adversely affect the husband, but he can ignore them, since relatively better options are always available. Let T^k stand for the complement set (with respect to $[\tau_{\min}, \tau_{\max}]$) to the set of a priori dominated transfers: $T^k = \left\{\tau : u_h(-\tau) \geq y_h^k,\ \tau_{\min} \leq \tau \leq \tau_{\max}\right\}$. T^k is a compact set. In the light of Theorem 1, it suffices to analyze transfers from T^k only.

To facilitate further analysis, I prove three lemmas that are based on properties of semicontinuous functions.[4]

Lemma 1 For any l, functions $1\begin{pmatrix} y_w^l > v_w^l \\ y_w^l > u_w(\tau) \end{pmatrix}$ and $1\begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix}$ are lower semicontinuous in τ.

Proof. Consider arbitrary l. The functions can be expressed as:

$$1\begin{pmatrix} y_w^l > v_w^l \\ y_w^l > u_w(\tau) \end{pmatrix} = 1\left(y_w^l > v_w^l\right) \cdot 1\left(y_w^l > u_w(\tau)\right),$$

$$1\begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix} = 1\left(v_w^l \geq y_w^l\right) \cdot 1\left(v_w^l > u_w(\tau)\right).$$

By continuity of $u_w(\tau)$ in τ, inequalities $y_w^l > u_w(\tau)$ and $v_w^l > u_w(\tau)$ define open sets. Then, $1\left(y_w^l > u_w(\tau)\right)$ and $1\left(v_w^l > u_w(\tau)\right)$ are lower semicontinuous as indicator functions of open sets.

$1\left(y_w^l > v_w^l\right)$ and $1\left(v_w^l \geq y_w^l\right)$ are nonnegative constants with respect to τ. Therefore, products $1\left(y_w^l > v_w^l\right) \cdot 1\left(y_w^l > u_w(\tau)\right)$ and $1\left(v_w^l \geq y_w^l\right) \cdot 1\left(v_w^l > u_w(\tau)\right)$ are lower semicontinuous in τ. ∎

[4] See Jost (2003) and Rudin (1987) for properties of semicontinuous functions.

Lemma 2 For any k, function:

$$f(\tau) = \left[v_h^k - u_h(-\tau)\right] \sum_l \delta^l 1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix}$$

is upper semicontinuous in τ.

Proof. Under the assumptions imposed on the payoffs, there exists τ^0 such that:

$$\begin{aligned} u_h\left(-\tau^0\right) &> v_h^H \geq v_h^k, \text{ for any } k, \text{ and} \\ u_w\left(\tau^0\right) &> v_w^H \geq v_w^l, \text{ for any } l. \end{aligned}$$

Consider arbitrary $\tau \geq \tau^0$. Since $u_w(\tau)$ increases in τ, it is the case that $u_w(\tau) > u_w(\tau^0) > v_w^H \geq v_w^l$. Therefore, $1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix} = 0$ for any l and $f(\tau) = 0$. Then, for $\tau \geq \tau^0$, $f(\tau)$ is trivially continuous and, thus, upper semicontinuous.

Next, consider arbitrary $\tau < \tau^0$. Since $u_h(-\tau)$ decreases in τ, it is the case that $u_h(-\tau) > u_h(-\tau^0) > v_h^H \geq v_h^k$. Since $u_h(-\tau)$ is also continuous in τ, it follows that for any k, $u_h(-\tau) - v_h^k$ is positive, continuous and, thus, lower semicontinuous.

Since $\delta^l \geq 0$, Lemma 1 implies that $\sum_l \delta^l 1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix}$ is a nonnegative lower semicontinuous function in τ. Then:

$$-f(\tau) = \left[u_h(-\tau) - v_h^k\right] \sum_l \delta^l 1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix}$$

is lower semicontinuous for $\tau < \tau^0$ as the product of two nonnegative lower semicontinuous functions. Then, function $f(\tau)$ is upper semicontinuous for $\tau < \tau^0$.

It follows that $f(\tau)$ is upper semicontinuous for all τ. ■

Lemma 3 For any k, function:

$$f(\tau) = \left[y_h^k - u_h(-\tau)\right] \sum_l \delta^l 1 \begin{pmatrix} y_w^l > v_w^l \\ y_w^l > u_w(\tau) \end{pmatrix}$$

is upper semicontinuous on T^k.

Proof. Since $\delta^l \geq 0$, Lemma 1 implies that $\sum_l \delta^l 1 \begin{pmatrix} y_w^l > v_w^l \\ y_w^l > u_w(\tau) \end{pmatrix}$ is a nonnegative lower semicontinuous function in τ.

Now, recall that for all $\tau \in T^k$, $u_h(-\tau) \geq y_h^k$. Thus, function $u_h(-\tau) - y_h^k$ is nonnegative on T^k. It is also continuous and, thus, lower semicontinuous.

Then, the product of two nonnegative lower semicontinuous functions:

$$-f(\tau) = \left[u_h(-\tau) - y_h^k\right] \cdot \sum_l \delta^l 1 \begin{pmatrix} y_w^l > v_w^l \\ y_w^l > u_w(\tau) \end{pmatrix}$$

is lower semicontinuous for all $\tau \in T^k$.

It follows that $f(\tau)$ is upper semicontinuous on T^k. ■

I can now establish an important property of $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ on set T^k.

Theorem 2 Function $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ is upper semicontinuous on T^k.

Proof. The proof is a simple application of Lemmas 2 and 3.

It is straightforward to express function $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ as:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) = u_h(-\tau) + \left[v_h^k - u_h(-\tau)\right] \sum_l \delta^l 1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau) \end{pmatrix} +$$

$$+ \left[y_h^k - u_h(-\tau)\right] \sum_l \delta^l 1 \begin{pmatrix} y_w^l > v_w^l \\ y_w^l > u_w(\tau) \end{pmatrix}.$$

The first summand above is continuous and, thus, upper semicontinuous for all τ. The second summand is upper semicontinuous for all τ by Lemma 2. The third summand is upper semicontinuous on T^k by Lemma 3. Hence, $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ is upper semicontinuous on T^k as a finite sum of upper semicontinuous functions. ■

Whenever T^k is not empty, $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ attains its maximum on T^k (Jost, 2003, Lemma 12.6), i.e., $\max_{\tau\in T^k}\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ is well defined.

Next, I prove a crucial dominance result for the husband's strategies.

Theorem 3 For any k:

(1) if T^k is empty, then $\hat{E}\mathcal{V}_h^k(\mathcal{D}) \geq \hat{E}\mathcal{V}_h^k(\mathcal{R})$;

(2) if T^k is not empty, then $\max_{\tau\in T^k}\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) \geq \hat{E}\mathcal{V}_h^k(\mathcal{R})$.

Proof. To prove part (1), observe that if T^k is empty, its complement set with respect to $[\tau_{\min},\tau_{\max}]$, $\left\{\tau : u_h(-\tau) < y_h^k\right\}$, coincides with set $[\tau_{\min},\tau_{\max}]$.

Recall that under the assumptions imposed on the payoffs, there exists transfer $\tau^0 \in [\tau_{\min},\tau_{\max}]$ such that:

$$y_h^k > u_h\left(-\tau^0\right) > v_h^H \geq v_h^k.$$

Then:

$$\hat{E}\mathcal{V}_h^k(\mathcal{D}) \equiv y_h^k = y_h^k \sum_l \delta^l 1\left(y_w^l > v_w^l\right) + y_h^k \sum_l \delta^l 1\left(v_w^l \geq y_w^l\right) \geq$$
$$\geq y_h^k \sum_l \delta^l 1\left(y_w^l > v_w^l\right) + v_h^k \sum_l \delta^l 1\left(v_w^l \geq y_w^l\right) = \hat{E}\mathcal{V}_h^k(\mathcal{R}).$$

To prove part (2), recall that by upper semicontinuity of function $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ on non-empty compact set T^k, $\max_{\tau\in T^k}\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ is attained.

Now, consider transfer $\tau_{\min}$. Under the assumptions imposed on the payoffs, for any l:

$$v_w^l \geq v_w^S > u_w(\tau_{\min}).$$

Then, $1\begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau_{\min}) \end{pmatrix} = 1\left(v_w^l \geq y_w^l\right)$ and $1\begin{pmatrix} u_w(\tau_{\min}) \geq y_w^l \\ u_w(\tau_{\min}) \geq v_w^l \end{pmatrix} = 0$ for any l.

Since $1\left(\begin{array}{c} y_w^l > v_w^l \\ y_w^l > u_w(\tau_{\min}) \end{array}\right) + 1\left(\begin{array}{c} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau_{\min}) \end{array}\right) + 1\left(\begin{array}{c} u_w(\tau_{\min}) \geq y_w^l \\ u_w(\tau_{\min}) \geq v_w^l \end{array}\right) = 1,$
by simple substitution:

$$1\left(\begin{array}{c} y_w^l > v_w^l \\ y_w^l > u_w(\tau_{\min}) \end{array}\right) = 1 - 1\left(v_w^l \geq y_w^l\right).$$

It follows that:

$$\hat{E}\mathcal{V}_h^k(\tau_{\min};\mathcal{C}) = y_h^k \sum_l \delta^l 1\left(\begin{array}{c} y_w^l > v_w^l \\ y_w^l > u_w(\tau_{\min}) \end{array}\right) +$$

$$+ v_h^k \sum_l \delta^l 1\left(\begin{array}{c} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau_{\min}) \end{array}\right) + u_h(-\tau_{\min}) \sum_l \delta^l 1\left(\begin{array}{c} u_w(\tau_{\min}) \geq y_w^l \\ u_w(\tau_{\min}) \geq v_w^l \end{array}\right) =$$

$$= y_h^k + \left[v_h^k - y_h^k\right] \sum_l \delta^l 1\left(v_w^l \geq y_w^l\right) =$$

$$= y_h^k \sum_l \delta^l 1\left(y_w^l > v_w^l\right) + v_h^k \sum_l \delta^l 1\left(v_w^l \geq y_w^l\right) = \hat{E}\mathcal{V}_h^k(\mathcal{R}).$$

Clearly, $\tau_{\min} \in T^k$. Then, $\max_{\tau \in T^k} \hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) \geq \hat{E}\mathcal{V}_h^k(\tau_{\min};\mathcal{C}) = \hat{E}\mathcal{V}_h^k(\mathcal{R})$. ■

If the outside option of the husband is so high that all transfers are a priori dominated and, thus, T^k is empty, there is no reason for him to stay married and incite a conflict. Otherwise, the husband need not consider inciting a conflict on his own as he can do better by offering some acceptable transfer. Therefore, I can simplify the game by dropping strategy $\mathcal{R}$ as shown in Figure 3.2.

Together, Theorems 1 and 3 imply that the husband's optimization problem (3.1) can be simplified as:

$$\max_{\{\mathcal{C},\,\mathcal{D}\}} \left\{ \max_{\tau \in T^k} \hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}),\, \hat{E}\mathcal{V}_h^k(\mathcal{D}) \right\}. \quad (3.2)$$

Finally, I establish the existence of an equilibrium.

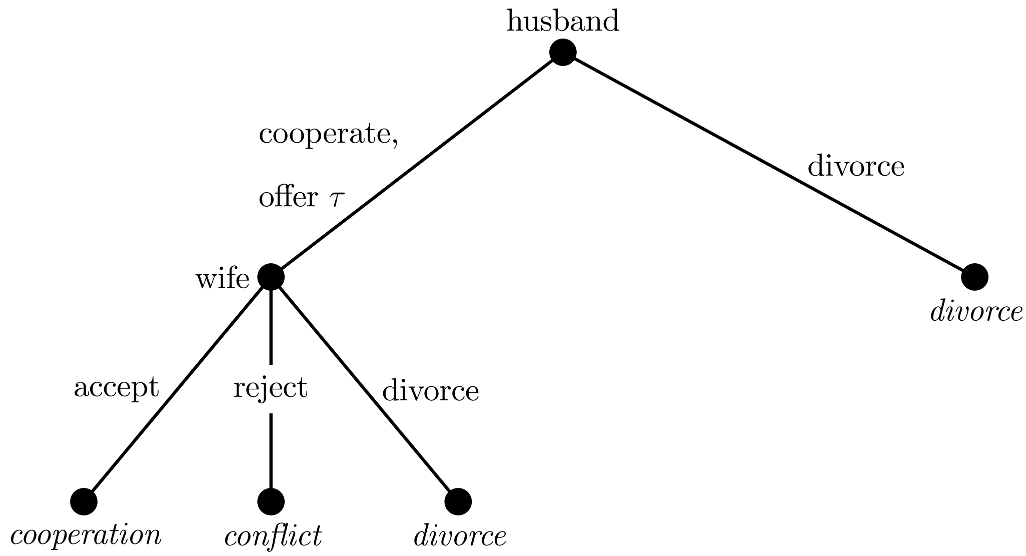

Figure 3.2: Simplified Structure of the Game

Theorem 4 An equilibrium of the game always exists.

Proof. The description of the wife's optimal strategy trivially implies that her best response function to any transfer offer τ is well defined for every wife's type l.[5]

Consider optimization problem (3.2) and let k be an arbitrary husband's type.

If set T^k is empty, Theorems 1 and 3 imply that $\mathcal{D}$ is the dominant strategy of the husband. Hence, he plays $\mathcal{D}$ and the game is over. An equilibrium can be specified in this case as the husband's strategy $\mathcal{D}$ and the wife's best response function (should the game ever reach an information set where the wife needs to move).

If set T^k is not empty, Theorem 2 implies that function $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ attains its maximum on T^k and, therefore, I can define $\tau^* = \arg\max_{\tau\in T^k} \hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$. Then, the husband's optimal strategic choice amounts to comparing two real numbers, $\hat{E}\mathcal{V}_h^k(\mathcal{D})$ and $\hat{E}\mathcal{V}_h^k(\tau^*;\mathcal{C})$. If on the basis of the comparison he picks $\mathcal{D}$, the game is over. Otherwise, an equilibrium comprises the husband's strategy $(\tau^*;\mathcal{C})$ and the wife's

[5] This best response function maps transfer offers into the set of wife's actions: (1) accept the offer, (2) reject the offer without separating, or (3) announce divorce.

best response function. ■

3.6 Role of Asymmetric Information

To illustrate the crucial role of asymmetric information in generating inefficient equilibrium outcomes, particularly the marital state of conflict, I consider an alternative model in which the spouses have private information about their own divorce prospects, but, unlike in the original model, perfectly observe the "bargaining strength" of their partners. I show that in this case the state of conflict cannot occur, unless husbands hold a specific set of degenerate beliefs about wives.

Since the "bargaining strength" of the partner is now common knowledge, the alternative model can be thought of as a special case of the original model with just two individual types: "optimist" (O) and "pessimist" (P). Then, for notational convenience, I can drop the type superscripts k and l on the spousal utilities in the state of conflict, v_h^k and v_w^l. In that case, v_h and v_w represent observable conflict payoffs of the husband and wife, respectively. It is straightforward to verify that Theorems 1 through 4 hold verbatim, except for this minor change in notation.

I can now establish an important property of the alternative model.

Theorem 5 The alternative model cannot generate the state of conflict if at least one of the following three conditions holds:

(1) $y_w^l > v_w$ for all l,

(2) husband's beliefs are such that $\delta^P > 0$,

(3) $y_w^l \leq v_w$ for all l.

Proof. Recall that the state of conflict occurs when the husband proposes cooperation and the wife rejects the offer, but abstains from divorcing. I show that each of the three conditions above rules out this outcome. In what follows, I can implicitly assume that set T^k is not empty. Otherwise, Theorems 1 and 3 imply that the husband plays strategy $\mathcal{D}$, which rules out conflict a priori.

Part I. The proof for condition (1) is trivial. If $y_w^l > v_w$ for all l, then announcing divorce is always a better choice for the wife than inciting conflict.

Part II. First, note that condition $y_w^l > v_w$ for all l can be either true or false. If it is true, then conflicts for all $\delta^P > 0$ are trivially ruled out by Part I.

Henceforth, suppose that condition $y_w^l > v_w$ for all l is false. Since $y_w^O > y_w^P$, this statement is equivalent to $v_w \geq y_w^P$.

Observe that if condition (2) holds:

$$\sum_l \delta^l 1\left(v_w \geq y_w^l\right) = \delta^O 1\left(v_w \geq y_w^O\right) + \delta^P 1\left(v_w \geq y_w^P\right) =$$

$$= \delta^O 1\left(v_w \geq y_w^O\right) + \delta^P \geq \delta^P > 0.$$

Next, recall that under the assumptions imposed on the payoffs, there exists transfer τ^0 such that $u_h\left(-\tau^0\right) > v_h$ and $u_w\left(\tau^0\right) > v_w$.

It is also true that $v_w > u_w\left(\tau_{\min}\right)$. Then, as $u_w\left(\tau\right)$ is continuous in τ, by the intermediate value theorem (Jost, 2003, Theorem 1.14), there exists transfer $\hat{\tau}$ such that $u_w\left(\hat{\tau}\right) = v_w$. Moreover, because $u_w\left(\tau\right)$ increases in τ, it must be that $\tau^0 > \hat{\tau}$. In turn, as $u_h\left(-\tau\right)$ decreases in τ, it follows that $u_h\left(-\hat{\tau}\right) > u_h\left(-\tau^0\right) > v_h$.

Clearly, the wife would not incite a conflict if transfer τ is such that $u_w\left(\tau\right) \geq v_w$.

Now, there are two mutually exclusive and exhaustive possibilities: either $\hat{\tau} \in T^k$, or $\hat{\tau} \notin T^k$.

Consider the case $\hat{\tau} \in T^k$. I show that the husband would offer such optimal transfer τ^* that the wife would not be willing to incite a conflict, i.e., $u_w\left(\tau^*\right) \geq v_w$.

By definition of T^k, $u_h\left(-\hat{\tau}\right) \geq y_h^k$ and, as $u_h\left(-\tau\right)$ decreases in τ, set $\left[\tau_{\min}, \hat{\tau}\right] \subset T^k$. Note that $u_w\left(\tau^*\right) \geq v_w = u_w\left(\hat{\tau}\right)$ is equivalent to $\tau^* \geq \hat{\tau}$.

It is straightforward to verify that:

$$\hat{E}\mathcal{V}_h^k(\hat{\tau};\mathcal{C}) = y_h^k \sum_l \delta^l 1\left(y_w^l > v_w\right) + u_h(-\hat{\tau}) \sum_l \delta^l 1\left(v_w \geq y_w^l\right) \geq y_h^k.$$

Hence, as $\max_{\tau \in T^k} \hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) \geq \hat{E}\mathcal{V}_h^k(\hat{\tau};\mathcal{C}) \geq y_h^k$, the husband makes an offer.

It is also easy to verify that for any $\tau \in [\tau_{\min}, \hat{\tau})$:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) = y_h^k \sum_l \delta^l 1\left(y_w^l > v_w\right) + v_h \sum_l \delta^l 1\left(v_w \geq y_w^l\right).$$

Then, since $u_h(-\hat{\tau}) > v_h$ and (as shown above) $\sum_l \delta^l 1\left(v_w \geq y_w^l\right) > 0$, it must be that for all $\tau \in [\tau_{\min}, \hat{\tau})$:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) < \hat{E}\mathcal{V}_h^k(\hat{\tau};\mathcal{C}) \leq \max_{\tau \in T^k} \hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}),$$

which shows that no $\tau \in [\tau_{\min}, \hat{\tau})$ can be $\arg\max_{\tau \in T^k} \hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ and, therefore, $\tau^* \geq \hat{\tau}$, as desired.

Next, consider the case $\hat{\tau} \notin T^k$. I show that the husband optimally decides to announce divorce.

Observe that as $\hat{\tau} \notin T^k$, it must be that $u_h(-\hat{\tau}) < y_h^k$. Since $u_h(-\tau)$ decreases in τ, and any $\tau \in T^k$ satisfies $u_h(-\tau) \geq y_h^k$, it immediately follows that for all $\tau \in T^k$: $\tau < \hat{\tau}$. Then (as has been just shown above for $\tau < \hat{\tau}$) for all $\tau \in T^k$:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) = y_h^k \sum_l \delta^l 1\left(y_w^l > v_w\right) + v_h \sum_l \delta^l 1\left(v_w \geq y_w^l\right).$$

Since $v_h < u_h(-\hat{\tau}) < y_h^k$ and $\sum_l \delta^l 1\left(v_w \geq y_w^l\right) > 0$, then for all $\tau \in T^k$:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) < y_h^k,$$

and, therefore:

$$\max_{\tau \in T^k} \hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) < \hat{E}\mathcal{V}_h^k(\mathcal{D}),$$

which means that the husband would play $\mathcal{D}$. Hence, irrespective of whether $\hat{\tau} \in T^k$ or $\hat{\tau} \notin T^k$, the state of conflict does not occur.

Part III. Observe that if $y_w^l \leq v_w$ for all l, then $\hat{E}\mathcal{V}_h^k(\tau; \mathcal{C})$ takes a particularly simple form:

$$\hat{E}\mathcal{V}_h^k(\tau; \mathcal{C}) = v_h 1(v_w > u_w(\tau)) + u_h(-\tau) 1(u_w(\tau) \geq v_w)$$

Recalling the definition of $\hat{\tau}$ from Part II, consider again the two mutually exclusive and exhaustive possibilities: $\hat{\tau} \in T^k$ and $\hat{\tau} \notin T^k$.

Suppose that $\hat{\tau} \in T^k$. Notice that since: (1) $u_h(-\hat{\tau}) > v_h$, (2) $u_w(\hat{\tau}) = v_w$, (3) $u_h(-\tau)$ decreases in τ, and (4) $u_w(\tau)$ increases in τ, function $\hat{E}\mathcal{V}_h^k(\tau; \mathcal{C})$ discontinuously increases from v_h to $u_h(-\hat{\tau})$ at $\tau = \hat{\tau}$ and decreases afterwards in τ, which implies that $\hat{\tau} = \arg\max_{\tau \in T^k} \hat{E}\mathcal{V}_h^k(\tau; \mathcal{C})$.

Moreover, $\hat{E}\mathcal{V}_h^k(\hat{\tau}; \mathcal{C}) = u_h(-\hat{\tau}) \geq y_h^k$ since $\hat{\tau} \in T^k$. So, the husband offers transfer $\hat{\tau}$, which is acceptable to the wife.

Next, consider the remaining case $\hat{\tau} \notin T^k$. As argued in Part II, for all $\tau \in T^k$: $\tau < \hat{\tau}$. Then, for all $\tau \in T^k$:

$$\hat{E}\mathcal{V}_h^k(\tau; \mathcal{C}) = v_h < y_h^k \equiv \hat{E}\mathcal{V}_h^k(\mathcal{D}),$$

due to the inequality $v_h < u_h(-\hat{\tau}) < y_h^k$, and, therefore, the husband plays $\mathcal{D}$. Hence, irrespective of whether $\hat{\tau} \in T^k$ or $\hat{\tau} \notin T^k$, the state of conflict does not occur. ■

Theorem 5 implies that the alternative model has a very limited ability to explain an occurrence of the marital state of conflict. A conflict can happen only if restrictions $y_w^P \leq v_w < y_w^O$ and $\delta^P = 0$ are simultaneously true.

The restriction $\delta^P = 0$ is particularly problematic. First, to explain the data, I would have to require that husbands in all couples in conflict have the same degenerate belief about their wives, which is implausible. Second, when parameterizing the unobservable vector of beliefs to the econometrician as having a continuous support, the state of conflict in the empirical application could only be sustained on a set with zero probability measure. Therefore, for the purpose of explaining the data and estimating structural parameters, the alternative model is unsatisfactory.

3.7 Numerical Examples

To give an intuitive understanding of how the model works, I provide two numerical examples for an actual couple in the NSFH.

The couple is specifically chosen with characteristics that are close to sample mean values. In particular, the spouses are high-school educated, Protestant whites who have been married for about 15 years. The husband and wife are 43 and 40 years old, respectively. They own their home, have one resident 12-year-old child, and live in a state with (1) no mandatory separation periods before divorce can be granted and (2) an almost 13% child support enforcement collection rate. The local marriage market availability ratios specific to the husband's and wife's race, age, and education are respectively 1.27 and 0.99.[6]

In both examples, I construct the payoffs using actual estimation results and assume that the spouses are "hard bargainer – pessimists."[7] The husband does not know his wife's type, but has beliefs about it, which are represented by vector $(\delta^{HO}, \delta^{HP}, \delta^{SO}, \delta^{SP})'$.

First, suppose that the husband is completely "uninformed" about the wife's type:

[6]The relatively high wife's availability ratio is primarily due to a disproportionately large number of males in age intervals that are adjacent to interval $[40, 44]$ in the couple's county of residence. See Chapter 4 for more details on data and variables.

[7]This assumption is made here for illustrative purposes. In the empirical application, I consider all possible spousal type combinations for a couple. The estimated model implies that about 14% of husbands and 25% of wives in the sample are "hard bargainer – pessimists."

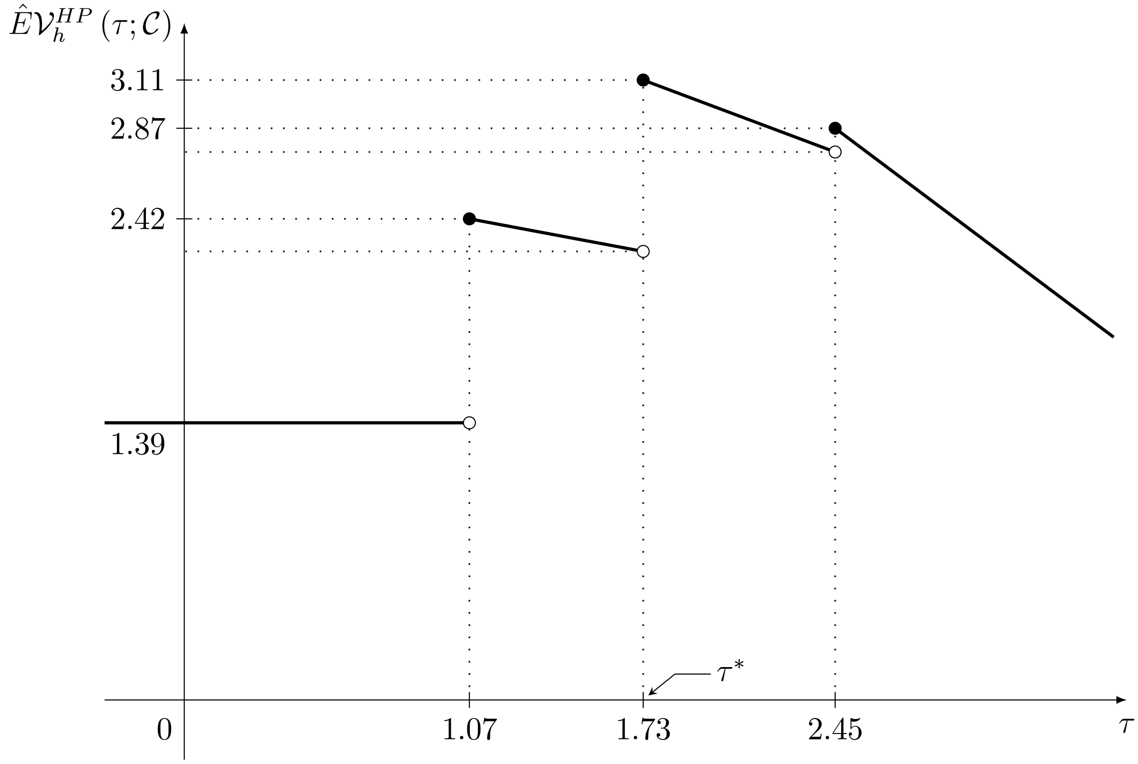

Figure 3.3: Example 1. Expected Utility of Cooperation

$\delta^{HO} = \delta^{HP} = \delta^{SO} = \delta^{SP} = 0.25$. In this case, the husband's expected utility of proposing cooperation (as a function of the transfer amount τ) takes the form shown in Figure 3.3.

Function $\hat{E}\mathcal{V}_h^{HP}(\tau;\mathcal{C})$ has three discontinuities in τ, each corresponding to a transfer level at which the wife is expected to switch between strategic responses to the husband's offers. Transfers below 1.07 are unacceptable to any type of wife, with "soft bargainer" wives announcing divorce and "hard bargainer" wives choosing conflict. At the transfer level of 1.07, a "soft bargainer – pessimist" wife switches from divorce to cooperation. However, a "soft bargainer – optimist" and a "hard bargainer" still prefer divorce and conflict, respectively, to accepting any $\tau \in [1.07, 1.73)$. At the transfer level of 1.73, a "soft bargainer – optimist" wife responds by accepting the offer. Thus, for any $\tau \in [1.73, 2.45)$, "soft bargainers" choose cooperation, whereas "hard bargainers" still prefer conflict. Finally, at the transfer level of 2.45, "hard bargainers" switch to cooperation and, henceforth, all transfers are acceptable to every type of wife.

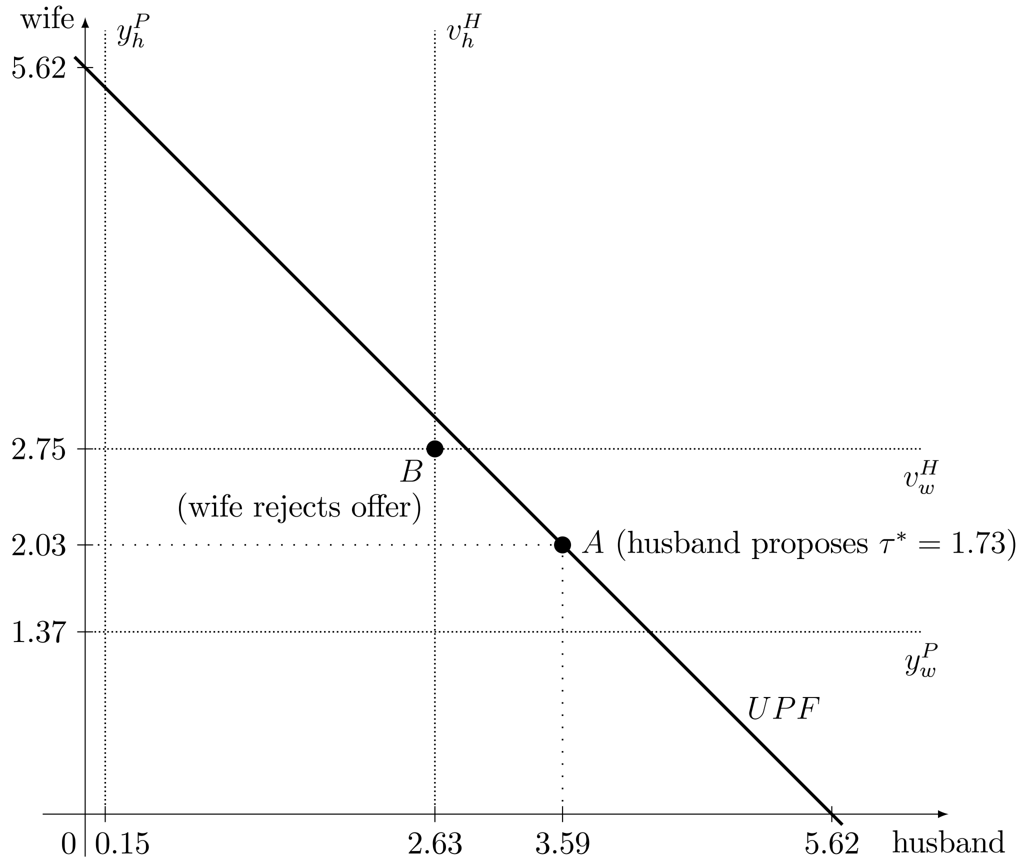

Figure 3.4: Example 1. Husband's Transfer Offer Is Rejected

One of the discontinuities of the expected utility function is the optimal transfer, $\tau^* = 1.73$. However, it is insufficient to ensure cooperation on part of the wife, given her true type "hard bargainer – optimist." To illustrate the wife's decision to reject the offer, I plot the utility possibility frontier that results from cooperation (UPF) along with spousal payoffs in the states of conflict and divorce in Figure 3.4.

In this example, the husband indeed chooses to offer $\tau^* = 1.73$ rather than to divorce, because $\hat{E}\mathcal{V}_h^{HP}(1.73;\mathcal{C}) = 3.11 > 0.15 = y_h^P$. The transfer corresponds to point A on the UPF, where the payoffs would turn out to be $u_h(-1.73) = 3.59$ and $u_w(1.73) = 2.03$. However, the wife would get a higher payoff in the state of conflict, $v_w^H = 2.75 > u_w(1.73)$. Thus, the wife rejects the offer and marital conflict ensues with payoffs $v_h^H = 2.63$ and $v_w^H = 2.75$, at point B. Divorce is not chosen by the wife, since $v_w^H > 1.37 = y_w^P$.

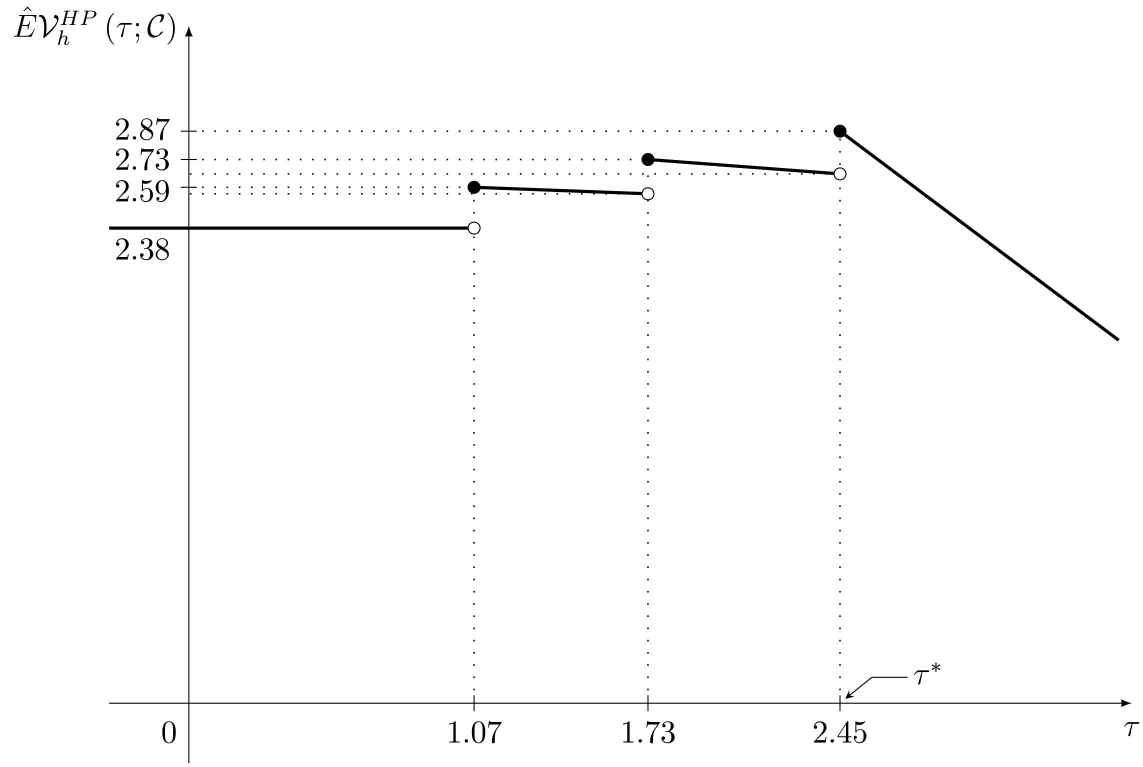

Figure 3.5: Example 2. Expected Utility of Cooperation

In the second example, suppose that the husband is better informed about the wife's type and let his beliefs be: $\delta^{HP} = 0.85$ and $\delta^{HO} = \delta^{SO} = \delta^{SP} = 0.05$. Figure 3.5 gives the expected utility of cooperation under the new beliefs. In this case, the husband wants to make a higher transfer offer, $\tau^* = 2.45$. The intuition is simple.

In the first example, the "uninformed" husband overestimated the wife's distaste for conflict and offered too low a transfer in the vain hope she would accept it. Better information makes the husband less "greedy."

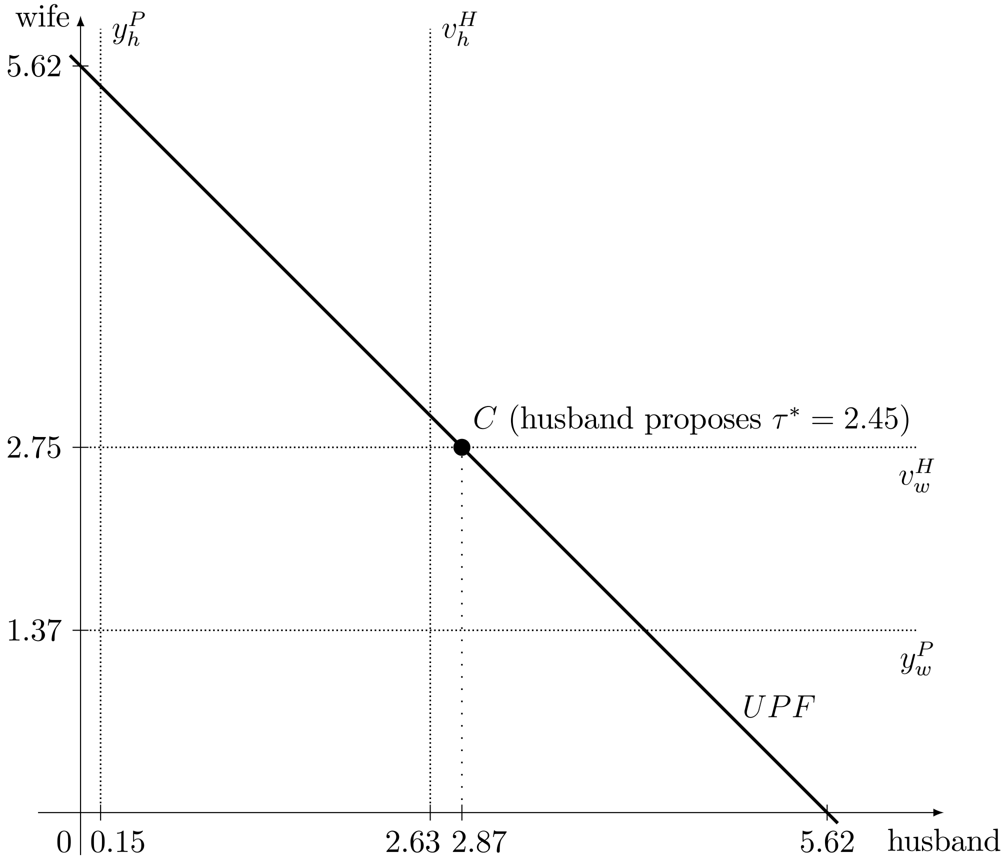

Figure 3.6: Example 2. Husband's Transfer Offer Is Accepted

The outcome of the game can be inferred from Figure 3.6. The husband chooses to offer $\tau^* = 2.45$ rather than to divorce, since $\hat{E}\mathcal{V}_h^{HP}(2.45; \mathcal{C}) = 2.87 > 0.15 = y_h^P$. Thus, he picks point C on the UPF with actual payoffs $u_h(-2.45) = 2.87$ and $u_w(2.45) = 2.75$. In turn, the wife agrees to cooperate, because $u_w(2.45) \geq 2.75 = v_w^H$ and $u_w(2.45) > 1.37 = y_w^P$.

Chapter 4

Data and Nonstructural Analysis

4.1 Estimation Sample

The primary data source for the empirical analysis in this dissertation is the National Survey of Families and Households (NSFH). The survey provides a broad range of information on family life to serve as a resource across disciplinary perspectives (Sweet et al., 1988). The NSFH is a probability sample of American households with oversampling of several minority and disadvantaged groups (e.g., Blacks and single-parent families). Married couples constitute a subsample of the survey and a set of weights is separately provided for them. Thus, I am able to compute representative statistics for the population of married couples when needed.

The NSFH collects data that are particularly suitable to this research project. I observe spousal reports on the frequency of disagreements and process of dispute resolution, which I use to determine when conflict occurs. I also observe information on one's own post-marital prospects and beliefs about the partner's well-being after hypothetical divorce. To my best knowledge, such data are not simultaneously available from other sources.[1] In addition, I employ information on demographic characteristics of couples, and the NSFH staff merged individual data with state- and county-level information that I describe later.

The NSFH is a panel survey with three completed waves of data collection. I use the first two waves to estimate structural parameters. The first wave was conducted in 1987-88 and included $13,007$ households. Of these original households, $10,005$ were found and agreed to be resurveyed during the second wave in 1992-94. The average time interval between the two interviews in the subsample of married couples is approximately 5.5 years.

Due to financial constraints, the third wave of the NSFH (2001-02) was completed only for a small selected subsample of families. Because of the nonrandom nature of these data, I do not employ them in the estimation. However, family histories collected in the third interview are used to evaluate the out-of-sample predictive power of the empirical model.

My estimation sample consists of 3,878 couples and is constructed as follows. I take all 5,270 married couples who participated in the first data collection wave. From these, I exclude 575 couples with missing data, 477 couples who were not reinterviewed during the second wave for reasons other than death, and 340 couples in which a spouse died between the first and second waves.

4.2 Analysis of Attrition

As noted above, to conduct the empirical analysis, I excluded 575 couples with missing data from the sample of 5,270 married couples who were initially interviewed

[1] For instance, the National Longitudinal Survey of Youth 1979 collects data on disagreement frequencies, but contains no information on the process of dispute resolution or beliefs about divorce prospects.

Table 4.1: Reasons for Attrition, Except Death

Reason	# of Cases	Fraction, %
Refusal to participate	257	53.88
Tracing exhausted	118	24.74
Incomplete interview	68	14.26
Too ill to participate	24	5.03
Other data problem	10	2.09
Total	477	100.00

in the NSFH. When processing and cleaning the data set, I discovered that information in the corresponding 575 records is typically absent in entire blocks (e.g., all individual attitudes, opinions, and beliefs are missing simultaneously). Since the structure of the NSFH is modular with very complex skip patterns, many such omissions are likely due to the interviewer oversight and, therefore, random in nature with respect to characteristics of a couple.

Henceforth, I focus on 4,695 couples with non-missing data and refer to them as "the sample before attrition." In this sample, 477 couples were not reinterviewed during the second wave for reasons other than death and 340 couples experienced death of a spouse between the first and second waves.

Table 4.1 gives the distribution of the 477 couples who were not reinterviewed during the second wave (for reasons other than death), by reason for attrition. The refusal to participate in the survey is the most important contributing factor, accounting for almost 54% of all attrition cases.

Tables 4.2, 4.3, 4.4, and 4.5 present summary statistics for selected demographic and socioeconomic characteristics of the sample before attrition, sample of couples who were not reinterviewed, sample of couples with spousal death, and estimation sample, respectively, as of the first NSFH wave. To facilitate the comparison with the

Table 4.2: Characteristics of Sample Before Attrition

	Husband		Wife	
Characteristic	Mean	Std. Dev.	Mean	Std. Dev.
age, years	45.58	(15.10)	42.89	(14.70)
black	0.07	(0.25)	0.06	(0.25)
hispanic	0.06	(0.23)	0.06	(0.23)
catholic	0.25	(0.43)	0.25	(0.43)
protestant	0.54	(0.50)	0.58	(0.49)
other religion	0.12	(0.32)	0.12	(0.32)
high school degree*	0.81	(0.39)	0.82	(0.38)
college degree*	0.32	(0.47)	0.26	(0.44)
graduate degree*	0.10	(0.31)	0.05	(0.23)
disability	0.04	(0.20)	0.03	(0.18)
employed	0.78	(0.42)	0.56	(0.50)
labor income, dollars†	645.54	(667.21)	349.21	(784.36)
# of resident children‡	1.99	(1.05)	1.99	(1.05)

Notes:

Sample includes 4,695 married couples. Summary statistics are weighted.

*Education indicators are not exclusive and represent a completed educational level.

†Typical weekly earnings as of the interview date. Statistics are for the subsamples of individuals with positive labor income (74% of husbands and 53% of wives).

‡Statistics are for the subsample of couples with resident children (54% of sample).

U.S. population of married couples in the following section, I computed the summary statistics using the NSFH sample weights.

Overall, I see no substantial difference between most characteristics of the sample before attrition and corresponding characteristics of couples who were not reinterviewed in the second wave. The only notable dissimilarity is that spouses in the latter sample are slightly less educated and tend to belong to minority groups. In particular, Hispanics appear to drop from the sample at a considerably higher rate

Table 4.3: Characteristics of Couples Who Were Not Reinterviewed

	Husband		Wife	
Characteristic	Mean	Std. Dev.	Mean	Std. Dev.
age, years	45.07	(15.56)	42.19	(15.28)
black	0.09	(0.28)	0.09	(0.28)
hispanic	0.16	(0.37)	0.16	(0.37)
catholic	0.31	(0.46)	0.31	(0.46)
protestant	0.49	(0.50)	0.56	(0.50)
other religion	0.09	(0.29)	0.08	(0.27)
high school degree*	0.74	(0.44)	0.73	(0.44)
college degree*	0.20	(0.40)	0.17	(0.38)
graduate degree*	0.05	(0.22)	0.04	(0.20)
disability	0.04	(0.20)	0.04	(0.19)
employed	0.77	(0.42)	0.53	(0.50)
labor income, dollars†	641.13	(1162.82)	331.34	(581.81)
# of resident children‡	2.01	(1.13)	2.01	(1.13)

Notes:

Sample includes 477 married couples. Summary statistics are weighted.

*Education indicators are not exclusive and represent a completed educational level.

†Typical weekly earnings as of the interview date. Statistics are for the subsamples of individuals with positive labor income (70% of husbands and 48% of wives).

‡Statistics are for the subsample of couples with resident children (55% of sample).

Table 4.4: Characteristics of Couples with Spousal Death

	Husband		Wife	
Characteristic	Mean	Std. Dev.	Mean	Std. Dev.
age, years	63.47	(13.36)	59.66	(13.84)
black	0.09	(0.29)	0.09	(0.29)
hispanic	0.03	(0.18)	0.04	(0.21)
catholic	0.16	(0.37)	0.17	(0.37)
protestant	0.60	(0.49)	0.67	(0.47)
other religion	0.13	(0.34)	0.13	(0.34)
high school degree*	0.57	(0.50)	0.62	(0.48)
college degree*	0.20	(0.40)	0.15	(0.35)
graduate degree*	0.07	(0.25)	0.02	(0.12)
disability	0.12	(0.33)	0.10	(0.31)
employed	0.35	(0.48)	0.26	(0.44)
labor income, dollars†	521.95	(367.59)	286.94	(230.71)
# of resident children‡	1.84	(0.93)	1.84	(0.93)

Notes:

Sample includes 340 married couples. Summary statistics are weighted.

*Education indicators are not exclusive and represent a completed educational level.

†Typical weekly earnings as of the interview date. Statistics are for the subsamples of individuals with positive labor income (34% of husbands and 23% of wives).

‡Statistics are for the subsample of couples with resident children (17% of sample).

Table 4.5: Characteristics of Estimation Sample

	Husband		Wife	
Characteristic	Mean	Std. Dev.	Mean	Std. Dev.
age, years	43.84	(14.00)	41.28	(13.61)
black	0.06	(0.24)	0.06	(0.24)
hispanic	0.05	(0.21)	0.04	(0.21)
catholic	0.25	(0.43)	0.25	(0.43)
protestant	0.54	(0.50)	0.57	(0.49)
other religion	0.12	(0.32)	0.12	(0.32)
high school degree*	0.85	(0.36)	0.86	(0.35)
college degree*	0.35	(0.48)	0.28	(0.45)
graduate degree*	0.11	(0.32)	0.06	(0.24)
disability	0.03	(0.18)	0.03	(0.16)
employed	0.82	(0.39)	0.59	(0.49)
labor income, dollars†	651.41	(595.04)	353.74	(817.54)
# of resident children‡	1.99	(1.04)	1.99	(1.04)

Notes:

Sample includes 3878 married couples. Summary statistics are weighted.

*Education indicators are not exclusive and represent a completed educational level.

†Typical weekly earnings as of the interview date. Statistics are for the subsamples of individuals with positive labor income (78% of husbands and 57% of wives).

‡Statistics are for the subsample of couples with resident children (57% of sample).

than Whites or Blacks. MaCurdy et al. (1998) report a similar finding for Hispanics in their analysis of attrition in the National Longitudinal Survey of Youth 1979.

Unsurprisingly, in comparison to the sample before attrition, spouses in couples with respondent's death tend to be considerably older, relatively less educated, in poorer health, and more often out of the labor force. They also have somewhat lower earnings, which may be attributed to the fact that individuals who decide to work past retirement age more often choose part-time employment.

Comparing the sample before attrition with the estimation sample, I find that the exclusions do not introduce any substantive distortions with respect to the observable characteristics. Spouses in the estimation sample tend to be marginally younger and more educated, but other summary statistics are numerically very close.

4.3 Population of Married Couples

In addition to the above brief analysis of the sample attrition, I compare selected characteristics of the estimation sample with characteristics of the corresponding population group in the U.S. This group consists of spouses in married-couple households covered by the 1990 Decennial Census for whom the following three conditions apply. First, husbands and wives must be at least 20 and 18 years old, respectively, as of April 1st, 1990. Second, every spouse must be a non-Hispanic White, a non-Hispanic Black, or a Hispanic of any racial descent.[2] Third, a couple must reside in a U.S. state (i.e., Census data from Puerto Rico are excluded).

I construct statistics for the population group on the basis of the 5% Public Use Microdata Sample (PUMS) for the 1990 Census, using provided PUMS weights. Since the NSFH and Census questionnaires are not identical, it is possible to compare only a

[2] Unlike in the 1990 Census, Hispanics did not provide racial self-identification in the NSFH.

few selected characteristics for which the corresponding questions match, as reported in Table 4.6.

There are a few slight dissimilarities between the characteristics of the estimation sample and population group. First, the estimation sample includes somewhat younger individuals. In part, the age discrepancy may be due to a sampling error. Another contributing factor is the exclusion of couples in which a spouse died between the first and second waves of the NSFH. Specifically, since the deceased tend to be relatively older (as shown in the previous section), by dropping couples with spousal death between April 1^{st}, 1990, and the second NSFH interview date I induce a lower average age in the estimation sample.

Next, the estimation sample appears to underrepresent Hispanics, which is likely due to a disproportionate growth in the Hispanic population in the U.S. that slightly changed the ethnic composition of the nation between 1987 and 1990.

Lastly, the NSFH respondents tend to have somewhat higher educational attainment. Taking the composition of the estimation sample with respect to age into account, this fact may be due to generally higher educational attainment of younger population cohorts in comparison to older cohorts. Likewise, a relatively lower incidence of disability and higher employment rate in the estimation sample may also be due to the age composition differential.

Overall, I conclude that the characteristics of the estimation sample are in line with the ones of the population group. There are some discrepancies, but they are relatively small and can be reasonably explained.

Table 4.6: Comparison of Estimation Sample and Population Group

	Estimation Sample				Population Group			
	Husband		Wife		Husband		Wife	
Characteristic	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
age, years	43.84	(14.00)	41.28	(13.61)	47.63	(15.57)	45.03	(15.14)
black	0.06	(0.24)	0.06	(0.24)	0.07	(0.25)	0.07	(0.25)
hispanic	0.05	(0.21)	0.04	(0.21)	0.07	(0.25)	0.07	(0.25)
high school degree*	0.85	(0.36)	0.86	(0.35)	0.78	(0.42)	0.80	(0.40)
college degree*	0.35	(0.48)	0.28	(0.45)	0.30	(0.46)	0.25	(0.43)
graduate degree*	0.11	(0.32)	0.06	(0.24)	0.09	(0.29)	0.06	(0.23)
disability	0.03	(0.18)	0.03	(0.16)	0.05	(0.23)	0.05	(0.22)
employed	0.82	(0.39)	0.59	(0.49)	0.76	(0.43)	0.56	(0.50)
earnings, dollars†	34,281	(39,652)	15,193	(25,480)	33,977	(30,412)	15,706	(14,435)

Notes:

Summary statistics are weighted.

*Education indicators are not exclusive and represent a completed educational level.

†Earnings refer to total labor income in 1986 for the estimation sample and 1989 for the population group. Statistics are for the subsamples of individuals with positive earnings (82% of husbands and 64% of wives in the estimation sample, 81% of husbands and 63% of wives in the population group).

4.4 Explanatory Variables

For convenience, I group the explanatory variables into three separate categories: (1) opinions and beliefs, (2) individual characteristics, and (3) location-specific information.

The first group consists of answers to selected questions from the first NSFH interview that provide information about spousal types. Specifically, all married respondents in the survey are asked: "Even though it may be very unlikely, think for a moment about how various areas of your life might be different if you separated. For each of the following areas, how do you think things would change?" I focus on the question about overall happiness, which is likely to be informative about the trait of "optimism" regarding the divorce prospects, and create (separately for husbands and wives) two indicator variables: for individuals who report that their happiness would be the same and for those who report that it would be better or much better (the base category comprises spouses who report that the happiness would be worse or much worse).

Spouses are also asked about the divorce opportunities of their partners: "How about your husband/wife? How do you think these various areas of life might be different for him/her if you separated?" I create an indicator for husbands who believe the overall happiness of their wives would be the same and an indicator for husbands who believe it would be better or much better.[3]

Additionally, all NSFH respondents provide their attitudes and opinions on a number of issues. In particular, every individual reports if he or she agrees with the statement: "I feel that I'm a person of worth, at least on an equal plane with others." I create an indicator for respondents who express strong agreement, which may be

[3]The beliefs of the wife do not affect the outcome of the game.

informative about the trait of "bargaining strength."

Summary statistics for the opinion and belief variables are given in Table 4.7. It is interesting that 23% of husbands and 22% of wives say their happiness would stay the same or even improve after divorce.

The group of individual characteristics includes variables specific to a couple (the number of children, marital duration, and an indicator for home ownership), as well as standard demographic data (age, race, religious affiliation, and education of spouses) as of the first NSFH wave. I use these variables to parameterize spousal utilities. Detailed variable descriptions and summary statistics are provided in Table 4.8.

Three comments are in order. First, since the effects of children[4] may vary with their age and relationship status, I split children into common children of the spouses vs. children of the wife (i.e., husband's step-children)[5] and, additionally, divide the offspring of both parents into children who are less than 6 years old and those who are 6 or older. Second, the NSFH collects very detailed information about religious beliefs. I group them by religious family (in accordance with the classification of Melton, 1977) and, then, create indicators for the husband's affiliation with the Roman Catholic Church and difference in the spousal religious affiliations (with respect to the religious family). Third, education categories are exclusive and reflect the highest completed level of schooling (the base category is "no high school degree").

Explanatory variables related to the geographical location of a couple appear in Table 4.9. I include proxies for the marriage market conditions that would be faced by a husband and wife if they choose to separate, as well as some specifics of the divorce legislation and enforcement of child support payments at a state level. These variables have an impact only on the payoffs attainable outside of marriage (McElroy,

[4]By a child I mean an individual who is at most 18 years old, is reported as a biological, adopted, or step-child of the household head, and resides full time in the couple's household.

[5]The fraction of couples with children of the husband (i.e., wife's step-children) is negligible.

Table 4.7: Explanatory Variables: Opinions and Beliefs

Variable	Mean	Std. Dev.	Min	Max	Description*
same happiness, husband	0.17	(0.38)	0	1	h says his own overall happiness would be the same after divorce
more happy, husband	0.06	(0.23)	0	1	h says his own overall happiness would be better or much better after divorce
worthy person, husband	0.38	(0.49)	0	1	h strongly agrees he is person of worth, at least on equal plane with others
same happiness, wife	0.15	(0.36)	0	1	w says her own overall happiness would be the same after divorce
more happy, wife	0.07	(0.26)	0	1	w says her own overall happiness would be better or much better after divorce
worthy person, wife	0.42	(0.49)	0	1	w strongly agrees she is person of worth, at least on equal plane with others
same happiness	0.19	(0.39)	0	1	h believes w's overall happiness would be the same after divorce
more happy	0.08	(0.27)	0	1	h believes w's overall happiness would be better or much better after divorce

Note:

*h stands for "husband" and w denotes "wife."

Table 4.8: Explanatory Variables: Individual Characteristics

Variable	Mean	Std. Dev.	Min	Max	Description*
children, <6 y.o.	0.45	(0.73)	0	5	# of common children who are <6 years old
children, ≥ 6 y.o.	0.57	(0.94)	0	5	# of common children who are ≥ 6 years old
children, wife's	0.14	(0.47)	0	5	# of children of w who are not children of h
marital duration†	14.51	(13.23)	0	64	duration in years
home ownership	0.75	(0.43)	0	1	h and w own a home
age, husband's†	41.02	(13.75)	17	90	age of h in years
age, abs. diff.†	3.62	(3.84)	0	38	absolute value of age difference
black husband	0.09	(0.29)	0	1	h is Black
catholic husband	0.23	(0.42)	0	1	h is Roman Catholic
religion, diff.	0.33	(0.47)	0	1	h and w report different affiliations
high sch., husband	0.51	(0.50)	0	1	h's highest education is high school
college, husband	0.33	(0.47)	0	1	h's highest education is (at least) college
high sch., wife‡	0.58	(0.49)	0	1	w's highest education is high school
college, wife‡	0.29	(0.45)	0	1	w's highest education is (at least) college
education, diff.	0.38	(0.48)	0	1	h and w report different attainments

Notes:

*h stands for "husband" and w denotes "wife."

†In estimation, marital duration and age variables are standardized.

‡Wife's education indicators are used only to parameterize the wife's divorce payoff.

Table 4.9: Explanatory Variables: Location-specific Information

Variable	Mean	Std. Dev.	Min	Max	Description*
male-specific avail. ratio	1.25	(0.24)	0.56	2.43	local marriage market availability ratio specific to h
female-specific avail. ratio	0.84	(0.15)	0.22	1.45	local marriage market availability ratio specific to w
separation, $\geq \frac{1}{2}$ year and ≤ 1 year	0.18	(0.39)	0	1	h and w reside in state with separation period between 6 months† and 1 year
separation, > 1 year	0.33	(0.47)	0	1	h and w reside in state with separation period that exceeds 1 year
collection rate‡	0.19	(0.06)	0.06	0.35	CSE collection rate interacted with indicator for presence of children

Notes:

*h stands for "husband" and w denotes "wife."

†6 months is the minimum period across states with separation requirements.

‡Statistics are for the subsample of couples with children.

1990).

Local marriage market conditions influence the ease of finding a new mate after divorce. As is common in the literature, I approximate the conditions with the availability ratio (Goldman et al., 1984). Simply put, it is the ratio of the number of marriageable individuals of one sex to the number of a corresponding group of the opposite sex. I compute availability ratios separately for each county covered by the NSFH on the basis of the 5% Public Use Microdata Sample (PUMS) for the 1990

Decennial Census. The ratios are specific to race, sex, age, and education. In Appendix A, I review the literature on the availability ratio and discuss in detail the methodology of calculating it for the NSFH respondents.

States in the U.S. stipulate different conditions that must be fulfilled before a couple can obtain a divorce decree. I focus on one aspect of divorce legislation, the existence of mandatory separation periods. Formally, a separation period is the requirement for spouses to live apart without any cohabitation for a specified period of time before a court may grant them a divorce from the bond of matrimony. Presumably, the existence of such a period increases divorce costs of separating spouses.[6] The impact of a period may vary with its length. Thus, I create two indicator variables: for individuals who live in a state with a separation period between 6 months and one year (18% of the sample) and for individuals in a state with a period lasting more than a year (33%). As of the first NSFH wave, some states (for instance, California) had no mandatory separation periods and in the states that did have them, the periods ranged from 6 months (e.g., Vermont) to 3 years (e.g., Utah).

If a divorced couple has children, the noncustodial parent (usually, the father) is typically required to make payments to the custodial parent for child support. In the late 1980s, the states substantially varied in how well they enforced child support payments (U.S. House. Committee on Ways and Means, 1991). Since the payments redistribute welfare between ex-spouses, the strength of the enforcement may influence divorce payoffs. To approximate the strength of child support enforcement (CSE) at a state level, I use the CSE collection rate reported by state enforcement agencies (Nixon, 1997). Specifically, I first calculate the rate by averaging the ratios of the number of cases with a collection to annual caseload over the fiscal years 1987 through

[6]Other divorce legislation focuses mostly on shifting property rights rather than altering divorce costs.

1994. Then, I assign this variable to spouses with children as of the first NSFH wave in accordance with the state of residence and set the value of the variable to 0 for childless couples (i.e., the CSE collection rate is interacted with the indicator for the presence of children).

4.5 Marital State

For estimation purposes, the marital state of a couple is the dependent variable. I assign its value on the basis of the second NSFH interview (1992-94).

In the model, the spousal game results in one of three mutually exclusive outcomes: cooperation, conflict, or divorce. A couple is in the state of divorce if the spouses are reported as divorced or separated in the second wave status file. The separation cases are included since a couple typically needs to go through a lengthy legal process before a court grants a divorce decree.

The assignment of the states of conflict and cooperation is a more challenging task. The psychology literature distinguishes between "constructive" and "destructive" disagreements in intact married couples (Grych and Fincham, 2001). "Constructive" disagreements involve disputes that are not intense and quickly resolved, while "destructive" disagreements are the ones that happen with high frequency and are not settled peacefully.

In the model, the state of conflict refers to situation in which an intact couple engages in "destructive," and hence inefficient, disagreements. I identify such instances using questions about frequencies of disputes and the process of conflict resolution. Specifically, all husbands and wives are asked: "The following is a list of subjects on which couples often have disagreements. How often, if at all, in the past year have you had open disagreements about each of the following: household tasks, money,

Table 4.10: Distribution of Marital State Variable

Marital State	Frequency	Fraction, %
Cooperation	2,948	76.02
Conflict	416	10.73
Divorce	514	13.25
Total	3,878	100.00

spending time together, sex, in-laws, the children?" The response categories for each disagreement area range from "never" to "almost every day." To mitigate the problem of underreporting, I infer the frequency for a couple as the maximum of the corresponding husband's and wife's frequencies. Additionally, spouses report how they deal with disputes: "There are various ways that married couples deal with serious disagreements. When you have a serious disagreement with your husband/wife, how often do you: discuss your disagreements calmly, argue heatedly or shout at each other?" Possible responses in each case range from "never" to "always."

I assign the state of conflict to a couple if the following conditions are met. First, the spouses must disagree about one or more aspects of their relationship with a frequency of several times a week or more often. Second, at least one spouse must admit that they seldom or never calmly discuss disagreements or often or always heatedly argue with each other. The remaining intact couples are in the state of cooperation.

The distribution of the marital state variable is given in Table 4.10. 76% of couples in the estimation sample are in the state of cooperation, 11% are in the state of conflict, and 13% are in the state of divorce.

4.6 Nonstructural Trinomial Model

To see how the explanatory variables covary with the marital state variable, I estimate a nonstructural trinomial probit model for the outcomes of cooperation, conflict, and divorce.

Most of the following notation is specific to this section. Let x_i stand for the vector of characteristics of couple i, as appears in Table 4.8. Additionally, vector x_i contains the availability ratios, indicators for separation period requirements, the CSE collection rate (see Table 4.9 for details), and interactions of the collection rate with husband's and wife's education. I use the interaction terms to account for the possibility of a varying effect of the strength of CSE across wealth groups (where education is a proxy for wealth).

The latent utility of couple i in marital state s is modeled as:

$$u_{i,s} = x_i' \beta_s + \epsilon_{i,s}, \tag{4.1}$$

where β_s is the vector of parameters to estimate.

The vector of random variables $\epsilon_i = (\epsilon_{i,\text{cooperation}}, \epsilon_{i,\text{conflict}}, \epsilon_{i,\text{divorce}})'$ is independently and identically distributed as a normal random vector, $\epsilon_i \sim i.i.d.\ N(0, \Delta)$, where Δ is the covariance matrix. The model is estimated by maximum likelihood. The utility associated with the state of cooperation is normalized to 0.

The estimated trinomial model is presented in Table 4.11. The effect of a variable in the marital states of conflict and divorce should be interpreted relative to the impact of the variable under cooperation.

Many estimated coefficients are statistically significant. In the state of conflict, I find negative effects (on $u_{i,\text{conflict}} - u_{i,\text{cooperation}}$) of husband's age and education, as well as of home ownership and separation period between 6 months and one year. Common children who are at least 6 years old, spousal absolute age difference, husband's

Table 4.11: Nonstructural Trinomial Model

	Conflict		*Divorce*	
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.
constant	-2.312^{**}	(0.558)	-2.668^{**}	(0.574)
children, $<$ 6 y.o.	0.038	(0.061)	-0.061	(0.061)
children, $\geq$ 6 y.o.	0.115^{**}	(0.048)	0.085	(0.052)
children, wife's	0.133	(0.083)	0.152^{**}	(0.077)
marital duration†	-0.006	(0.007)	-0.035^{**}	(0.007)
home ownership	-0.220^{**}	(0.091)	-0.272^{**}	(0.086)
age, husband's†	-0.025^{**}	(0.007)	-0.027^{**}	(0.007)
age, abs. diff.†	0.029^{**}	(0.012)	0.047^{**}	(0.012)
black husband	0.404^{**}	(0.135)	0.425^{**}	(0.140)
catholic husband	0.169^{*}	(0.090)	-0.121	(0.093)
religion, diff.	0.127	(0.082)	0.159^{**}	(0.080)
high sch., husband‡	-0.298^{*}	(0.167)	-0.091	(0.187)
college, husband‡	-0.353^{*}	(0.186)	-0.409^{**}	(0.201)
education, diff.	0.130	(0.081)	0.170^{**}	(0.081)
male-specific avail. ratio	0.862^{**}	(0.281)	0.538^{*}	(0.302)
female-specific avail. ratio	-0.315	(0.383)	0.710^{*}	(0.372)
$\frac{1}{2}$ year $\leq$ separation $\leq$ 1 year	-0.181^{*}	(0.110)	-0.101	(0.105)
separation $>$ 1 year	0.021	(0.086)	-0.211^{**}	(0.087)
collection rate	2.215^{*}	(1.235)	2.505^{**}	(1.264)
coll. rate $\times$ high sch., husband‡	-0.442	(1.153)	-1.215	(1.211)
coll. rate $\times$ college, husband‡	-0.453	(1.297)	-0.533	(1.342)
coll. rate $\times$ high sch., wife‡	-0.973	(0.853)	-1.377^{*}	(0.827)
coll. rate $\times$ college, wife‡	-1.612^{*}	(0.970)	-1.652^{*}	(0.935)

Notes:

†Variable is standardized in estimation. I report the impact of a one-year increase.

‡The omitted education category is "no high school degree."

* and ** denote significance at 10 and 5% level, respectively.

In estimation, Δ is set to the identity matrix. Sample log-likelihood is -2525.62.

race and Catholic religious affiliation, and favorable marriage market conditions for males have a positive impact. The effect of the CSE collection rate is positive, but appears to dissipate with better education. Overall, the coefficients in the state of conflict are not particularly intuitive. Some estimated parameters may reflect the impact of the explanatory variables on the utility, whereas others might capture the role of bargaining. To illustrate, the positive coefficient for Catholics (0.169) may indicate a preference for intact marriage, even with sustained conflict, since the Catholic Church strongly opposes divorce. However, the positive coefficient of the male-specific availability ratio (0.862) might reflect more aggressive and conflict-prone bargaining by husbands when their outside options are high. Moreover, the trinomial model does not allow me to evaluate the impact of a variable separately on husbands and wives. A structural model may be better suited to disentangle these various effects.

In the state of divorce, I detect a negative impact (on $u_{i,\text{divorce}} - u_{i,\text{cooperation}}$) of marital duration, husband's age and college education, home ownership, and separation period of longer than a year. Wife's children, husband's Black race, differences in spousal ages, religious affiliations, and education levels, as well as favorable marriage market conditions for both sexes have positive effects. The impact of the CSE collection rate is positive when spouses have no high school degree, but weakens at higher education levels. These coefficients are in line with the predictions of traditional theories of marriage. According to Becker et al. (1977), the negative impact of duration (-0.035) is expected, because marriage-specific capital accumulates with time and makes marriage more attractive. Discrepancies between the husband's and wife's traits indicate a nonoptimal spousal sorting, which makes marriage less desirable. Thus, the positive effects of differences in spousal ages (0.047), religious affiliations (0.159), and education levels (0.170) are expected, as well.

Several explanatory variables in the trinomial model, namely, common children, marital duration, and home ownership, are potentially endogenous. For instance, spouses with low marital match quality may anticipate separation and choose to have fewer children in order to reduce future divorce costs. I reestimated the trinomial model excluding these variables. The results for the remaining variables (see Table C.1 in Appendix C) are practically the same as before, which suggests that the scope of a potential bias is limited.

Chapter 5

Parameterization and Estimation

5.1 Parameterized Payoffs

In the model, the payoffs of every player type in each marital state are common knowledge. However, the econometrician does not observe many factors that may affect spousal utilities, for instance, love and physical attractiveness. Therefore, I specify that, to the econometrician, a payoff has a deterministic component and an unobservable error component.

In what follows, I typically suppress the index of a couple to keep notation simple. Let x stand for the vector of spousal characteristics as of the first NSFH wave related to children, marriage duration, demographics, etc. (see Table 4.8 for a complete list), and a constant term. In the state of cooperation, payoffs do not depend on player types but are affected by the transfer, τ. The payoffs of the husband and wife are parameterized as:

$$u_h(-\tau) = x'\alpha_h - \tau + \theta_1 \text{ and } u_w(\tau) = x'\alpha_w + \tau + \theta_3,$$

where α_h and α_w are the coefficients on the deterministic components of utilities and

θ_1 and θ_3 are error terms. As explained in detail later in this Chapter, I can identify only the sum $\alpha = \alpha_h + \alpha_w$.

In the state of conflict, payoffs depend on the "bargaining strength" of a player. The payoffs of the "soft bargainer" husband and wife are:

$$v_h^S = x'\beta_h + \theta_2 \text{ and } v_w^S = x'\beta_w + \theta_4,$$

where β_h and β_w represent the coefficients and θ_2 and θ_4 are error terms.

By definition, a "hard bargainer" is relatively better off in the state of conflict that a "soft bargainer." For convenience, I specify that the corresponding utilities differ by a positive constant. Thus, the payoffs of the "hard bargainer" husband and wife become:

$$v_h^H = v_h^S + \beta_h^H \text{ and } v_w^H = v_w^S + \beta_w^H,$$

where $\beta_h^H > 0$ and $\beta_w^H > 0$ are the constants to estimate.

The vector of error terms $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)'$ is assumed to be independently and identically distributed across couples as a normal random vector, $\theta \sim i.i.d.\ N(0, \Sigma)$.

For identification reasons to be explained in this Chapter, I cannot employ vector x in the specification of the divorce payoffs. Instead, I use the location-specific variables (Table 4.9), which should affect the spousal utilities after the break-up, but not in an intact marriage. Let vector z_h consist of the male-specific availability ratio, indicators for legal separation periods, as well as the collection rate and its interactions with husband's education. Vector z_w contains the same variables, with some specific to females. The interaction terms in z_h and z_w help account for potentially varying effect of the strength of CSE across wealth groups (with education proxying for spousal wealth after divorce).

In the state of divorce, the payoffs depend on the personal trait of "optimism" about the outside option. The payoffs of the "pessimistic" husband and wife are parameterized as:

$$y_h^P = z_h' \gamma_h \text{ and } y_w^P = z_w' \gamma_w,$$

where γ_h and γ_w are the coefficients.

By definition, an "optimist" is better off than a "pessimist" after divorce. I specify that the corresponding utilities differ by a positive constant. Hence, the payoffs of the "optimistic" husband and wife become:

$$y_h^O = y_h^P + \gamma_h^O \text{ and } y_w^O = y_w^P + \gamma_w^O,$$

where $\gamma_h^O > 0$ and $\gamma_w^O > 0$ are the constants to estimate.

5.2 Parameterized Type Probabilities and Beliefs

Every player in the game knows his or her own true type with certainty. However, the econometrician observes only discrete answers about divorce prospects and selected opinions. Hence, I infer the spousal types probabilistically given the available data from the first NSFH interview.

In modeling the type probabilities, I extend the approach of Degan and Merlo (2006). Let vector a_h include a constant term and the following three indicators: if the husband thinks his overall happiness would be the same after divorce, if he thinks it would be better or much better, and if he strongly agrees that he is a person of worth. Let vector a_w contain analogous indicators for the wife. I specify that the husband is of type k and wife is of type l with probabilities:

$$\pi_h^k = \frac{\exp\left(a_h' \lambda_h^k\right)}{\sum_j \exp\left(a_h' \lambda_h^j\right)} \text{ and } \pi_w^l = \frac{\exp\left(a_w' \lambda_w^l\right)}{\sum_j \exp\left(a_w' \lambda_w^j\right)},$$

where λ_h^k and λ_w^l for $k, l \in \{HO, HP, SO\}$ represent the parameters to estimate and $\lambda_h^{SP} = \lambda_w^{SP} = 0$ by normalization.

The chosen form is convenient because it restricts the values of the type probabilities to the standard 3-simplex: $\pi_h^k \geq 0$, $\pi_w^l \geq 0$ for any k, l and $\sum_k \pi_h^k = \sum_l \pi_w^l = 1$.

The parameterization of the husband's beliefs about the wife is similar. In the model, the husband knows his beliefs, but the econometrician only observes his discrete answers about the wife's expected divorce opportunities. Therefore, to the econometrician, vector $\left(\delta^{HO}, \delta^{HP}, \delta^{SO}, \delta^{SP}\right)'$ is randomly distributed on the standard 3-simplex. Let vector b include a constant term and the following two indicators: if the husband believes his wife's overall happiness would be the same after divorce and if he believes it would be better or much better. I specify the probability that the husband assigns to the event his wife is of type l as:

$$\delta^l = \frac{\exp\left(b'\rho^l + \eta^l\right)}{\sum_j \exp\left(b'\rho^j + \eta^j\right)},$$

where ρ^l for $l \in \{HO, HP, SO\}$ stands for the vector of coefficients to estimate, η^l for $l \in \{HO, HP, SO\}$ is an error term, and $\rho^{SP} = 0$ and $\eta^{SP} = 0$ by normalization.

Random vector $\eta = \left(\eta^{HO}, \eta^{HP}, \eta^{SO}\right)$ accounts for the information that the husband has about his wife's type but does not reveal in the NSFH interview. I assume that it is independently and identically distributed across couples as a normal random vector, $\eta \sim i.i.d.\ N\left(0, \Omega\right)$.

5.3 Estimation Strategy

My estimation strategy comprises three steps. First, I identify the conditions on the spousal payoffs, types, and beliefs under which each marital state is an equilibrium outcome of the game. Second, I utilize the parameterized functional forms in the explanatory variables as of the first NSFH wave for a couple to solve for the probability of observing the actual marital state in the second wave. Third, I estimate the structural parameters by the simulated maximum likelihood method.

A practical implementation of the strategy is contingent on finding an easy-to-compute expression for the likelihood contribution of a couple. I devise an algorithm to transform the likelihood contribution in such a way that it can be simulated by the GHK method, which is known to have high accuracy and desirable properties for numerical optimization (Börsch-Supan and Hajivassiliou, 1993; Hajivassiliou et al., 1996).

5.4 Likelihood Contributions

Let $X = (x, z_h, z_w, a_h, a_w, b)$ represent the entire data array of a couple (including information about spousal characteristics, location, types, and beliefs) as of the first NSFH wave, and let s index the marital states as of the second NSFH wave. Also, let $\Gamma = \left(\alpha, \beta_h, \beta_h^H, \beta_w, \beta_w^H, \gamma_h, \gamma_h^O, \gamma_w, \gamma_w^O, \left\{\lambda_h^j, \lambda_w^j, \rho^j\right\}_{j\in\{HO,HP,SO\}}, \Sigma, \Omega\right)$ stand for the array of all parameters of the model. The likelihood contribution of a couple in marital state s is the probability that s is an equilibrium outcome of the game. Since the three states are mutually exclusive and exhaustive, it suffices to solve for the likelihood contributions when $s =$ cooperation and $s =$ conflict.

The first step is to express the probability of a marital state in terms of the conditional probabilities given spousal types. I infer the spousal types independently on the basis of the available data. The probabilities of the states of cooperation and conflict are obtained by integrating the types out:

$$\Pr\left[s = \text{cooperation}|X, \Gamma\right] = \tag{5.1}$$
$$= \sum_{k,l} \pi_h^k\left(X, \Gamma\right) \pi_w^l\left(X, \Gamma\right) \Pr\left[s = \text{cooperation}|k, l, X, \Gamma\right],$$

$$\Pr\left[s = \text{conflict}|X, \Gamma\right] = \sum_{k,l} \pi_h^k\left(X, \Gamma\right) \pi_w^l\left(X, \Gamma\right) \Pr\left[s = \text{conflict}|k, l, X, \Gamma\right]. \tag{5.2}$$

The second step is to use the structure of the game in order to express the conditional probabilities given some fixed spousal types k and l. I employ the results in Chapter 3 to specify the restrictions under which cooperation and conflict are equilibrium outcomes. Then, each conditional probability can be calculated as the measure of the set of the error terms for which the corresponding restrictions are satisfied.

Consider the marital state of cooperation, which occurs if the husband offers an acceptable transfer to the wife. In equilibrium, the husband chooses the transfer that is best for him. I can formally write the conditional probability as:

$$\Pr\left[s = \text{cooperation} | k, l, X, \Gamma\right] = \tag{5.3}$$
$$= E_{\theta,\eta} 1(\tau^* = \arg \max_{\tau \in T^k(\theta, X, \Gamma)} \hat{E}\mathcal{V}_h^k\left(\tau; \mathcal{C}, \theta, \eta, X, \Gamma\right), \hat{E}\mathcal{V}_h^k\left(\tau^*; \mathcal{C}, \theta, \eta, X, \Gamma\right) \geq y_h^k\left(X, \Gamma\right),$$
$$u_w\left(\tau^*; \theta, X, \Gamma\right) \geq v_w^l\left(\theta, X, \Gamma\right), u_w\left(\tau^*; \theta, X, \Gamma\right) \geq y_w^l\left(X, \Gamma\right))',$$

where the dependence on the errors, data, and parameters is explicitly shown.

The first and second conditions in the indicator function in equation (5.3) mean that, for the husband of type k, transfer τ^* maximizes his expected utility of cooperating, and the husband decides to offer τ^* rather than to announce divorce. The third and fourth conditions mean that the utility that the wife of type l gets from accepting τ^* is at least as high as the utility from rejecting the offer or announcing divorce. The expected value of the indicator function is the probability measure of the set of the error terms for which the state of cooperation is the equilibrium outcome.

The conditional probability of the marital state of conflict is obtained analogously. This state happens if the husband offers a transfer, but the wife rejects it without announcing divorce. Again, in equilibrium, the husband must offer the transfer that maximizes his expected utility. The conditional probability is:

$$\Pr\left[s = \text{conflict}|k, l, X, \Gamma\right] = \tag{5.4}$$

$$= E_{\theta,\eta} 1(\tau^* = \arg\max_{\tau \in T^k(\theta,X,\Gamma)} \hat{E}\mathcal{V}_h^k\left(\tau; \mathcal{C}, \theta, \eta, X, \Gamma\right), \hat{E}\mathcal{V}_h^k\left(\tau^*; \mathcal{C}, \theta, \eta, X, \Gamma\right) \geq y_h^k\left(X, \Gamma\right),$$

$$v_w^l\left(\theta, X, \Gamma\right) > u_w\left(\tau^*; \theta, X, \Gamma\right), v_w^l\left(\theta, X, \Gamma\right) \geq y_w^l\left(X, \Gamma\right))',$$

and the third and fourth conditions now imply that the wife of type l decides to reject τ^* (but abstains from separating) rather than to accept the offer or announce divorce.

Once the conditional probabilities (5.3) and (5.4) have been evaluated in the estimation algorithm for all possible combinations of spousal types, it is straightforward to compute the likelihood contribution of couple i, $L_i\left(\Gamma\right) = \Pr\left[s_i|X_i, \Gamma\right]$, using equations (5.1) and (5.2). Then, the parameters of the model can be estimated by maximizing the sample log-likelihood function $\mathcal{L}\left(\Gamma\right) = \sum_i \ln L_i\left(\Gamma\right)$.

5.5 Transformation Algorithm

The computation of the likelihood contributions requires the evaluation of 7-dimensional integrals of the indicator functions in equations (5.3) and (5.4).[1] I transform the problem of evaluating these integrals by analytically solving for the boundaries of the corresponding integration regions of the joint density of θ and η. The transformation algorithm consists of several steps.

First, I partition the domain of error θ_4 into intervals. The interval boundaries are specifically chosen so that the expected value function of the husband of type k, $\hat{E}\mathcal{V}_h^k\left(\tau; \mathcal{C}\right)$, has a simple closed form expression on each interval.

Second, I study the properties of $\hat{E}\mathcal{V}_h^k\left(\tau; \mathcal{C}\right)$ on every interval of θ_4. It turns out that the expected value function has discontinuities in τ and is non-increasing in τ

[1] The integrals are 7-dimensional because the error terms θ and η are 4×1 and 3×1 vectors, respectively.

between adjacent discontinuity points. Thus, the optimal transfer, τ^*, is always at one of the discontinuities. Figures 3.3 and 3.5 for the numerical examples in Chapter 3 provide graphical illustrations of this important fact.

Third, given a specific wife's type l, I determine which transfers would be accepted by the wife and which ones would be rejected (without the divorce announcement). Acceptable transfers would result in the state of cooperation and the unacceptable ones would lead to the state of conflict.

Fourth, for every such transfer, I write out a system of inequalities underlying the decision of the husband of type k to offer it and, then, solve the system for integration bounds. In the solution, the errors are systematically arranged in a convenient way for further simulation.

The same steps are repeated for all intervals of θ_4 and all possible spousal types k and l.

Effectively, the algorithm transforms the problem of evaluating the expected value in equation (5.3) or equation (5.4) into the problem of computing several integrals of the form:

$$\int_{\mathbb{R}^3} \int_{f_1(\eta)}^{f_2(\eta)} \int_{f_3(\theta_4,\eta)}^{f_4(\theta_4,\eta)} \int_{f_5(\theta_3,\theta_4,\eta)}^{f_6(\theta_3,\theta_4,\eta)} \int_{f_7(\theta_2,\theta_3,\theta_4,\eta)}^{f_8(\theta_2,\theta_3,\theta_4,\eta)} \psi\left(\theta,\eta\right) d\theta_1 d\theta_2 d\theta_3 d\theta_4 d\eta,$$

where $\psi\left(\cdot\right)$ is the density of the errors and functions $f_1\left(\cdot\right)$, $f_2\left(\cdot\right)$,..., $f_8\left(\cdot\right)$ are the integration bounds.

Given the structure of the integrals, it is straightforward to simulate them with the GHK method. I give a complete analytical solution for integration bounds in Appendix B.

5.6 Identification

To identify the parameters of the model, I exploit the cross-sectional covariation of the explanatory variables with marital states observed for the NSFH couples. The variation in the spousal demographic characteristics helps to identify the coefficients of the payoff functions in the states of cooperation and conflict, if some characteristics are observed more commonly among spouses in conflict than in cooperation, for example. The variation in the location-specific variables helps to identify the parameters of the divorce utilities, if some characteristics (like longer separation periods, perhaps) are correlated with, e.g., a lower incidence of divorce. Differential responses of spouses about their own overall happiness after hypothetical marital break-up and about being a person of worth identify the coefficients of the type probability functions. Differential responses of husbands about the overall happiness of their wives after potential divorce are helpful in the identification of the parameters of the belief functions.

Two important remarks about the identification of the payoff coefficients are in order. First, I cannot separately estimate α_h and α_w in the specifications of $u_h(-\tau)$ and $u_w(\tau)$. Instead, I compute their sum, $\alpha = \alpha_h + \alpha_w$, which is the impact of x on the joint spousal value of the cooperative marriage relative to divorce. Mathematically, the reason is that the boundaries of the sets of error terms for which the states of cooperation and divorce are equilibrium outcomes depend on $x'(\alpha_h + \alpha_w)$ and not separately on $x'\alpha_h$ and $x'\alpha_w$. Intuitively, the occurrence of a marital state depends on the relative position of the utility possibility frontier that results from cooperation (see Figure 3.4 for a graphical illustration of the frontier). As the husband's and wife's payoffs are linear in the transfer, the location of the frontier depends on the sum of spousal utilities. If the husband's payoff rises, the frontier shifts to the right, but this change is observationally equivalent to an upward shift after an increase in the wife's payoff.

Second, I cannot estimate how divorce utilities depend on spousal characteristics x. If vector x were included in the deterministic components of the divorce payoffs along with the cooperation and conflict payoffs, the corresponding effects could not be simultaneously estimated. The reason is that an equilibrium in the game is invariant with respect to a positive affine transformation of the payoffs and, consequently, the spousal utility levels are not identifiable. An analogous identification issue is present in much simpler multinomial probit models, in which the value of a baseline alternative needs to be set to zero and the estimated coefficients for other alternatives are interpreted relative to the baseline. The exclusion restriction does not mean that spousal characteristics x are irrelevant after separation, but rather that the effect of x in the states of cooperation and conflict should be interpreted as being relative to the impact in the state of divorce.

The structural error terms η (reflecting the discreteness of husbands' reported beliefs about wives' type) and θ (reflecting unobserved factors influencing cooperation and conflict payoffs relative to divorce) are identified via the occurrence of marital states that cannot be otherwise explained using observable data. The model implies that if, for two couples with identical values of spousal payoffs, one ended up in the state of cooperation and the other one in the state of conflict, it must be that the husbands held different beliefs. The model also implies that if, for two couples with identical observable characteristics and belief vectors, one ended up in the state of cooperation and the other one in the state of divorce, it must be that the couples differed in their stochastic utility components.

A last restriction is needed to estimate the model. Since the game equilibrium is invariant to a positive affine transformation of the payoffs, I cannot identify the scale of the husband's and wife's utilities. Thus, in estimation, I normalize the corresponding diagonal elements of covariance matrix Σ to 1.

Chapter 6

Results

In this Chapter, I first analyze the estimated effects of spousal characteristics in the states of cooperation and conflict and of location-specific variables in the state of divorce. Then, I compute sample averages for the player type probabilities and beliefs, quantify the disutility impact of conflict, and conduct policy experiments. Lastly, I discuss the robustness of the results to the exclusion of potentially endogenous variables, perform goodness-of-fit and specification tests, and evaluate the out-of-sample predictive power of the model.

6.1 Payoff Parameters

As discussed in Chapter 5, in the state of cooperation, I can identify the impact of a variable on the sum of the husband's and wife's utilities. Moreover, the estimated coefficients in the states of cooperation and conflict show the effect of the couple's characteristics relative to their impact in the state of divorce. To facilitate further analysis of the relative importance of the explanatory variables, I define one util as the standard deviation of the normally distributed error that represents the stochastic component of a spousal payoff. This definition is due to the normalization in covariance matrix Σ.

6.1.1 Cooperation

Table 6.1 shows the parameters that determine by how much spousal characteristics contribute to payoffs from cooperation. The constant term is positive and significant (4.70). A positive sign is expected, since due to the chosen specification of the payoffs the constant absorbs divorce costs that cannot be attributed to other explanatory variables (such as separation requirements).

The effects of children differ by child age and relationship to the husband. A common child who is less than 6 years old is associated with an increase in the sum of the husband's and wife's payoffs by approximately 0.27 utils relative to the utility impact in the state of divorce. On the contrary, a wife's child decreases the joint surplus by 0.26 utils, and no significant effect can be attributed to common children who are 6 years old or older.

Additionally, I detect a positive role of marital duration, husband's age, and race. Increases in marital duration and husband's age by one year above their corresponding means are associated with extra 0.09 utils and 0.03 utils, respectively.

If the husband is black, it appears to raise the sum of spousal utilities by 0.54 utils. I also find positive effects of the husband being Catholic (0.18 utils) and college educated (0.20 utils), but they are not precisely estimated.

Indicators of marital heterogamy tend to decrease the marital surplus. Namely, a rise in spousal absolute age difference by one year lowers the sum of the payoffs by 0.04 utils. A difference of the educational attainments is associated with a decrease by 0.38 utils. However, the estimated impact of a difference in religious affiliations is

Table 6.1: Utility Parameters in State of Cooperation

Variable	Coeff.	Std. Err.
constant	4.702**	(0.303)
children, $<$ 6 y.o.	0.274**	(0.102)
children, $\geq$ 6 y.o.	-0.055	(0.072)
children, wife's	-0.261**	(0.107)
marital duration†	0.093**	(0.014)
home ownership	-0.134	(0.127)
age, husband's†	0.033**	(0.010)
age, abs. diff.†	-0.041**	(0.018)
black husband	0.543**	(0.254)
catholic husband	0.182	(0.125)
religion, diff.	0.067	(0.096)
high sch., husband‡	0.010	(0.048)
college, husband‡	0.195	(0.145)
education, diff.	-0.378**	(0.113)

Notes:

Coefficients denote the effect of corresponding variables on the sum of spousal utilities relative to the impact in the state of divorce.

†Variable is standardized in estimation. I report the impact of a one-year increase.

‡The omitted education category is "no high school degree."

* and ** denote significance at 10 and 5% levels, respectively.

not significant.

6.1.2 Conflict

Table 6.2 lists the coefficients that determine by how much spousal characteristics contribute to payoffs in the state of conflict (husband's and wife's payoffs are separately identifiable). I find strong and sizeable effects associated with the trait of "bargaining strength." If the husband is a "hard bargainer," his conflict payoff rises by 2.39 utils and the corresponding increase for the wife is 4.10 utils.

The constant terms of the husband's and wife's payoffs are negative and significant (-2.62 and -1.62, respectively). The negative signs are indicative of the utility losses associated with persistent disputes.[1]

I find that the presence of children tends to mitigate the negative impact of conflict by increasing spousal payoffs (relative to the divorce utilities). A common child who is less than 6 years old raises the husband's and wife's payoffs by 0.62 and 0.55 utils, respectively. A common child who is at least 6 increases the corresponding utilities by 0.45 and 0.50 utils. The effects of a wife's child are slightly smaller, but still significant. Such a child is associated with extra 0.31 and 0.41 utils of the husband's and wife's payoffs, respectively.

The effect of marital duration considerably differs from its impact in the state of cooperation. I do not detect a significant utility increase for husbands. Moreover, the effect on wives is negative: a rise in marital duration by one year lowers the wife's conflict payoff by 0.02 utils. Perhaps, spousal disputes escalate with the duration of marriage and the corresponding disutility effect outweighs the positive influence of accumulated marital capital (and it may also be that the capital matters only in the state of cooperation).

[1] The disutility impact of conflict is more rigorously quantified later in this Chapter.

Table 6.2: Utility Parameters in State of Conflict

	Husband		Wife	
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.
constant	-2.624^{**}	(0.678)	-1.620^{**}	(0.319)
children, $<$ 6 y.o.	0.623^{**}	(0.108)	0.554^{**}	(0.095)
children, $\geq$ 6 y.o.	0.453^{**}	(0.070)	0.498^{**}	(0.057)
children, wife's	0.310^{**}	(0.108)	0.406^{**}	(0.148)
marital duration†	0.015	(0.011)	-0.017^{**}	(0.006)
home ownership	1.544^{**}	(0.233)	-0.261^{*}	(0.150)
age, husband's†	0.113^{**}	(0.011)	0.000	(0.002)
age, abs. diff.†	-0.224^{**}	(0.027)	-0.002	(0.007)
black husband	-1.274^{**}	(0.367)	0.593^{**}	(0.228)
catholic husband	0.495^{**}	(0.150)	0.367^{**}	(0.131)
religion, diff.	-0.929^{**}	(0.199)	-0.019	(0.053)
high sch., husband‡	0.238^{*}	(0.141)	-0.500^{**}	(0.147)
college, husband‡	0.009	(0.042)	-0.960^{**}	(0.175)
education, diff.	-0.066	(0.095)	0.259^{**}	(0.116)
hard barg. constant	2.391^{**}	(0.529)	4.101^{**}	(0.125)

Notes:

Coefficients denote the effect of corresponding variables on spousal utilities relative to the impact in the state of divorce.

†Variable is standardized in estimation. I report the impact of a one-year increase.

‡The omitted education category is "no high school degree."

* and ** denote significance at 10 and 5% levels, respectively.

The estimation reveals differential impacts of home ownership and race. Home ownership raises the husband's conflict payoff by 1.54 utils and decreases the wife's payoff by 0.26 utils. This result is not surprising, since the utility effects are identified relative to the state of divorce and courts typically granted ownership of the marital home to wives in separating couples in early 1990s. If the husband is black, it appears to lower his utility from conflict (relative to divorce) by 1.27 utils and to raise the wife's payoff by 0.59 utils.

Additionally, I estimate a positive effect of the husband's age, as well as negative effects of the spousal absolute age difference and difference in religious affiliations on the husband's conflict payoff. An increase in the age by one year above the mean augments the utility of the husband by 0.11 utils. On the contrary, a rise in the absolute age difference by a year lowers the payoff by 0.22 utils and the difference in religious affiliations decreases it by 0.93 utils. The corresponding effects for wives are small and imprecisely estimated.

Interestingly, the husband being Catholic appears to partially mitigate the disutility impact of conflict on both spouses by adding 0.50 and 0.37 utils to the husband's and wife's payoffs, respectively. The positive coefficients may reflect higher separation costs of Catholics or their higher tolerance of disagreements in intact marriages, since the Catholic Church strongly opposes divorce.

The estimated coefficients do not indicate that better education of the husband considerably improves his conflict payoff. On the other hand, there is strong evidence that the husband's education decreases the utility of the wife. Specifically, the high school degree lowers her payoff by 0.50 utils and college degree is associated with a decline by 0.96 utils. The disutility effect is attenuated (or may even be reversed if the husband has no high school degree) by 0.26 utils when the spouses have different educational attainments.

Table 6.3: Utility Parameters in State of Divorce

	Husband		Wife	
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.
male-specific avail. ratio	0.264	(0.244)	–	
female-specific avail. ratio	–		1.369^{**}	(0.342)
$\frac{1}{2}$ year $\leq$ separation $\leq$ 1 year	-0.269^{*}	(0.158)	0.032	(0.099)
separation $>$ 1 year	-0.309^{**}	(0.134)	-0.162	(0.114)
collection rate	0.165	(0.253)	1.938^{**}	(0.819)
coll. rate $\times$ high sch., husband†	-1.633^{**}	(0.653)	–	
coll. rate $\times$ college, husband†	-0.819	(0.565)	–	
coll. rate $\times$ high sch., wife†	–		-1.802^{**}	(0.713)
coll. rate $\times$ college, wife†	–		-0.894	(0.626)
optimist's constant	3.710^{**}	(0.295)	0.655^{**}	(0.103)

Notes:

†The omitted education category is "no high school degree."

* and ** denote significance at 10 and 5% levels, respectively.

6.1.3 Divorce

Table 6.3 provides the estimated impact of the explanatory variables on spousal payoffs in the state of divorce (husband's and wife's payoffs are separately identifiable). I find strong and sizeable effects associated with the trait of "optimism." If the husband is an "optimist," his divorce payoff rises by 3.71 utils and the corresponding increase for the wife is 0.65 utils.

The results for the location-specific explanatory variables are in line with intuition. I find that better marriage market conditions tend to improve the divorce utilities. For instance, a 10 percentage point rise in the female-specific availability ratio increases the wife's payoff by 0.14 utils. The impact of the male-specific availability ratio on the husband's utility is positive but not precisely estimated. A relatively large coefficient for wives may reflect the fact that women typically experience unfavorable marriage market conditions and, thus, face a more pronounced marginal effect of partner availability.

As expected, I see that separation period requirements tend to decrease the divorce payoffs. Moreover, facing a separation period of longer than a year matters more than a shorter-term period. A period between 6 months and a year and a period of more than a year reduce the husband's utility by 0.27 and 0.31 utils, respectively. I do not find any impact of the shorter term period on the wife, but the longer term period decreases her utility by 0.16 utils (the effect is significantly negative at 8% level in a one-sided test). Also, I reject (at 10% significance level) the hypothesis of equal effects of the two periods on the sum of spousal payoffs in favor of a more negative effect of a separation period that is longer than a year.

As argued by Nixon (1997), the utility effect of better child support enforcement (CSE) is theoretically hard to quantify because of many concomitant factors, such as the amount of child support payments, individual's marginal utility of income, potential loss of welfare benefits, etc. Thus, the magnitude of the impact is an empirical matter. Interestingly, my estimates imply that the effect of the strength of CSE varies considerably with spousal education level, which may proxy for wealth. I detect no significant influence of the collection rate on husbands without a high school degree. However, higher collection rate decreases the divorce payoffs of husbands with high school and college degrees.[2] The corresponding reductions in the utility due to a 10 percentage point rise in the collection rate are 0.15 and 0.07 utils. In addition, I find no impact of the strength of CSE on wives with high school degrees (the statistical hypothesis of no effect cannot be rejected at a conventional significance level), while better enforcement of the payments positively affects the divorce payoffs

[2]Statistical hypotheses of no effect are respectively rejected at 1 and 10% significance levels in favor of a negative impact.

Table 6.4: Sample Means of Type Probabilities and Beliefs

Spousal Type		True Types† Husband	Wife	Beliefs‡ Husband
HO	(hard bargainer – optimist)	0.106	0.040	0.170
HP	(hard bargainer – pessimist)	0.141	0.249	0.027
SO	(soft bargainer – optimist)	0.019	0.048	0.112
SP	(soft bargainer – pessimist)	0.734	0.663	0.691

Notes:

†A cell represents the sample mean probability of the event that a spouse is of a corresponding type.

‡A cell represents the simulated sample mean probability which a husband assigns to the event that his wife is of a corresponding type.

In estimation, Σ and Ω are diagonal matrices. Sample log-likelihood: -2377.04.

of wives without high school degree and with college degree.[3] If the collection rate rose by 10 percentage points, the corresponding utilities would increase by 0.19 and 0.10 utils. Thus, stronger CSE overall tends to reduce the well-being of ex-husbands and improve the welfare of ex-wives.

6.2 Average Type Probabilities and Beliefs

I use the estimated parameters of the player type probabilities to infer the sample means of the probabilities for both spouses. Additionally, I employ the computed coefficients of the belief functions and the distribution of the unobservable belief component η to simulate husbands' beliefs and calculate the sample mean of the simulated belief vectors. The results are reported in Table 6.4.

[3]Statistical hypotheses of no effect are respectively rejected at 1 and 5% significance levels in favor of a positive impact.

The estimates indicate that almost three-fourths of husbands and two-thirds of wives in the sample are of the type "soft bargainer – pessimist." Hence, there is evidence that a large majority of individuals are deeply hurt by marital conflict and have a relatively low assessment of their own post-divorce opportunities, which contributes to a high rate of cooperation.

In the last two columns of Table 6.4, the mean type probabilities for wives somewhat differ from the mean husbands' beliefs.[4] Specifically, husbands tend to overestimate the prevalence of wives' "optimism" about the outside option and underestimate the prevalence of "hard bargaining." This result indicates the information asymmetries between spouses that may lead to inefficient outcomes.

6.3 Disutility Impact of Conflict

Given the estimates of structural parameters, it is possible to assess the disutility arising from persistent marital disputes by evaluating the expected difference between the sum of spousal payoffs in the state of cooperation and the sum of payoffs in the state of conflict, $E\left[u_h + u_w - v_h^k - v_w^l\right]$. This expected value shows by how much the joint utility of husband and wife of types k and l, respectively, would fall if the couple failed to reach an agreement about the transfer and stayed intact.

Since the true spousal types are unobservable, I cannot quantify the disutility effect as a single number for any actual couple. Nevertheless, it is straightforward to calculate the lower and upper bounds of the impact. Specifically, the solution for the boundaries of the set of the error terms for which the state of conflict is an equilibrium outcome (see Appendix B) implies that conflict can occur only when either both spouses are "hard bargainers," or the husband is a "soft bargainer" and

[4] I ignore wives' beliefs since they do not affect the outcome of the game.

wife is a "hard bargainer." Hence, the lower and upper bounds are:

$$
\begin{aligned}
LB &= E\left[u_h + u_w - v_h^H - v_w^H\right], \\
UB &= E\left[u_h + u_w - v_h^S - v_w^H\right].
\end{aligned}
$$

I evaluated the lower and upper bounds of the effect for every couple in the sample and calculated the corresponding means. I find that, on average, a couple would experience a loss between 1.45 and 3.84 utils should the spouses fail to agree on the transfer. Judging from the magnitudes of the effects of the explanatory variables on the payoffs, I conclude that conflict has a sizeable disutility impact on married couples.[5]

6.4 Policy Experiments

It is difficult to imagine that government action can address the root causes of inefficient outcomes in this model, which are generated by asymmetric information. Instead, I consider two types of public policies that have been subject to change in recent decades and that can alter the incidence of cooperation, conflict, and divorce among married couples. First, I analyze the impact of reducing separation period requirements. Second, I examine the effect of better enforcement of child support payments.

All counterfactual experiments are conducted on 100,000 couples that are randomly drawn with replacement from the NSFH sample. I adjust the explanatory variables in the divorce payoff functions in accordance with a proposed policy change, compute the probabilities of the three marital states under the new payoff structure,

[5]It is impossible to determine the relative magnitude of the disutility from conflict (i.e., a percentage utility loss in comparison to cooperation), because the level of utility is not identifiable.

and draw a realization of the state from the implied trinomial distribution. The fractions of couples in each marital state are calculated using the NSFH sample weights and, thus, the results are applicable to the U.S. population of married couples.

6.4.1 Separation Period Requirements

Separation periods are an important element of divorce legislation in a number of U.S. states, and state legislatures sometimes consider proposals to repeal or reduce the requirements.[6] I consider three hypothetical scenarios involving a relaxation of separation requirements. In the first scenario ("experiment 1"), states with separation periods of longer than a year replace them with periods between 6 months and a year and there are no other changes. In the second scenario ("experiment 2"), all separation periods are shortened. Specifically, states with short separation periods (6 months to 1 year) repeal them completely and states with long periods (more than a year) make them short. In the third scenario ("experiment 3"), all separation periods are eliminated.

The results of the three policy simulations are reported in Table 6.5. A counterfactual distribution of the marital states can be compared to their incidence before any policy change ("baseline"). Reducing separation period requirements leads to a moderately higher incidence of divorce (primarily by reducing the incidence of conflict for the marginal changes and the incidence of cooperation under the third scenario). For instance, if all separation periods were completely eliminated ("experiment 3"), the incidence of divorce would rise by about 0.9 percentage points (or 8.4% of the baseline rate). The corresponding effect on the occurrence of conflict is negative, but considerably smaller in absolute and relative size. Namely, the incidence of conflict

[6] Although it is uncommon, a separation period may also be extended. In 2006, Louisiana prolonged the period from 6 months to 1 year for couples with minor children.

Table 6.5: Policy Experiments: Changes in Separation Period Requirements

Marital State	Baseline	Experiment 1*	Experiment 2†	Experiment 3‡
Cooperation	78.65	78.81	78.53	77.97
Conflict	10.27	9.85	9.89	10.02
Divorce	11.08	11.34	11.58	12.01
Total	100.00	100.00	100.00	100.00

Notes:

All statistics are calculated using the NSFH sample weights.

A cell represents the weighted fraction of couples (in %) in a corresponding marital state.

*Separation periods of longer than 1 year are replaced by periods between 6 months and a year.

†Separation periods between 6 months and a year are eliminated and separation periods of longer than 1 year are replaced by periods between 6 months and a year.

‡All separation periods are eliminated.

would fall by at most 0.4 percentage points (or 4.1% of the baseline rate), if long periods were replaced with short ones ("experiment 1").

The structural estimation results imply that a shortening of separation periods improves the divorce payoffs of both spouses. It leads to a higher divorce rate, because conflict-ridden and poor match quality marriages become more prone to dissolution. However, the negative impact on the incidence of conflict is small and decreasing in absolute magnitude from the first to the third experiment. This decline occurs because a sizeable improvement in the husband's outside option in the second and third experiments provides him with an incentive to bargain more "aggressively," therefore, creating a concurrent positive effect on the probability of a negotiation failure. More generally, simultaneous changes in the spousal divorce payoffs trigger an adjustment of the bargaining strategy that is difficult to predict with mere intuition.

6.4.2 Child Support Enforcement

The nonpayment of court ordered child support is a matter of great public concern in the U.S., and both the federal and state governments devote substantial resources to the enforcement of child support obligations. For instance, according to the Office of Child Support Enforcement, the combined federal and state expenditures on the CSE program were $5.6 billion in the fiscal year 2006.[7] I analyze three counterfactual scenarios with a stronger enforcement of child support payments. In the first scenario ("experiment 4"), the CSE collection rate is doubled in every state. In the second scenario ("experiment 5"), the rate is uniformly increased to 50%. The 50% rate is the actual rate that the state CSE agencies had achieved on average by 2004, 10 years after the second NSFH wave. The third scenario ("experiment 6") considers the world of a perfect enforcement of child support obligations, in which the CSE

[7] http://www.acf.hhs.gov/programs/cse/pubs/2007/preliminary_report/.

Table 6.6: Policy Experiments: Changes in Strength of Child Support Enforcement

Marital State	Baseline	Experiment 4*	Experiment 5†	Experiment 6‡
Cooperation	78.65	79.42	79.95	81.56
Conflict	10.27	9.85	9.52	8.38
Divorce	11.08	10.73	10.53	10.06
Total	100.00	100.00	100.00	100.00

Notes:

All statistics are calculated using the NSFH sample weights.

A cell represents the weighted fraction of couples (in %) in a corresponding marital state.

*The CSE collection rate is increased two times in each state.

†The CSE collection rate is uniformly increased to 50% in each state.

‡The CSE collection rate is uniformly increased to 100% in each state.

collection rate is 100%.

The results of the three policy experiments are presented in Table 6.6. Interestingly, stronger CSE affects the incidence of marital disputes and dissolution in the same direction. An increase of the collection rate to 50% ("experiment 5") leads to a reduction of the fractions of couples in conflict and divorce by approximately 0.8 and 0.6 percentage points, respectively (or 7.3 and 5% of the corresponding baseline rates). Potentially, perfect CSE ("experiment 6") may decrease the incidence of conflict and divorce by almost 1.9 and about 1 percentage points (or 18.4 and 9.2% of the baseline rates).

The structural results indicate that better enforcement of child support payments tends to raise the divorce payoff of the wife and lower the payoff of the husband. Intuitively, since the effects have opposite signs, the impact on the incidence of marital dissolution among poor match quality couples should be small. However, a considerable decline in the husband's outside option induces him to bargain less "aggressively," which can explain a net reduction of the fractions of couples in the conflict and divorce states.

6.5 Potentially Endogenous Variables

Some explanatory variables in the spousal payoff functions, namely, common children, marital duration, and home ownership in the cooperation and conflict utilities, are potentially endogenous because they may reflect decisions about investments in marriage. For example, couples with an unfavorable realization of the stochastic utility component θ may perceive separation as a likely event and choose not to have children or not to purchase a house in order to reduce future divorce costs. In case the variables are endogenous, the structural parameters are inconsistently estimated.

I reestimated the structural model excluding common children, marital duration, and home ownership from the specification of the cooperation and conflict payoffs. Except for a change in the magnitude of coefficients for a few variables,[8] the reestimation results (see Tables C.2 through C.5 in Appendix C) are overall very similar to the ones obtained for the full model. This finding suggests that the scope of a potential endogeneity bias is limited.

6.6 Goodness-of-fit Tests

To evaluate how well the structural model fits the data in the estimation sample, I perform a series of conventional χ^2 goodness-of-fit tests (Bartoszyński and

[8]For example, the coefficient on husband's age considerably increases in the specification of the cooperation utility, which may be attributed to a high correlation between age and excluded marital duration.

Table 6.7: χ^2 Goodness-of-fit Tests: Sample Is Split by Predicted Marital State Probability

	Groups					
Partition Basis	1st	2nd	3rd	4th	5th	Sample
$\hat{P}_{cooperation}$	1.985	1.268	6.494**	2.323	1.848	13.919**
$\hat{P}_{conflict}$	3.768*	3.745*	0.920	0.013	0.700	9.147*
$\hat{P}_{divorce}$	0.979	3.775*	0.890	0.267	2.238	8.150*
H_0 distribution	$\chi^2(1)$	$\chi^2(1)$	$\chi^2(1)$	$\chi^2(1)$	$\chi^2(1)$	$\chi^2(4)$
# observations	776	775	776	775	776	3878

Note:

* and ** denote values of a statistic that are above 10 and 5% upper quantiles of a corresponding χ^2 distribution, respectively.

Niewiadomska-Bugaj, 1996, Section 17.2). The tests allow me to determine whether the probability distribution of the three marital states induced by the model could be the distribution that generated the data.

First, I use the estimated structural parameters to generate predicted probabilities of the three marital states for every observation in the estimation sample. Then, for each marital state, the sample is ordered according to a corresponding marital state probability and partitioned into five equally sized groups. Lastly, using actual and predicted counts of observations in the marital states, I compute χ^2 statistics for every group and the entire sample. Table 6.7 reports the results.

Under the null hypothesis (H_0) that the actual and predicted counts of observations are equal (i.e., the probability distribution of the marital states induced by the model could, indeed, be the distribution that generated the data), the test statistic is χ^2-distributed with 1 degree of freedom in a group and 4 degrees of freedom in the sample.

The test results indicate that the null hypothesis must be rejected at 5% signif-

Table 6.8: Details on 3rd Group in $\hat{P}_{cooperation}$ Partition

	Observations	
Marital State	Actual Count	Predicted Count
Cooperation	639	610
Conflict	72	85
Divorce	65	81
Total	776	776

icance level for the sample as a whole when it is split according to the predicted probability of cooperation. However, they also reveal that, with one exception, the structural model provides an adequate fit for every group in the three sample partitions and the corresponding test statistics are, at worst, only marginally significant.

Table 6.8 provides an example by listing actual and predicted counts of observations in the third group from the sample partitioning by the predicted cooperation probability (the null hypothesis for this group is rejected at 5% level). Within this group, the structural model underpredicts cooperation, while overpredicting conflict and divorce. However, the predicted counts still appear reasonably close to the actual ones.

Additionally, I perform χ^2 goodness-of-fit tests by partitioning the sample into 9 subsets according to the following scheme. The sample is first split in 3 equally sized groups with respect to the predicted probability of cooperation. Then, each such group is ordered by the predicted probability of conflict and partitioned into 3 equally sized subgroups. Table 6.9 provides values of the test statistic.

The test results indicate that the null hypothesis of the equality between the actual and predicted observation counts must be rejected at 5% significance level for the sample as a whole. However, the null hypothesis is decisively rejected for merely one subset and the test statistic is, at worst, only marginally significant in all other

Table 6.9: χ^2 Goodness-of-fit Tests: Sample Is Split by $\hat{P}_{cooperation}$ and $\hat{P}_{conflict}$

	$\hat{P}_{conflict}$					
	1st Subgroup		2nd Subgroup		3rd Subgroup	
$\hat{P}_{cooperation}$	Statistic	(# Obs.)	Statistic	(# Obs.)	Statistic	(# Obs.)
1st Group	1.284	(431)	6.229**	(431)	2.795*	(431)
2nd Group	2.184	(431)	1.997	(430)	3.619*	(431)
3rd Group	0.508	(431)	3.399*	(431)	2.970*	(431)
			Sample χ^2 statistic:		24.983**	(3878)

Notes:

Under the null hypothesis, χ^2 statistic for each cell is $\chi^2(1)$ and χ^2 statistic for the entire sample is $\chi^2(8)$.

* and ** denote values of a statistic that are above 10 and 5% upper quantiles of a corresponding χ^2 distribution, respectively.

Table 6.10: Details on 2nd $\hat{P}_{conflict}$ Subgroup of 1st $\hat{P}_{cooperation}$ Group

	Observations	
Marital State	Actual Count	Predicted Count
Cooperation	247	259
Conflict	78	60
Divorce	106	112
Total	431	431

cases.

Table 6.10 gives actual and predicted counts of observations in the second subgroup of the first group in the sample partition (the null hypothesis for this subgroup is rejected at 5% level). Within this subgroup, the structural model underpredicts conflict, while overpredicting cooperation and divorce. Nevertheless, the predicted counts look more or less in line with the actual ones.

Overall, I conclude that, while the fit of the structural model to the data is not perfect, in most instances the model performs reasonably well in predicting observable cooperation, conflict, and divorce in the estimation sample.

6.7 Specification Tests

The structural model employs a relatively parsimonious parameterization of the payoff functions, which most likely results in an omission of many potentially relevant variables from the estimated utility specifications. In this Section, I perform standard Lagrange multiplier tests (Hayashi, 2000, Section 7.4) to assess the impact on the spousal outside options of omitted variables pertaining to a state's property division regime and to the differential impact of CSE with respect to the number of a couple's children.

States in the U.S. have different legal regimes of property allocation after divorce. "Community property" states (for example, California) mandate an equal division of assets acquired during marriage between an ex-husband and ex-wife, while explicitly allowing the ex-spouses to keep the property they brought into marriage, including gifts and inheritance. In "common law" states (e.g., Mississippi), assets are allocated according to who has legal title to them. "Equitable distribution" states (for instance, Iowa) allow courts substantial discretion in dividing property "fairly" between the ex-spouses, conditional on their earning potentials, previous contributions to the property and marriage, and many other factors the court may deem relevant (Gray, 1998; Stevenson, 2007).[9]

Gray (1998) shows that the property division regime mediates the impact of the

[9] The legal literature and online reference sources typically group the common law and equitable distribution regimes together, categorizing states as either "community property," or "equitable distribution common-law" jurisdictions (see Freed and Walker, 1991). This classification is due to the fact that almost all formerly common law states presently have many equitable distribution provisions in the divorce statutes. Still, since such states (e.g., Virginia) have often retained at least some common law clauses, I follow the trinomial categorization scheme of Gray (1998, Table 1) and Stevenson (2007, Table 1).

Table 6.11: Specification Tests: Impact of Property Division Regimes

Test	Divorce Utilities Additionally Include:	H_0 Distribution	Statistic
1	Common law regime indicator	$\chi^2\,(2)$	1.599
2	Community property regime indicator	$\chi^2\,(2)$	1.484
3	Common law and community property regime indicators†	$\chi^2\,(4)$	3.715
4	Interaction of home ownership indicator with common law regime indicator	$\chi^2\,(2)$	5.355*
5	Interaction of home ownership indicator with community property regime indicator	$\chi^2\,(2)$	3.390
6	Interactions of home ownership indicator with common law and community property regime indicators†	$\chi^2\,(4)$	7.592

Notes:

†The omitted category is "equitable distribution regime."

* denotes value of a statistic that is above 10% upper quantile of a corresponding χ^2 distribution.

unilateral divorce adoption on married women's labor supply, and Stevenson (2007) demonstrates that the regime matters for home ownership among married couples. Thus, it is conceivable that the legal regime of asset allocation may affect the spousal bargaining positions via its impact on the divorce payoffs. I explore this possibility by including indicators for the property division regimes and their interactions with the home ownership dummy in the specification of the husband's and wife's divorce utilities and performing Lagrange multiplier tests for coefficients on the additional variables. Under the null hypothesis (H_0) of no impact on the divorce payoffs, the coefficients are 0. Table 6.11 outlines extensions to the specification of the utilities and reports test statistics.

The results indicate that the property division regime per se has no statistically detectable impact on the spousal outside options (i.e., H_0 of no impact cannot be rejected at a conventional significance level). Moreover, only the common law regime has an effect in interaction with the home ownership indicator, and the effect itself is merely marginal. Thus, I conclude that there is no strong evidence for a misspecification of the structural model as far as the asset allocation regime after divorce is concerned.

Previous estimation and policy simulation results (Tables 6.3 and 6.6) reveal important effects of a proxy for the strength of CSE, the collection rate, on the spousal utilities and incidence of marital outcomes. However, since the collection rate is not a monetary measure specific to a particular couple's circumstances, the functional forms of the husband's and wife's divorce payoffs, which include the rate and its interactions with education indicators, may not be fully capturing the impact of CSE on the utilities. I explore this possibility by including interactions of an indicator for the presence of 2 or more common children in a household with the collection rate[10] and with products of the rate and education dummies as additional variables in the divorce payoffs and performing Lagrange multiplier tests for corresponding coefficients. The idea of the tests is that the number of children is likely correlated with an expected amount of child support and better enforcement may more strongly affect parents with larger obligations. Under the null hypothesis of no impact on the divorce payoffs, the coefficients are 0. Table 6.12 outlines extensions to the specification of the utilities and reports test statistics.

The results indicate that the null hypothesis of no additional interactions of the collection rate in the divorce payoffs must be rejected at 5% significance level. The estimated structural model, indeed, does not fully capture the impact of the strength of CSE on the outside options. However, attempting to do so via inclusion of many

[10]Recall that the collection rate is positive in case a couple has at least *one* common child and is set to 0 otherwise.

Table 6.12: Specification Tests: Impact of Strength of CSE and Number of Children

Test	Divorce Utilities Additionally Include:	H_0 Distribution	Statistic
1	Interaction of indicator for presence of 2 or more children with collection rate	$\chi^2\,(2)$	11.812^{**}
2	Interactions of indicator for presence of 2 or more children with collection rate and products of collection rate with high school and college education indicators†	$\chi^2\,(6)$	14.153^{**}

Notes:

†The omitted education category is "no high school degree."

** denotes value of a statistic that is above 5% upper quantile of a corresponding χ^2 distribution.

extra terms in the parameterized utilities is likely to substantially slow down numerical optimization and may reduce the precision of other estimates. Thus, I defer a more refined modeling of the effects of the collection rate to future research work.

6.8 Out-of-sample Predictions

I evaluate the out-of-sample performance of the estimated structural model by predicting the incidence of divorce after the second NSFH wave in a sample of couples with available marital history records as of the third wave (2001-02).[11] This sample is referred to as "the prediction sample."

Because of funding constraints, the third survey was completed for a selected subset of original households. It did not include respondents under age 45 as of January 2000, unless in the first wave they had a child eligible for the second wave

[11]Data collected in the third NSFH interview was not used in the estimation of the model.

Table 6.13: Opinions and Beliefs in Prediction Sample

Variable	Mean	Std. Dev.	Min	Max
same happiness, husband	0.18	(0.38)	0	1
more happy, husband	0.06	(0.24)	0	1
worthy person, husband	0.39	(0.49)	0	1
same happiness, wife	0.14	(0.35)	0	1
more happy, wife	0.09	(0.28)	0	1
worthy person, wife	0.39	(0.49)	0	1
same happiness	0.22	(0.41)	0	1
more happy	0.06	(0.24)	0	1

Note:

See Table 4.7 for definitions of variables.

interview. Other techniques to save on interviewing costs were implemented, as well.[12] In total, I am able to use family history data for 2,002 couples who were married as of the second wave and did not experience death of a spouse afterwards.

Tables 6.13, 6.14, and 6.15 provide summary statistics for opinions and beliefs, individual characteristics, and location-specific variables, respectively, in the prediction sample as of the second wave. Most notably, spouses in this sample are on average older than respondents in the estimation sample (mean husband's age is 49 vs. 41 years) and have been married for a longer time (mean marital duration is 20 vs. 15 years). Also, they are slightly better educated, have fewer children who are less than 6 years old, and are more likely to be homeowners.

Since the average time lag between the second and third interviews is more than 50% longer than the one between the first and second interviews (roughly 8.5 vs. 5.5 years, respectively), the actual marital state of a couple as of the third wave is not helpful for evaluating the out-of-sample predictive power of the estimated model.

[12] For instance, new spouses and minor children were no longer interviewed.

Table 6.14: Individual Characteristics in Prediction Sample

Variable	Mean	Std. Dev.	Min	Max
children, $<$ 6 y.o.	0.19	(0.51)	0	4
children, $\geq$ 6 y.o.	0.68	(1.02)	0	8
children, wife's	0.14	(0.47)	0	4
marital duration	20.36	(12.95)	0	61
home ownership	0.90	(0.30)	0	1
age, husband's	48.78	(11.03)	20	90
age, abs. diff.	3.89	(3.83)	0	25
black husband	0.08	(0.27)	0	1
catholic husband	0.23	(0.42)	0	1
religion, diff.	0.29	(0.45)	0	1
high sch., husband	0.49	(0.50)	0	1
college, husband	0.41	(0.49)	0	1
high sch., wife	0.56	(0.50)	0	1
college, wife	0.35	(0.48)	0	1
education, diff.	0.37	(0.48)	0	1

Note:

See Table 4.8 for definitions of variables.

Table 6.15: Location-specific Information in Prediction Sample

Variable	Mean	Std. Dev.	Min	Max
male-specific avail. ratio	1.27	(0.22)	0.45	2.30
female-specific avail. ratio	0.81	(0.12)	0.29	1.32
$\frac{1}{2}$ year $\leq$ separation $\leq$ 1 year	0.18	(0.38)	0	1
separation, $>$ 1 year	0.30	(0.46)	0	1
collection rate*	0.20	(0.08)	0.07	0.37

Notes:

See Table 4.9 for definitions of variables.

*Statistics are for the subsample of couples with children.

Table 6.16: Divorce Incidence in Prediction Sample

Marital Status	Actual Rate, %	Predicted Rate*, %
Divorce	7.99	9.25

Note:

*Sample mean of predicted divorce probability.

Instead, I employ data from union history records, which contain starting and ending dates of all respondents' marriages, to infer whether couples in the prediction sample were still intact 5.5 years after the second NSFH wave.[13] Thus, I can assess the out-of-sample performance of the structural model in terms of its ability to predict the incidence of divorce. Table 6.16 presents the result of this exercise.

The structural model somewhat overpredicts divorce, since the predicted rate of 9.25% is 1.26 percentage points higher than the actual rate of 7.99%. However, the relative magnitude of the overprediction is on the order of only 15%. Thus, I conclude that, given a relatively parsimonious specification of the payoffs,[14] the model has a good out-of-sample predictive ability.

[13]Technically, I use here the average time lag between the first and second interviews in the estimation sample.

[14]Notably, the functional forms of the utilities are *linear* in husband's age and marital duration.

Chapter 7

Conclusion

In this dissertation, I develop and estimate a structural game theoretic model to explain why some married couples are observed to have persistent disputes but keep living together, while other couples cooperate, and the rest divorce.

In comparison to the existing family economics literature, this research project has a number of novel features. First, I treat marital conflict as a third distinct equilibrium outcome that can result from spousal negotiations. Second, family bargaining is modeled as a noncooperative game, which allows me to endogenize Pareto inferior outcomes and incorporate asymmetric information. Third, I exploit the informational richness of the NSFH to construct an indicator of conflict that encompasses both the frequency and intensity of disputes. Lastly, I estimate how policies related to separation periods and child support enforcement affect the outside options and simulate the impact of substantial changes in these policies on the incidence of marital cooperation, conflict, and divorce.

The model fits the data well and has a good out-of-sample predictive ability. The estimation results are mostly in line with intuition. Conflict has a sizeable disutility impact on spouses. Marital heterogamy indicators, e.g., a difference in spousal ages, tend to have a negative impact on payoffs when the marriage is intact. Mandatory separation periods adversely affect divorce utilities, whereas the effect of child support enforcement varies with educational attainment. The estimates also indicate two types of information asymmetries between spouses (related to differential effects of conflict and divorce), which can generate inefficient outcomes. Eliminating separation requirements increases the fraction of divorced couples by 0.9 percentage points (or 8.4% of the observed rate) and serves as a weak deterrent to persistent disputes. Contrastingly, strong child support enforcement has a potential to reduce the incidence of divorce and conflict by as much as 1 and 1.9 percentage points (or 9.2 and 18.4% of the respective rates). Thus, child support enforcement may be a promising avenue to explore for policy makers who want to target the phenomena of dysfunctional marriage and divorce simultaneously. Specification tests, however, suggest that a more refined approach to modeling the impact of the strength of CSE than the one pursued in this dissertation may be needed to fully grasp the effects of enforcement of child support obligations.

I conclude by pointing out two general directions for future research. First, the NSFH data on disagreements includes more information than can be completely captured by just two states of an intact marriage, with spouses reporting disputes over six distinct areas of the marital relationship. It may be interesting to see if existing multi-issue bargaining models (e.g., Lang and Rosenthal, 2001; Busch and Horstmann, 2002) can be extended to address the multidimensionality of family negotiations.

Second, for reasons of tractability and data availability, I constructed a relatively simple static model of spousal interactions. As more data are collected, a structural dynamic model may be needed to understand the evolution of negotiation strategies and private information, as well as possible interdependence between the search for a mate and marital bargaining.

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Appendix A

Availability Ratio

A.1 Terminology

The *sex ratio* is most simply defined as the ratio of the number individuals of one sex to the number of individuals of the opposite sex in a population. This concept is usually considered too coarse for the purpose of approximating conditions in a marriage market. In that case, the sex ratio is more narrowly defined as the ratio of the number of *suitable* marriageable individuals of one sex to the size of an *appropriate* group of individuals of the opposite sex. Methodological papers and empirical applications substantially differ as to whom they consider "suitable" and "appropriate." The *availability ratio*, which is the most refined form of the sex ratio concept existing to date, simplifies the task of determining "suitability" and "appropriateness" by utilizing weighting schemes that are based on observable bivariate distributions of characteristics of husbands and wives.

A couple is said to be married *endogamously* with respect to some characteristic if the spouses share it (e.g., both husband and wife are Black). Otherwise, the couple is married *exogamously*.

A.2 Literature

Demographers and sociologists have long been using the sex ratio, but the research agenda was typically limited to its effect on marriage rates. The two earliest examples are a study by Groves and Ogburn (1928), who analyze sex ratios for select cities, states, and ethnic groups in the U.S., and a paper by Cox (1940), who considers the geographical variation in the sex ratio for Blacks.

Empirical studies are often motivated by the need to explain large mate selection differentials between Whites and Blacks in the U.S. Schoen and Kluegel (1988) tabulate sex ratios specific to race, age, and education to determine the impact of population composition on the difference between marriage rates of Whites and Blacks in North Carolina and Virginia. Lichter et al. (1991) calculate race-specific ratios for each labor market area and conduct regression analysis to isolate the impact of spousal availability on the inter-area variation in female marriage rates. Brien (1997) constructs a whole spectrum of sex ratios to explore the difference in the timing of marriage between the two racial groups.

Grossbard-Shechtman (1993) summarizes sociological and economic theories that have led to the derivation of hypotheses regarding the impact of the sex ratio on household bargaining, welfare incidence, and labor supply. A number of recent papers elaborate on and test such hypotheses. Chiappori et al. (2002) outline a collective household model in which the sex ratio is an exogenous "distribution factor" that affects spousal bargaining strength. Fitzgerald (1991) investigates the impact of spousal availability and potential spousal quality on the length of the AFDC program spells. In a similar vein, Dickert-Conlin and Houser (1998) employ age- and race-specific sex ratios in a model that quantifies marriage disincentives arising from the interaction between the welfare system and federal income tax. Angrist (2002) uses variation in immigrant flows to identify the effect of the ratio on the labor force participation of children and grandchildren of immigrants.

The sorting of men and women into marriage has a close similarity to the matching of employers and employees. Presumably, individuals look for the best match among available candidates and a local marriage market serves as a spatial arena for "hiring" suitable mates (Lichter et al., 1992). This methodological approach has guided the development of many search-theoretic models of mating and marriage, ranging from simple nonstructural models (e.g., Lichter et al., 1995) to full-scale structural models (e.g., Brien et al., 2006). Within the search-theoretic framework, the availability of potential partners affects the reservation "quality" of a prospective mate, as well as the length of time one keeps searching.

A.3 Methodology

The concept of the availability ratio was developed by Goldman et al. (1984). Fossett and Kiecolt (1991) compare it to other forms of the sex ratio and conclude that the availability ratio is methodologically superior for the following reasons. First, it more accurately accounts for the opportunity sets of potential mates. Second, the ratio attempts to incorporate preferences over characteristics of a prospective spouse. Third, it can more readily deal with multiple status characteristics. Lastly, the ratio adequately registers competition on part of individuals from similar status groups.

Following the literature, I construct availability ratios that are specific to geographical location and selected individual characteristics. Namely, I limit the scope of a marriage market to county of residence and race and use a weighting scheme for age and education.

I introduce a simple age- and education-specific sex ratio first and subsequently generalize it to the availability ratio. Consider a population of individuals of a particular race in a given locality. Let $w^{i,e}_{j,k}$ stand for the weight assigned to age-education group (j,k) of men by women from age-education group (i,e). The weighted age- and education-specific sex ratio for a woman from group (i,e) is defined as:

$$SR_{i,e} = \frac{\sum\limits_{(j,k)} w^{i,e}_{j,k} M_{j,k}}{F_{i,e}}, \tag{A.1}$$

where $F_{i,e}$ is the number of women in group (i,e), $M_{j,k}$ is the number of men in group (j,k), and non-negative weights satisfy $\sum\limits_{(j,k)} w^{i,e}_{j,k} = 1$.

The numerator of the sex ratio formula $(A.1)$ can be interpreted as the expected number of suitable men to a woman from group (i,e). In practice, it is hardly ever the case that only women from group (i,e) "compete" for men $\sum\limits_{(j,k)} w^{i,e}_{j,k} M_{j,k}$. Marital partners suitable to these men may belong to other groups of women. The idea of the availability ratio is to modify the denominator in the sex ratio formula to account for the "competition" on part of such population groups.

Let $\tilde{w}^{j,k}_{l,m}$ be the weight that is assigned to age-education group (l,m) of women by men from age-education group (j,k). Clearly, the expected number of suitable women for a man from group (j,k) is $\sum\limits_{(l,m)} \tilde{w}^{j,k}_{l,m} F_{l,m}$, and, therefore, the expected number of female "competitors" faced by a woman from group (i,e) becomes $\sum\limits_{(j,k)} w^{i,e}_{j,k} \sum\limits_{(l,m)} \tilde{w}^{j,k}_{l,m} F_{l,m}$.

Then, the availability ratio for a woman in age-education group (i,e) can be defined as:

$$AR_{i,e} = \frac{\sum\limits_{(j,k)} w^{i,e}_{j,k} M_{j,k}}{\sum\limits_{(j,k)} \sum\limits_{(l,m)} w^{i,e}_{j,k} \tilde{w}^{j,k}_{l,m} F_{l,m}}. \tag{A.2}$$

The availability ratios for men are defined analogously.

A.4 Data

I calculate availability ratios for husbands and wives in the NSFH on the basis of the 5% PUMS for the 1990 Decennial Census. I limit the analysis to the population group of married-couple households defined in Chapter 4 and use provided PUMS weights to impute population characteristics.

The geographical unit in the PUMS is a Public Use Microdata Area (PUMA). By design, a PUMA encompasses a county with 100,000 to 200,000 inhabitants and never crosses state borders. Smaller adjacent counties are typically grouped together in one PUMA, while counties with more than 200,000 residents are split into several PUMAs. In case a PUMA consists of several counties, I calculate the availability ratios for the PUMA and assign the results to every constituent county. In case a county is split into several PUMAs, I pool the data from all such PUMAs, calculate the ratios, and assign the results to the county.

A.5 Marital Endogamy with Respect to Race

Table A.1 presents simple measures of marital endogamy by race and ethnicity.[1] The extent of the endogamy considerably varies across the three racial/ethnic groups. While the incidence of intermarriage between Whites and Blacks or Blacks and Hispanics is relatively low, it is nonnegligible between Whites and Hispanics.

The relatively high incidence of marital exogamy for Hispanics warrants a more detailed analysis. Table A.2 gives the marginal distributions of marriages in this ethnic group by partner's race and descent. Hispanics appear to marry endogamously with respect to race and Hispanic origin, but the tendency is often fairly weak. Roughly 22% of marriages of Hispanic Whites are to non-Hispanic Whites. The intermarriage

[1]Hispanics are grouped in a separate category.

Table A.1: Marital Endogamy by Race and Ethnicity, %

Variable*	Husbands	Wives
$\frac{\text{White married to White}}{\text{Married White}}$	98.47	98.34
$\frac{\text{White married to Black}}{\text{Married White}}$	0.11	0.34
$\frac{\text{White married to Hispanic}}{\text{Married White}}$	1.42	1.32
$\frac{\text{Black married to Black}}{\text{Married Black}}$	94.68	97.90
$\frac{\text{Black married to White}}{\text{Married Black}}$	4.19	1.40
$\frac{\text{Black married to Hispanic}}{\text{Married Black}}$	1.13	0.70
$\frac{\text{Hispanic married to Hispanic}}{\text{Married Hispanic}}$	82.12	80.73
$\frac{\text{Hispanic married to White}}{\text{Married Hispanic}}$	17.18	18.11
$\frac{\text{Hispanic married to Black}}{\text{Married Hispanic}}$	0.70	1.16

Notes:

Statistics are based on weighted counts from the 1990 Census PUMS.

*The numerator and denominator represent the number of individuals of a given sex for whom the specified conditions apply. For instance, variable $\frac{\text{White married to Black}}{\text{Married White}}$ in the "Husbands" column is the ratio of the number of White husbands who are married to a Black wife to the total number of White husbands. In the "Wives" column, it is the ratio of the number of White wives who are married to a Black husband to the total number of White wives.

Table A.2: Marriages of Hispanics by Partner's Race and Descent, %

	Hispanic Husbands			Hispanic Wives		
Spouse's Race	White	Black	Other	White	Black	Other
non-Hispanic:						
White	20.95	4.10	12.86	23.46	2.75	11.47
Black	0.18	19.57	0.56	0.41	24.16	1.18
Hispanic:						
White	77.41	7.79	2.11	74.32	4.88	1.69
Black	0.17	63.34	0.20	0.25	63.53	0.23
Other	1.29	5.20	84.27	1.56	4.68	85.43
Total	100.00	100.00	100.00	100.00	100.00	100.00

Notes:

Statistics are based on weighted counts from the 1990 Census PUMS.

"White," "Black," and "Other" exclusively refer to race.

rate between Hispanics of "Other" racial descent and non-Hispanic Whites is lower, but still nonnegligible (about 12%). Hispanic Blacks tend to actively intermarry with non-Hispanic Blacks.

I conclude that, as an approximation, Whites and Hispanics not of Black race may be treated as participants in the same marriage market, whereas Blacks (including Black Hispanics) participate in a separate market.

A.6 Marital Endogamy with Respect to Age

To compute the joint distribution of spousal ages, after some experimentation, I chose the following age intervals for husbands: [20, 29], [30, 34], [35, 39], [40, 44], [45, 54], [55, 64], [65, ∞). The intervals for wives are: [18, 27], [28, 32], [33, 37], [38, 42], [43, 52], [53, 62], [63, ∞). This partitioning precludes any age group from having disproportionate size.

Table A.3 presents the joint probability mass function (p.m.f.) of spousal ages for

Table A.3: Joint P.M.F. of Spousal Ages for Whites and Hispanics, %

	wife							
husband	$[18, 27]$	$[28, 32]$	$[33, 37]$	$[38, 42]$	$[43, 52]$	$[53, 62]$	$[63, \infty)$	Σ
$[20, 29]$	8.32	2.61	0.38	0.10	0.03	0.02	0.04	11.50
$[30, 34]$	2.12	6.96	2.89	0.43	0.11	0.03	0.01	12.55
$[35, 39]$	0.49	2.45	6.76	2.50	0.38	0.03	0.01	12.62
$[40, 44]$	0.14	0.68	2.47	6.35	2.26	0.06	0.01	11.97
$[45, 54]$	0.07	0.28	0.86	2.69	12.54	1.49	0.08	18.01
$[55, 64]$	0.02	0.05	0.11	0.28	3.00	10.16	1.74	15.35
$[65, \infty)$	0.03	0.01	0.02	0.04	0.35	2.80	14.76	18.01
Σ	11.19	13.03	13.48	12.39	18.68	14.59	16.64	100.00

Notes:

Statistics are based on weighted counts from the 1990 Census PUMS.

Numbers may not add up to 100% due to rounding.

Table A.4: Joint P.M.F. of Spousal Ages for Blacks, %

	wife							
husband	$[18, 27]$	$[28, 32]$	$[33, 37]$	$[38, 42]$	$[43, 52]$	$[53, 62]$	$[63, \infty)$	Σ
$[20, 29]$	7.80	2.66	0.48	0.10	0.04	0.04	0.06	11.18
$[30, 34]$	2.11	7.09	3.13	0.54	0.13	0.03	0.01	13.05
$[35, 39]$	0.54	2.66	7.08	2.84	0.48	0.05	0.02	13.67
$[40, 44]$	0.16	0.81	2.81	6.44	2.39	0.11	0.02	12.74
$[45, 54]$	0.10	0.38	1.22	3.26	12.51	1.95	0.16	19.57
$[55, 64]$	0.05	0.08	0.22	0.46	3.49	8.89	1.75	14.95
$[65, \infty)$	0.04	0.03	0.06	0.13	0.68	3.23	10.68	14.84
Σ	10.78	13.71	15.00	13.77	19.73	14.31	12.69	100.00

Notes:

Statistics are based on weighted counts from the 1990 Census PUMS.

Numbers may not add up to 100% due to rounding.

Table A.5: Age Interval Weights for White and Hispanic Men

	wife							
husband	$[18, 27]$	$[28, 32]$	$[33, 37]$	$[38, 42]$	$[43, 52]$	$[53, 62]$	$[63, \infty)$	Σ
$[20, 29]$	0.72	0.28	–	–	–	–	–	1.00
$[30, 34]$	0.17	0.55	0.28	–	–	–	–	1.00
$[35, 39]$	–	0.23	0.54	0.23	–	–	–	1.00
$[40, 44]$	–	–	0.27	0.54	0.19	–	–	1.00
$[45, 54]$	–	–	–	0.22	0.70	0.08	–	1.00
$[55, 64]$	–	–	–	–	0.23	0.66	0.11	1.00
$[65, \infty)$	–	–	–	–	–	0.18	0.82	1.00

Table A.6: Age Interval Weights for Black Men

	wife							
husband	$[18, 27]$	$[28, 32]$	$[33, 37]$	$[38, 42]$	$[43, 52]$	$[53, 62]$	$[63, \infty)$	Σ
$[20, 29]$	0.70	0.30	–	–	–	–	–	1.00
$[30, 34]$	0.16	0.54	0.30	–	–	–	–	1.00
$[35, 39]$	–	0.23	0.52	0.25	–	–	–	1.00
$[40, 44]$	–	–	0.30	0.50	0.20	–	–	1.00
$[45, 54]$	–	–	–	0.25	0.64	0.11	–	1.00
$[55, 64]$	–	–	–	–	0.29	0.59	0.12	1.00
$[65, \infty)$	–	–	–	–	–	0.28	0.72	1.00

non-Hispanic Whites and Hispanics (not of Black race) and Table A.4 gives the joint p.m.f. for Blacks (including Black Hispanics). Clearly, individuals tend to marry endogamously with respect to age and there are statistically few marriages in which husband's and wife's ages vastly differ. However, the number of mates who belong to adjacent age intervals is nonnegligible.

To obtain age interval weights, I first compute conditional age distributions using the joint distributions and then assign weights to at most three adjacent intervals. The age interval weights for men are given in Tables A.5 and A.6. The weights for women are presented in Tables A.7 and A.8.

Table A.7: Age Interval Weights for White and Hispanic Women

	wife						
husband	$[18, 27]$	$[28, 32]$	$[33, 37]$	$[38, 42]$	$[43, 52]$	$[53, 62]$	$[63, \infty)$
$[20, 29]$	0.74	0.20	–	–	–	–	–
$[30, 34]$	0.26	0.53	0.24	–	–	–	–
$[35, 39]$	–	0.27	0.50	0.24	–	–	–
$[40, 44]$	–	–	0.26	0.52	0.15	–	–
$[45, 54]$	–	–	–	0.24	0.67	0.11	–
$[55, 64]$	–	–	–	–	0.18	0.70	0.11
$[65, \infty)$	–	–	–	–	–	0.19	0.89
Σ	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table A.8: Age Interval Weights for Black Women

	wife						
husband	$[18, 27]$	$[28, 32]$	$[33, 37]$	$[38, 42]$	$[43, 52]$	$[53, 62]$	$[63, \infty)$
$[20, 29]$	0.72	0.19	–	–	–	–	–
$[30, 34]$	0.28	0.52	0.24	–	–	–	–
$[35, 39]$	–	0.29	0.47	0.25	–	–	–
$[40, 44]$	–	–	0.29	0.47	0.15	–	–
$[45, 54]$	–	–	–	0.28	0.64	0.15	–
$[55, 64]$	–	–	–	–	0.21	0.62	0.16
$[65, \infty)$	–	–	–	–	–	0.23	0.84
Σ	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table A.9: Joint P.M.F. of Spousal Education Levels for Whites and Hispanics, %

	wife			
husband	no HS	HS only	College +	Σ
no HS	11.63	8.93	1.04	21.60
HS only	6.42	33.93	7.55	47.89
College +	1.02	12.63	16.86	30.51
Σ	19.06	55.48	25.45	100.00

Notes:

Statistics are based on weighted counts from the 1990 Census PUMS.

Numbers may not add up to 100% due to rounding.

Table A.10: Joint P.M.F. of Spousal Education Levels for Blacks, %

	wife			
husband	no HS	HS only	College +	Σ
no HS	20.57	12.69	2.35	35.61
HS only	6.94	31.18	8.54	46.66
College +	0.99	6.92	9.82	17.73
Σ	28.50	50.79	20.71	100.00

Notes:

Statistics are based on weighted counts from the 1990 Census PUMS.

Numbers may not add up to 100% due to rounding.

A.7 Marital Endogamy with Respect to Education

Similar to the case of spousal age, I calculate the joint p.m.f.'s of educational attainments. The three mutually exclusive and exhaustive categories are "no HS" (no high school degree), "HS only" (high school as the highest completed level), and "College +" (the highest level is at least college).

The joint p.m.f.'s are given in Tables A.9 and A.10. Expectedly, marriages tend to be endogamous with respect to education. The intermarriage rate between high school drop-outs and college graduates is very low. However, individuals from these

Table A.11: Education Weights for White and Hispanic Men

	wife			
husband	no HS	HS only	College +	Σ
no HS	0.54	0.46	–	1.00
HS only	0.13	0.71	0.16	1.00
College +	–	0.45	0.55	1.00

Table A.12: Education Weights for Black Men

	wife			
husband	no HS	HS only	College +	Σ
no HS	0.58	0.42	–	1.00
HS only	0.15	0.67	0.18	1.00
College +	–	0.45	0.55	1.00

two groups frequently marry high school graduates.

Thus, I can obtain education weights by calculating the conditional p.m.f.'s and ignoring intermarriages between the "no HS" and "College +" categories. The weights are reported in Tables A.11, A.12, A.13, and A.14.

A.8 Age-Education Weights

The weight assigned to age-education group (j, k) of men by women from age-education group (i, e) is defined as:

$$w_{j,k}^{i,e} = w_{\,j}^{\,i} \cdot w_k^e, \tag{A.3}$$

where $w_{\,j}^{\,i}$ is the weight assigned to age interval j by women from age interval i and w_k^e is the weight assigned to education level k by women with education level e.

While using products is not as precise as a scheme in which the weights are specific to joint age-education categories, the approach is less computationally burdensome and employs a total of $4\times(19+7) = 104$ weights (instead of $4\times 19\times 7 = 532$ weights).

Table A.13: Education Weights for White and Hispanic Women

	wife		
husband	no HS	HS only	College +
no HS	0.61	0.16	–
HS only	0.39	0.61	0.34
College +	–	0.23	0.66
Σ	1.00	1.00	1.00

Table A.14: Education Weights for Black Women

	wife		
husband	no HS	HS only	College +
no HS	0.72	0.25	–
HS only	0.28	0.61	0.53
College +	–	0.14	0.47
Σ	1.00	1.00	1.00

Moreover, the more refined scheme is unreliable, because it requires a tabulation of very narrowly defined subsets of married couples in the PUMS and, thus, is prone to substantial extrapolation error.

A.9 Example: Availability Ratios in the U.S.

Table A.15 presents availability ratios for the United States that are computed in accordance with formula $(A.2)$.

The pattern of the ratios is characteristic of the mate selection differentials and marriage market outcomes long noted in the literature. First, marital opportunities of women deteriorate with age as older female cohorts face less favorable availability ratios in comparison to younger cohorts. This observation is typically attributed to higher male mortality rates. Second, there are considerable differences between the races. Black women typically have less favorable ratios than White women. The out-

Table A.15: Availability Ratios in the U.S.

	Men			Women		
education	age	White or Hispanic	Black	age	White or Hispanic	Black
no HS	[20, 29]	1.01	1.33	[18, 27]	0.92	0.78
HS only	[20, 29]	1.21	1.45	[18, 27]	1.13	0.98
College+	[20, 29]	1.03	1.09	[18, 27]	0.97	0.87
no HS	[30, 34]	0.89	1.18	[28, 32]	0.78	0.68
HS only	[30, 34]	1.10	1.33	[28, 32]	0.97	0.83
College+	[30, 34]	1.00	1.08	[28, 32]	0.89	0.76
no HS	[35, 39]	0.90	1.17	[33, 37]	0.68	0.61
HS only	[35, 39]	1.12	1.36	[33, 37]	0.88	0.73
College+	[35, 39]	1.03	1.15	[33, 37]	0.87	0.68
no HS	[40, 44]	0.98	1.23	[38, 42]	0.67	0.68
HS only	[40, 44]	1.21	1.42	[38, 42]	0.85	0.74
College+	[40, 44]	1.08	1.22	[38, 42]	0.86	0.67
no HS	[45, 54]	1.15	1.34	[43, 52]	0.74	0.80
HS only	[45, 54]	1.38	1.48	[43, 52]	0.86	0.77
College+	[45, 54]	1.19	1.29	[43, 52]	0.84	0.66
no HS	[55, 64]	1.24	1.42	[53, 62]	0.69	0.79
HS only	[55, 64]	1.40	1.41	[53, 62]	0.72	0.65
College+	[55, 64]	1.14	1.20	[53, 62]	0.66	0.49
no HS	$[65, \infty)$	1.76	1.82	$[63, \infty)$	0.67	0.79
HS only	$[65, \infty)$	1.80	1.48	$[63, \infty)$	0.63	0.58
College+	$[65, \infty)$	1.42	1.16	$[63, \infty)$	0.53	0.36

come is believed to result from a disproportionately large fraction of institutionalized Black males, as well as a low sex ratio at birth (1.02 for Blacks vs. 1.05 for Whites) and, to a lesser degree, a differential Census undercount. Third, Black women who are college graduates have poor marital opportunities and the prospects are particularly bad for such women in older cohorts.

Appendix B

Analytical Solution for Integration Bounds

B.1 Notation

To save space in this Appendix, I use the notation from Chapter 5 with two minor extensions. First, the deterministic components of the payoffs in the states of cooperation and conflict are denoted with bars:

$$u_h(-\tau) = \bar{u}_h - \tau + \theta_1 \text{ and } u_w(\tau) = \bar{u}_w + \tau + \theta_3,$$

$$v_h^k = \bar{v}_h^k + \theta_2 \text{ and } v_w^l = \bar{v}_w^l + \theta_4 \text{ for any } k, l,$$

where a summand with a bar is a function of the data and parameters, for instance, $\bar{v}_h^H = x'\beta_h + \beta_h^H$.

Second, $\hat{E}\mathcal{V}_h^k(\mathcal{C})$ denotes for the maximized value of the expected utility of cooperation for the husband of type k.

B.2 Partition of θ_4 Domain

Recall that the expected utility of the husband of type k of proposing cooperation and offering transfer τ is:

$$\begin{aligned}\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) &= \sum_l \delta^l \left[u_h(-\tau)\cdot 1\begin{pmatrix} u_w(\tau)\geq y_w^l \\ u_w(\tau)\geq v_w^l \end{pmatrix} + v_h^k \cdot 1 \begin{pmatrix} v_w^l \geq y_w^l \\ v_w^l > u_w(\tau)\end{pmatrix} + \right. \\ & \left. + y_h^k \cdot 1 \begin{pmatrix} y_w^l > v_w^l \\ y_w^l > u_w(\tau)\end{pmatrix}\right].\end{aligned}$$

The value of structural error θ_4 determines the relative position of the wife's payoffs in the states of conflict and divorce, v_w^l and y_w^l. If the domain of θ_4 is appropriately partitioned, it is possible to considerably simplify the expression for $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ on each interval. In total, I consider six cases.

Case I $y_w^O > y_w^P > v_w^H > v_w^S$

It can be shown that the corresponding region for θ_4 is $\left(-\infty, y_w^P - \bar{v}_w^H\right)$.

Define $\tau^1 : u_w(\tau^1) = y_w^P$, $\tau^2 : u_w(\tau^2) = y_w^O$. Clearly, $\tau^1 < \tau^2$. It is straightforward to simplify the expression for $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ as:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) = \begin{cases} y_h^k, & \text{if } \tau\in[\tau_{\min},\tau^1), \\ \left(\delta^{HO}+\delta^{SO}\right)y_h^k + \left(\delta^{HP}+\delta^{SP}\right)u_h(-\tau), & \text{if } \tau\in[\tau^1,\tau^2), \\ u_h(-\tau), & \text{if } \tau\in[\tau^2,\tau_{\max}]. \end{cases}$$

Case II $y_w^O > v_w^H \geq y_w^P > v_w^S$

The corresponding region for θ_4 depends on the relative size of $\bar{v}_w^H - \bar{v}_w^S$ and $y_w^O - y_w^P$. If $\bar{v}_w^H - \bar{v}_w^S < y_w^O - y_w^P$, then $\theta_4 \in \left[y_w^P - \bar{v}_w^H, y_w^P - \bar{v}_w^S\right)$. If $\bar{v}_w^H - \bar{v}_w^S > y_w^O - y_w^P$, then

$\theta_4 \in \left[y_w^P - \bar{v}_w^H, y_w^O - \bar{v}_w^H\right)$. If $\bar{v}_w^H - \bar{v}_w^S = y_w^O - y_w^P$, then $\theta_4 \in \left[y_w^P - \bar{v}_w^H, y_w^P - \bar{v}_w^S\right)$ or, equivalently, $\theta_4 \in \left[y_w^P - \bar{v}_w^H, y_w^O - \bar{v}_w^H\right)$. Rearranging these expressions, the region for θ_4 is $\left[y_w^P - \bar{v}_w^H, \min\left\{y_w^P - \bar{v}_w^S, y_w^O - \bar{v}_w^H\right\}\right)$.

Let $\tau^1 : u_w(\tau^1) = y_w^P$, $\tau^2 : u_w(\tau^2) = v_w^H$, $\tau^3 : u_w(\tau^3) = y_w^O$. Clearly, $\tau^1 \leq \tau^2 < \tau^3$. The expression for $\hat{E}\mathcal{V}_h^k(\tau; \mathcal{C})$ is:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) = \begin{cases} \left(\delta^{HO} + \delta^{SO} + \delta^{SP}\right) y_h^k + \delta^{HP} v_h^k, & \text{if } \tau \in [\tau_{\min}, \tau^1), \\ \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h(-\tau), & \text{if } \tau \in [\tau^1, \tau^2), \\ \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) u_h(-\tau), & \text{if } \tau \in [\tau^2, \tau^3), \\ u_h(-\tau), & \text{if } \tau \in [\tau^3, \tau_{\max}]. \end{cases}$$

Case III $y_w^O > v_w^H > v_w^S \geq y_w^P$

This case occurs provided that $\bar{v}_w^H - \bar{v}_w^S < y_w^O - y_w^P$ and the corresponding region for θ_4 is $\left[y_w^P - \bar{v}_w^S, y_w^O - \bar{v}_w^H\right)$.

Let $\tau^1 : u_w(\tau^1) = v_w^S$, $\tau^2 : u_w(\tau^2) = v_w^H$, $\tau^3 : u_w(\tau^3) = y_w^O$. Clearly, $\tau^1 < \tau^2 < \tau^3$. The expression for $\hat{E}\mathcal{V}_h^k(\tau; \mathcal{C})$ is:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) = \begin{cases} \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) v_h^k, & \text{if } \tau \in [\tau_{\min}, \tau^1), \\ \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h(-\tau), & \text{if } \tau \in [\tau^1, \tau^2), \\ \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) u_h(-\tau), & \text{if } \tau \in [\tau^2, \tau^3), \\ u_h(-\tau), & \text{if } \tau \in [\tau^3, \tau_{\max}]. \end{cases}$$

Case IV $v_w^H \geq y_w^O > y_w^P > v_w^S$

This case occurs under condition $y_w^O - y_w^P < \bar{v}_w^H - \bar{v}_w^S$ and the corresponding region for θ_4 is $\left[y_w^O - \bar{v}_w^H, y_w^P - \bar{v}_w^S\right)$.

Let $\tau^1 : u_w(\tau^1) = y_w^P$, $\tau^2 : u_w(\tau^2) = y_w^O$, $\tau^3 : u_w(\tau^3) = v_w^H$. Clearly, $\tau^1 < \tau^2 \leq \tau^3$.

The expression for $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ is:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) = \begin{cases} \left(\delta^{SO}+\delta^{SP}\right)y_h^k + \left(\delta^{HO}+\delta^{HP}\right)v_h^k, & \text{if } \tau\in[\tau_{\min},\tau^1), \\ \delta^{SO}y_h^k + \left(\delta^{HO}+\delta^{HP}\right)v_h^k + \delta^{SP}u_h(-\tau), & \text{if } \tau\in[\tau^1,\tau^2), \\ \left(\delta^{HO}+\delta^{HP}\right)v_h^k + \left(\delta^{SO}+\delta^{SP}\right)u_h(-\tau), & \text{if } \tau\in[\tau^2,\tau^3), \\ u_h(-\tau), & \text{if } \tau\in[\tau^3,\tau_{\max}]. \end{cases}$$

Case V $v_w^H \geq y_w^O > v_w^S \geq y_w^P$

The corresponding region for θ_4 depends on the relative size of $\bar{v}_w^H - \bar{v}_w^S$ and $y_w^O - y_w^P$. If $\bar{v}_w^H - \bar{v}_w^S < y_w^O - y_w^P$, then $\theta_4 \in \left[y_w^O - \bar{v}_w^H, y_w^O - \bar{v}_w^S\right)$. If $\bar{v}_w^H - \bar{v}_w^S > y_w^O - y_w^P$, then $\theta_4 \in \left[y_w^P - \bar{v}_w^S, y_w^O - \bar{v}_w^S\right)$. If $\bar{v}_w^H - \bar{v}_w^S = y_w^O - y_w^P$, then $\theta_4 \in \left[y_w^O - \bar{v}_w^H, y_w^O - \bar{v}_w^S\right)$ or, equivalently, $\theta_4 \in \left[y_w^P - \bar{v}_w^S, y_w^O - \bar{v}_w^S\right)$. Rearranging these expressions, the region for θ_4 is $\left[\max\left\{y_w^O - \bar{v}_w^H, y_w^P - \bar{v}_w^S\right\}, y_w^O - \bar{v}_w^S\right)$.

Let $\tau^1 : u_w(\tau^1) = v_w^S$, $\tau^2 : u_w(\tau^2) = y_w^O$, $\tau^3 : u_w(\tau^3) = v_w^H$. Clearly, $\tau^1 < \tau^2 \leq \tau^3$. The expression for $\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ is:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) = \begin{cases} \delta^{SO}y_h^k + \left(\delta^{HO}+\delta^{HP}+\delta^{SP}\right)v_h^k, & \text{if } \tau\in[\tau_{\min},\tau^1), \\ \delta^{SO}y_h^k + \left(\delta^{HO}+\delta^{HP}\right)v_h^k + \delta^{SP}u_h(-\tau), & \text{if } \tau\in[\tau^1,\tau^2), \\ \left(\delta^{HO}+\delta^{HP}\right)v_h^k + \left(\delta^{SO}+\delta^{SP}\right)u_h(-\tau), & \text{if } \tau\in[\tau^2,\tau^3), \\ u_h(-\tau), & \text{if } \tau\in[\tau^3,\tau_{\max}]. \end{cases}$$

Case VI $v_w^H > v_w^S \geq y_w^O > y_w^P$

The corresponding region for θ_4 is $\left[y_w^O - \bar{v}_w^S, +\infty\right)$.

Let $\tau^1 : u_w(\tau^1) = v_w^S$, $\tau^2 : u_w(\tau^2) = v_w^H$. Clearly, $\tau^1 < \tau^2$. The expression for

$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C})$ is:

$$\hat{E}\mathcal{V}_h^k(\tau;\mathcal{C}) = \begin{cases} v_h^k, & \text{if } \tau \in [\tau_{\min}, \tau^1), \\ \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h(-\tau), & \text{if } \tau \in [\tau^1, \tau^2), \\ u_h(-\tau), & \text{if } \tau \in [\tau^2, \tau_{\max}]. \end{cases}$$

B.3 Integration Bounds

Prior to solving for the bounds of integration, I establish a condition that must be satisfied by vector θ in all six cases.

Proposition 1 The assumption that the payoffs in the state of conflict are inside the utility possibility frontier resulting from cooperation is equivalent to condition:

$$\theta_1 > -\bar{u}_h - \bar{u}_w + \bar{v}_h^H + \bar{v}_w^H + \theta_2 - \theta_3 + \theta_4 \text{ for all } \theta.$$

Proof. Suppose the assumption holds. Then, there exists transfer τ^0 such that $\bar{u}_h + \theta_1 - \tau^0 > \bar{v}_h^H + \theta_2$ and $\bar{u}_w + \theta_3 + \tau^0 > \bar{v}_w^H + \theta_4$ for any $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)'$. These inequalities imply that $\bar{u}_h - \bar{v}_h^H + \theta_1 - \theta_2 > \tau^0 > -\bar{u}_w + \bar{v}_w^H - \theta_3 + \theta_4$ and, thus, $\theta_1 > -\bar{u}_h - \bar{u}_w + \bar{v}_h^H + \bar{v}_w^H + \theta_2 - \theta_3 + \theta_4$.

Conversely, suppose that the condition holds. Then, it must be that case that $\bar{u}_h - \bar{v}_h^H + \theta_1 - \theta_2 > -\bar{u}_w + \bar{v}_w^H - \theta_3 + \theta_4$ for any θ. Define transfer τ^0 as:

$$\tau^0 = \bar{u}_h - \bar{v}_h^H + \theta_1 - \theta_2 - \frac{\epsilon}{2} = -\bar{u}_w + \bar{v}_w^H - \theta_3 + \theta_4 + \frac{\epsilon}{2},$$

where $\epsilon = \left[\bar{u}_h - \bar{v}_h^H + \theta_1 - \theta_2\right] - \left[-\bar{u}_w + \bar{v}_w^H - \theta_3 + \theta_4\right] > 0$.

Then, for any θ it is the case that $\bar{u}_h - \bar{v}_h^H + \theta_1 - \theta_2 > \bar{u}_h - \bar{v}_h^H + \theta_1 - \theta_2 - \frac{\epsilon}{2}$ or, equivalently, $\bar{u}_h + \theta_1 - \tau^0 > \bar{v}_h^H + \theta_2$. Also, $-\bar{u}_w + \bar{v}_w^H - \theta_3 + \theta_4 + \frac{\epsilon}{2} > -\bar{u}_w + \bar{v}_w^H - \theta_3 + \theta_4$ or, equivalently, $\bar{u}_w + \theta_3 + \tau^0 > \bar{v}_w^H + \theta_4$, which shows that the assumption holds. ■

It is easier to first solve for integration bounds ignoring the condition in Proposition 1 and, then, modify the bounds accounting for the condition.

Case I $y_w^O > y_w^P > v_w^H > v_w^S$

In this case, $\tau^1 = y_w^P - \bar{u}_w - \theta_3$ and $\tau^2 = y_w^O - \bar{u}_w - \theta_3$. Also, since $u_h(-\tau)$ is decreasing in τ:

$$\hat{E}\mathcal{V}_h^k(\mathcal{C}) = \max\left\{y_h^k, \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) u_h\left(-\tau^1\right), u_h\left(-\tau^2\right)\right\}.$$

Cooperation

The state of cooperation obtains provided that the following conditions are satisfied:

$$\begin{aligned}
\hat{E}\mathcal{V}_h^k(\mathcal{C}) &\geq y_h^k, \\
u_w(\tau^*) &\geq \max\left\{v_w^l, y_w^l\right\}, \\
u_h(-\tau^*) &\geq y_h^k, \hat{E}\mathcal{V}_h^k(\tau^*, \mathcal{C}) = \hat{E}\mathcal{V}_h^k(\mathcal{C}).
\end{aligned}$$

In present case, $\hat{E}\mathcal{V}_h^k(\mathcal{C}) \geq y_h^k$ and $y_w^l > v_w^l$. Therefore, the set of conditions can be simplified as:

$$\begin{aligned}
u_h(-\tau^*) &\geq y_h^k, u_w(\tau^*) \geq y_w^l, \\
\hat{E}\mathcal{V}_h^k(\tau^*, \mathcal{C}) &= \hat{E}\mathcal{V}_h^k(\mathcal{C}).
\end{aligned}$$

If $l \in \{HO, SO\}$, the set of conditions can only hold when $\tau^* = \tau^2$. Then, the set of conditions becomes:

$$\begin{aligned}
u_h\left(-\tau^2\right) &\geq y_h^k, \\
u_h\left(-\tau^2\right) &\geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) u_h\left(-\tau^1\right).
\end{aligned}$$

Equivalently:

$$\begin{aligned}
\theta_1 &\geq y_h^k + y_w^O - \bar{u}_h - \bar{u}_w - \theta_3, \\
\theta_1 &\geq y_h^k + \frac{y_w^O - \left(\delta^{HP} + \delta^{SP}\right) y_w^P}{\delta^{HO} + \delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3.
\end{aligned}$$

Since $\frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right) y_w^P}{\delta^{HO}+\delta^{SO}} > y_w^O$, the conditions can be simplified as:

$$\theta_1 \geq y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right) y_w^P}{\delta^{HO}+\delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3.$$

Incorporating the condition in Proposition 1:

$$\theta_1 \in \left(\max \left\{ \begin{array}{c} y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right) y_w^P}{\delta^{HO}+\delta^{SO}}, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, +\infty \right).$$

If $l \in \{HP, SP\}$, the set of inequalities can only hold when $\tau^* = \tau^1$ or $\tau^* = \tau^2$. That is:

$$\begin{aligned} u_h\left(-\tau^1\right) &\geq y_h^k, \\ \left(\delta^{HO}+\delta^{SO}\right) y_h^k + \left(\delta^{HP}+\delta^{SP}\right) u_h\left(-\tau^1\right) &\geq y_h^k, \\ \left(\delta^{HO}+\delta^{SO}\right) y_h^k + \left(\delta^{HP}+\delta^{SP}\right) u_h\left(-\tau^1\right) &\geq u_h\left(-\tau^2\right) \end{aligned}$$

or:

$$\begin{aligned} u_h\left(-\tau^2\right) &\geq y_h^k, \\ u_h\left(-\tau^2\right) &\geq \left(\delta^{HO}+\delta^{SO}\right) y_h^k + \left(\delta^{HP}+\delta^{SP}\right) u_h\left(-\tau^1\right). \end{aligned}$$

Equivalently:

$$\begin{aligned} \theta_1 &\geq y_h^k + y_w^P - \bar{u}_h - \bar{u}_w - \theta_3, \\ \theta_1 &\leq y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right) y_w^P}{\delta^{HO}+\delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 \end{aligned}$$

or:

$$\theta_1 \geq y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right) y_w^P}{\delta^{HO}+\delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3.$$

Merging and simplifying these conditions and noting that $y_w^P < \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right) y_w^P}{\delta^{HO}+\delta^{SO}}$:

$$\theta_1 \geq y_h^k + y_w^P - \bar{u}_h - \bar{u}_w - \theta_3.$$

Incorporating the condition in Proposition 1:

$$\theta_1 \in \left(\max \left\{ \begin{array}{c} y_h^k + y_w^P, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, +\infty \right).$$

Conflict

Observe that in this case, $y_w^l > v_w^l$ for all l. Thus, the state of conflict is ruled out.

Case II $y_w^O > v_w^H \geq y_w^P > v_w^S$

In this case, $\tau^1 = y_w^P - \bar{u}_w - \theta_3$, $\tau^2 = \bar{v}_w^H - \bar{u}_w - \theta_3 + \theta_4$, and $\tau^3 = y_w^O - \bar{u}_w - \theta_3$. Also:

$$\hat{E}\mathcal{V}_h^k(\mathcal{C}) = \max \left\{ \begin{array}{c} \left(\delta^{HO} + \delta^{SO} + \delta^{SP}\right) y_h^k + \delta^{HP} v_h^k, \\ \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right), \\ \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) u_h\left(-\tau^2\right), \\ u_h\left(-\tau^3\right) \end{array} \right\}.$$

Cooperation

The state of cooperation obtains provided that the following conditions are satisfied:

$$\begin{array}{rcl} \hat{E}\mathcal{V}_h^k(\mathcal{C}) & \geq & y_h^k, \\ u_w(\tau^*) & \geq & \max\left\{v_w^l, y_w^l\right\}, \\ u_h(-\tau^*) & \geq & y_h^k, \\ \hat{E}\mathcal{V}_h^k(\tau^*, \mathcal{C}) & = & \hat{E}\mathcal{V}_h^k(\mathcal{C}). \end{array}$$

If $l \in \{HO, SO\}$, then $\max\left\{v_w^l, y_w^l\right\} = y_w^O$. The state of cooperation obtains if

$\tau^* = \tau^3$. The set of conditions specializes as:

$$\begin{aligned} u_h\left(-\tau^3\right) &\geq y_h^k, \\ u_h\left(-\tau^3\right) &\geq \left(\delta^{HO} + \delta^{SO} + \delta^{SP}\right) y_h^k + \delta^{HP} v_h^k, \\ u_h\left(-\tau^3\right) &\geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right), \\ u_h\left(-\tau^3\right) &\geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) u_h\left(-\tau^2\right). \end{aligned}$$

Equivalently:

$$\begin{aligned} \theta_1 &\geq y_h^k - \bar{u}_h + \tau^3, \\ \theta_1 &\geq \left(1 - \delta^{HP}\right) y_h^k - \bar{u}_h + \delta^{HP} v_h^k + \tau^3, \\ \theta_1 &\geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k - \bar{u}_h + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right) + \tau^3, \\ \theta_1 &\geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k - \bar{u}_h + \left(\delta^{HP} + \delta^{SP}\right) u_h\left(-\tau^2\right) + \tau^3, \end{aligned}$$

or, equivalently:

$$\begin{aligned} \theta_1 &\geq y_h^k + y_w^O - \bar{u}_h - \bar{u}_w - \theta_3, \\ \theta_1 &\geq \left(1 - \delta^{HP}\right) y_h^k + y_w^O + \delta^{HP} \bar{v}_h^k - \bar{u}_h - \bar{u}_w + \delta^{HP} \theta_2 - \theta_3, \\ \theta_1 &\geq \frac{\left(\delta^{HO} + \delta^{SO}\right) y_h^k + y_w^O - \delta^{SP} y_w^P + \delta^{HP} \bar{v}_h^k}{1 - \delta^{SP}} - \bar{u}_h - \bar{u}_w + \frac{\delta^{HP}}{1 - \delta^{SP}} \theta_2 - \theta_3, \\ \theta_1 &\geq y_h^k + \frac{y_w^O - \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H}{\delta^{HO} + \delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\left(\delta^{HP} + \delta^{SP}\right)}{\delta^{HO} + \delta^{SO}} \theta_4. \end{aligned}$$

It can be shown that:

$$y_h^k + \frac{y_w^O - \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H}{\delta^{HO} + \delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\left(\delta^{HP} + \delta^{SP}\right)}{\delta^{HO} + \delta^{SO}} \theta_4 \geq y_h^k + y_w^O - \bar{u}_h - \bar{u}_w - \theta_3.$$

The inequality follows from $\theta_4 \leq y_w^O - \bar{v}_w^H$, which is true here, since if $\bar{v}_w^H - \bar{v}_w^S < y_w^O - y_w^P$, then $\theta_4 < y_w^P - \bar{v}_w^S < y_w^O - \bar{v}_w^H \leq y_w^O - \bar{v}_w^H$, whereas if $\bar{v}_w^H - \bar{v}_w^S > y_w^O - y_w^P$, then $\theta_4 < y_w^O - \bar{v}_w^H \leq y_w^O - \bar{v}_w^H$, as well.

The set of inequalities becomes:

$$\begin{aligned}
\theta_1 &\geq \left(1-\delta^{HP}\right) y_h^k + y_w^O + \delta^{HP}\bar{v}_h^k - \bar{u}_h - \bar{u}_w + \delta^{HP}\theta_2 - \theta_3, \\
\theta_1 &\geq \frac{\left(\delta^{HO}+\delta^{SO}\right) y_h^k + y_w^O - \delta^{SP} y_w^P + \delta^{HP}\bar{v}_h^k}{1-\delta^{SP}} - \bar{u}_h - \bar{u}_w + \frac{\delta^{HP}}{1-\delta^{SP}}\theta_2 - \theta_3, \\
\theta_1 &\geq y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HO}+\delta^{SO}}\theta_4.
\end{aligned}$$

Equivalently:

$$\theta_1 \geq \max\left\{\begin{array}{c}
\left(1-\delta^{HP}\right) y_h^k + y_w^O + \delta^{HP}\bar{v}_h^k + \delta^{HP}\theta_2, \\
\frac{\left(\delta^{HO}+\delta^{SO}\right) y_h^k + y_w^O - \delta^{SP} y_w^P + \delta^{HP}\bar{v}_h^k}{1-\delta^{SP}} + \frac{\delta^{HP}}{1-\delta^{SP}}\theta_2, \\
y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \frac{\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HO}+\delta^{SO}}\theta_4
\end{array}\right\} - \bar{u}_h - \bar{u}_w - \theta_3.$$

Incorporating the condition in Proposition 1:

$$\theta_1 \in \left(\max\left\{\begin{array}{c}
\left(1-\delta^{HP}\right) y_h^k + y_w^O + \delta^{HP}\bar{v}_h^k + \delta^{HP}\theta_2, \\
\frac{\left(\delta^{HO}+\delta^{SO}\right) y_h^k + y_w^O - \delta^{SP} y_w^P + \delta^{HP}\bar{v}_h^k}{1-\delta^{SP}} + \frac{\delta^{HP}}{1-\delta^{SP}}\theta_2, \\
y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \frac{\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HO}+\delta^{SO}}\theta_4, \\
\bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4
\end{array}\right\} - \bar{u}_h - \bar{u}_w - \theta_3, +\infty\right).$$

If $l = HP$ then $\max\left\{v_w^l, y_w^l\right\} = v_w^H$. The state of cooperation obtains if $\tau^* = \tau^3$ or $\tau^* = \tau^2$. The case of $\tau^* = \tau^3$ has been solved above.

Consider $\tau^* = \tau^2$. The set of conditions specializes as:

$$\begin{aligned}
u_h\left(-\tau^2\right) &\geq y_h^k, \\
\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \left(\delta^{HP}+\delta^{SP}\right) u_h\left(-\tau^2\right) &\geq \left(\delta^{HO}+\delta^{SO}+\delta^{SP}\right) y_h^k + \delta^{HP} v_h^k, \\
\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \left(\delta^{HP}+\delta^{SP}\right) u_h\left(-\tau^2\right) &\geq \left(\delta^{HO}+\delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \\
&\quad + \delta^{SP} u_h\left(-\tau^1\right), \\
\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \left(\delta^{HP}+\delta^{SP}\right) u_h\left(-\tau^2\right) &\geq u_h\left(-\tau^3\right).
\end{aligned}$$

Equivalently:

$$\begin{aligned}
\theta_1 &\geq y_h^k - \bar{u}_h + \tau^2, \\
\theta_1 &\geq \frac{\delta^{SP}}{\delta^{HP} + \delta^{SP}} y_h^k + \frac{\delta^{HP}}{\left(\delta^{HP} + \delta^{SP}\right)} v_h^k - \bar{u}_h + \tau^2, \\
\theta_1 &\geq v_h^k - \bar{u}_h - \frac{\delta^{SP}}{\delta^{HP}} \tau^1 + \frac{\left(\delta^{HP} + \delta^{SP}\right)}{\delta^{HP}} \tau^2, \\
\theta_1 &\leq y_h^k - \bar{u}_h - \frac{\delta^{HP} + \delta^{SP}}{\delta^{HO} + \delta^{SO}} \tau^2 + \frac{1}{\delta^{HO} + \delta^{SO}} \tau^3.
\end{aligned}$$

Equivalently:

$$\begin{aligned}
\theta_1 &\geq y_h^k + \bar{v}_w^H - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\
\theta_1 &\geq \frac{\delta^{SP} y_h^k + \delta^{HP} \bar{v}_h^k}{\delta^{HP} + \delta^{SP}} + \bar{v}_w^H - \bar{u}_h - \bar{u}_w + \frac{\delta^{HP}}{\delta^{HP} + \delta^{SP}} \theta_2 - \theta_3 + \theta_4, \\
\theta_1 &\geq \bar{v}_h^k + \frac{\left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H - \delta^{SP} y_w^P}{\delta^{HP}} - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \frac{\delta^{HP} + \delta^{SP}}{\delta^{HP}} \theta_4, \\
\theta_1 &\leq y_h^k + \frac{y_w^O - \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H}{\delta^{HO} + \delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\delta^{HP} + \delta^{SP}}{\delta^{HO} + \delta^{SO}} \theta_4.
\end{aligned}$$

That is:

$$\theta_1 \in \left(\begin{array}{c} \max \left\{ \begin{array}{c} y_h^k + \bar{v}_w^H + \theta_4, \\ \frac{\delta^{SP} y_h^k + \delta^{HP} \bar{v}_h^k}{\delta^{HP} + \delta^{SP}} + \bar{v}_w^H + \frac{\delta^{HP}}{\delta^{HP} + \delta^{SP}} \theta_2 + \theta_4, \\ \bar{v}_h^k + \frac{\left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H - \delta^{SP} y_w^P}{\delta^{HP}} + \theta_2 + \frac{\delta^{HP} + \delta^{SP}}{\delta^{HP}} \theta_4 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, \\ y_h^k + \frac{y_w^O - \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H}{\delta^{HO} + \delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\delta^{HP} + \delta^{SP}}{\delta^{HO} + \delta^{SO}} \theta_4 \end{array} \right).$$

Depending on θ_2, the above interval for θ_1 may be an empty set. The interval is non-empty provided that the following inequalities hold:

$$y_h^k + \bar{v}_w^H + \theta_4 < y_h^k + \frac{y_w^O - \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H}{\delta^{HO} + \delta^{SO}} - \frac{\delta^{HP} + \delta^{SP}}{\delta^{HO} + \delta^{SO}} \theta_4,$$

$$\frac{\delta^{SP}y_h^k+\delta^{HP}\bar{v}_h^k}{\delta^{HP}+\delta^{SP}}+\bar{v}_w^H+\frac{\delta^{HP}}{\delta^{HP}+\delta^{SP}}\theta_2+\theta_4<y_h^k+\frac{y_w^O-\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}}-$$
$$-\frac{\delta^{HP}+\delta^{SP}}{\delta^{HO}+\delta^{SO}}\theta_4,$$
$$\bar{v}_h^k+\frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H-\delta^{SP}y_w^P}{\delta^{HP}}+\theta_2+\frac{\delta^{HP}+\delta^{SP}}{\delta^{HP}}\theta_4<y_h^k+\frac{y_w^O-\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}}-$$
$$-\frac{\delta^{HP}+\delta^{SP}}{\delta^{HO}+\delta^{SO}}\theta_4.$$

Note that in this case, if $\bar{v}_w^H-\bar{v}_w^S<y_w^O-y_w^P$, then $\theta_4<y_w^P-\bar{v}_w^S<y_w^O-\bar{v}_w^H$, whereas if $\bar{v}_w^H-\bar{v}_w^S>y_w^O-y_w^P$, $\theta_4<y_w^O-\bar{v}_w^H$, as well. Thus, it is always true here that $\theta_4<y_w^O-\bar{v}_w^H$.

The first inequality can be simplified as $\bar{v}_w^H-y_w^O<-\theta_4$. Thus, it always holds.

The second and third inequalities are equivalent to:

$$\begin{aligned}
\theta_2 &< y_h^k-\bar{v}_h^k+\frac{\delta^{HP}+\delta^{SP}}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)}\left(y_w^O-\bar{v}_w^H\right)-\frac{\delta^{HP}+\delta^{SP}}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)}\theta_4,\\
\theta_2 &< y_h^k-\bar{v}_h^k+\frac{\delta^{HP}y_w^O-\left(1-\delta^{SP}\right)\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H+\left(\delta^{HO}+\delta^{SO}\right)\delta^{SP}y_w^P}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)}-\\
&\quad-\frac{\left(1-\delta^{SP}\right)\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)}\theta_4.
\end{aligned}$$

Notice that the third inequality implies the second one:

$$\frac{\delta^{HP}y_w^O-\left(1-\delta^{SP}\right)\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H+\left(\delta^{HO}+\delta^{SO}\right)\delta^{SP}y_w^P}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)}-$$
$$-\frac{\left(1-\delta^{SP}\right)\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)}\theta_4\leq\frac{\delta^{HP}+\delta^{SP}}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)}\left(y_w^O-\bar{v}_w^H\right)-\frac{\delta^{HP}+\delta^{SP}}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)}\theta_4,$$

since it can be rearranged as:

$$\frac{y_w^O-\left(\delta^{HO}+\delta^{SO}\right)y_w^P}{\left(\delta^{HP}+\delta^{SP}\right)}-\bar{v}_w^H\geq\theta_4.$$

The inequality is true because $\theta_4<y_w^O-\bar{v}_w^H\leq\frac{y_w^O-\left(\delta^{HO}+\delta^{SO}\right)y_w^P}{\left(\delta^{HP}+\delta^{SP}\right)}-\bar{v}_w^H$, which follows from $0\leq\left(\delta^{HO}+\delta^{SO}\right)\left[y_w^O-y_w^P\right]$.

Thus, the interval for θ_1 is non-empty provided that:

$$\begin{aligned}\theta_2 &< y_h^k - \bar{v}_h^k + \frac{\delta^{HP} y_w^O - \left(1-\delta^{SP}\right)\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H + \left(\delta^{HO}+\delta^{SO}\right)\delta^{SP} y_w^P}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)} - \\ &\quad - \frac{\left(1-\delta^{SP}\right)\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)}\theta_4.\end{aligned}$$

Now, incorporating the condition in Proposition 1:

$$\theta_1 \in \left(\begin{array}{c} \max\left\{ \begin{array}{c} y_h^k + \bar{v}_w^H + \theta_4, \\ \frac{\delta^{SP} y_h^k + \delta^{HP} \bar{v}_h^k}{\delta^{HP}+\delta^{SP}} + \bar{v}_w^H + \frac{\delta^{HP}}{\delta^{HP}+\delta^{SP}}\theta_2 + \theta_4, \\ \bar{v}_h^k + \frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H - \delta^{SP} y_w^P}{\delta^{HP}} + \theta_2 + \frac{\delta^{HP}+\delta^{SP}}{\delta^{HP}}\theta_4, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, \\ y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\delta^{HP}+\delta^{SP}}{\delta^{HO}+\delta^{SO}}\theta_4 \end{array} \right).$$

This interval is non-empty if the following additional condition regarding θ_2 holds:

$$\bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 < y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \frac{\delta^{HP}+\delta^{SP}}{\delta^{HO}+\delta^{SO}}\theta_4,$$

or, equivalently:

$$\theta_2 < y_h^k - \bar{v}_h^H + \frac{y_w^O - \bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \frac{1}{\delta^{HO}+\delta^{SO}}\theta_4.$$

Thus, the interval for θ_1 is non-empty provided that θ_2 belongs to:

$$\left(-\infty, \min\left\{ \begin{array}{c} -\bar{v}_h^k + \frac{\delta^{HP} y_w^O - \left(1-\delta^{SP}\right)\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H + \left(\delta^{HO}+\delta^{SO}\right)\delta^{SP} y_w^P}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)} - \\ - \frac{\left(1-\delta^{SP}\right)\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)}\theta_4, \\ -\bar{v}_h^H + \frac{y_w^O - \bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \frac{1}{\delta^{HO}+\delta^{SO}}\theta_4 \end{array} \right\} + y_h^k \right).$$

If $l = SP$, then $\max\left\{v_w^l, y_w^l\right\} = y_w^P$. The state of cooperation occurs if $\tau^* = \tau^3$, or $\tau^* = \tau^2$, or $\tau^* = \tau^1$. The cases $\tau^* = \tau^3$ and $\tau^* = \tau^2$ have been solved above.

Consider $\tau^* = \tau^1$. The set of conditions specializes as:

$$\begin{aligned}
u_h\left(-\tau^1\right) &\geq y_h^k, \\
\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq y_h^k, \\
\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq \left(\delta^{HO}+\delta^{SO}\right) y_h^k + \\
&\quad + \left(\delta^{HP}+\delta^{SP}\right) u_h\left(-\tau^2\right), \\
\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq u_h\left(-\tau^3\right).
\end{aligned}$$

Equivalently:

$$\begin{aligned}
\theta_1 &\geq y_h^k + y_w^P - \bar{u}_h - \bar{u}_w - \theta_3, \\
\theta_1 &\geq \frac{\left(\delta^{HP}+\delta^{SP}\right) y_h^k - \delta^{HP}\bar{v}_h^k}{\delta^{SP}} + y_w^P - \bar{u}_h - \bar{u}_w - \frac{\delta^{HP}}{\delta^{SP}}\theta_2 - \theta_3, \\
\theta_1 &\leq \bar{v}_h^k + \frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H - \delta^{SP} y_w^P}{\delta^{HP}} - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \frac{\delta^{HP}+\delta^{SP}}{\delta^{HP}}\theta_4, \\
\theta_1 &\leq \frac{\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \delta^{HP}\bar{v}_h^k + y_w^O - \delta^{SP} y_w^P}{1-\delta^{SP}} - \bar{u}_h - \bar{u}_w + \frac{\delta^{HP}}{1-\delta^{SP}}\theta_2 - \theta_3.
\end{aligned}$$

That is:

$$\theta_1 \in \left(\begin{array}{c} \max\left\{ \begin{array}{c} y_h^k + y_w^P, \\ \frac{\left(\delta^{HP}+\delta^{SP}\right) y_h^k - \delta^{HP}\bar{v}_h^k}{\delta^{SP}} + y_w^P - \frac{\delta^{HP}}{\delta^{SP}}\theta_2 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, \\ \min\left\{ \begin{array}{c} \bar{v}_h^k + \frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H - \delta^{SP} y_w^P}{\delta^{HP}} + \theta_2 + \frac{\delta^{HP}+\delta^{SP}}{\delta^{HP}}\theta_4, \\ \frac{\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \delta^{HP}\bar{v}_h^k + y_w^O - \delta^{SP} y_w^P}{1-\delta^{SP}} + \frac{\delta^{HP}}{1-\delta^{SP}}\theta_2 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3 \end{array} \right).$$

Depending on θ_2, this interval for θ_1 may be an empty set. The interval is non-empty provided that the following four inequalities hold:

$$\begin{aligned}
y_h^k + y_w^P - \frac{\delta^{HP}+\delta^{SP}}{\delta^{HP}}\theta_4 &< \bar{v}_h^k + \frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H - \delta^{SP} y_w^P}{\delta^{HP}} + \theta_2, \\
y_h^k + y_w^P - \frac{\delta^{HP}}{1-\delta^{SP}}\theta_2 &< \frac{\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \delta^{HP}\bar{v}_h^k + y_w^O - \delta^{SP} y_w^P}{1-\delta^{SP}},
\end{aligned}$$

$$\frac{\left(\delta^{HP}+\delta^{SP}\right)y_h^k-\delta^{HP}\bar{v}_h^k}{\delta^{SP}}+y_w^P<\bar{v}_h^k+\frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H-\delta^{SP}y_w^P}{\delta^{HP}}+\theta_2+$$
$$+\frac{\delta^{HP}}{\delta^{SP}}\theta_2+\frac{\delta^{HP}+\delta^{SP}}{\delta^{HP}}\theta_4,$$
$$\frac{\left(\delta^{HP}+\delta^{SP}\right)y_h^k-\delta^{HP}\bar{v}_h^k}{\delta^{SP}}+y_w^P<\frac{\left(\delta^{HO}+\delta^{SO}\right)y_h^k+\delta^{HP}\bar{v}_h^k+y_w^O-\delta^{SP}y_w^P}{1-\delta^{SP}}+$$
$$+\frac{\delta^{HP}}{\delta^{SP}}\theta_2+\frac{\delta^{HP}}{1-\delta^{SP}}\theta_2.$$

Equivalently:

$$\begin{aligned}
y_h^k-\bar{v}_h^k+\frac{\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HP}}\left(y_w^P-\bar{v}_w^H\right)-\frac{\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HP}}\theta_4 &< \theta_2,\\
y_h^k-\bar{v}_h^k+\frac{y_w^P-y_w^O}{\delta^{HP}} &< \theta_2,\\
y_h^k-\bar{v}_h^k+\frac{\delta^{SP}}{\delta^{HP}}\left(y_w^P-\bar{v}_w^H\right)-\frac{\delta^{SP}}{\delta^{HP}}\theta_4 &< \theta_2,\\
y_h^k-\bar{v}_h^k+\frac{\delta^{SP}}{\delta^{HP}}\left(y_w^P-y_w^O\right) &< \theta_2.
\end{aligned}$$

It is easy to verify that the first inequality implies the second one:

$$y_h^k-\bar{v}_h^k+\frac{\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HP}}\left(y_w^P-\bar{v}_w^H\right)-\frac{\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HP}}\theta_4>y_h^k-\bar{v}_h^k+\frac{y_w^P-y_w^O}{\delta^{HP}},$$

or, equivalently:

$$y_w^P-\bar{v}_w^H+\frac{y_w^O-y_w^P}{\delta^{HP}+\delta^{SP}}>\theta_4,$$

which is true because:

$$y_w^P-\bar{v}_w^H+\frac{y_w^O-y_w^P}{\delta^{HP}+\delta^{SP}}\geq y_w^P-\bar{v}_w^H+y_w^O-y_w^P=y_w^O-\bar{v}_w^H>\theta_4.$$

Also, the third inequality implies the fourth one:

$$y_h^k-\bar{v}_h^k+\frac{\delta^{SP}}{\delta^{HP}}\left(y_w^P-\bar{v}_w^H\right)-\frac{\delta^{SP}}{\delta^{HP}}\theta_4\geq y_h^k-\bar{v}_h^k+\frac{\delta^{SP}}{\delta^{HP}}\left(y_w^P-y_w^O\right),$$

equivalently, $y_w^O-\bar{v}_w^H\geq\theta_4$, which holds in this case.

Lastly, the third inequality implies the first one:

$$y_h^k-\bar{v}_h^k+\frac{\delta^{SP}}{\delta^{HP}}\left(y_w^P-\bar{v}_w^H\right)-\frac{\delta^{SP}}{\delta^{HP}}\theta_4 \geq y_h^k-\bar{v}_h^k+\frac{\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HP}}\left(y_w^P-\bar{v}_w^H\right)-\frac{\left(\delta^{HP}+\delta^{SP}\right)}{\delta^{HP}}\theta_4,$$

equivalently, $\theta_4 \geq y_w^P - \bar{v}_w^H$, which holds in this case, as well.

Therefore, when $l = SP$ the interval for θ_1 is non-empty provided that:

$$\theta_2 > y_h^k - \bar{v}_h^k + \frac{\delta^{SP}}{\delta^{HP}}\left(y_w^P - \bar{v}_w^H\right) - \frac{\delta^{SP}}{\delta^{HP}}\theta_4.$$

Incorporating the condition in Proposition 1:

$$\theta_1 \in \left(\begin{array}{c} \max\left\{\begin{array}{c} y_h^k + y_w^P, \\ \frac{\left(\delta^{HP}+\delta^{SP}\right)y_h^k-\delta^{HP}\bar{v}_h^k}{\delta^{SP}} + y_w^P - \frac{\delta^{HP}}{\delta^{SP}}\theta_2, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 \end{array}\right\} - \bar{u}_h - \bar{u}_w - \theta_3, \\ \min\left\{\begin{array}{c} \bar{v}_h^k + \frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H-\delta^{SP}y_w^P}{\delta^{HP}} + \theta_2 + \frac{\delta^{HP}+\delta^{SP}}{\delta^{HP}}\theta_4, \\ \frac{\left(\delta^{HO}+\delta^{SO}\right)y_h^k+\delta^{HP}\bar{v}_h^k+y_w^O-\delta^{SP}y_w^P}{1-\delta^{SP}} + \frac{\delta^{HP}}{1-\delta^{SP}}\theta_2 \end{array}\right\} - \bar{u}_h - \bar{u}_w - \theta_3 \end{array}\right).$$

This interval is non-empty if in addition to $\theta_2 > y_h^k - \bar{v}_h^k + \frac{\delta^{SP}}{\delta^{HP}}\left(y_w^P - \bar{v}_w^H\right) - \frac{\delta^{SP}}{\delta^{HP}}\theta_4$, the following conditions are satisfied:

$$\begin{aligned}
\bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 - \bar{u}_h - \bar{u}_w - \theta_3 \quad &< \quad \bar{v}_h^k + \frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H - \delta^{SP}y_w^P}{\delta^{HP}} + \theta_2 + \\
&\quad + \frac{\delta^{HP}+\delta^{SP}}{\delta^{HP}}\theta_4 - \bar{u}_h - \bar{u}_w - \theta_3, \\
\bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 - \bar{u}_h - \bar{u}_w - \theta_3 \quad &< \quad \frac{\left(\delta^{HO}+\delta^{SO}\right)y_h^k + \delta^{HP}\bar{v}_h^k + y_w^O - \delta^{SP}y_w^P}{1-\delta^{SP}} + \\
&\quad + \frac{\delta^{HP}}{1-\delta^{SP}}\theta_2 - \bar{u}_h - \bar{u}_w - \theta_3.
\end{aligned}$$

Simplifying:

$$\begin{aligned}
\theta_4 \quad &> \quad \frac{\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) + y_w^P - \bar{v}_w^H, \\
\theta_2 \quad &< \quad y_h^k + \frac{\delta^{HP}\bar{v}_h^k - \left(1-\delta^{SP}\right)\bar{v}_h^H + y_w^O - \delta^{SP}y_w^P - \left(1-\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \frac{\left(1-\delta^{SP}\right)}{\delta^{HO}+\delta^{SO}}\theta_4.
\end{aligned}$$

Thus, for θ_1 interval to be non-empty, it must be the case that:

$$\theta_4 > \frac{\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) + y_w^P - \bar{v}_w^H$$

and:

$$\theta_2 \in \left(\begin{array}{c} y_h^k - \bar{v}_h^k + \frac{\delta^{SP}}{\delta^{HP}}\left(y_w^P - \bar{v}_w^H\right) - \frac{\delta^{SP}}{\delta^{HP}}\theta_4, \\ y_h^k + \frac{\delta^{HP}\bar{v}_h^k - \left(1-\delta^{SP}\right)\bar{v}_h^H + y_w^O - \delta^{SP} y_w^P - \left(1-\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \frac{\left(1-\delta^{SP}\right)}{\delta^{HO}+\delta^{SO}}\theta_4 \end{array} \right).$$

In turn, to ensure that both intervals (for θ_1 and θ_2) are non-empty, θ_4 must satisfy:

$$\begin{gathered} \frac{\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) + y_w^P - \bar{v}_w^H < \theta_4, \\ y_h^k - \bar{v}_h^k + \frac{\delta^{SP}}{\delta^{HP}}\left(y_w^P - \bar{v}_w^H\right) - \frac{\delta^{SP}}{\delta^{HP}}\theta_4 < y_h^k + \\ + \frac{\delta^{HP}\bar{v}_h^k - \left(1-\delta^{SP}\right)\bar{v}_h^H + y_w^O - \delta^{SP} y_w^P - \left(1-\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \frac{\left(1-\delta^{SP}\right)}{\delta^{HO}+\delta^{SO}}\theta_4, \end{gathered}$$

or, equivalently:

$$\begin{aligned} \theta_4 &> \frac{\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) + y_w^P - \bar{v}_w^H, \\ \left[\delta^{HP} - \delta^{SP}\left(1-\delta^{SP}\right)\right]\theta_4 &< \left(1-\delta^{SP}\right)\delta^{HP}\left(\bar{v}_h^k - \bar{v}_h^H\right) + \\ &\quad + \delta^{HP}\left(y_w^O - y_w^P\right) + \left[\delta^{HP} - \delta^{SP}\left(1-\delta^{SP}\right)\right]\left(y_w^P - \bar{v}_w^H\right). \end{aligned}$$

Now, note that since $\bar{v}_h^H \geq \bar{v}_h^k$, $\frac{\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) + y_w^P - \bar{v}_w^H \geq y_w^P - \bar{v}_w^H$. There are three possibilities.

If $\delta^{HP} > \delta^{SP}\left(1-\delta^{SP}\right)$, the interval for θ_4 specializes as:

$$\theta_4 \in \left(\frac{\delta^{HP}\left(\bar{v}_h^H - \bar{v}_h^k\right)}{\delta^{SP}} + y_w^P - \bar{v}_w^H, \min\left\{ \begin{array}{c} y_w^P - \bar{v}_w^S, y_w^O - \bar{v}_w^H, \left(y_w^P - \bar{v}_w^H\right) + \\ + \frac{\left(1-\delta^{SP}\right)\delta^{HP}\left(\bar{v}_h^k - \bar{v}_h^H\right) + \delta^{HP}\left(y_w^O - y_w^P\right)}{\delta^{HP} - \delta^{SP}\left(1-\delta^{SP}\right)} \end{array} \right\} \right).$$

If $\delta^{HP} < \delta^{SP}\left(1-\delta^{SP}\right)$, the interval for θ_4 specializes as:

$$\theta_4 \in \left(\max\left\{ \begin{array}{c} \frac{\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right), \\ \frac{\left(1-\delta^{SP}\right)\delta^{HP}\left(\bar{v}_h^k - \bar{v}_h^H\right) + \delta^{HP}\left(y_w^O - y_w^P\right)}{\delta^{HP} - \delta^{SP}\left(1-\delta^{SP}\right)} \end{array} \right\} + y_w^P - \bar{v}_w^H, \min\left\{ \begin{array}{c} y_w^P - \bar{v}_w^S, \\ y_w^O - \bar{v}_w^H \end{array} \right\} \right).$$

If $\delta^{HP} = \delta^{SP}\left(1-\delta^{SP}\right)$, the interval for θ_4 specializes simply as:

$$\theta_4 \in \left(\frac{\delta^{HP}}{\delta^{SP}} \left(\bar{v}_h^H - \bar{v}_h^k\right) + y_w^P - \bar{v}_w^H, \min \left\{ \begin{array}{c} y_w^P - \bar{v}_w^S, \\ y_w^O - \bar{v}_w^H \end{array} \right\} \right),$$

but for the non-emptiness of the modified intervals for θ_1 and θ_2 it is also necessary that $\left(1-\delta^{SP}\right)\left(\bar{v}_h^H - \bar{v}_h^k\right) < y_w^O - y_w^P$.

Conflict

The state of conflict occurs whenever the following conditions hold:

$$\begin{aligned}
\hat{E}\mathcal{V}_h^k\left(\mathcal{C}\right) &\geq y_h^k, \\
v_w^l &> u_w\left(\tau^*\right), \\
v_w^l &\geq y_w^l, \\
\hat{E}\mathcal{V}_h^k\left(\tau^*, \mathcal{C}\right) &= \hat{E}\mathcal{V}_h^k\left(\mathcal{C}\right),
\end{aligned}$$

and the optimal transfer τ^* must satisfy $u_h\left(-\tau^*\right) \geq y_h^k$.

In this case, if $l \in \{HO, SO, SP\}$, then $y_w^l > v_w^l$ and the state of conflict cannot occur.

If $l = HP$, the state of conflict occurs if $\tau^* < \tau^1$ or $\tau^* = \tau^1$. The case $\tau^* = \tau^1$ has already been solved above.

Consider $\tau^* < \tau^1$. The set of conditions specializes as:

$$\begin{aligned}
\left(\delta^{HO} + \delta^{SO} + \delta^{SP}\right) y_h^k + \delta^{HP} v_h^k &\geq y_h^k, \\
\left(\delta^{HO} + \delta^{SO} + \delta^{SP}\right) y_h^k + \delta^{HP} v_h^k &\geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right), \\
\left(\delta^{HO} + \delta^{SO} + \delta^{SP}\right) y_h^k + \delta^{HP} v_h^k &\geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) u_h\left(-\tau^2\right), \\
\left(\delta^{HO} + \delta^{SO} + \delta^{SP}\right) y_h^k + \delta^{HP} v_h^k &\geq u_h\left(-\tau^3\right).
\end{aligned}$$

Equivalently:

$$\begin{aligned}
\theta_2 &\geq y_h^k - \bar{v}_h^k, \\
\theta_1 &\leq y_h^k + y_w^P - \bar{u}_h - \bar{u}_w - \theta_3, \\
\theta_1 &\leq \frac{\delta^{SP} y_h^k + \delta^{HP} \bar{v}_h^k}{\delta^{HP} + \delta^{SP}} + \bar{v}_w^H - \bar{u}_h - \bar{u}_w + \frac{\delta^{HP}}{\delta^{HP} + \delta^{SP}} \theta_2 - \theta_3 + \theta_4, \\
\theta_1 &\leq \left(1 - \delta^{HP}\right) y_h^k + \delta^{HP} \bar{v}_h^k + y_w^O - \bar{u}_h - \bar{u}_w + \delta^{HP} \theta_2 - \theta_3.
\end{aligned}$$

This system can be simplified. It can be shown that given $\theta_2 \geq y_h^k - \bar{v}_h^k$:

$$y_h^k + y_w^P - \bar{u}_h - \bar{u}_w - \theta_3 \leq \frac{\delta^{SP} y_h^k + \delta^{HP} \bar{v}_h^k}{\delta^{HP} + \delta^{SP}} + \bar{v}_w^H - \bar{u}_h - \bar{u}_w + \frac{\delta^{HP}}{\delta^{HP} + \delta^{SP}} \theta_2 - \theta_3 + \theta_4,$$

or, equivalently:

$$\delta^{HP} y_h^k - \delta^{HP} \bar{v}_h^k + \left(\delta^{HP} + \delta^{SP}\right) y_w^P - \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H - \delta^{HP} \theta_2 \leq \left(\delta^{HP} + \delta^{SP}\right) \theta_4.$$

The above inequality is true since $-\delta^{HP} \theta_2 \leq -\delta^{HP} \left(y_h^k - \bar{v}_h^k\right)$ and $\theta_4 \geq y_w^P - \bar{v}_w^H$. Then:

$$\begin{aligned}
\delta^{HP} y_h^k - \delta^{HP} \bar{v}_h^k + \left(\delta^{HP} + \delta^{SP}\right) y_w^P - \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H - \delta^{HP} \theta_2 &\leq \\
\leq \left(\delta^{HP} + \delta^{SP}\right) y_w^P - \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H &\leq \\
\leq \left(\delta^{HP} + \delta^{SP}\right) \theta_4.&
\end{aligned}$$

It can also be shown that given $\theta_2 \geq y_h^k - \bar{v}_h^k$:

$$y_h^k + y_w^P - \bar{u}_h - \bar{u}_w - \theta_3 \leq \left(1 - \delta^{HP}\right) y_h^k + \delta^{HP} \bar{v}_h^k + y_w^O - \bar{u}_h - \bar{u}_w + \delta^{HP} \theta_2 - \theta_3,$$

equivalently, $y_h^k - \bar{v}_h^k \leq \frac{y_w^O - y_w^P}{\delta^{HP}} + \theta_2$.

Note that the inequality is true, since $\frac{y_w^O - y_w^P}{\delta^{HP}} + \theta_2 > \theta_2 \geq y_h^k - \bar{v}_h^k$.

Therefore, the set of conditions becomes:

$$\begin{aligned}
\theta_2 &\in \left(y_h^k - \bar{v}_h^k, +\infty\right), \\
\theta_1 &\in \left(-\infty, y_h^k + y_w^P - \bar{u}_h - \bar{u}_w - \theta_3\right).
\end{aligned}$$

Now, incorporating the condition in Proposition 1, the interval for θ_1 becomes:

$$\theta_1 \in \left(\bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 - \bar{u}_h - \bar{u}_w - \theta_3, y_h^k + y_w^P - \bar{u}_h - \bar{u}_w - \theta_3\right).$$

It is non-empty provided that $\theta_2 < y_h^k - \bar{v}_h^H + y_w^P - \bar{v}_w^H - \theta_4$.

Thus, it must be that $\theta_2 \in \left(y_h^k - \bar{v}_h^k, y_h^k - \bar{v}_h^H + y_w^P - \bar{v}_w^H - \theta_4\right)$. However, this interval for θ_2 is itself non-empty provided that $\theta_4 < \bar{v}_h^k - \bar{v}_h^H + y_w^P - \bar{v}_w^H$ and observe that no $\theta_4 \in \left[y_w^P - \bar{v}_w^H, \min\left\{y_w^P - \bar{v}_w^S, y_w^O - \bar{v}_w^H\right\}\right)$ can satisfy the inequality, because $\bar{v}_h^k - \bar{v}_h^H + y_w^P - \bar{v}_w^H \leq y_w^P - \bar{v}_w^H$.

Case III $y_w^O > v_w^H > v_w^S \geq y_w^P$

In this case, $\tau^1 = \bar{v}_w^S - \bar{u}_w - \theta_3 + \theta_4$, $\tau^2 = \bar{v}_w^H - \bar{u}_w - \theta_3 + \theta_4$, and $\tau^3 = y_w^O - \bar{u}_w - \theta_3$. Also:

$$\hat{E}\mathcal{V}_h^k(\mathcal{C}) = \max \left\{ \begin{array}{c} \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) v_h^k, \\ \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right), \\ \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) u_h\left(-\tau^2\right), \\ u_h\left(-\tau^3\right) \end{array} \right\}.$$

Cooperation

If $l \in \{HO, SO\}$, then $\max\left\{v_w^l, y_w^l\right\} = y_w^O$. The state of cooperation obtains if $\tau^* = \tau^3$. The set of conditions specializes as:

$$\begin{aligned} u_h\left(-\tau^3\right) &\geq y_h^k, \\ u_h\left(-\tau^3\right) &\geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) v_h^k, \end{aligned}$$

$$u_h\left(-\tau^3\right) \geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right),$$

$$u_h\left(-\tau^3\right) \geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) u_h\left(-\tau^2\right).$$

Equivalently:

$$\begin{aligned}
\theta_1 &\geq y_h^k + y_w^O - \bar{u}_h - \bar{u}_w - \theta_3, \\
\theta_1 &\geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_h^k + y_w^O - \bar{u}_h - \bar{u}_w + \left(\delta^{HP} + \delta^{SP}\right) \theta_2 - \theta_3, \\
\theta_1 &\geq \frac{\left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} \bar{v}_h^k + y_w^O - \delta^{SP} \bar{v}_w^S}{1 - \delta^{SP}} - \bar{u}_h - \bar{u}_w + \frac{\delta^{HP}}{1 - \delta^{SP}} \theta_2 - \theta_3 - \\
&\quad - \frac{\delta^{SP}}{1 - \delta^{SP}} \theta_4, \\
\theta_1 &\geq y_h^k + \frac{y_w^O - \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H}{\delta^{HO} + \delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\delta^{HP} + \delta^{SP}}{\delta^{HO} + \delta^{SO}} \theta_4.
\end{aligned}$$

The fourth inequality above implies the first one:

$$y_h^k + \frac{y_w^O - \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H}{\delta^{HO} + \delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\left(\delta^{HP} + \delta^{SP}\right) \theta_4}{\delta^{HO} + \delta^{SO}} \geq y_h^k + y_w^O - \bar{u}_h - \bar{u}_w - \theta_3,$$

equivalently, $y_w^O - \bar{v}_w^H \geq \theta_4$, which is true since $\theta_4 \in \left[y_w^P - \bar{v}_w^S, y_w^O - \bar{v}_w^H\right)$.

Hence, the set of inequalities corresponds to interval:

$$\theta_1 \in \left(\max \left\{ \begin{array}{c} \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_h^k + y_w^O + \\ + \left(\delta^{HP} + \delta^{SP}\right) \theta_2, \\ \frac{\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \delta^{HP} \bar{v}_h^k + y_w^O - \delta^{SP} \bar{v}_w^S}{1-\delta^{SP}} + \frac{\delta^{HP}}{1-\delta^{SP}} \theta_2 - \\ - \frac{\delta^{SP}}{1-\delta^{SP}} \theta_4, \\ y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right) \bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \frac{\delta^{HP}+\delta^{SP}}{\delta^{HO}+\delta^{SO}} \theta_4 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, +\infty \right).$$

Now, incorporating the condition in Proposition 1, the interval for θ_1 is:

$$\left(\max \left\{ \begin{array}{c} \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) \bar{v}_h^k + y_w^O + \\ + \left(\delta^{HP} + \delta^{SP}\right) \theta_2, \\ \frac{\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \delta^{HP} \bar{v}_h^k + y_w^O - \delta^{SP} \bar{v}_w^S}{1-\delta^{SP}} + \frac{\delta^{HP} \theta_2}{1-\delta^{SP}} - \frac{\delta^{SP} \theta_4}{1-\delta^{SP}}, \\ y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right) \bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \frac{\delta^{HP}+\delta^{SP}}{\delta^{HO}+\delta^{SO}} \theta_4, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, +\infty \right).$$

If $l = HP$, the state of cooperation obtains if $\tau^* = \tau^3$ or $\tau^* = \tau^2$. The case $\tau^* = \tau^3$ has been solved above.

Consider $\tau^{*}=\tau^{2}$. The set of conditions is:

$$\begin{aligned}
u_h\left(-\tau^2\right) &\geq y_h^k,\\
\left(\delta^{HO}+\delta^{SO}\right)y_h^k+\left(\delta^{HP}+\delta^{SP}\right)u_h\left(-\tau^2\right) &\geq y_h^k,\\
\left(\delta^{HO}+\delta^{SO}\right)y_h^k+\left(\delta^{HP}+\delta^{SP}\right)u_h\left(-\tau^2\right) &\geq \left(\delta^{HO}+\delta^{SO}\right)y_h^k+\left(\delta^{HP}+\delta^{SP}\right)v_h^k,\\
\left(\delta^{HO}+\delta^{SO}\right)y_h^k+\left(\delta^{HP}+\delta^{SP}\right)u_h\left(-\tau^2\right) &\geq \left(\delta^{HO}+\delta^{SO}\right)y_h^k+\delta^{HP}v_h^k+\\
&\quad +\delta^{SP}u_h\left(-\tau^1\right),\\
\left(\delta^{HO}+\delta^{SO}\right)y_h^k+\left(\delta^{HP}+\delta^{SP}\right)u_h\left(-\tau^2\right) &\geq u_h\left(-\tau^3\right),
\end{aligned}$$

which can be simplified as:

$$\begin{aligned}
u_h\left(-\tau^2\right) &\geq y_h^k,\\
u_h\left(-\tau^2\right) &\geq v_h^k,\\
\left(\delta^{HP}+\delta^{SP}\right)u_h\left(-\tau^2\right) &\geq \delta^{HP}v_h^k+\delta^{SP}u_h\left(-\tau^1\right),\\
\left(\delta^{HO}+\delta^{SO}\right)y_h^k+\left(\delta^{HP}+\delta^{SP}\right)u_h\left(-\tau^2\right) &\geq u_h\left(-\tau^3\right).
\end{aligned}$$

Equivalently:

$$\begin{aligned}
\theta_1 &\geq y_h^k+\bar{v}_w^H-\bar{u}_h-\bar{u}_w-\theta_3+\theta_4,\\
\theta_1 &\geq \bar{v}_h^k+\bar{v}_w^H-\bar{u}_h-\bar{u}_w+\theta_2-\theta_3+\theta_4,\\
\theta_1 &\geq \bar{v}_h^k+\frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H-\delta^{SP}\bar{v}_w^S}{\delta^{HP}}-\bar{u}_h-\bar{u}_w+\theta_2-\theta_3+\theta_4,\\
\theta_1 &\leq y_h^k+\frac{y_w^O-\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}}-\bar{u}_h-\bar{u}_w-\theta_3-\frac{\delta^{HP}+\delta^{SP}}{\delta^{HO}+\delta^{SO}}\theta_4.
\end{aligned}$$

It is easy to show that the third inequality implies the second one:

$$\bar{v}_h^k+\frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H-\delta^{SP}\bar{v}_w^S}{\delta^{HP}}-\bar{u}_h-\bar{u}_w+\theta_2-\theta_3+\theta_4\geq$$
$$\geq\bar{v}_h^k+\bar{v}_w^H-\bar{u}_h-\bar{u}_w+\theta_2-\theta_3+\theta_4,$$

equivalently, $\bar{v}_w^H-\bar{v}_w^S\geq 0$, which is always true.

Thus, the set of conditions corresponds to interval:

$$\theta_1 \in \left(\begin{array}{c} \max\left\{ \begin{array}{c} y_h^k + \bar{v}_w^H, \\ \bar{v}_h^k + \frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H - \delta^{SP}\bar{v}_w^S}{\delta^{HP}} + \theta_2 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\ y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\delta^{HP}+\delta^{SP}}{\delta^{HO}+\delta^{SO}}\theta_4 \end{array} \right).$$

Depending on θ_2, this interval may be an empty set. The set is non-empty provided that the following conditions hold:

$$y_h^k + \bar{v}_w^H + \theta_4 < y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \frac{\delta^{HP}+\delta^{SP}}{\delta^{HO}+\delta^{SO}}\theta_4,$$

$$\bar{v}_h^k + \frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H - \delta^{SP}\bar{v}_w^S}{\delta^{HP}} + \theta_2 + \theta_4 < y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - $$
$$-\frac{\delta^{HP}+\delta^{SP}}{\delta^{HO}+\delta^{SO}}\theta_4.$$

Equivalently:

$$\begin{aligned} \theta_4 &< y_w^O - \bar{v}_w^H, \\ \theta_2 &< y_h^k - \bar{v}_h^k + \frac{\delta^{HP}y_w^O - \left(1-\delta^{SP}\right)\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H + \left(\delta^{HO}+\delta^{SO}\right)\delta^{SP}\bar{v}_w^S}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)} - \\ &\quad -\frac{1}{\delta^{HO}+\delta^{SO}}\theta_4, \end{aligned}$$

where the first inequality holds.

Thus, the interval for θ_1 is non-empty when θ_2 belongs to:

$$\left(\begin{array}{c} -\infty, y_h^k - \bar{v}_h^k + \frac{\delta^{HP}y_w^O - \left(1-\delta^{SP}\right)\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H + \left(\delta^{HO}+\delta^{SO}\right)\delta^{SP}\bar{v}_w^S}{\delta^{HP}\left(\delta^{HO}+\delta^{SO}\right)} - \\ -\frac{1}{\delta^{HO}+\delta^{SO}}\theta_4 \end{array} \right).$$

Now, incorporating the condition in Proposition 1, the interval for θ_1 becomes:

$$\left(\begin{array}{c} \max\left\{ \begin{array}{c} y_h^k + \bar{v}_w^H, \\ \bar{v}_h^k + \frac{\left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H - \delta^{SP}\bar{v}_w^S}{\delta^{HP}} + \theta_2, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\ y_h^k + \frac{y_w^O - \left(\delta^{HP}+\delta^{SP}\right)\bar{v}_w^H}{\delta^{HO}+\delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\delta^{HP}+\delta^{SP}}{\delta^{HO}+\delta^{SO}}\theta_4 \end{array} \right).$$

This set is non-empty under the following additional condition regarding θ_2:

$$\bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 < y_h^k + \frac{y_w^O - \left(\delta^{HP} + \delta^{SP}\right)\bar{v}_w^H}{\delta^{HO} + \delta^{SO}} - \frac{\delta^{HP} + \delta^{SP}}{\delta^{HO} + \delta^{SO}}\theta_4,$$

equivalently, $\theta_2 < y_h^k - \bar{v}_h^H + \frac{y_w^O - \bar{v}_w^H}{\delta^{HO} + \delta^{SO}} - \frac{1}{\delta^{HO} + \delta^{SO}}\theta_4$.

Thus, the modified interval for θ_1 is non-empty provided that θ_2 belongs to:

$$\left(-\infty, y_h^k - \frac{\theta_4}{\delta^{HO} + \delta^{SO}} + \min\left\{\begin{array}{c} -\bar{v}_h^H + \frac{y_w^O - \bar{v}_w^H}{\delta^{HO} + \delta^{SO}}, -\bar{v}_h^k + \\ + \frac{\delta^{HP} y_w^O - \left(1 - \delta^{SP}\right)\left(\delta^{HP} + \delta^{SP}\right)\bar{v}_w^H + \left(\delta^{HO} + \delta^{SO}\right)\delta^{SP}\bar{v}_w^S}{\delta^{HP}\left(\delta^{HO} + \delta^{SO}\right)} \end{array}\right\}\right).$$

If $l = SP$, the state of cooperation obtains if $\tau^* = \tau^3$, or $\tau^* = \tau^2$, or $\tau^* = \tau^1$. The cases $\tau^* = \tau^3$ and $\tau^* = \tau^2$ have been solved above.

Consider $\tau^* = \tau^1$. The set of conditions is:

$$\begin{aligned}
u_h\left(-\tau^1\right) &\geq y_h^k, \\
\left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq y_h^k, \\
\left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \left(\delta^{HP} + \delta^{SP}\right) v_h^k, \\
\left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq \left(\delta^{HO} + \delta^{SO}\right) y_h^k + \\
&\quad + \left(\delta^{HP} + \delta^{SP}\right) u_h\left(-\tau^2\right), \\
\left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq u_h\left(-\tau^3\right),
\end{aligned}$$

which can be simplified as:

$$\begin{aligned}
u_h\left(-\tau^1\right) &\geq y_h^k, \\
\delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq \left(\delta^{HP} + \delta^{SP}\right) y_h^k, \\
u_h\left(-\tau^1\right) &\geq v_h^k, \\
\delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq \left(\delta^{HP} + \delta^{SP}\right) u_h\left(-\tau^2\right), \\
\left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq u_h\left(-\tau^3\right).
\end{aligned}$$

Equivalently:

$$\begin{aligned}
\theta_1 &\geq y_h^k + \bar{v}_w^S - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\
\theta_1 &\geq \frac{\left(\delta^{HP} + \delta^{SP}\right) y_h^k - \delta^{HP} \bar{v}_h^k}{\delta^{SP}} + \bar{v}_w^S - \bar{u}_h - \bar{u}_w - \frac{\delta^{HP}}{\delta^{SP}} \theta_2 - \theta_3 + \theta_4, \\
\theta_1 &\geq \bar{v}_h^k + \bar{v}_w^S - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4, \\
\theta_1 &\leq \bar{v}_h^k + \frac{\left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{\delta^{HP}} - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4, \\
\theta_1 &\leq \frac{\left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} \bar{v}_h^k + y_w^O - \delta^{SP} \bar{v}_w^S}{1 - \delta^{SP}} - \bar{u}_h - \bar{u}_w + \frac{\delta^{HP}}{1 - \delta^{SP}} \theta_2 - \theta_3 - \\
&\quad - \frac{\delta^{SP}}{1 - \delta^{SP}} \theta_4.
\end{aligned}$$

Thus, the set of conditions corresponds to the following interval for θ_1:

$$\left(\begin{array}{c} \max \left\{ \begin{array}{c} y_h^k + \bar{v}_w^S, \\ \frac{\left(\delta^{HP}+\delta^{SP}\right) y_h^k - \delta^{HP} \bar{v}_h^k}{\delta^{SP}} + \bar{v}_w^S - \frac{\delta^{HP}}{\delta^{SP}} \theta_2, \\ \bar{v}_h^k + \bar{v}_w^S + \theta_2 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\ \min \left\{ \begin{array}{c} \bar{v}_h^k + \frac{\left(\delta^{HP}+\delta^{SP}\right) \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{\delta^{HP}} + \theta_2 + \theta_4, \\ \frac{\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \delta^{HP} \bar{v}_h^k + y_w^O - \delta^{SP} \bar{v}_w^S}{1-\delta^{SP}} + \frac{\delta^{HP} \theta_2}{1-\delta^{SP}} - \frac{\delta^{SP} \theta_4}{1-\delta^{SP}} \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3 \end{array} \right).$$

Depending on θ_2, this interval for θ_1 may be an empty set. It is non-empty if the following inequalities hold:

$$y_h^k + \bar{v}_w^S < \bar{v}_h^k + \frac{\left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{\delta^{HP}} + \theta_2,$$

$$y_h^k + \bar{v}_w^S + \theta_4 < \frac{\left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} \bar{v}_h^k + y_w^O - \delta^{SP} \bar{v}_w^S}{1 - \delta^{SP}} + \frac{\delta^{HP}}{1 - \delta^{SP}} \theta_2 - \frac{\delta^{SP}}{1 - \delta^{SP}} \theta_4,$$

$$\frac{\left(\delta^{HP} + \delta^{SP}\right) y_h^k - \delta^{HP} \bar{v}_h^k}{\delta^{SP}} + \bar{v}_w^S - \frac{\delta^{HP}}{\delta^{SP}} \theta_2 < \bar{v}_h^k + \frac{\left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{\delta^{HP}} + \theta_2,$$

$$\begin{aligned}
\frac{\left(\delta^{HP} + \delta^{SP}\right) y_h^k - \delta^{HP} \bar{v}_h^k}{\delta^{SP}} + \bar{v}_w^S &< \frac{\left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} \bar{v}_h^k + y_w^O - \delta^{SP} \bar{v}_w^S}{1 - \delta^{SP}} + \\
&+ \frac{\delta^{HP}}{\delta^{SP}} \theta_2 + \frac{\delta^{HP}}{1 - \delta^{SP}} \theta_2 - \theta_4 - \frac{\delta^{SP}}{1 - \delta^{SP}} \theta_4,
\end{aligned}$$

$$\bar{v}_h^k + \bar{v}_w^S + \theta_2 + \theta_4 < \bar{v}_h^k + \frac{\left(\delta^{HP} + \delta^{SP}\right)\bar{v}_w^H - \delta^{SP}\bar{v}_w^S}{\delta^{HP}} + \theta_2 + \theta_4,$$

$$\bar{v}_h^k + \bar{v}_w^S + \theta_2 + \theta_4 < \frac{\left(\delta^{HO} + \delta^{SO}\right)y_h^k + \delta^{HP}\bar{v}_h^k + y_w^O - \delta^{SP}\bar{v}_w^S}{1 - \delta^{SP}} + \frac{\delta^{HP}\theta_2}{1 - \delta^{SP}} - \frac{\delta^{SP}\theta_4}{1 - \delta^{SP}}.$$

Equivalently:

$$\begin{aligned}
y_h^k - \bar{v}_h^k - \frac{\left(\delta^{HP} + \delta^{SP}\right)\left(\bar{v}_w^H - \bar{v}_w^S\right)}{\delta^{HP}} &< \theta_2,\\
y_h^k - \bar{v}_h^k + \frac{\bar{v}_w^S - y_w^O}{\delta^{HP}} + \frac{1}{\delta^{HP}}\theta_4 &< \theta_2,\\
y_h^k - \bar{v}_h^k - \frac{\delta^{SP}}{\delta^{HP}}\left(\bar{v}_w^H - \bar{v}_w^S\right) &< \theta_2,\\
y_h^k - \bar{v}_h^k + \frac{\delta^{SP}}{\delta^{HP}}\left(\bar{v}_w^S - y_w^O\right) + \frac{\delta^{SP}}{\delta^{HP}}\theta_4 &< \theta_2,\\
\bar{v}_w^S &< \bar{v}_w^H,\\
y_h^k - \bar{v}_h^k + \frac{y_w^O - \bar{v}_w^S}{\delta^{HO} + \delta^{SO}} - \frac{1}{\delta^{HO} + \delta^{SO}}\theta_4 &> \theta_2,
\end{aligned}$$

where the fifth inequality is always true.

The third inequality implies the first, second, and fourth inequalities:

$$y_h^k - \bar{v}_h^k - \frac{\delta^{SP}}{\delta^{HP}}\left(\bar{v}_w^H - \bar{v}_w^S\right) \geq y_h^k - \bar{v}_h^k - \frac{\left(\delta^{HP} + \delta^{SP}\right)\left(\bar{v}_w^H - \bar{v}_w^S\right)}{\delta^{HP}},$$

equivalently, $0 \leq 1$.

Next:

$$y_h^k - \bar{v}_h^k - \frac{\delta^{SP}}{\delta^{HP}}\left(\bar{v}_w^H - \bar{v}_w^S\right) \geq y_h^k - \bar{v}_h^k + \frac{\bar{v}_w^S - y_w^O}{\delta^{HP}} + \frac{1}{\delta^{HP}}\theta_4,$$

equivalently, $y_w^O - \bar{v}_w^S - \delta^{SP}\left(\bar{v}_w^H - \bar{v}_w^S\right) \geq \theta_4$, which is true because:

$$y_w^O - \bar{v}_w^S - \delta^{SP}\left(\bar{v}_w^H - \bar{v}_w^S\right) \geq y_w^O - \bar{v}_w^S - \left(\bar{v}_w^H - \bar{v}_w^S\right) = y_w^O - \bar{v}_w^H > \theta_4.$$

Lastly:

$$y_h^k - \bar{v}_h^k - \frac{\delta^{SP}}{\delta^{HP}}\left(\bar{v}_w^H - \bar{v}_w^S\right) \geq y_h^k - \bar{v}_h^k + \frac{\delta^{SP}}{\delta^{HP}}\left(\bar{v}_w^S - y_w^O\right) + \frac{\delta^{SP}}{\delta^{HP}}\theta_4,$$

equivalently, $y_w^O - \bar{v}_w^H \geq \theta_4$.

Thus, the interval for θ_1 is non-empty provided that:

$$\theta_2 \in \left(y_h^k - \bar{v}_h^k - \frac{\delta^{SP}}{\delta^{HP}} \left(\bar{v}_w^H - \bar{v}_w^S \right), y_h^k - \bar{v}_h^k + \frac{y_w^O - \bar{v}_w^S}{\delta^{HO} + \delta^{SO}} - \frac{1}{\delta^{HO} + \delta^{SO}} \theta_4 \right).$$

Now, noting that $\bar{v}_h^H + \bar{v}_w^H > \bar{v}_h^k + \bar{v}_w^S$ and incorporating the condition in Proposition 1, the interval for θ_1 becomes:

$$\left(\begin{array}{c} \max \left\{ \begin{array}{c} y_h^k + \bar{v}_w^S, \\ \frac{\left(\delta^{HP}+\delta^{SP}\right) y_h^k - \delta^{HP} \bar{v}_h^k}{\delta^{SP}} + \bar{v}_w^S - \frac{\delta^{HP}}{\delta^{SP}} \theta_2, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\ \min \left\{ \begin{array}{c} \bar{v}_h^k + \frac{\left(\delta^{HP}+\delta^{SP}\right) \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{\delta^{HP}} + \theta_2 + \theta_4, \\ \frac{\left(\delta^{HO}+\delta^{SO}\right) y_h^k + \delta^{HP} \bar{v}_h^k + y_w^O - \delta^{SP} \bar{v}_w^S}{1-\delta^{SP}} + \frac{\delta^{HP}\theta_2}{1-\delta^{SP}} - \frac{\delta^{SP}\theta_4}{1-\delta^{SP}} \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3 \end{array} \right).$$

This interval for θ_1 is non-empty provided that the following additional conditions hold:

$$\begin{aligned} \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 &< \bar{v}_h^k + \frac{\left(\delta^{HP} + \delta^{SP}\right) \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{\delta^{HP}} + \theta_2 + \theta_4, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 &< \frac{\left(\delta^{HO} + \delta^{SO}\right) y_h^k + \delta^{HP} \bar{v}_h^k + y_w^O - \delta^{SP} \bar{v}_w^S}{1 - \delta^{SP}} + \frac{\delta^{HP}}{1 - \delta^{SP}} \theta_2 - \\ &\quad - \frac{\delta^{SP}}{1 - \delta^{SP}} \theta_4. \end{aligned}$$

Equivalently:

$$\delta^{HP} \left(\bar{v}_h^H - \bar{v}_h^k \right) < \delta^{SP} \left(\bar{v}_w^H - \bar{v}_w^S \right),$$

$$\theta_2 < y_h^k + \frac{\delta^{HP} \bar{v}_h^k - \left(1 - \delta^{SP}\right) \bar{v}_h^H + y_w^O - \bar{v}_w^H + \delta^{SP} \left(\bar{v}_w^H - \bar{v}_w^S \right)}{\delta^{HO} + \delta^{SO}} - \frac{1}{\delta^{HO} + \delta^{SO}} \theta_4.$$

First, it can be shown that:

$$\begin{aligned} y_h^k + \frac{\delta^{HP} \bar{v}_h^k - \left(1 - \delta^{SP}\right) \bar{v}_h^H + y_w^O - \bar{v}_w^H + \delta^{SP} \left(\bar{v}_w^H - \bar{v}_w^S \right)}{\delta^{HO} + \delta^{SO}} - \frac{1}{\delta^{HO} + \delta^{SO}} \theta_4 < y_h^k - \\ - \bar{v}_h^k + \frac{y_w^O - \bar{v}_w^S}{\delta^{HO} + \delta^{SO}} - \frac{1}{\delta^{HO} + \delta^{SO}} \theta_4, \end{aligned}$$

since the inequality can be simplified as $\bar{v}_h^k - \bar{v}_h^H < \bar{v}_w^H - \bar{v}_w^S$. This inequality is true because $\bar{v}_h^k - \bar{v}_h^H \leq 0 < \bar{v}_w^H - \bar{v}_w^S$.

Second, assuming that condition $\delta^{HP}\left(\bar{v}_h^H - \bar{v}_h^k\right) < \delta^{SP}\left(\bar{v}_w^H - \bar{v}_w^S\right)$ holds:

$$\begin{aligned} y_h^k - \bar{v}_h^k - \frac{\delta^{SP}}{\delta^{HP}}\left(\bar{v}_w^H - \bar{v}_w^S\right) \quad < \quad & \frac{\delta^{HP}\bar{v}_h^k - \left(1-\delta^{SP}\right)\bar{v}_h^H + y_w^O - \bar{v}_w^H + \delta^{SP}\left(\bar{v}_w^H - \bar{v}_w^S\right)}{\delta^{HO}+\delta^{SO}} + \\ & + y_h^k - \frac{1}{\delta^{HO}+\delta^{SO}}\theta_4, \end{aligned}$$

since the inequality simplifies as:

$$\theta_4 < \left(1-\delta^{SP}\right)\left(\bar{v}_h^k - \bar{v}_h^H\right) + y_w^O - \bar{v}_w^H + \frac{\delta^{SP}\left(1-\delta^{SP}\right)}{\delta^{HP}}\left(\bar{v}_w^H - \bar{v}_w^S\right).$$

The inequality is true, since in this case $\theta_4 < y_w^O - \bar{v}_w^H$ and:

$$y_w^O - \bar{v}_w^H < \left(1-\delta^{SP}\right)\left(\bar{v}_h^k - \bar{v}_h^H\right) + y_w^O - \bar{v}_w^H + \frac{\delta^{SP}\left(1-\delta^{SP}\right)}{\delta^{HP}}\left(\bar{v}_w^H - \bar{v}_w^S\right),$$

because the latter simplifies as $\delta^{HP}\left(\bar{v}_h^H - \bar{v}_h^k\right) < \delta^{SP}\left(\bar{v}_w^H - \bar{v}_w^S\right)$.

Thus, the interval for θ_1 is non-empty if $\delta^{HP}\left(\bar{v}_h^H - \bar{v}_h^k\right) < \delta^{SP}\left(\bar{v}_w^H - \bar{v}_w^S\right)$ holds and θ_2 satisfies:

$$\theta_2 \in \left(\begin{array}{c} y_h^k - \bar{v}_h^k - \frac{\delta^{SP}}{\delta^{HP}}\left(\bar{v}_w^H - \bar{v}_w^S\right), \\ y_h^k + \frac{\delta^{HP}\bar{v}_h^k - \left(1-\delta^{SP}\right)\bar{v}_h^H + y_w^O - \bar{v}_w^H + \delta^{SP}\left(\bar{v}_w^H - \bar{v}_w^S\right)}{\delta^{HO}+\delta^{SO}} - \frac{1}{\delta^{HO}+\delta^{SO}}\theta_4 \end{array}\right).$$

Conflict

First, note that if $l \in \{HO, SO\}$, then $\max\left\{v_w^l, y_w^l\right\} = y_w^l = y_w^O$ and the state of conflict cannot occur.

If $l = SP$, the state of conflict occurs if $\tau^* < \tau^1$. The set of conditions is:

$$\begin{aligned} \left(\delta^{HO}+\delta^{SO}\right)y_h^k + \left(\delta^{HP}+\delta^{SP}\right)v_h^k \quad & \geq \quad y_h^k, \\ \left(\delta^{HO}+\delta^{SO}\right)y_h^k + \left(\delta^{HP}+\delta^{SP}\right)v_h^k \quad & \geq \quad \left(\delta^{HO}+\delta^{SO}\right)y_h^k + \delta^{HP}v_h^k + \delta^{SP}u_h\left(-\tau^1\right), \\ \left(\delta^{HO}+\delta^{SO}\right)y_h^k + \left(\delta^{HP}+\delta^{SP}\right)v_h^k \quad & \geq \quad \left(\delta^{HO}+\delta^{SO}\right)y_h^k + \left(\delta^{HP}+\delta^{SP}\right)u_h\left(-\tau^2\right), \\ \left(\delta^{HO}+\delta^{SO}\right)y_h^k + \left(\delta^{HP}+\delta^{SP}\right)v_h^k \quad & \geq \quad u_h\left(-\tau^3\right), \end{aligned}$$

which can be simplified as:

$$
\begin{aligned}
v_h^k &\geq y_h^k, \\
v_h^k &\geq u_h\left(-\tau^1\right), \\
\left(\delta^{HO}+\delta^{SO}\right) y_h^k+\left(\delta^{HP}+\delta^{SP}\right) v_h^k &\geq u_h\left(-\tau^3\right),
\end{aligned}
$$

since $u_h\left(-\tau^1\right) > u_h\left(-\tau^2\right)$.

Equivalently:

$$
\begin{aligned}
\theta_2 &\geq y_h^k-\bar{v}_h^k, \\
\theta_1 &\leq \bar{v}_h^k+\bar{v}_w^S-\bar{u}_h-\bar{u}_w+\theta_2-\theta_3+\theta_4, \\
\theta_1 &\leq \left(\delta^{HO}+\delta^{SO}\right) y_h^k+\left(\delta^{HP}+\delta^{SP}\right) \bar{v}_h^k+y_w^O-\bar{u}_h-\bar{u}_w+\left(\delta^{HP}+\delta^{SP}\right) \theta_2-\theta_3,
\end{aligned}
$$

Thus, the set of conditions corresponds to interval:

$$
\theta_2 \in \left(y_h^k-\bar{v}_h^k,+\infty\right),
$$

$$
\theta_1 \in \left(-\infty, \min\left\{\begin{array}{c}
\bar{v}_h^k+\bar{v}_w^S+\theta_2+\theta_4, \\
\left(\delta^{HO}+\delta^{SO}\right) y_h^k+\left(\delta^{HP}+\delta^{SP}\right) \bar{v}_h^k+ \\
+y_w^O+\left(\delta^{HP}+\delta^{SP}\right) \theta_2
\end{array}\right\}-\bar{u}_h-\bar{u}_w-\theta_3\right).
$$

Now, given the condition in Proposition 1, the modified interval for θ_1 is an empty set, since:

$$
-\bar{u}_h-\bar{u}_w+\bar{v}_h^H+\bar{v}_w^H+\theta_2-\theta_3+\theta_4 > \bar{v}_h^k+\bar{v}_w^S+\theta_2+\theta_4-\bar{u}_h-\bar{u}_w-\theta_3,
$$

equivalently, $\bar{v}_h^H+\bar{v}_w^H > \bar{v}_h^k+\bar{v}_w^S$, which is always true.

If $l=HP$, the state of conflict occurs if $\tau^* < \tau^1$ or $\tau^* = \tau^1$. Both cases have been solved above.

Case IV $v_w^H \geq y_w^O > y_w^P > v_w^S$

In this case, $\tau^1 = y_w^P - \bar{u}_w - \theta_3$, $\tau^2 = y_w^O - \bar{u}_w - \theta_3$, $\tau^3 = \bar{v}_w^H - \bar{u}_w - \theta_3 + \theta_4$. Also:

$$\hat{E}\mathcal{V}_h^k(\mathcal{C}) = \max\left\{\begin{array}{c} \left(\delta^{SO}+\delta^{SP}\right)y_h^k + \left(\delta^{HO}+\delta^{HP}\right)v_h^k, \\ \delta^{SO}y_h^k + \left(\delta^{HO}+\delta^{HP}\right)v_h^k + \delta^{SP}u_h\left(-\tau^1\right), \\ \left(\delta^{HO}+\delta^{HP}\right)v_h^k + \left(\delta^{SO}+\delta^{SP}\right)u_h\left(-\tau^2\right), \\ u_h\left(-\tau^3\right) \end{array}\right\}.$$

Cooperation

If $l \in \{HO, HP\}$, then $\max\left\{v_w^l, y_w^l\right\} = v_w^H$ and the state of cooperation obtains provided that $\tau^* = \tau^3$. The set of conditions is:

$$\begin{aligned} u_h\left(-\tau^3\right) &\geq y_h^k, \\ u_h\left(-\tau^3\right) &\geq \left(\delta^{SO}+\delta^{SP}\right)y_h^k + \left(\delta^{HO}+\delta^{HP}\right)v_h^k, \\ u_h\left(-\tau^3\right) &\geq \delta^{SO}y_h^k + \left(\delta^{HO}+\delta^{HP}\right)v_h^k + \delta^{SP}u_h\left(-\tau^1\right), \\ u_h\left(-\tau^3\right) &\geq \left(\delta^{HO}+\delta^{HP}\right)v_h^k + \left(\delta^{SO}+\delta^{SP}\right)u_h\left(-\tau^2\right). \end{aligned}$$

Equivalently:

$$\begin{aligned} \theta_1 &\geq y_h^k + \bar{v}_w^H - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\ \theta_1 &\geq \left(\delta^{SO}+\delta^{SP}\right)y_h^k + \left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k + \bar{v}_w^H - \bar{u}_h - \bar{u}_w + \left(\delta^{HO}+\delta^{HP}\right)\theta_2 - \\ &\quad -\theta_3 + \theta_4, \\ \theta_1 &\geq \frac{\delta^{SO}y_h^k + \left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k + \bar{v}_w^H - \delta^{SP}y_w^P}{1-\delta^{SP}} - \bar{u}_h - \bar{u}_w + \frac{\left(\delta^{HO}+\delta^{HP}\right)}{1-\delta^{SP}}\theta_2 - \\ &\quad -\theta_3 + \frac{1}{1-\delta^{SP}}\theta_4, \\ \theta_1 &\geq \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO}+\delta^{SP}\right)y_w^O}{\delta^{HO}+\delta^{HP}} - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \frac{1}{\delta^{HO}+\delta^{HP}}\theta_4. \end{aligned}$$

Thus, the set of conditions implies:

$$\theta_1 \in \left(\max \left\{ \begin{array}{c} y_h^k + \bar{v}_w^H + \theta_4, \\ \left(\delta^{SO} + \delta^{SP}\right) y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \\ + \bar{v}_w^H + \left(\delta^{HO} + \delta^{HP}\right) \theta_2 + \theta_4, \\ \frac{\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \delta^{SP} y_w^P}{1 - \delta^{SP}} + \\ + \frac{\left(\delta^{HO} + \delta^{HP}\right)}{1 - \delta^{SP}} \theta_2 + \frac{1}{1 - \delta^{SP}} \theta_4, \\ \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right) y_w^O}{\delta^{HO} + \delta^{HP}} + \theta_2 + \frac{\theta_4}{\delta^{HO} + \delta^{HP}} \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, +\infty \right).$$

Now, incorporating the condition in Proposition 1, the modified interval for θ_1 is:

$$\theta_1 \in \left(\max \left\{ \begin{array}{c} y_h^k + \bar{v}_w^H + \theta_4, \\ \left(\delta^{SO} + \delta^{SP}\right) y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \\ + \bar{v}_w^H + \left(\delta^{HO} + \delta^{HP}\right) \theta_2 + \theta_4, \\ \frac{\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \delta^{SP} y_w^P}{1 - \delta^{SP}} + \\ + \frac{\left(\delta^{HO} + \delta^{HP}\right)}{1 - \delta^{SP}} \theta_2 + \frac{1}{1 - \delta^{SP}} \theta_4, \\ \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right) y_w^O}{\delta^{HO} + \delta^{HP}} + \theta_2 + \frac{\theta_4}{\delta^{HO} + \delta^{HP}}, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, +\infty \right).$$

If $l = SO$, then $\max\left\{v_w^l, y_w^l\right\} = y_w^O$ and the state of cooperation obtains provided that $\tau^* = \tau^3$ or $\tau^* = \tau^2$. The case $\tau^* = \tau^3$ has been solved above.

Consider $\tau^* = \tau^2$. The set of conditions is:

$$\begin{array}{rcl} u_h\left(-\tau^2\right) & \geq & y_h^k, \\ \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right) & \geq & y_h^k, \\ \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right) & \geq & \left(\delta^{SO} + \delta^{SP}\right) y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k, \\ \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right) & \geq & \delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \\ & & + \delta^{SP} u_h\left(-\tau^1\right), \\ \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right) & \geq & u_h\left(-\tau^3\right), \end{array}$$

which can be simplified as:

$$\begin{aligned}
u_h\left(-\tau^2\right) &\geq y_h^k, \\
\left(\delta^{HO}+\delta^{HP}\right) v_h^k+\left(\delta^{SO}+\delta^{SP}\right) u_h\left(-\tau^2\right) &\geq y_h^k, \\
\left(\delta^{SO}+\delta^{SP}\right) u_h\left(-\tau^2\right) &\geq \delta^{SO} y_h^k+\delta^{SP} u_h\left(-\tau^1\right), \\
\left(\delta^{HO}+\delta^{HP}\right) v_h^k+\left(\delta^{SO}+\delta^{SP}\right) u_h\left(-\tau^2\right) &\geq u_h\left(-\tau^3\right).
\end{aligned}$$

Equivalently:

$$\begin{aligned}
\theta_1 &\geq y_h^k+y_w^O-\bar{u}_h-\bar{u}_w-\theta_3, \\
\theta_1 &\geq \frac{y_h^k-\left(\delta^{HO}+\delta^{HP}\right) \bar{v}_h^k}{\delta^{SO}+\delta^{SP}}+y_w^O-\bar{u}_h-\bar{u}_w-\frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}} \theta_2-\theta_3, \\
\theta_1 &\geq y_h^k+\frac{\left(\delta^{SO}+\delta^{SP}\right) y_w^O-\delta^{SP} y_w^P}{\delta^{SO}}-\bar{u}_h-\bar{u}_w-\theta_3, \\
\theta_1 &\leq \bar{v}_h^k+\frac{\bar{v}_w^H-\left(\delta^{SO}+\delta^{SP}\right) y_w^O}{\delta^{HO}+\delta^{HP}}-\bar{u}_h-\bar{u}_w+\theta_2-\theta_3+\frac{1}{\delta^{HO}+\delta^{HP}} \theta_4 .
\end{aligned}$$

It is easy to show that the third inequality implies the first one:

$$y_h^k+\frac{\left(\delta^{SO}+\delta^{SP}\right) y_w^O-\delta^{SP} y_w^P}{\delta^{SO}}-\bar{u}_h-\bar{u}_w-\theta_3 \geq y_h^k+y_w^O-\bar{u}_h-\bar{u}_w-\theta_3,$$

equivalently, $\delta^{SP}\left[y_w^O-y_w^P\right] \geq 0$, which is always true.

Then, the set of conditions can be simplified as:

$$\theta_1 \in\left(\begin{array}{c}
\max \left\{\begin{array}{c}
\frac{y_h^k-\left(\delta^{HO}+\delta^{HP}\right) \bar{v}_h^k}{\delta^{SO}+\delta^{SP}}+y_w^O-\frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}} \theta_2, \\
y_h^k+\frac{\left(\delta^{SO}+\delta^{SP}\right) y_w^O-\delta^{SP} y_w^P}{\delta^{SO}}
\end{array}\right\}-\bar{u}_h-\bar{u}_w-\theta_3, \\
\bar{v}_h^k+\frac{\bar{v}_w^H-\left(\delta^{SO}+\delta^{SP}\right) y_w^O}{\delta^{HO}+\delta^{HP}}-\bar{u}_h-\bar{u}_w+\theta_2-\theta_3+\frac{1}{\delta^{HO}+\delta^{HP}} \theta_4
\end{array}\right).$$

The interval is non-empty provided that:

$$y_h^k+\frac{\left(\delta^{SO}+\delta^{SP}\right) y_w^O-\delta^{SP} y_w^P}{\delta^{SO}}<\bar{v}_h^k+\frac{\bar{v}_w^H-\left(\delta^{SO}+\delta^{SP}\right) y_w^O}{\delta^{HO}+\delta^{HP}}+\theta_2+\frac{1}{\delta^{HO}+\delta^{HP}} \theta_4,$$

$$\begin{aligned}
\frac{y_h^k-\left(\delta^{HO}+\delta^{HP}\right) \bar{v}_h^k}{\delta^{SO}+\delta^{SP}}+y_w^O-\frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}} \theta_2<\bar{v}_h^k+\frac{\bar{v}_w^H-\left(\delta^{SO}+\delta^{SP}\right) y_w^O}{\delta^{HO}+\delta^{HP}}+\theta_2+ \\
+\frac{1}{\delta^{HO}+\delta^{HP}} \theta_4 .
\end{aligned}$$

Equivalently:

$$\begin{aligned} y_h^k - \bar{v}_h^k + \frac{\delta^{SP}\left(y_w^O - y_w^P\right)}{\delta^{SO}} + \frac{y_w^O - \bar{v}_w^H}{\delta^{HO} + \delta^{HP}} - \frac{1}{\delta^{HO} + \delta^{HP}}\theta_4 &< \theta_2, \\ y_h^k - \bar{v}_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right)\left(y_w^O - \bar{v}_w^H\right)}{\delta^{HO} + \delta^{HP}} - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{HO} + \delta^{HP}}\theta_4 &< \theta_2. \end{aligned}$$

Thus, the interval for θ_1 is non-empty provided that:

$$\theta_2 \in \left(\max\left\{ \begin{array}{c} \frac{\left(\delta^{SO}+\delta^{SP}\right)\left(y_w^O-\bar{v}_w^H\right)}{\delta^{HO}+\delta^{HP}} - \frac{\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{HO}+\delta^{HP}}\theta_4, \\ \frac{\delta^{SP}\left(y_w^O-y_w^P\right)}{\delta^{SO}} + \frac{y_w^O-\bar{v}_w^H}{\delta^{HO}+\delta^{HP}} - \frac{1}{\delta^{HO}+\delta^{HP}}\theta_4 \end{array} \right\} + y_h^k - \bar{v}_h^k, +\infty \right).$$

Now, incorporating the condition in Proposition 1, the modified interval for θ_1 is:

$$\theta_1 \in \left(\begin{array}{c} \max\left\{ \begin{array}{c} \frac{y_h^k - \left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k}{\delta^{SO}+\delta^{SP}} + y_w^O - \frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}}\theta_2, \\ y_h^k + \frac{\left(\delta^{SO}+\delta^{SP}\right)y_w^O - \delta^{SP}y_w^P}{\delta^{SO}}, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, \\ \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO}+\delta^{SP}\right)y_w^O}{\delta^{HO}+\delta^{HP}} - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \frac{1}{\delta^{HO}+\delta^{HP}}\theta_4 \end{array} \right).$$

The modified interval for θ_1 is non-empty under the following additional condition:

$$\bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 < \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right) y_w^O}{\delta^{HO} + \delta^{HP}} + \theta_2 + \frac{1}{\delta^{HO} + \delta^{HP}}\theta_4,$$

equivalently, $\theta_4 > \frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) + y_w^O - \bar{v}_w^H$.

Note that since $\bar{v}_h^H \geq \bar{v}_h^k$, then $\frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) + y_w^O - \bar{v}_w^H \geq y_w^O - \bar{v}_w^H$. In this case, $\theta_4 \in \left[y_w^O - \bar{v}_w^H, y_w^P - \bar{v}_w^S\right)$ and, thus, the interval for θ_4 needs to be restricted as:

$$\left(\frac{\left(\delta^{HO} + \delta^{HP}\right)}{\delta^{SO} + \delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) + y_w^O - \bar{v}_w^H, y_w^P - \bar{v}_w^S \right).$$

If $l = SP$, $\max\left\{v_w^l, y_w^l\right\} = y_w^P$ and the state of cooperation obtains if $\tau^* = \tau^3$, or $\tau^* = \tau^2$, or $\tau^* = \tau^1$. The cases $\tau^* = \tau^3$ and $\tau^* = \tau^2$ have been solved above.

Consider $\tau^* = \tau^1$. The set of conditions is:

$$\begin{aligned} u_h\left(-\tau^1\right) &\geq y_h^k, \\ \delta^{SO}y_h^k + \left(\delta^{HO} + \delta^{HP}\right)v_h^k + \delta^{SP}u_h\left(-\tau^1\right) &\geq y_h^k, \end{aligned}$$

$$
\begin{aligned}
\delta^{SO}y_h^k+\left(\delta^{HO}+\delta^{HP}\right)v_h^k+\delta^{SP}u_h\left(-\tau^1\right) &\geq \left(\delta^{SO}+\delta^{SP}\right)y_h^k+\left(\delta^{HO}+\delta^{HP}\right)v_h^k,\\
\delta^{SO}y_h^k+\left(\delta^{HO}+\delta^{HP}\right)v_h^k+\delta^{SP}u_h\left(-\tau^1\right) &\geq \left(\delta^{HO}+\delta^{HP}\right)v_h^k+\\
&\quad+\left(\delta^{SO}+\delta^{SP}\right)u_h\left(-\tau^2\right),\\
\delta^{SO}y_h^k+\left(\delta^{HO}+\delta^{HP}\right)v_h^k+\delta^{SP}u_h\left(-\tau^1\right) &\geq u_h\left(-\tau^3\right),
\end{aligned}
$$

which can be simplified as:

$$
\begin{aligned}
u_h\left(-\tau^1\right) &\geq y_h^k,\\
\left(\delta^{HO}+\delta^{HP}\right)v_h^k+\delta^{SP}u_h\left(-\tau^1\right) &\geq \left(1-\delta^{SO}\right)y_h^k,\\
\delta^{SO}y_h^k+\delta^{SP}u_h\left(-\tau^1\right) &\geq \left(\delta^{SO}+\delta^{SP}\right)u_h\left(-\tau^2\right),\\
\delta^{SO}y_h^k+\left(\delta^{HO}+\delta^{HP}\right)v_h^k+\delta^{SP}u_h\left(-\tau^1\right) &\geq u_h\left(-\tau^3\right).
\end{aligned}
$$

Equivalently:

$$
\begin{aligned}
\theta_1 &\geq y_h^k+y_w^P-\bar{u}_h-\bar{u}_w-\theta_3,\\
\theta_1 &\geq \frac{\left(1-\delta^{SO}\right)y_h^k-\left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k}{\delta^{SP}}+y_w^P-\bar{u}_h-\bar{u}_w-\frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SP}}\theta_2-\theta_3,\\
\theta_1 &\leq y_h^k+\frac{\left(\delta^{SO}+\delta^{SP}\right)y_w^O-\delta^{SP}y_w^P}{\delta^{SO}}-\bar{u}_h-\bar{u}_w-\theta_3,\\
\theta_1 &\leq \frac{\delta^{SO}y_h^k+\left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k+\bar{v}_w^H-\delta^{SP}y_w^P}{1-\delta^{SP}}-\bar{u}_h-\bar{u}_w+\frac{\left(\delta^{HO}+\delta^{HP}\right)}{1-\delta^{SP}}\theta_2-\\
&\quad-\theta_3+\frac{1}{1-\delta^{SP}}\theta_4.
\end{aligned}
$$

Thus, the set of conditions implies that the interval for θ_1 is:

$$
\left(\begin{array}{c}
\max\left\{\begin{array}{c} y_h^k+y_w^P,\\ \frac{\left(1-\delta^{SO}\right)y_h^k-\left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k}{\delta^{SP}}+y_w^P-\frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SP}}\theta_2\end{array}\right\}-\bar{u}_h-\bar{u}_w-\theta_3,\\
\min\left\{\begin{array}{c} y_h^k+\frac{\left(\delta^{SO}+\delta^{SP}\right)y_w^O-\delta^{SP}y_w^P}{\delta^{SO}},\\ \frac{\delta^{SO}y_h^k+\left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k+\bar{v}_w^H-\delta^{SP}y_w^P}{1-\delta^{SP}}+\frac{\left(\delta^{HO}+\delta^{HP}\right)\theta_2+\theta_4}{1-\delta^{SP}}\end{array}\right\}-\bar{u}_h-\bar{u}_w-\theta_3
\end{array}\right).
$$

Depending on θ_2, the above interval for θ_1 may be an empty set. The set is non-empty provided that the following inequalities hold:

$$y_h^k + y_w^P < y_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right) y_w^O - \delta^{SP} y_w^P}{\delta^{SO}},$$

$$y_h^k + y_w^P < \frac{\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \delta^{SP} y_w^P}{1 - \delta^{SP}} + \frac{\left(\delta^{HO} + \delta^{HP}\right)}{1 - \delta^{SP}} \theta_2 + \frac{1}{1 - \delta^{SP}} \theta_4,$$

$$\frac{\left(1 - \delta^{SO}\right) y_h^k - \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k}{\delta^{SP}} + y_w^P - \frac{\left(\delta^{HO} + \delta^{HP}\right)}{\delta^{SP}} \theta_2 < \frac{\left(\delta^{SO} + \delta^{SP}\right) y_w^O}{\delta^{SO}} + $$
$$+ y_h^k - \frac{\delta^{SP}}{\delta^{SO}} y_w^P,$$

$$\frac{\left(1 - \delta^{SO}\right) y_h^k - \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k}{\delta^{SP}} + y_w^P - \frac{\left(\delta^{HO} + \delta^{HP}\right)}{\delta^{SP}} \theta_2 < \frac{\left(\delta^{HO} + \delta^{HP}\right) \theta_2 + \theta_4}{1 - \delta^{SP}} + $$
$$+ \frac{\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \delta^{SP} y_w^P}{1 - \delta^{SP}}.$$

Equivalently:

$$\begin{aligned}
y_w^P &< y_w^O, \\
y_h^k - \bar{v}_h^k + \frac{y_w^P - \bar{v}_w^H}{\delta^{HO} + \delta^{HP}} - \frac{1}{\delta^{HO} + \delta^{HP}} \theta_4 &< \theta_2, \\
y_h^k - \bar{v}_h^k - \frac{\delta^{SP} \left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO} \left(\delta^{HO} + \delta^{HP}\right)} \left(y_w^O - y_w^P\right) &< \theta_2, \\
y_h^k - \bar{v}_h^k + \frac{\delta^{SP}}{\delta^{HO} + \delta^{HP}} \left(y_w^P - \bar{v}_w^H\right) - \frac{\delta^{SP}}{\delta^{HO} + \delta^{HP}} \theta_4 &< \theta_2.
\end{aligned}$$

Notice that the first inequality is always true.

It is easy to show that the fourth inequality implies the second one:

$$y_h^k - \bar{v}_h^k + \frac{\delta^{SP}}{\delta^{HO} + \delta^{HP}} \left(y_w^P - \bar{v}_w^H\right) - \frac{\delta^{SP}}{\delta^{HO} + \delta^{HP}} \theta_4 \geq y_h^k - \bar{v}_h^k + \frac{y_w^P - \bar{v}_w^H}{\delta^{HO} + \delta^{HP}} - \frac{\theta_4}{\delta^{HO} + \delta^{HP}},$$

equivalently, $\theta_4 \geq y_w^P - \bar{v}_w^H$, which is true since $\theta_4 \geq y_w^O - \bar{v}_w^H > y_w^P - \bar{v}_w^H$.

Thus, the interval for θ_1 is non-empty provided that:

$$\theta_2 \in \left(\max \left\{ \begin{array}{c} -\frac{\delta^{SP}\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)} \left(y_w^O - y_w^P\right), \\ \frac{\delta^{SP}}{\delta^{HO}+\delta^{HP}} \left(y_w^P - \bar{v}_w^H\right) - \frac{\delta^{SP}}{\delta^{HO}+\delta^{HP}} \theta_4 \end{array} \right\} + y_h^k - \bar{v}_h^k, +\infty \right).$$

Now, incorporating the condition in Proposition 1, the modified interval for θ_1 is:

$$\left(\begin{array}{c} \max\left\{\begin{array}{c} y_h^k + y_w^P, \\ \frac{\left(1-\delta^{SO}\right)y_h^k - \left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k}{\delta^{SP}} + y_w^P - \frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SP}}\theta_2, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 \end{array}\right\} - \bar{u}_h - \bar{u}_w - \theta_3, \\ \min\left\{\begin{array}{c} y_h^k + \frac{\left(\delta^{SO}+\delta^{SP}\right)y_w^O - \delta^{SP}y_w^P}{\delta^{SO}}, \\ \frac{\delta^{SO}y_h^k + \left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k + \bar{v}_w^H - \delta^{SP}y_w^P}{1-\delta^{SP}} + \frac{\left(\delta^{HO}+\delta^{HP}\right)\theta_2 + \theta_4}{1-\delta^{SP}} \end{array}\right\} - \bar{u}_h - \bar{u}_w - \theta_3 \end{array}\right).$$

The modified interval for θ_1 is non-empty if the following extra conditions hold:

$$\begin{aligned} \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 &< y_h^k + \frac{\left(\delta^{SO}+\delta^{SP}\right)y_w^O - \delta^{SP}y_w^P}{\delta^{SO}}, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 &< \frac{\delta^{SO}y_h^k + \left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k + \bar{v}_w^H - \delta^{SP}y_w^P}{1-\delta^{SP}} + \\ &\quad + \frac{\left(\delta^{HO}+\delta^{HP}\right)}{1-\delta^{SP}}\theta_2 + \frac{1}{1-\delta^{SP}}\theta_4. \end{aligned}$$

Equivalently:

$$\begin{aligned} \theta_2 &< y_h^k - \bar{v}_h^k + \bar{v}_h^k - \bar{v}_h^H + \frac{\left(\delta^{SO}+\delta^{SP}\right)y_w^O - \delta^{SP}y_w^P}{\delta^{SO}} - \bar{v}_w^H - \theta_4, \\ \theta_2 &< y_h^k - \bar{v}_h^k + \frac{\left(1-\delta^{SP}\right)}{\delta^{SO}}\left(\bar{v}_h^k - \bar{v}_h^H\right) + \frac{\delta^{SP}}{\delta^{SO}}\left(\bar{v}_w^H - y_w^P\right) + \frac{\delta^{SP}}{\delta^{SO}}\theta_4. \end{aligned}$$

Thus, the modified interval for θ_2 is:

$$\left(\begin{array}{c} \max\left\{\begin{array}{c} -\frac{\delta^{SP}\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\left(y_w^O - y_w^P\right), \\ \frac{\delta^{SP}}{\delta^{HO}+\delta^{HP}}\left(y_w^P - \bar{v}_w^H\right) - \frac{\delta^{SP}}{\delta^{HO}+\delta^{HP}}\theta_4 \end{array}\right\} + y_h^k - \bar{v}_h^k, \\ \min\left\{\begin{array}{c} \bar{v}_h^k - \bar{v}_h^H + \frac{\left(\delta^{SO}+\delta^{SP}\right)y_w^O - \delta^{SP}y_w^P}{\delta^{SO}} - \bar{v}_w^H - \theta_4, \\ \frac{\left(1-\delta^{SP}\right)}{\delta^{SO}}\left(\bar{v}_h^k - \bar{v}_h^H\right) + \frac{\delta^{SP}}{\delta^{SO}}\left(\bar{v}_w^H - y_w^P\right) + \frac{\delta^{SP}}{\delta^{SO}}\theta_4 \end{array}\right\} + y_h^k - \bar{v}_h^k \end{array}\right).$$

Now, the modified interval for θ_2 is non-empty under the following conditions:

$$\frac{\delta^{SP}\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\left(y_w^P - y_w^O\right) < \bar{v}_h^k - \bar{v}_h^H + \frac{\left(\delta^{SO}+\delta^{SP}\right)y_w^O - \delta^{SP}y_w^P}{\delta^{SO}} - \bar{v}_w^H - \theta_4,$$

$$\frac{\delta^{SP}\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\left(y_w^P - y_w^O\right) < \frac{\left(1-\delta^{SP}\right)}{\delta^{SO}}\left(\bar{v}_h^k - \bar{v}_h^H\right) + \frac{\delta^{SP}}{\delta^{SO}}\left(\bar{v}_w^H - y_w^P\right) + \frac{\delta^{SP}\theta_4}{\delta^{SO}},$$

$$
\begin{aligned}
\frac{\delta^{SP}\left(y_w^P-\bar{v}_w^H\right)}{\delta^{HO}+\delta^{HP}}-\frac{\delta^{SP}\theta_4}{\delta^{HO}+\delta^{HP}} &< \bar{v}_h^k-\bar{v}_h^H+\frac{\left(\delta^{SO}+\delta^{SP}\right)y_w^O-\delta^{SP}y_w^P}{\delta^{SO}}-\bar{v}_w^H-\theta_4,\\
\frac{\delta^{SP}\left(y_w^P-\bar{v}_w^H\right)}{\delta^{HO}+\delta^{HP}}-\frac{\delta^{SP}\theta_4}{\delta^{HO}+\delta^{HP}} &< \frac{\left(1-\delta^{SP}\right)}{\delta^{SO}}\left(\bar{v}_h^k-\bar{v}_h^H\right)+\frac{\delta^{SP}}{\delta^{SO}}\left(\bar{v}_w^H-y_w^P\right)+\frac{\delta^{SP}\theta_4}{\delta^{SO}}.
\end{aligned}
$$

Equivalently:

$$
\begin{aligned}
\theta_4 &< \bar{v}_h^k-\bar{v}_h^H+y_w^O-\bar{v}_w^H+\frac{\delta^{SP}}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\left(y_w^O-y_w^P\right),\\
\theta_4 &> \frac{\left(1-\delta^{SP}\right)}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H-\frac{\delta^{SO}+\delta^{SP}}{\delta^{HO}+\delta^{HP}}\left(y_w^O-y_w^P\right),\\
\frac{\left(\delta^{HO}+\delta^{HP}-\delta^{SP}\right)\theta_4}{\delta^{HO}+\delta^{HP}} &< \bar{v}_h^k-\bar{v}_h^H+\frac{\left(\delta^{SO}+\delta^{SP}\right)\left(y_w^O-y_w^P\right)}{\delta^{SO}}+\\
&\quad+\frac{\left(\delta^{HO}+\delta^{HP}-\delta^{SP}\right)\left(y_w^P-\bar{v}_w^H\right)}{\delta^{HO}+\delta^{HP}},\\
\theta_4 &> \frac{\delta^{HO}+\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H.
\end{aligned}
$$

There are three possibilities. If $\delta^{HO}+\delta^{HP}>\delta^{SP}$, the set of conditions is:

$$
\begin{aligned}
\theta_4 &< \bar{v}_h^k-\bar{v}_h^H+y_w^O-\bar{v}_w^H+\frac{\delta^{SP}}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\left(y_w^O-y_w^P\right),\\
\theta_4 &> \frac{\left(1-\delta^{SP}\right)}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H-\frac{\delta^{SO}+\delta^{SP}}{\delta^{HO}+\delta^{HP}}\left(y_w^O-y_w^P\right),\\
\theta_4 &< \frac{\left(\delta^{HO}+\delta^{HP}\right)\left(\bar{v}_h^k-\bar{v}_h^H\right)}{\delta^{HO}+\delta^{HP}-\delta^{SP}}+y_w^P-\bar{v}_w^H+\frac{\left(\delta^{HO}+\delta^{HP}\right)\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}-\delta^{SP}\right)}\left(y_w^O-y_w^P\right),\\
\theta_4 &> \frac{\delta^{HO}+\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H,
\end{aligned}
$$

and the interval for θ_4 specializes as:

$$
\left(\begin{array}{c}
\max\left\{\begin{array}{c}
y_w^O-\bar{v}_w^H,\\
\frac{\left(1-\delta^{SP}\right)}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H-\frac{\delta^{SO}+\delta^{SP}}{\delta^{HO}+\delta^{HP}}\left(y_w^O-y_w^P\right),\\
\frac{\delta^{HO}+\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H
\end{array}\right\},\\
\min\left\{\begin{array}{c}
y_w^P-\bar{v}_w^S,\bar{v}_h^k-\bar{v}_h^H+y_w^O-\bar{v}_w^H+\frac{\delta^{SP}\left(y_w^O-y_w^P\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)},\\
\frac{\left(\delta^{HO}+\delta^{HP}\right)\left(\bar{v}_h^k-\bar{v}_h^H\right)}{\delta^{HO}+\delta^{HP}-\delta^{SP}}+y_w^P-\bar{v}_w^H+\frac{\left(\delta^{HO}+\delta^{HP}\right)\left(\delta^{SO}+\delta^{SP}\right)\left(y_w^O-y_w^P\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}-\delta^{SP}\right)}
\end{array}\right\}
\end{array}\right).
$$

If $\delta^{HO}+\delta^{HP}<\delta^{SP}$, the set of conditions is:

$$\begin{aligned}
\theta_4 &< \bar{v}_h^k-\bar{v}_h^H+y_w^O-\bar{v}_w^H+\frac{\delta^{SP}}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\left(y_w^O-y_w^P\right), \\
\theta_4 &> \frac{\left(1-\delta^{SP}\right)}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H-\frac{\delta^{SO}+\delta^{SP}}{\delta^{HO}+\delta^{HP}}\left(y_w^O-y_w^P\right), \\
\theta_4 &> \frac{\left(\delta^{HO}+\delta^{HP}\right)\left(\bar{v}_h^k-\bar{v}_h^H\right)}{\delta^{HO}+\delta^{HP}-\delta^{SP}}+y_w^P-\bar{v}_w^H+\frac{\left(\delta^{HO}+\delta^{HP}\right)\left(\delta^{SO}+\delta^{SP}\right)\left(y_w^O-y_w^P\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}-\delta^{SP}\right)}, \\
\theta_4 &> \frac{\delta^{HO}+\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H,
\end{aligned}$$

and the interval for θ_4 specializes as:

$$\left(\begin{array}{c}\max\left\{\begin{array}{c}\frac{\left(1-\delta^{SP}\right)}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H-\frac{\delta^{SO}+\delta^{SP}}{\delta^{HO}+\delta^{HP}}\left(y_w^O-y_w^P\right), \\ \frac{\left(\delta^{HO}+\delta^{HP}\right)\left(\bar{v}_h^k-\bar{v}_h^H\right)}{\delta^{HO}+\delta^{HP}-\delta^{SP}}+y_w^P-\bar{v}_w^H+\frac{\left(\delta^{HO}+\delta^{HP}\right)\left(\delta^{SO}+\delta^{SP}\right)\left(y_w^O-y_w^P\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}-\delta^{SP}\right)}, \\ y_w^O-\bar{v}_w^H, \frac{\delta^{HO}+\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H\end{array}\right\}, \\ \min\left\{y_w^P-\bar{v}_w^S, \bar{v}_h^k-\bar{v}_h^H+y_w^O-\bar{v}_w^H+\frac{\delta^{SP}\left(y_w^O-y_w^P\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\right\}\end{array}\right).$$

If $\delta^{HO}+\delta^{HP}=\delta^{SP}$, the set of conditions is:

$$\begin{aligned}
\theta_4 &< \bar{v}_h^k-\bar{v}_h^H+y_w^O-\bar{v}_w^H+\frac{\delta^{SP}}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\left(y_w^O-y_w^P\right), \\
\theta_4 &> \frac{\left(1-\delta^{SP}\right)}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H-\frac{\delta^{SO}+\delta^{SP}}{\delta^{HO}+\delta^{HP}}\left(y_w^O-y_w^P\right), \\
\delta^{SO}\left(\bar{v}_h^H-\bar{v}_h^k\right) &< \left(\delta^{SO}+\delta^{SP}\right)\left(y_w^O-y_w^P\right), \\
\theta_4 &> \frac{\delta^{HO}+\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H.
\end{aligned}$$

Thus, thus interval for θ_4 specializes simply as:

$$\left(\begin{array}{c}\max\left\{\begin{array}{c}\frac{\left(1-\delta^{SP}\right)\left(\bar{v}_h^H-\bar{v}_h^k\right)}{\delta^{SP}}+y_w^P-\bar{v}_w^H-\frac{\left(\delta^{SO}+\delta^{SP}\right)\left(y_w^O-y_w^P\right)}{\delta^{HO}+\delta^{HP}}, \\ y_w^O-\bar{v}_w^H, \frac{\delta^{HO}+\delta^{HP}}{\delta^{SP}}\left(\bar{v}_h^H-\bar{v}_h^k\right)+y_w^P-\bar{v}_w^H\end{array}\right\}, \\ \min\left\{y_w^P-\bar{v}_w^S, \bar{v}_h^k-\bar{v}_h^H+y_w^O-\bar{v}_w^H+\frac{\delta^{SP}\left(y_w^O-y_w^P\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\right\}\end{array}\right),$$

but inequality $\delta^{SO}\left(\bar{v}_h^H-\bar{v}_h^k\right)<\left(\delta^{SO}+\delta^{SP}\right)\left(y_w^O-y_w^P\right)$ must additionally hold.

Conflict

If $l \in \{SO, SP\}$, then $y_w^l > v_w^l$ and the state of conflict cannot occur.

If $l \in \{HO, HP\}$, then $\max\left\{v_w^l, y_w^l\right\} = v_w^H$. The state of conflict occurs if $\tau^* < \tau^1$, or $\tau^* = \tau^1$, or $\tau^* = \tau^2$. The cases $\tau^* = \tau^1$ and $\tau^* = \tau^2$ have already been solved.

Consider $\tau^* < \tau^1$. The set of conditions is:

$$\begin{aligned}
\left(\delta^{SO} + \delta^{SP}\right) y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k &\geq y_h^k, \\
\left(\delta^{SO} + \delta^{SP}\right) y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k &\geq \delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \delta^{SP} u_h\left(-\tau^1\right), \\
\left(\delta^{SO} + \delta^{SP}\right) y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k &\geq \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right), \\
\left(\delta^{SO} + \delta^{SP}\right) y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k &\geq u_h\left(-\tau^3\right),
\end{aligned}$$

which can be simplified as:

$$\begin{aligned}
v_h^k &\geq y_h^k, \\
y_h^k &\geq u_h\left(-\tau^1\right), \\
\left(\delta^{SO} + \delta^{SP}\right) y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k &\geq u_h\left(-\tau^3\right),
\end{aligned}$$

since $u_h\left(-\tau^1\right) > u_h\left(-\tau^2\right)$.

Equivalently:

$$\begin{aligned}
\theta_2 &\geq y_h^k - \bar{v}_h^k, \\
\theta_1 &\leq y_h^k + y_w^P - \bar{u}_h - \bar{u}_w - \theta_3, \\
\theta_1 &\leq \left(\delta^{SO} + \delta^{SP}\right) y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \bar{u}_h - \bar{u}_w + \left(\delta^{HO} + \delta^{HP}\right) \theta_2 - \\
&\quad -\theta_3 + \theta_4.
\end{aligned}$$

It can be shown that given $\theta_2 \geq y_h^k - \bar{v}_h^k$, the second inequality above implies the third inequality. In particular:

$$\begin{aligned}
y_h^k + y_w^P - \bar{u}_h - \bar{u}_w - \theta_3 &\leq \left(\delta^{SO} + \delta^{SP}\right) y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \bar{u}_h - \bar{u}_w + \\
&\quad + \left(\delta^{HO} + \delta^{HP}\right) \theta_2 - \theta_3 + \theta_4,
\end{aligned}$$

equivalently, $y_h^k - \bar{v}_h^k + \frac{y_w^P - \bar{v}_w^H}{\left(\delta^{HO}+\delta^{HP}\right)} - \frac{1}{\left(\delta^{HO}+\delta^{HP}\right)}\theta_4 \leq \theta_2$.

Given $\theta_2 \geq y_h^k - \bar{v}_h^k$, the above inequality is true since:

$$y_h^k - \bar{v}_h^k + \frac{y_w^P - \bar{v}_w^H}{\left(\delta^{HO} + \delta^{HP}\right)} - \frac{1}{\left(\delta^{HO} + \delta^{HP}\right)}\theta_4 \leq y_h^k - \bar{v}_h^k,$$

equivalently, $y_w^P - \bar{v}_w^H \leq \theta_4$, which is true since $y_w^P - \bar{v}_w^H < y_w^O - \bar{v}_w^H \leq \theta_4$.

Thus, the set of conditions becomes:

$$\theta_2 \in \left(y_h^k - \bar{v}_h^k, +\infty\right),$$

$$\theta_1 \in \left(-\infty, y_h^k + y_w^P - \bar{u}_h - \bar{u}_w - \theta_3\right).$$

Now, incorporating the condition in Proposition 1, the modified interval for θ_1 is:

$$\left(\bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 - \bar{u}_h - \bar{u}_w - \theta_3, y_h^k + y_w^P - \bar{u}_h - \bar{u}_w - \theta_3\right),$$

which is non-empty under condition $\theta_2 < y_h^k - \bar{v}_h^H + y_w^P - \bar{v}_w^H - \theta_4$.

Thus, the modified interval for θ_2 is:

$$\left(y_h^k - \bar{v}_h^k, y_h^k - \bar{v}_h^H + y_w^P - \bar{v}_w^H - \theta_4\right).$$

However, for $\theta_4 \in \left[y_w^O - \bar{v}_w^H, y_w^P - \bar{v}_w^S\right)$, the modified interval for θ_2 is empty because:

$$\begin{aligned} y_h^k - \bar{v}_h^H + y_w^P - \bar{v}_w^H - \theta_4 &\leq y_h^k - \bar{v}_h^H + y_w^P - \bar{v}_w^H - \left(y_w^O - \bar{v}_w^H\right) = \\ &= y_h^k - \bar{v}_h^H - \left(y_w^O - y_w^P\right) < \\ &< y_h^k - \bar{v}_h^H \leq y_h^k - \bar{v}_h^k. \end{aligned}$$

Case V $v_w^H \geq y_w^O > v_w^S \geq y_w^P$

In this case, $\tau^1 = \bar{v}_w^S - \bar{u}_w - \theta_3 + \theta_4$, $\tau^2 = y_w^O - \bar{u}_w - \theta_3$, $\tau^3 = \bar{v}_w^H - \bar{u}_w - \theta_3 + \theta_4$, and:

$$\hat{E}\mathcal{V}_h^k(\mathcal{C}) = \max \left\{ \begin{array}{c} \delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP} + \delta^{SP}\right) v_h^k, \\ \delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \delta^{SP} u_h\left(-\tau^1\right), \\ \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right), \\ u_h\left(-\tau^3\right) \end{array} \right\}.$$

Cooperation

If $l \in \{HO, HP\}$, then $\max\left\{v_w^l, y_w^l\right\} = v_w^H$ and the state of cooperation occurs provided that $\tau^* = \tau^3$. The set of conditions is:

$$\begin{aligned} u_h\left(-\tau^3\right) &\geq y_h^k, \\ u_h\left(-\tau^3\right) &\geq \delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP} + \delta^{SP}\right) v_h^k, \\ u_h\left(-\tau^3\right) &\geq \delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \delta^{SP} u_h\left(-\tau^1\right), \\ u_h\left(-\tau^3\right) &\geq \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right). \end{aligned}$$

Equivalently:

$$\begin{aligned} \theta_1 &\geq y_h^k + \bar{v}_w^H - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\ \theta_1 &\geq \delta^{SO} y_h^k + \left(1 - \delta^{SO}\right) \bar{v}_h^k + \bar{v}_w^H - \bar{u}_h - \bar{u}_w + \left(1 - \delta^{SO}\right) \theta_2 - \theta_3 + \theta_4, \\ \theta_1 &\geq \frac{\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{1 - \delta^{SP}} - \bar{u}_h - \bar{u}_w + \frac{\left(\delta^{HO} + \delta^{HP}\right)}{1 - \delta^{SP}} \theta_2 - \\ &\quad -\theta_3 + \theta_4, \\ \theta_1 &\geq \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right) y_w^O}{\delta^{HO} + \delta^{HP}} - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \frac{1}{\delta^{HO} + \delta^{HP}} \theta_4. \end{aligned}$$

It follows that the set of conditions corresponds to the following interval for θ_1:

$$\left(\max \left\{ \begin{array}{c} y_h^k + \bar{v}_w^H + \theta_4, \\ \delta^{SO} y_h^k + \left(1 - \delta^{SO}\right) \bar{v}_h^k + \bar{v}_w^H + \left(1 - \delta^{SO}\right) \theta_2 + \theta_4, \\ \frac{\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{1 - \delta^{SP}} + \frac{\left(\delta^{HO} + \delta^{HP}\right) \theta_2}{1 - \delta^{SP}} + \theta_4, \\ \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right) y_w^O}{\delta^{HO} + \delta^{HP}} + \theta_2 + \frac{1}{\delta^{HO} + \delta^{HP}} \theta_4 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, +\infty \right).$$

Now, incorporating the condition in Proposition 1, the modified interval for θ_1 is:

$$\left(\max \left\{ \begin{array}{c} y_h^k + \bar{v}_w^H + \theta_4, \\ \delta^{SO} y_h^k + \left(1 - \delta^{SO}\right) \bar{v}_h^k + \bar{v}_w^H + \left(1 - \delta^{SO}\right) \theta_2 + \theta_4, \\ \frac{\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{1 - \delta^{SP}} + \frac{\left(\delta^{HO} + \delta^{HP}\right) \theta_2}{1 - \delta^{SP}} + \theta_4, \\ \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right) y_w^O}{\delta^{HO} + \delta^{HP}} + \theta_2 + \frac{1}{\delta^{HO} + \delta^{HP}} \theta_4, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, +\infty \right).$$

If $l = SO$, then $\max \left\{ v_w^l, y_w^l \right\} = y_w^O$ and the state of cooperation obtains provided that either $\tau^* = \tau^3$, or $\tau^* = \tau^2$. The case $\tau^* = \tau^3$ has been solved above.

Consider $\tau^* = \tau^2$. The set of conditions is:

$$\begin{aligned} u_h\left(-\tau^2\right) &\geq y_h^k, \\ \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right) &\geq y_h^k, \\ \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right) &\geq \delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP} + \delta^{SP}\right) v_h^k, \\ \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right) &\geq \delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \\ &\quad + \delta^{SP} u_h\left(-\tau^1\right), \\ \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right) &\geq u_h\left(-\tau^3\right). \end{aligned}$$

Equivalently:

$$\begin{aligned} \theta_1 &\geq y_h^k + y_w^O - \bar{u}_h - \bar{u}_w - \theta_3, \\ \theta_1 &\geq \frac{y_h^k - \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k}{\delta^{SO} + \delta^{SP}} + y_w^O - \bar{u}_h - \bar{u}_w - \frac{\left(\delta^{HO} + \delta^{HP}\right)}{\delta^{SO} + \delta^{SP}} \theta_2 - \theta_3, \end{aligned}$$

$$\begin{aligned}
\theta_1 &\geq \frac{\delta^{SO} y_h^k + \delta^{SP} \bar{v}_h^k}{\delta^{SO} + \delta^{SP}} + y_w^O - \bar{u}_h - \bar{u}_w + \frac{\delta^{SP}}{\delta^{SO} + \delta^{SP}} \theta_2 - \theta_3, \\
\theta_1 &\geq y_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right) y_w^O - \delta^{SP} \bar{v}_w^S}{\delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\delta^{SP}}{\delta^{SO}} \theta_4, \\
\theta_1 &\leq \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right) y_w^O}{\delta^{HO} + \delta^{HP}} - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \frac{1}{\delta^{HO} + \delta^{HP}} \theta_4.
\end{aligned}$$

It can be shown that the fourth inequality implies the first one:

$$y_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right) y_w^O - \delta^{SP} \bar{v}_w^S}{\delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\delta^{SP}}{\delta^{SO}} \theta_4 \geq y_h^k + y_w^O - \bar{u}_h - \bar{u}_w - \theta_3,$$

equivalently, $y_w^O - \bar{v}_w^S \geq \theta_4$, which is always true in this case.

Thus, the set of conditions corresponds to interval:

$$\theta_1 \in \left(\begin{array}{c} \max \left\{ \begin{array}{c} \frac{y_h^k - \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k}{\delta^{SO} + \delta^{SP}} + y_w^O - \frac{\left(\delta^{HO} + \delta^{HP}\right) \theta_2}{\delta^{SO} + \delta^{SP}}, \\ \frac{\delta^{SO} y_h^k + \delta^{SP} \bar{v}_h^k}{\delta^{SO} + \delta^{SP}} + y_w^O + \frac{\delta^{SP}}{\delta^{SO} + \delta^{SP}} \theta_2, \\ y_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right) y_w^O - \delta^{SP} \bar{v}_w^S}{\delta^{SO}} - \frac{\delta^{SP}}{\delta^{SO}} \theta_4 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3, \\ \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right) y_w^O}{\delta^{HO} + \delta^{HP}} - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \frac{1}{\delta^{HO} + \delta^{HP}} \theta_4 \end{array} \right).$$

Depending on θ_2, the above interval may be an empty set. The interval is non-empty provided that the following inequalities hold:

$$\begin{aligned}
\frac{y_h^k - \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k}{\delta^{SO} + \delta^{SP}} + y_w^O - \frac{\left(\delta^{HO} + \delta^{HP}\right)}{\delta^{SO} + \delta^{SP}} \theta_2 &< \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right) y_w^O}{\delta^{HO} + \delta^{HP}} + \\
&\quad + \theta_2 + \frac{1}{\delta^{HO} + \delta^{HP}} \theta_4, \\
\frac{\delta^{SO} y_h^k + \delta^{SP} \bar{v}_h^k}{\delta^{SO} + \delta^{SP}} + y_w^O + \frac{\delta^{SP}}{\delta^{SO} + \delta^{SP}} \theta_2 &< \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right) y_w^O}{\delta^{HO} + \delta^{HP}} + \\
&\quad + \theta_2 + \frac{1}{\delta^{HO} + \delta^{HP}} \theta_4, \\
y_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right) y_w^O - \delta^{SP} \bar{v}_w^S}{\delta^{SO}} - \frac{\delta^{SP}}{\delta^{SO}} \theta_4 &< \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right) y_w^O}{\delta^{HO} + \delta^{HP}} + \\
&\quad + \theta_2 + \frac{1}{\delta^{HO} + \delta^{HP}} \theta_4.
\end{aligned}$$

Equivalently:

$$y_h^k - \bar{v}_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{HO} + \delta^{HP}} \left(y_w^O - \bar{v}_w^H\right) - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{HO} + \delta^{HP}} \theta_4 < \theta_2,$$

$$y_h^k - \bar{v}_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO} + \delta^{HP}\right)}\left(y_w^O - \bar{v}_w^H\right) - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO} + \delta^{HP}\right)}\theta_4 \quad < \quad \theta_2,$$

$$y_h^k - \bar{v}_h^k + \frac{y_w^O - \bar{v}_w^H}{\delta^{HO} + \delta^{HP}} + \frac{\delta^{SP}\left(y_w^O - \bar{v}_w^S\right)}{\delta^{SO}} - \frac{\delta^{SP}\left(\delta^{HO} + \delta^{HP}\right) + \delta^{SO}}{\delta^{SO}\left(\delta^{HO} + \delta^{HP}\right)}\theta_4 \quad < \quad \theta_2.$$

It can be shown that the first inequality implies the second one:

$$y_h^k - \bar{v}_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{HO} + \delta^{HP}}\left(y_w^O - \bar{v}_w^H\right) - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{HO} + \delta^{HP}}\theta_4 \geq y_h^k - \bar{v}_h^k + \\ + \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO} + \delta^{HP}\right)}\left(y_w^O - \bar{v}_w^H\right) - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO} + \delta^{HP}\right)}\theta_4,$$

equivalently, $\theta_4 \geq y_w^O - \bar{v}_w^H$, which is true here since $\theta_4 \geq y_w^O - \bar{v}_w^H$.

Thus, the interval for θ_1 is non-empty provided that:

$$\theta_2 \in \left(\max\left\{\begin{array}{c} \frac{\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{HO}+\delta^{HP}}\left(y_w^O - \bar{v}_w^H\right) - \frac{\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{HO}+\delta^{HP}}\theta_4, \\ \frac{y_w^O - \bar{v}_w^H}{\delta^{HO}+\delta^{HP}} + \frac{\delta^{SP}\left(y_w^O - \bar{v}_w^S\right)}{\delta^{SO}} - \frac{\delta^{SP}\left(\delta^{HO}+\delta^{HP}\right)+\delta^{SO}}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\theta_4 \end{array}\right\} + y_h^k - \bar{v}_h^k, +\infty\right).$$

Now, incorporating the condition in Proposition 1, the modified interval for θ_1 is:

$$\left(\begin{array}{c} \max\left\{\begin{array}{c} \frac{y_h^k - \left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k}{\delta^{SO}+\delta^{SP}} + y_w^O - \frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}}\theta_2, \\ \frac{\delta^{SO} y_h^k + \delta^{SP}\bar{v}_h^k}{\delta^{SO}+\delta^{SP}} + y_w^O + \frac{\delta^{SP}}{\delta^{SO}+\delta^{SP}}\theta_2, \\ y_h^k + \frac{\left(\delta^{SO}+\delta^{SP}\right)y_w^O - \delta^{SP}\bar{v}_w^S}{\delta^{SO}} - \frac{\delta^{SP}}{\delta^{SO}}\theta_4, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 \end{array}\right\} - \bar{u}_h - \bar{u}_w - \theta_3, \\ \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO}+\delta^{SP}\right)y_w^O}{\delta^{HO}+\delta^{HP}} - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \frac{1}{\delta^{HO}+\delta^{HP}}\theta_4 \end{array}\right).$$

The modified interval for θ_1 is non-empty under the following additional condition:

$$\bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 < \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right)y_w^O}{\delta^{HO} + \delta^{HP}} + \theta_2 + \frac{1}{\delta^{HO} + \delta^{HP}}\theta_4,$$

equivalently, $\theta_4 > \frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) + \left(y_w^O - \bar{v}_w^H\right)$.

Since $\bar{v}_h^H \geq \bar{v}_h^k$, then $\frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) + \left(y_w^O - \bar{v}_w^H\right) \geq \left(y_w^O - \bar{v}_w^H\right)$. Thus, the interval for θ_4 specializes as:

$$\left(\max\left\{\begin{array}{c} y_w^P - \bar{v}_w^S, \\ \frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) + \left(y_w^O - \bar{v}_w^H\right) \end{array}\right\}, y_w^O - \bar{v}_w^S\right).$$

If $l = SP$, then $\max\left\{v_w^l, y_w^l\right\} = v_w^S$ and the state of cooperation occurs provided that $\tau^* = \tau^3$, or $\tau^* = \tau^2$, or $\tau^* = \tau^1$. The cases $\tau^* = \tau^3$ and $\tau^* = \tau^2$ have been solved.

Consider $\tau^* = \tau^1$. The set of conditions is:

$$\begin{aligned}
u_h\left(-\tau^1\right) &\geq y_h^k, \\
\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq y_h^k, \\
\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq \delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP} + \delta^{SP}\right) v_h^k, \\
\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \\
&\quad + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right), \\
\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \delta^{SP} u_h\left(-\tau^1\right) &\geq u_h\left(-\tau^3\right).
\end{aligned}$$

Equivalently:

$$\begin{aligned}
\theta_1 &\geq y_h^k + \bar{v}_w^S - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\
\theta_1 &\geq \frac{\left(1 - \delta^{SO}\right) y_h^k - \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k}{\delta^{SP}} + \bar{v}_w^S - \bar{u}_h - \bar{u}_w - \frac{\left(\delta^{HO} + \delta^{HP}\right) \theta_2}{\delta^{SP}} - \theta_3 + \theta_4, \\
\theta_1 &\geq \bar{v}_h^k + \bar{v}_w^S - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4, \\
\theta_1 &\leq y_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right) y_w^O - \delta^{SP} \bar{v}_w^S}{\delta^{SO}} - \bar{u}_h - \bar{u}_w - \theta_3 - \frac{\delta^{SP}}{\delta^{SO}} \theta_4, \\
\theta_1 &\leq \frac{\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{1 - \delta^{SP}} - \bar{u}_h - \bar{u}_w + \frac{\left(\delta^{HO} + \delta^{HP}\right) \theta_2}{1 - \delta^{SP}} - \theta_3 + \\
&\quad + \theta_4.
\end{aligned}$$

Hence, the set of conditions corresponds to the following interval for θ_1:

$$\left(\begin{array}{l}
\max\left\{\begin{array}{c}
y_h^k + \bar{v}_w^S, \\
\frac{\left(1-\delta^{SO}\right) y_h^k - \left(\delta^{HO}+\delta^{HP}\right) \bar{v}_h^k}{\delta^{SP}} + \bar{v}_w^S - \frac{\left(\delta^{HO}+\delta^{HP}\right)\theta_2}{\delta^{SP}}, \\
\bar{v}_h^k + \bar{v}_w^S + \theta_2
\end{array}\right\} - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\
\min\left\{\begin{array}{c}
y_h^k + \frac{\left(\delta^{SO}+\delta^{SP}\right) y_w^O - \delta^{SP} \bar{v}_w^S}{\delta^{SO}} - \frac{\delta^{SP}}{\delta^{SO}} \theta_4, \\
\frac{\delta^{SO} y_h^k + \left(\delta^{HO}+\delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{1-\delta^{SP}} + \frac{\left(\delta^{HO}+\delta^{HP}\right)}{1-\delta^{SP}} \theta_2 + \theta_4
\end{array}\right\} - \bar{u}_h - \bar{u}_w - \theta_3
\end{array}\right).$$

Depending on θ_2, the above interval may be an empty set. The interval is non-empty provided that:

$$y_h^k + \bar{v}_w^S + \theta_4 < y_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right) y_w^O - \delta^{SP} \bar{v}_w^S}{\delta^{SO}} - \frac{\delta^{SP}}{\delta^{SO}} \theta_4,$$

$$y_h^k + \bar{v}_w^S < \frac{\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{1 - \delta^{SP}} + \frac{\left(\delta^{HO} + \delta^{HP}\right)}{1 - \delta^{SP}} \theta_2,$$

$$\frac{\left(1 - \delta^{SO}\right) y_h^k - \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k}{\delta^{SP}} - \frac{\left(\delta^{HO} + \delta^{HP}\right)}{\delta^{SP}} \theta_2 + \bar{v}_w^S + \theta_4 < y_h^k - \frac{\delta^{SP}}{\delta^{SO}} \theta_4 +$$
$$+ \frac{\left(\delta^{SO} + \delta^{SP}\right) y_w^O - \delta^{SP} \bar{v}_w^S}{\delta^{SO}},$$

$$\frac{\left(1 - \delta^{SO}\right) y_h^k - \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k}{\delta^{SP}} - \frac{\left(\delta^{HO} + \delta^{HP}\right)}{\delta^{SP}} \theta_2 + \bar{v}_w^S < \frac{\left(\delta^{HO} + \delta^{HP}\right)}{1 - \delta^{SP}} \theta_2 +$$
$$+ \frac{\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{1 - \delta^{SP}},$$

$$\bar{v}_h^k + \theta_2 + \bar{v}_w^S + \theta_4 < y_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right) y_w^O - \delta^{SP} \bar{v}_w^S}{\delta^{SO}} - \frac{\delta^{SP}}{\delta^{SO}} \theta_4,$$

$$\bar{v}_h^k + \theta_2 + \bar{v}_w^S < \frac{\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) \bar{v}_h^k + \bar{v}_w^H - \delta^{SP} \bar{v}_w^S}{1 - \delta^{SP}} + \frac{\left(\delta^{HO} + \delta^{HP}\right)}{1 - \delta^{SP}} \theta_2.$$

Equivalently:

$$\begin{aligned}
\theta_4 &< y_w^O - \bar{v}_w^S, \\
y_h^k - \bar{v}_h^k - \frac{\bar{v}_w^H - \bar{v}_w^S}{\delta^{HO} + \delta^{HP}} &< \theta_2, \\
y_h^k - \bar{v}_h^k - \frac{\delta^{SP}\left(\delta^{SO} + \delta^{SP}\right)\left(y_w^O - \bar{v}_w^S\right)}{\delta^{SO}\left(\delta^{HO} + \delta^{HP}\right)} + \frac{\delta^{SP}\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO} + \delta^{HP}\right)} \theta_4 &< \theta_2, \\
y_h^k - \bar{v}_h^k - \frac{\delta^{SP}\left(\bar{v}_w^H - \bar{v}_w^S\right)}{\delta^{HO} + \delta^{HP}} &< \theta_2, \\
y_h^k - \bar{v}_h^k + \frac{\left(\delta^{SO} + \delta^{SP}\right)\left(y_w^O - \bar{v}_w^S\right)}{\delta^{SO}} - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO}} \theta_4 &> \theta_2, \\
y_h^k - \bar{v}_h^k + \frac{\bar{v}_w^H - \bar{v}_w^S}{\delta^{SO}} &> \theta_2.
\end{aligned}$$

The first inequality above is always true in this case.

Next, it can be shown that the fourth inequality implies the second one:

$$y_h^k - \bar{v}_h^k - \frac{\delta^{SP}\left(\bar{v}_w^H - \bar{v}_w^S\right)}{\delta^{HO} + \delta^{HP}} \geq y_h^k - \bar{v}_h^k - \frac{\bar{v}_w^H - \bar{v}_w^S}{\delta^{HO} + \delta^{HP}},$$

equivalently, $1 \geq \delta^{SP}$, which is always true.

Also, the fifth inequality implies the sixth one:

$$y_h^k - \bar{v}_h^k + \frac{\left(\delta^{SO}+\delta^{SP}\right)\left(y_w^O - \bar{v}_w^S\right)}{\delta^{SO}} - \frac{\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{SO}}\theta_4 \leq y_h^k - \bar{v}_h^k + \frac{\bar{v}_w^H - \bar{v}_w^S}{\delta^{SO}},$$

equivalently, $y_w^O - \bar{v}_w^S - \frac{\bar{v}_w^H - \bar{v}_w^S}{\delta^{SO}+\delta^{SP}} \leq \theta_4$, which is true because:

$$y_w^O - \bar{v}_w^S - \frac{\bar{v}_w^H - \bar{v}_w^S}{\delta^{SO}+\delta^{SP}} \leq y_w^O - \bar{v}_w^S - \left(\bar{v}_w^H - \bar{v}_w^S\right) = y_w^O - \bar{v}_w^H \leq \theta_4.$$

Thus, the interval for θ_1 is non-empty provided that:

$$\theta_2 \in \left(\begin{array}{c} \max \left\{ \begin{array}{c} -\frac{\delta^{SP}\left(\delta^{SO}+\delta^{SP}\right)\left(y_w^O-\bar{v}_w^S\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)} + \frac{\delta^{SP}\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\theta_4, \\ -\frac{\delta^{SP}\left(\bar{v}_w^H-\bar{v}_w^S\right)}{\delta^{HO}+\delta^{HP}} \end{array} \right\} + y_h^k - \bar{v}_h^k, \\ y_h^k - \bar{v}_h^k + \frac{\left(\delta^{SO}+\delta^{SP}\right)\left(y_w^O-\bar{v}_w^S\right)}{\delta^{SO}} - \frac{\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{SO}}\theta_4 \end{array} \right).$$

Now, note that since $\bar{v}_h^H \geq \bar{v}_h^k$ and $\bar{v}_w^H > \bar{v}_w^S$, then $\bar{v}_h^H + \bar{v}_w^H + \theta_2 > \bar{v}_h^k + \bar{v}_w^S + \theta_2$.

Thus, incorporating the condition in Proposition 1, the modified interval for θ_1 is:

$$\left(\begin{array}{c} \max \left\{ \begin{array}{c} \frac{\left(1-\delta^{SO}\right)y_h^k - \left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k}{\delta^{SP}} + \bar{v}_w^S - \frac{\left(\delta^{HO}+\delta^{HP}\right)\theta_2}{\delta^{SP}}, \\ y_h^k + \bar{v}_w^S, \bar{v}_h^H + \bar{v}_w^H + \theta_2 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\ \min \left\{ \begin{array}{c} y_h^k + \frac{\left(\delta^{SO}+\delta^{SP}\right)y_w^O - \delta^{SP}\bar{v}_w^S}{\delta^{SO}} - \frac{\delta^{SP}}{\delta^{SO}}\theta_4, \\ \frac{\delta^{SO}y_h^k + \left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k + \bar{v}_w^H - \delta^{SP}\bar{v}_w^S}{1-\delta^{SP}} + \frac{\left(\delta^{HO}+\delta^{HP}\right)\theta_2}{1-\delta^{SP}} + \theta_4 \end{array} \right\} - \bar{u}_h - \bar{u}_w - \theta_3 \end{array} \right).$$

Now, the modified interval for θ_1 is non-empty under the following additional conditions:

$$\begin{aligned} \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 &< y_h^k + \frac{\left(\delta^{SO}+\delta^{SP}\right)y_w^O - \delta^{SP}\bar{v}_w^S}{\delta^{SO}} - \frac{\delta^{SP}}{\delta^{SO}}\theta_4, \\ \bar{v}_h^H + \bar{v}_w^H + \theta_2 + \theta_4 &< \frac{\delta^{SO}y_h^k + \left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k + \bar{v}_w^H - \delta^{SP}\bar{v}_w^S}{1-\delta^{SP}} + \\ &\quad + \frac{\left(\delta^{HO}+\delta^{HP}\right)}{1-\delta^{SP}}\theta_2 + \theta_4. \end{aligned}$$

Equivalently:

$$\begin{aligned} \theta_2 &< y_h^k - \bar{v}_h^k + \bar{v}_h^k - \bar{v}_h^H + y_w^O - \bar{v}_w^H + \frac{\delta^{SP}}{\delta^{SO}}\left(y_w^O - \bar{v}_w^S\right) - \frac{\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{SO}}\theta_4, \\ \theta_2 &< y_h^k - \bar{v}_h^k + \frac{\left(1-\delta^{SP}\right)}{\delta^{SO}}\left(\bar{v}_h^k - \bar{v}_h^H\right) + \frac{\delta^{SP}}{\delta^{SO}}\left(\bar{v}_w^H - \bar{v}_w^S\right). \end{aligned}$$

It can be seen that:

$$y_h^k - \bar{v}_h^k + \bar{v}_h^k - \bar{v}_h^H + y_w^O - \bar{v}_w^H + \frac{\delta^{SP}}{\delta^{SO}}\left(y_w^O - \bar{v}_w^S\right) - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO}}\theta_4 < y_h^k - \bar{v}_h^k + \\ + \frac{\left(\delta^{SO} + \delta^{SP}\right)\left(y_w^O - \bar{v}_w^S\right)}{\delta^{SO}} - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO}}\theta_4,$$

equivalently, $\bar{v}_h^k - \bar{v}_h^H < \bar{v}_w^H - \bar{v}_w^S$, which is true because $\bar{v}_h^k - \bar{v}_h^H \leq 0 < \bar{v}_w^H - \bar{v}_w^S$.

Thus, the modified interval for θ_2 is:

$$\left(\begin{array}{c} \max\left\{\begin{array}{c} -\frac{\delta^{SP}\left(\delta^{SO}+\delta^{SP}\right)\left(y_w^O-\bar{v}_w^S\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)} + \frac{\delta^{SP}\left(\delta^{SO}+\delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\theta_4, \\ -\frac{\delta^{SP}\left(\bar{v}_w^H-\bar{v}_w^S\right)}{\delta^{HO}+\delta^{HP}} \end{array}\right\} + y_h^k - \bar{v}_h^k, \\ \min\left\{\begin{array}{c} \bar{v}_h^k - \bar{v}_h^H + y_w^O - \bar{v}_w^H + \frac{\delta^{SP}}{\delta^{SO}}\left(y_w^O - \bar{v}_w^S\right) - \frac{\left(\delta^{SO}+\delta^{SP}\right)\theta_4}{\delta^{SO}}, \\ \frac{\left(1-\delta^{SP}\right)}{\delta^{SO}}\left(\bar{v}_h^k - \bar{v}_h^H\right) + \frac{\delta^{SP}}{\delta^{SO}}\left(\bar{v}_w^H - \bar{v}_w^S\right) \end{array}\right\} + y_h^k - \bar{v}_h^k \end{array}\right).$$

In turn, the modified interval for θ_2 is non-empty if the following conditions hold:

$$-\frac{\delta^{SP}\left(\delta^{SO} + \delta^{SP}\right)\left(y_w^O - \bar{v}_w^S\right)}{\delta^{SO}\left(\delta^{HO} + \delta^{HP}\right)} + \frac{\delta^{SP}\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO} + \delta^{HP}\right)}\theta_4 < \bar{v}_h^k - \bar{v}_h^H + y_w^O - \bar{v}_w^H + \\ + \frac{\delta^{SP}}{\delta^{SO}}\left(y_w^O - \bar{v}_w^S\right) - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO}}\theta_4,$$

$$-\frac{\delta^{SP}\left(\delta^{SO} + \delta^{SP}\right)\left(y_w^O - \bar{v}_w^S\right)}{\delta^{SO}\left(\delta^{HO} + \delta^{HP}\right)} + \frac{\delta^{SP}\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO}\left(\delta^{HO} + \delta^{HP}\right)}\theta_4 < \frac{\left(1 - \delta^{SP}\right)}{\delta^{SO}}\left(\bar{v}_h^k - \bar{v}_h^H\right) + \\ + \frac{\delta^{SP}}{\delta^{SO}}\left(\bar{v}_w^H - \bar{v}_w^S\right),$$

$$-\frac{\delta^{SP}\left(\bar{v}_w^H - \bar{v}_w^S\right)}{\delta^{HO} + \delta^{HP}} < \bar{v}_h^k - \bar{v}_h^H + y_w^O - \bar{v}_w^H + \frac{\delta^{SP}}{\delta^{SO}}\left(y_w^O - \bar{v}_w^S\right) - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{SO}}\theta_4,$$

$$-\frac{\delta^{SP}\left(\bar{v}_w^H - \bar{v}_w^S\right)}{\delta^{HO} + \delta^{HP}} < \frac{\left(1 - \delta^{SP}\right)}{\delta^{SO}}\left(\bar{v}_h^k - \bar{v}_h^H\right) + \frac{\delta^{SP}}{\delta^{SO}}\left(\bar{v}_w^H - \bar{v}_w^S\right).$$

Equivalently:

$$\begin{aligned} \theta_4 &< -\frac{\delta^{SO}\left(\delta^{HO} + \delta^{HP}\right)}{\left(1 - \delta^{SO}\right)\left(\delta^{SO} + \delta^{SP}\right)}\left(\bar{v}_h^H - \bar{v}_h^k + \bar{v}_w^H - \bar{v}_w^S\right) + y_w^O - \bar{v}_w^S, \\ \theta_4 &< -\frac{\left(1 - \delta^{SP}\right)\left(\delta^{HO} + \delta^{HP}\right)}{\delta^{SP}\left(\delta^{SO} + \delta^{SP}\right)}\left(\bar{v}_h^H - \bar{v}_h^k\right) + \frac{\left(\delta^{HO} + \delta^{HP}\right)}{\delta^{SO} + \delta^{SP}}\left(\bar{v}_w^H - \bar{v}_w^S\right) + y_w^O - \bar{v}_w^S, \end{aligned}$$

$$\theta_4 < \frac{\delta^{SO}\delta^{SP}\left(\bar{v}_w^H - \bar{v}_w^S\right)}{\left(\delta^{HO}+\delta^{HP}\right)\left(\delta^{SO}+\delta^{SP}\right)} - \frac{\delta^{SO}\left(\bar{v}_h^H - \bar{v}_h^k + \bar{v}_w^H - \bar{v}_w^S\right)}{\left(\delta^{SO}+\delta^{SP}\right)} + y_w^O - \bar{v}_w^S,$$

$$\bar{v}_w^H - \bar{v}_w^S > \frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right).$$

Under condition $\bar{v}_w^H - \bar{v}_w^S > \frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right)$, it can be shown that the first inequality above implies the second and third inequalities.

Namely:

$$-\frac{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}{\left(1-\delta^{SO}\right)\left(\delta^{SO}+\delta^{SP}\right)}\left(\bar{v}_h^H - \bar{v}_h^k + \bar{v}_w^H - \bar{v}_w^S\right) + y_w^O - \bar{v}_w^S < y_w^O - \bar{v}_w^S -$$
$$-\frac{\left(1-\delta^{SP}\right)\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SP}\left(\delta^{SO}+\delta^{SP}\right)}\left(\bar{v}_h^H - \bar{v}_h^k\right) + \frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}}\left(\bar{v}_w^H - \bar{v}_w^S\right),$$

equivalently, $\frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) < \bar{v}_w^H - \bar{v}_w^S$.

Likewise:

$$-\frac{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}{\left(1-\delta^{SO}\right)\left(\delta^{SO}+\delta^{SP}\right)}\left(\bar{v}_h^H - \bar{v}_h^k + \bar{v}_w^H - \bar{v}_w^S\right) + y_w^O - \bar{v}_w^S < y_w^O - \bar{v}_w^S -$$
$$-\frac{\delta^{SO}}{\left(\delta^{SO}+\delta^{SP}\right)}\left(\bar{v}_h^H - \bar{v}_h^k + \bar{v}_w^H - \bar{v}_w^S\right) + \frac{\delta^{SO}\delta^{SP}}{\left(\delta^{HO}+\delta^{HP}\right)\left(\delta^{SO}+\delta^{SP}\right)}\left(\bar{v}_w^H - \bar{v}_w^S\right),$$

equivalently, $\frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right) < \bar{v}_w^H - \bar{v}_w^S$.

Thus, the modified set for θ_2 is non-empty under the following two conditions:

$$\begin{aligned}\theta_4 &< -\frac{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}{\left(1-\delta^{SO}\right)\left(\delta^{SO}+\delta^{SP}\right)}\left(\bar{v}_h^H - \bar{v}_h^k + \bar{v}_w^H - \bar{v}_w^S\right) + y_w^O - \bar{v}_w^S,\\ \bar{v}_w^H - \bar{v}_w^S &> \frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SP}}\left(\bar{v}_h^H - \bar{v}_h^k\right).\end{aligned}$$

Now, note that since $\bar{v}_h^H \geq \bar{v}_h^k$ and $\bar{v}_w^H - \bar{v}_w^S > 0$:

$$-\frac{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}{\left(1-\delta^{SO}\right)\left(\delta^{SO}+\delta^{SP}\right)}\left(\bar{v}_h^H - \bar{v}_h^k + \bar{v}_w^H - \bar{v}_w^S\right) + y_w^O - \bar{v}_w^S < y_w^O - \bar{v}_w^S.$$

Thus, the modified interval for θ_4 specializes as:

$$\left(\max\left\{\begin{array}{c} y_w^O - \bar{v}_w^H, \\ y_w^P - \bar{v}_w^S \end{array}\right\}, -\frac{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)\left(\bar{v}_h^H - \bar{v}_h^k + \bar{v}_w^H - \bar{v}_w^S\right)}{\left(1-\delta^{SO}\right)\left(\delta^{SO}+\delta^{SP}\right)} + y_w^O - \bar{v}_w^S\right),$$

provided that $\bar{v}_w^H - \bar{v}_w^S > \frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SP}} \left(\bar{v}_h^H - \bar{v}_h^k\right)$ also holds.

Conflict

If $l = SO$, then $\max\left\{v_w^l, y_w^l\right\} = y_w^O$ and the state of conflict cannot occur.

If $l = SP$, then $\max\left\{v_w^l, y_w^l\right\} = v_w^S$ and the state of conflict occurs if $\tau^* < \tau^1$. The set of conditions is:

$$\begin{aligned}
\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP} + \delta^{SP}\right) v_h^k &\geq y_h^k, \\
\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP} + \delta^{SP}\right) v_h^k &\geq \delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \delta^{SP} u_h\left(-\tau^1\right), \\
\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP} + \delta^{SP}\right) v_h^k &\geq \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right), \\
\delta^{SO} y_h^k + \left(\delta^{HO} + \delta^{HP} + \delta^{SP}\right) v_h^k &\geq u_h\left(-\tau^3\right),
\end{aligned}$$

which can be simplified as:

$$\begin{aligned}
v_h^k &\geq y_h^k, \\
v_h^k &\geq u_h\left(-\tau^1\right), \\
\delta^{SO} y_h^k + \delta^{SP} v_h^k &\geq \left(\delta^{SO} + \delta^{SP}\right) u_h\left(-\tau^2\right), \\
\delta^{SO} y_h^k + \left(1 - \delta^{SO}\right) v_h^k &\geq u_h\left(-\tau^3\right).
\end{aligned}$$

Equivalently:

$$\begin{aligned}
\theta_2 &\geq y_h^k - \bar{v}_h^k, \\
\theta_1 &\leq \bar{v}_h^k + \bar{v}_w^S - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4, \\
\theta_1 &\leq \frac{\delta^{SO} y_h^k + \delta^{SP} \bar{v}_h^k}{\delta^{SO} + \delta^{SP}} + y_w^O - \bar{u}_h - \bar{u}_w + \frac{\delta^{SP}}{\delta^{SO} + \delta^{SP}} \theta_2 - \theta_3, \\
\theta_1 &\leq \delta^{SO} y_h^k + \left(1 - \delta^{SO}\right) \bar{v}_h^k + \bar{v}_w^H - \bar{u}_h - \bar{u}_w + \left(1 - \delta^{SO}\right) \theta_2 - \theta_3 + \theta_4.
\end{aligned}$$

It can be shown that given $\theta_2 \geq y_h^k - \bar{v}_h^k$, the third inequality implies the fourth one:

$$\frac{\delta^{SO}y_h^k+\delta^{SP}\bar{v}_h^k}{\delta^{SO}+\delta^{SP}}+y_w^O-\bar{u}_h-\bar{u}_w+\frac{\delta^{SP}}{\delta^{SO}+\delta^{SP}}\theta_2-\theta_3\leq\delta^{SO}y_h^k+\left(1-\delta^{SO}\right)\bar{v}_h^k+$$
$$+\bar{v}_w^H-\bar{u}_h-\bar{u}_w+\left(1-\delta^{SO}\right)\theta_2-\theta_3+\theta_4,$$

or, equivalently:

$$y_h^k-\bar{v}_h^k+\frac{\delta^{SO}+\delta^{SP}}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\left(y_w^O-\bar{v}_w^H\right)-\frac{\delta^{SO}+\delta^{SP}}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\theta_4\leq\theta_2.$$

The above inequality is true given $\theta_2\geq y_h^k-\bar{v}_h^k$, because:

$$y_h^k-\bar{v}_h^k+\frac{\delta^{SO}+\delta^{SP}}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\left(y_w^O-\bar{v}_w^H\right)-\frac{\delta^{SO}+\delta^{SP}}{\delta^{SO}\left(\delta^{HO}+\delta^{HP}\right)}\theta_4\leq y_h^k-\bar{v}_h^k,$$

equivalently, $y_w^O-\bar{v}_w^H\leq\theta_4$, which holds in this case.

Thus, the set of conditions can be simplified as:

$$\theta_2\in\left(y_h^k-\bar{v}_h^k,+\infty\right),$$

$$\theta_1\in\left(-\infty,\min\left\{\begin{array}{c}\bar{v}_h^k+\bar{v}_w^S+\theta_2+\theta_4,\\ \frac{\delta^{SO}y_h^k+\delta^{SP}\bar{v}_h^k}{\delta^{SO}+\delta^{SP}}+y_w^O+\frac{\delta^{SP}\theta_2}{\delta^{SO}+\delta^{SP}}\end{array}\right\}-\bar{u}_h-\bar{u}_w-\theta_3\right).$$

Now, incorporating the condition in Proposition 1, θ_1 must simultaneously satisfy:

$$\begin{aligned}\theta_1 &> \bar{v}_h^H+\bar{v}_w^H+\theta_2+\theta_4-\bar{u}_h-\bar{u}_w-\theta_3,\\ \theta_1 &< \bar{v}_h^k+\bar{v}_w^S+\theta_2+\theta_4-\bar{u}_h-\bar{u}_w-\theta_3,\end{aligned}$$

which is impossible because $\bar{v}_h^H\geq\bar{v}_h^k$, $\bar{v}_w^H>\bar{v}_w^S$ and, therefore:

$$\bar{v}_h^k+\bar{v}_w^S+\theta_2+\theta_4-\bar{u}_h-\bar{u}_w-\theta_3<\bar{v}_h^H+\bar{v}_w^H+\theta_2+\theta_4-\bar{u}_h-\bar{u}_w-\theta_3.$$

If $l\in\{HO,HP\}$, then $\max\left\{v_w^l,y_w^l\right\}=v_w^H$ and the state of conflict occurs provided that $\tau^*<\tau^1$, or $\tau^*=\tau^1$, or $\tau^*=\tau^2$. All these three cases have already been solved.

Case VI $v_w^H > v_w^S \geq y_w^O > y_w^P$

In this case, $\tau^1 = \bar{v}_w^S - \bar{u}_w - \theta_3 + \theta_4$, $\tau^2 = \bar{v}_w^H - \bar{u}_w - \theta_3 + \theta_4$, and:

$$\hat{E}\mathcal{V}_h^k(\mathcal{C}) = \max\left\{\begin{array}{c} v_h^k, \\ \left(\delta^{HO}+\delta^{HP}\right)v_h^k + \left(\delta^{SO}+\delta^{SP}\right)u_h\left(-\tau^1\right), \\ u_h\left(-\tau^2\right) \end{array}\right\}.$$

Cooperation

If $l \in \{HO, HP\}$, then $\max\left\{v_w^l, y_w^l\right\} = v_w^H$ and the state of cooperation occurs provided that $\tau^* = \tau^2$. The set of conditions is:

$$\begin{aligned} u_h\left(-\tau^2\right) &\geq y_h^k, \\ u_h\left(-\tau^2\right) &\geq v_h^k, \\ u_h\left(-\tau^2\right) &\geq \left(\delta^{HO}+\delta^{HP}\right)v_h^k + \left(\delta^{SO}+\delta^{SP}\right)u_h\left(-\tau^1\right). \end{aligned}$$

Equivalently:

$$\begin{aligned} \theta_1 &\geq y_h^k + \bar{v}_w^H - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\ \theta_1 &\geq \bar{v}_h^k + \bar{v}_w^H - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4, \\ \theta_1 &\geq \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO}+\delta^{SP}\right)\bar{v}_w^S}{\delta^{HO}+\delta^{HP}} - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4. \end{aligned}$$

It can be shown that the third inequality implies the second one:

$$\bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO}+\delta^{SP}\right)\bar{v}_w^S}{\delta^{HO}+\delta^{HP}} - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4 \geq \bar{v}_h^k + \bar{v}_w^H - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4,$$

equivalently, $\bar{v}_w^H - \bar{v}_w^S \geq 0$, which always holds.

Thus, the set of conditions can be simplified as:

$$\theta_1 \in \left(\max\left\{\begin{array}{c} y_h^k + \bar{v}_w^H, \\ \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO}+\delta^{SP}\right)\bar{v}_w^S}{\delta^{HO}+\delta^{HP}} + \theta_2 \end{array}\right\} - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, +\infty\right).$$

Now, incorporating the condition in Proposition 1, the modified interval for θ_1 is:

$$\left(\max\left\{\begin{array}{c} y_h^k+\bar{v}_w^H, \\ \bar{v}_h^k+\frac{\bar{v}_w^H-\left(\delta^{SO}+\delta^{SP}\right)\bar{v}_w^S}{\delta^{HO}+\delta^{HP}}+\theta_2, \\ \bar{v}_h^H+\bar{v}_w^H+\theta_2 \end{array}\right\}-\bar{u}_h-\bar{u}_w-\theta_3+\theta_4,+\infty\right).$$

If $l\in\{SO,SP\}$, then $\max\left\{v_w^l,y_w^l\right\}=v_w^S$ and the state of cooperation obtains provided that either $\tau^*=\tau^2$, or $\tau^*=\tau^1$. The case $\tau^*=\tau^2$ has been solved above.

Consider $\tau^*=\tau^1$. The set of conditions is:

$$\begin{array}{rcl} u_h\left(-\tau^1\right) & \geq & y_h^k, \\ \left(\delta^{HO}+\delta^{HP}\right)v_h^k+\left(\delta^{SO}+\delta^{SP}\right)u_h\left(-\tau^1\right) & \geq & y_h^k, \\ \left(\delta^{HO}+\delta^{HP}\right)v_h^k+\left(\delta^{SO}+\delta^{SP}\right)u_h\left(-\tau^1\right) & \geq & v_h^k, \\ \left(\delta^{HO}+\delta^{HP}\right)v_h^k+\left(\delta^{SO}+\delta^{SP}\right)u_h\left(-\tau^1\right) & \geq & u_h\left(-\tau^2\right). \end{array}$$

Equivalently:

$$\begin{array}{rcl} \theta_1 & \geq & y_h^k+\bar{v}_w^S-\bar{u}_h-\bar{u}_w-\theta_3+\theta_4, \\ \theta_1 & \geq & \dfrac{y_h^k-\left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k}{\delta^{SO}+\delta^{SP}}+\bar{v}_w^S-\bar{u}_h-\bar{u}_w-\dfrac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}}\theta_2-\theta_3+\theta_4, \\ \theta_1 & \geq & \bar{v}_h^k+\bar{v}_w^S-\bar{u}_h-\bar{u}_w+\theta_2-\theta_3+\theta_4, \\ \theta_1 & \leq & \bar{v}_h^k+\dfrac{\bar{v}_w^H-\left(\delta^{SO}+\delta^{SP}\right)\bar{v}_w^S}{\delta^{HO}+\delta^{HP}}-\bar{u}_h-\bar{u}_w+\theta_2-\theta_3+\theta_4. \end{array}$$

Thus, the interval for θ_1 is:

$$\left(\begin{array}{c} \max\left\{\begin{array}{c} y_h^k+\bar{v}_w^S, \\ \frac{y_h^k-\left(\delta^{HO}+\delta^{HP}\right)\bar{v}_h^k}{\delta^{SO}+\delta^{SP}}+\bar{v}_w^S-\frac{\left(\delta^{HO}+\delta^{HP}\right)\theta_2}{\delta^{SO}+\delta^{SP}}, \\ \bar{v}_h^k+\bar{v}_w^S+\theta_2 \end{array}\right\}-\bar{u}_h-\bar{u}_w-\theta_3+\theta_4, \\ \bar{v}_h^k+\frac{\bar{v}_w^H-\left(\delta^{SO}+\delta^{SP}\right)\bar{v}_w^S}{\delta^{HO}+\delta^{HP}}-\bar{u}_h-\bar{u}_w+\theta_2-\theta_3+\theta_4 \end{array}\right).$$

The above interval is non-empty provided that:

$$\begin{aligned}
y_h^k + \bar{v}_w^S &< \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right)\bar{v}_w^S}{\delta^{HO} + \delta^{HP}} + \theta_2, \\
\frac{y_h^k - \left(\delta^{HO} + \delta^{HP}\right)\bar{v}_h^k}{\delta^{SO} + \delta^{SP}} + \bar{v}_w^S - \frac{\left(\delta^{HO} + \delta^{HP}\right)}{\delta^{SO} + \delta^{SP}}\theta_2 &< \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right)\bar{v}_w^S}{\delta^{HO} + \delta^{HP}} + \theta_2, \\
\bar{v}_h^k + \bar{v}_w^S + \theta_2 &< \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right)\bar{v}_w^S}{\delta^{HO} + \delta^{HP}} + \theta_2.
\end{aligned}$$

Equivalently:

$$\begin{aligned}
y_h^k - \bar{v}_h^k - \frac{\bar{v}_w^H - \bar{v}_w^S}{\delta^{HO} + \delta^{HP}} &< \theta_2, \\
y_h^k - \bar{v}_h^k - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{HO} + \delta^{HP}}\left(\bar{v}_w^H - \bar{v}_w^S\right) &< \theta_2, \\
0 &< \bar{v}_w^H - \bar{v}_w^S.
\end{aligned}$$

Given the assumption of the model, the third inequality above is always true. It can also be shown that the second inequality implies the first one:

$$y_h^k - \bar{v}_h^k - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{HO} + \delta^{HP}}\left(\bar{v}_w^H - \bar{v}_w^S\right) \geq y_h^k - \bar{v}_h^k - \frac{\bar{v}_w^H - \bar{v}_w^S}{\delta^{HO} + \delta^{HP}},$$

equivalently, $\delta^{SO} + \delta^{SP} \leq 1$, which is always true.

Thus, the interval for θ_1 is non-empty provided that:

$$\theta_2 \in \left(y_h^k - \bar{v}_h^k - \frac{\left(\delta^{SO} + \delta^{SP}\right)}{\delta^{HO} + \delta^{HP}}\left(\bar{v}_w^H - \bar{v}_w^S\right), +\infty\right).$$

Now, since $\bar{v}_h^H \geq \bar{v}_h^k$ and $\bar{v}_w^H > \bar{v}_w^S$:

$$\bar{v}_h^H + \bar{v}_w^H + \theta_2 - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4 > \bar{v}_h^k + \bar{v}_w^S + \theta_2 - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4.$$

Thus, incorporating the condition in Proposition 1, the modified interval for θ_1 is:

$$\left(\begin{array}{c}
\max\left\{\begin{array}{c}
y_h^k + \bar{v}_w^S, \\
\frac{y_h^k - \left(\delta^{HO} + \delta^{HP}\right)\bar{v}_h^k}{\delta^{SO} + \delta^{SP}} + \bar{v}_w^S - \frac{\left(\delta^{HO} + \delta^{HP}\right)\theta_2}{\delta^{SO} + \delta^{SP}}, \\
\bar{v}_h^H + \bar{v}_w^H + \theta_2
\end{array}\right\} - \bar{u}_h - \bar{u}_w - \theta_3 + \theta_4, \\
\bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right)\bar{v}_w^S}{\delta^{HO} + \delta^{HP}} - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4
\end{array}\right).$$

This modified interval is non-empty under the following additional condition:

$$\bar{v}_h^H + \bar{v}_w^H < \bar{v}_h^k + \frac{\bar{v}_w^H - \left(\delta^{SO} + \delta^{SP}\right) \bar{v}_w^S}{\delta^{HO} + \delta^{HP}},$$

equivalently, $\bar{v}_w^H - \bar{v}_w^S > \frac{\left(\delta^{HO}+\delta^{HP}\right)}{\delta^{SO}+\delta^{SP}} \left(\bar{v}_h^H - \bar{v}_h^k\right)$.

Conflict

If $l \in \{SO, SP\}$, then $\max\left\{v_w^l, y_w^l\right\} = v_w^S$ and the state of conflict occurs provided that $\tau^* < \tau^1$. The set of conditions is:

$$\begin{aligned} v_h^k &\geq y_h^k, \\ v_h^k &\geq \left(\delta^{HO} + \delta^{HP}\right) v_h^k + \left(\delta^{SO} + \delta^{SP}\right) u_h \left(-\tau^1\right), \\ v_h^k &\geq u_h \left(-\tau^2\right). \end{aligned}$$

Since $u_h(-\tau^1) > u_h(-\tau^2)$, the conditions become $v_h^k \geq y_h^k$ and $v_h^k \geq u_h(-\tau^1)$.

Equivalently:

$$\begin{aligned} \theta_2 &\geq y_h^k - \bar{v}_h^k, \\ \theta_1 &\leq \bar{v}_h^k + \bar{v}_w^S - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4. \end{aligned}$$

Thus, the set of conditions can be simplified as:

$$\theta_2 \in \left(y_h^k - \bar{v}_h^k, +\infty\right),$$

$$\theta_1 \in \left(-\infty, \bar{v}_h^k + \bar{v}_w^S - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4\right).$$

Under the condition in Proposition 1, θ_1 must also satisfy:

$$\begin{aligned} \theta_1 &< \bar{v}_h^k + \bar{v}_w^S - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4, \\ \theta_1 &> \bar{v}_h^H + \bar{v}_w^H - \bar{u}_h - \bar{u}_w + \theta_2 - \theta_3 + \theta_4. \end{aligned}$$

However, $\bar{v}_h^H + \bar{v}_w^H > \bar{v}_h^k + \bar{v}_w^S$ and, thus, the modified interval for θ_1 is empty.

If $l \in \{HO, HP\}$, then $\max\left\{v_w^l, y_w^l\right\} = v_w^H$ and the state of conflict occurs if $\tau^* < \tau^1$ or $\tau^* = \tau^1$. Both cases have been solved above.

Appendix C

Additional Estimation Results

This Appendix presents estimation results for the nonstructural trinomial and structural models in which I excluded from the list of explanatory variables the following four potentially endogenous characteristics of a couple: common children less than 6 years old, common children at least 6 years old, marital duration, and home ownership.

Table C.1: Nonstructural Trinomial Model: Potentially Endogenous Variables Are Excluded

	Conflict		*Divorce*	
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.
constant	-2.369^{**}	(0.543)	-3.013^{**}	(0.554)
children, $<$ 6 y.o.	–		–	
children, $\geq$ 6 y.o.	–		–	
children, wife's	0.104	(0.078)	0.235^{**}	(0.071)
marital duration	–		–	
home ownership	–		–	
age, husband's†	-0.032^{**}	(0.005)	-0.052^{**}	(0.005)
age, abs. diff.†	0.035^{**}	(0.010)	0.074^{**}	(0.010)
black husband	0.451^{**}	(0.133)	0.460^{**}	(0.137)
catholic husband	0.175^{**}	(0.089)	-0.108	(0.092)
religion, diff.	0.118	(0.081)	0.195^{**}	(0.078)
high sch., husband‡	-0.272^{*}	(0.164)	0.005	(0.180)
college, husband‡	-0.348^{*}	(0.184)	-0.293	(0.196)
education, diff.	0.120	(0.081)	0.171^{**}	(0.080)
male-specific avail. ratio	0.818^{**}	(0.276)	0.565^{*}	(0.294)
female-specific avail. ratio	-0.345	(0.380)	0.813^{**}	(0.365)
$\frac{1}{2}$ year $\leq$ separation $\leq$ 1 year	-0.200^{*}	(0.109)	-0.108	(0.103)
separation $>$ 1 year	0.019	(0.085)	-0.201^{**}	(0.086)
collection rate	3.290^{**}	(1.159)	3.083^{**}	(1.190)
coll. rate $\times$ high sch., husband‡	-0.820	(1.139)	-1.857	(1.183)
coll. rate $\times$ college, husband‡	-0.714	(1.286)	-1.177	(1.318)
coll. rate $\times$ high sch., wife‡	-1.262	(0.842)	-1.633^{**}	(0.818)
coll. rate $\times$ college, wife‡	-2.049^{**}	(0.949)	-1.878^{**}	(0.918)

Notes:

†Variable is standardized in estimation. I report the impact of a one-year increase.

‡The omitted education category is "no high school degree."

* and ** denote significance at 10 and 5% level, respectively.

In estimation, Δ is set to the identity matrix. Sample log-likelihood is -2549.15.

Table C.2: Utility Parameters in State of Cooperation: Potentially Endogenous Variables Are Excluded

Variable	Coeff.	Std. Err.
constant	4.496**	(0.689)
children, $<$ 6 y.o.	–	
children, $\geq$ 6 y.o.	–	
children, wife's	−0.451**	(0.168)
marital duration	–	
home ownership	–	
age, husband's†	0.090**	(0.014)
age, abs. diff.†	−0.111**	(0.029)
black husband	0.435	(0.319)
catholic husband	0.287	(0.203)
religion, diff.	−0.033	(0.103)
high sch., husband‡	0.067	(0.147)
college, husband‡	0.120	(0.222)
education, diff.	−0.231	(0.167)

Notes:

Coefficients denote the effect of corresponding variables on the sum of spousal utilities relative to the impact in the state of divorce.

†Variable is standardized in estimation. I report the impact of a one-year increase.

‡The omitted education category is "no high school degree."

* and ** denote significance at 10 and 5% levels, respectively.

Table C.3: Utility Parameters in State of Conflict: Potentially Endogenous Variables Are Excluded

	Husband		Wife	
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.
constant	-2.522^{**}	(0.753)	-1.170^{**}	(0.592)
children, <6 y.o.	–		–	
children, ≥ 6 y.o.	–		–	
children, wife's	0.333^{**}	(0.162)	0.643^{**}	(0.175)
marital duration	–		–	
home ownership	–		–	
age, husband's†	0.102^{**}	(0.019)	-0.033^{**}	(0.008)
age, abs. diff.†	-0.113^{**}	(0.041)	0.061^{**}	(0.024)
black husband	-0.982^{*}	(0.584)	0.821^{**}	(0.287)
catholic husband	0.641^{*}	(0.344)	0.218	(0.160)
religion, diff.	-0.799^{**}	(0.360)	0.215	(0.149)
high sch., husband‡	0.144	(0.193)	-0.416^{**}	(0.207)
college, husband‡	0.251	(0.275)	-0.818^{**}	(0.235)
education, diff.	-0.164	(0.204)	0.162	(0.141)
hard barg. constant	2.274^{**}	(0.657)	3.503^{**}	(0.396)

Notes:

Coefficients denote the effect of corresponding variables on spousal utilities relative to the impact in the state of divorce.

†Variable is standardized in estimation. I report the impact of a one-year increase.

‡The omitted education category is "no high school degree."

* and ** denote significance at 10 and 5% levels, respectively.

Table C.4: Utility Parameters in State of Divorce: Potentially Endogenous Variables Are Excluded

	Husband		Wife	
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.
male-specific avail. ratio	0.321	(0.334)	–	
female-specific avail. ratio	–		0.946**	(0.481)
$\frac{1}{2}$ year $\leq$ separation $\leq$ 1 year	−0.229	(0.163)	0.081	(0.150)
separation > 1 year	−0.178	(0.132)	−0.256	(0.159)
collection rate	−0.162	(0.263)	1.989**	(0.901)
coll. rate × high sch., husband‡	−1.645**	(0.734)	–	
coll. rate × college, husband‡	−0.888	(0.652)	–	
coll. rate × high sch., wife‡	–		−1.820**	(0.823)
coll. rate × college, wife‡	–		−0.829	(0.669)
optimist's constant	3.750**	(0.411)	0.668**	(0.160)

Notes:

‡The omitted education category is "no high school degree."

* and ** denote significance at 10 and 5% levels, respectively.

Table C.5: Sample Means of Type Probabilities and Beliefs: Potentially Endogenous Variables Are Excluded

Spousal Type		True Types† Husband	Wife	Beliefs‡ Husband
HO	(hard bargainer – optimist)	0.097	0.038	0.148
HP	(hard bargainer – pessimist)	0.148	0.222	0.037
SO	(soft bargainer – optimist)	0.020	0.053	0.119
SP	(soft bargainer – pessimist)	0.735	0.687	0.696

Notes:

†A cell represents the sample mean probability of the event that a spouse is of a corresponding type.

‡A cell represents the simulated sample mean probability which a husband assigns to the event that his wife is of a corresponding type.

In estimation, Σ and Ω are diagonal matrices. Sample log-likelihood: -2454.01.