

# ACE Market Game Examples

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# Outline

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- ◆ Example 1: ACE double-auction trading game
- ◆ Example 2: ACE posted-auction trading game

# Example 1: ACE Double-Auction Trading Game

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- ◆ J. Nicolaisen, V. Petrov, L. Tesfatsion, *IEEE Transactions on Evolutionary Computation*, 5(5), 2001, pp. 504-523  
<https://www2.econ.iastate.edu/tesfatsi/mpeieee.pdf>

- ◆ **Key Issue Addressed:**

Relative role of structure vs. learning in determining the performance of a double-auction design for a day-ahead electricity market.

# Two Key Issues Addressed

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\* Sensitivity of market performance to changes in *market structure*:

**RCON** =: Relative seller/buyer **concentration**

**RCAP** =: Relative demand/supply **capacity**

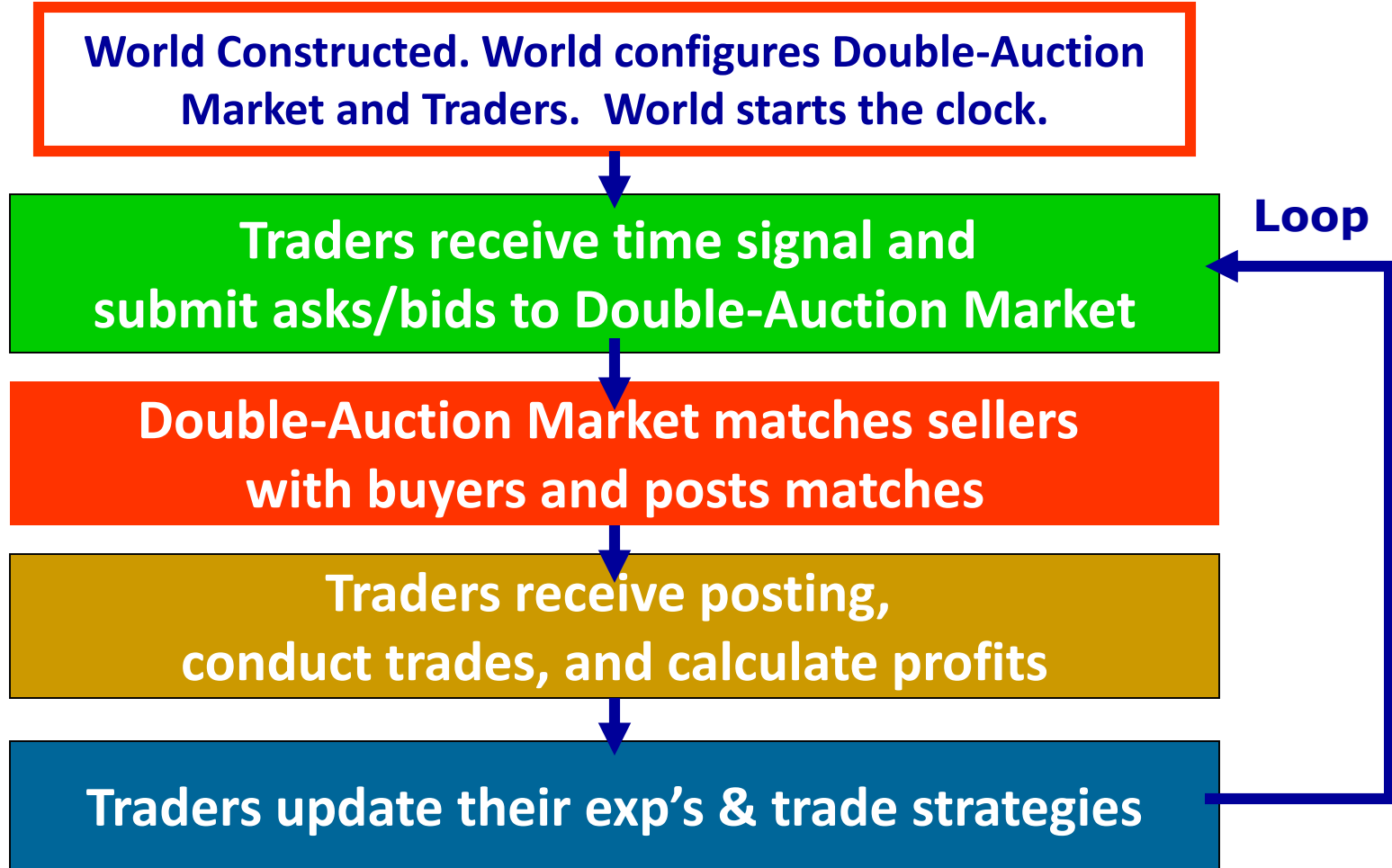
\* Sensitivity of market performance to changes in *trader learning methods*:

-- **Learning Treatment 1**: Individual **Reinforcement Learning (RL)**

-- **Learning Treatment 2**: Social **Genetic Algorithm (GA)** learning

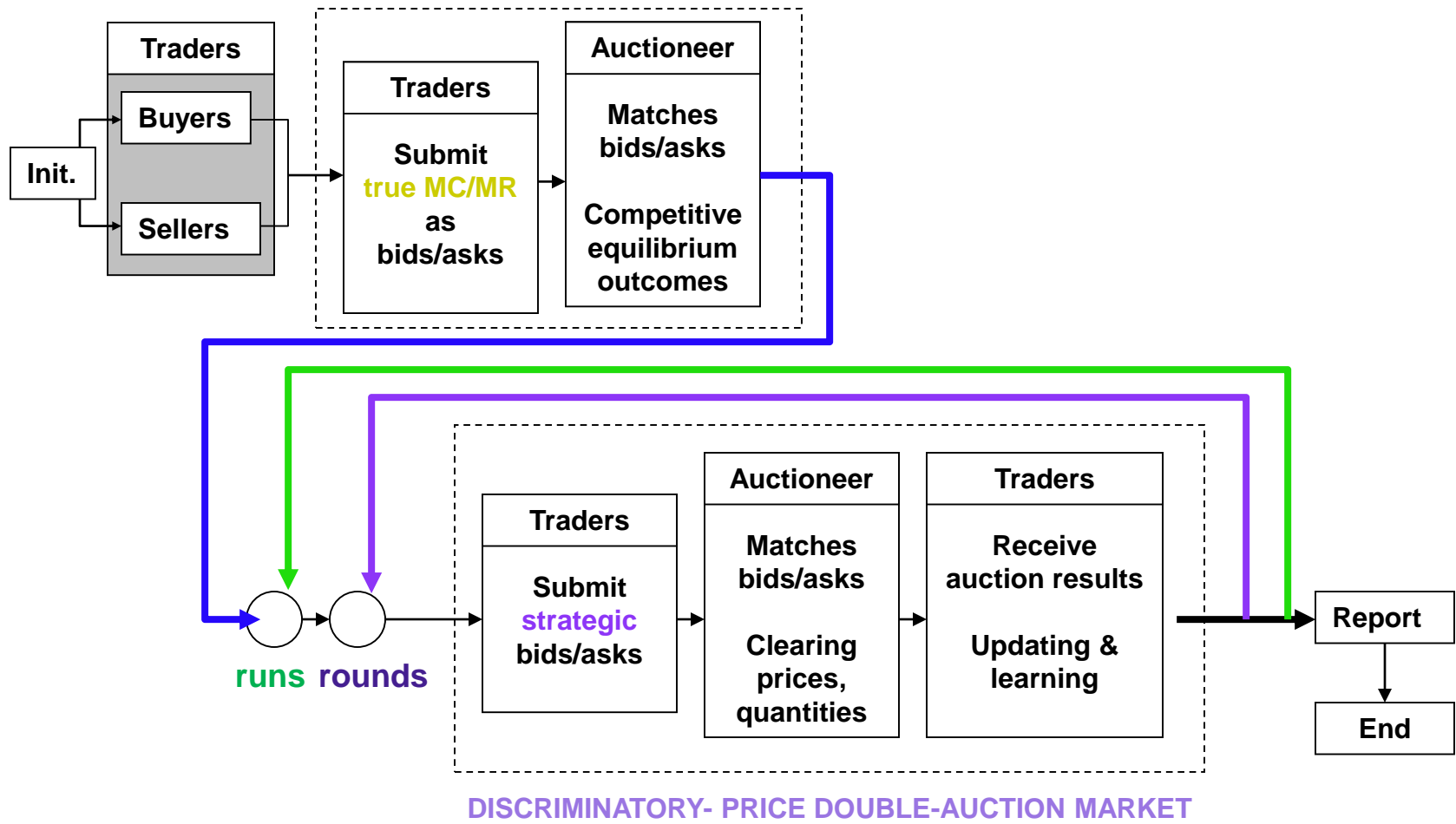
# Dynamic Flow of the Double-Auction World

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# Dynamic Activity Flow for the Double-Auction Market

## COMPETITIVE EQUILIBRIUM BENCHMARK (Calculated Off-Line)



# Nine Structural Treatments Tested for Each Learning Treatment

Each Structural Treatment Consists of Four Market Structural Settings  
 Together with **\*\*True\*\*** Trader Demand and Supply Schedules

Ns = Number of Sellers  
 Nb = Number of Buyers  
 Cs = Seller Supply Capacity  
 Cb = Buyer Demand Capacity

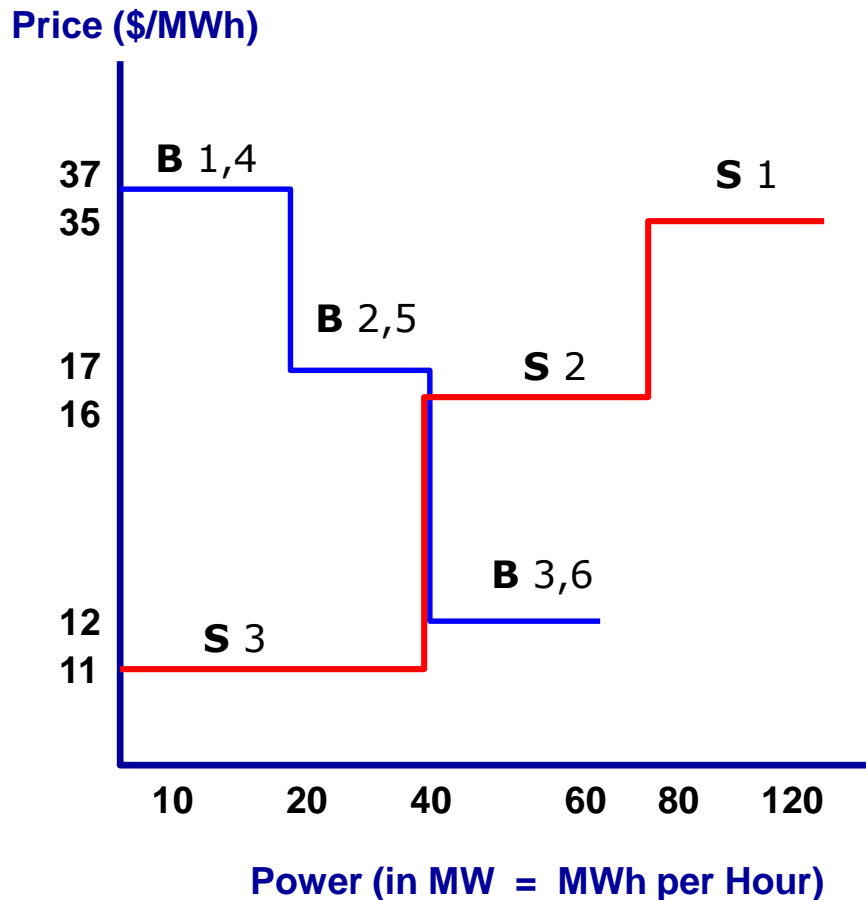
		RCAP		
		1/2	1	2
R C O N	2	Ns = 6 Nb = 3 Cs = 10 Cb = 10	Ns = 6 Nb = 3 Cs = 10 Cb = 20	Ns = 6 Nb = 3 Cs = 10 Cb = 40
	1	Ns = 3 Nb = 3 Cs = 20 Cb = 10	Ns = 3 Nb = 3 Cs = 10 Cb = 10	Ns = 3 Nb = 3 Cs = 10 Cb = 20
	1/2	Ns = 3 Nb = 6 Cs = 40 Cb = 10	Ns = 3 Nb = 6 Cs = 20 Cb = 10	Ns = 3 Nb = 6 Cs = 10 Cb = 10

Cell (3,1)

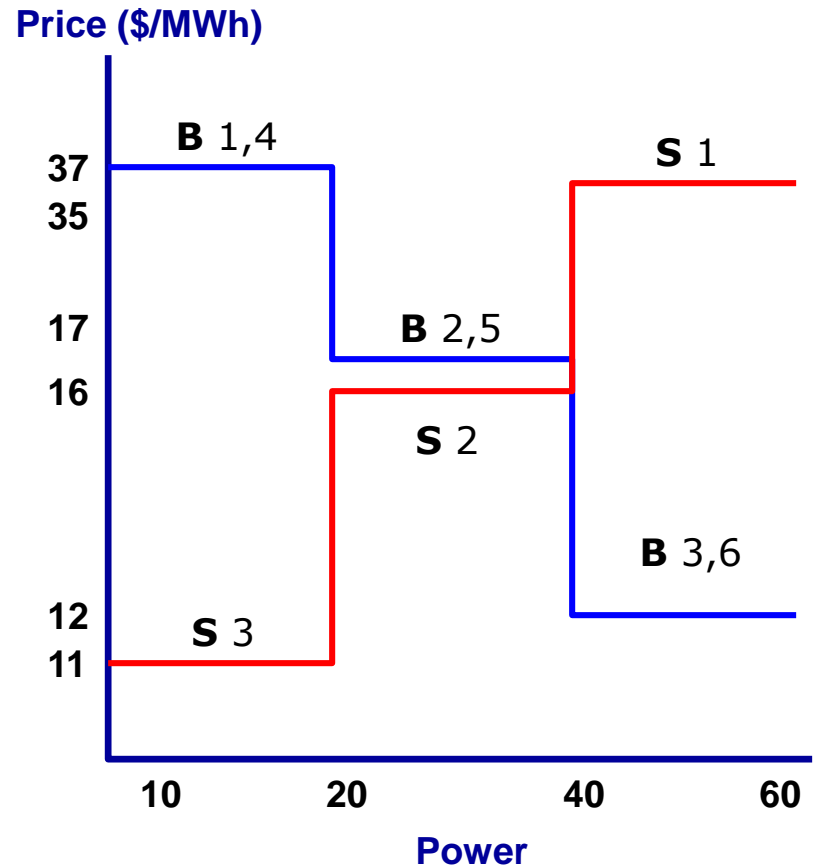
Cell (3,2)

# True Aggregate Demand and Supply Schedule Specifications are Illustrated below for Structural Treatments (3,1) and (3,2)

Cell (3,1)



Cell (3,2)





# The Double-Auction World Agent

## Public Access:

### // **Public Methods**

The *World Event Schedule*, i.e., a system clock that permits inhabitants to time and synchronize activities (e.g., submission of asks/bids into the DA market);  
Protocols governing trader collusion;  
Protocols governing trader insolvency;  
Methods for receiving data;  
Methods for retrieving World data.

## Private Access:

### // **Private Methods**

Methods for gathering, storing, and sending data;

### // **Private Data**

World attributes (e.g., spatial configuration);  
World inhabitants (DA market, buyers, sellers);  
World inhabitants' methods and data.

# The Double-Auction Market Agent

## Public Access:

### // **Public Methods**

getWorldEventSchedule(clock time);

Protocols governing the public posting of bids/offers;

Protocols governing matching, trades, and settlements;

Methods for receiving data;

Methods for retrieving Market data.

## Private Access:

### // **Private Methods**

Methods for gathering, storing, and sending data.

### // **Private Data**

Data recorded about sellers (e.g., seller offers);

Data recorded about buyers (e.g., buyer bids);

Address book (communication links).

# A Double-Auction Trader Agent

## Public Access:

### // **Public Methods**

getWorldEventSchedule(clock time);  
getWorldProtocols (collusion, insolvency);  
getMarketProtocols (posting, matching, trade, settlement);  
Methods for receiving data;  
Methods for retrieving Trader data.

## Private Access:

### // **Private Methods**

Methods for gathering, storing, and sending data;  
Methods for calculating expected & actual profit outcomes;  
Method for updating my bid/offer strategy (**LEARNING**).

### // **Private Data**

Data about me (history, profit function, current wealth,...);  
Data about external world (rivals' bids/offers, ...);  
Address book (communication links).

# What Do Seller & Buyer Traders Learn?

## **\*\*Strategically Reported\*\*** Supply Offers and Demand Bids

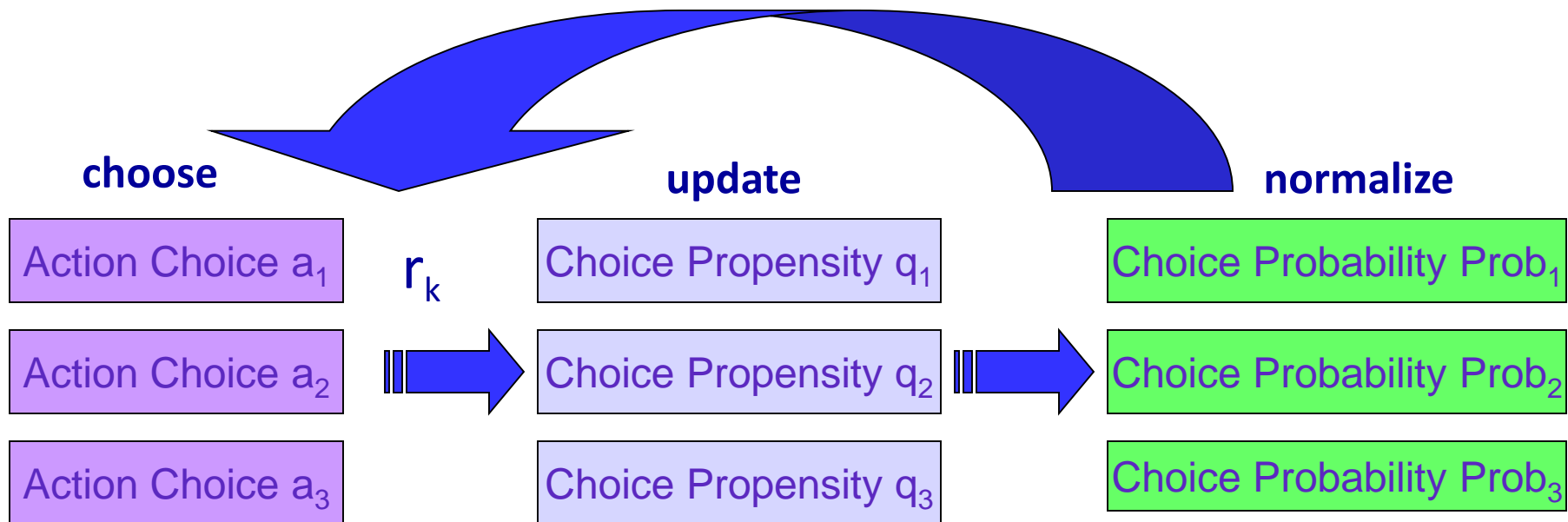
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- Offer for each Seller  $i$  = **reported** supply  $q_i^S$  of real power measured in units of Megawatts (MWs) together with a **reported** unit price  $p_i$  for real power measured in U.S. dollars \$ per MW.
- Bid for each Buyer  $j$  = **reported** demand  $q_j^D$  (MWs) for real power together with a **reported** unit price  $p_j$  (\$/MW).
- **Action choice set for sellers** = Their *possible reported offers*
- **Action choice set for buyers** = Their *possible reported bids*

# Reactive Reinforcement Method Used for Learning Treatment 1:

## MRE Reactive Reinforcement Learning

(MRE = Modified Roth-Erev, see Nicolaisen et al., 2001)



- Each trader maintains action choice propensities  $q$ , normalized to action choice probabilities Prob, to choose actions. A good (bad) profit  $r_k$  for action  $a_k$  results in a strengthening (weakening) of the propensity  $q_k$  for  $a_k$  and hence in the probability of choosing  $a_k$ .

# Modified Roth-Erev Reactive Reinforcement Learning (MRE RRL)

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1. **Initialize** action propensities to an initial propensity value.
2. **Generate** choice probabilities for all actions using current propensities.
3. **Choose** an action according to the current choice probability distribution.
4. **Update** propensities for all actions using the reward for the last chosen action.
5. **Repeat** from Step 2.

# MRE RRL: Updating of Action Propensities

## Parameters:

- $q_j(1)$  Initial propensity
- $\epsilon$  Experimentation
- $\phi$  Recency (forgetting)

## Variables:

- $a_j$  Current action choice
- $q_j$  Propensity for action  $a_j$
- $a_k$  Last action chosen
- $r_k$  Reward for action  $a_k$
- $t$  Current time step
- $N$  Number of actions

$$q_j(t + 1) = [1 - \phi]q_j(t) + E_j(\epsilon, N, k, t)$$

$$E_j(\epsilon, N, k, t) = \begin{cases} r_k(t)[1 - \epsilon] & \text{if } j = k \\ q_j(t) \frac{\epsilon}{N-1} & \text{if } j \neq k \end{cases}$$

# MRE RRL: From Propensities to Probabilities

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$$p_j(t) = \frac{q_j(t)}{\sum_{j=0}^{N-1} q_j(t)}$$

$p_j(t)$  = Probability of choosing action  $j$  at time  $t$

$N$  = Number of available actions at each time  $t$



# Table of Experimental Results for Learning Treatment 1: MRE Reactive Reinforcement Learning

TABLE VI  
EXPERIMENTAL MARKET POWER AND EFFICIENCY OUTCOMES FOR THE BEST FIT MRE ALGORITHM WITH 1000 AUCTION ROUNDS AND PARAMETER VALUES  
 $s(1) = 9.00$ ,  $r = 0.10$ , AND  $c = 0.20$

	1/2		Relative Capacity 1		2		
	MP	StdDev	MP	StdDev	MP	StdDev	
2	All Buyers:	-0.13* (0.09)	All Buyers:	-0.15* (0.09)	All Buyers:	0.10 (0.30)	
	All Sellers:	0.55* (0.38)	All Sellers:	0.38* (0.33)	All Sellers:	-0.10 (0.25)	
	Buyer[1]:	-0.12* (0.08)	Buyer[1]:	-0.13* (0.10)	Buyer[1]:	0.10 (0.30)	
	Buyer[2]:	-0.20 (0.40)	Buyer[2]:	-0.75* (0.33)	Buyer[2]:	ZP (0.00)	
	Buyer[3]:	ZP (0.00)	Buyer[3]:	ZP (0.00)	Buyer[3]:	ZP (0.00)	
	Seller[1]:	ZP (0.00)	Seller[1]:	ZP (0.00)	Seller[1]:	ZP (0.00)	
	Seller[2]:	ZP (0.00)	Seller[2]:	-0.50 (1.34)	Seller[2]:	-0.12 (0.34)	
	Seller[3]:	0.54 (0.63)	Seller[3]:	0.45* (0.40)	Seller[3]:	-0.10 (0.22)	
	Seller[4]:	ZP (0.00)	Seller[4]:	ZP (0.00)	Seller[4]:	ZP (0.00)	
	Seller[5]:	ZP (0.00)	Seller[5]:	-0.42 (1.67)	Seller[5]:	-0.08 (0.36)	
	Seller[6]:	0.55 (0.60)	Seller[6]:	0.46* (0.41)	Seller[6]:	-0.09 (0.24)	
	Efficiency:	99.81 (0.02)	Efficiency:	96.30 (0.05)	Efficiency:	99.88 (0.06)	
Relative Concentration 1	MP	StdDev	MP	StdDev	MP	StdDev	
	All Buyers:	-0.22* (0.12)	All Buyers:	-0.13* (0.10)	All Buyers:	0.13 (0.33)	
	All Sellers:	0.80* (0.53)	All Sellers:	0.28 (0.35)	All Sellers:	-0.10 (0.26)	
	Buyer[1]:	-0.21* (0.11)	Buyer[1]:	-0.11* (0.10)	Buyer[1]:	0.13 (0.33)	
	Buyer[2]:	-0.31 (0.44)	Buyer[2]:	-0.80* (0.40)	Buyer[2]:	ZP (0.00)	
	Buyer[3]:	ZP (0.00)	Buyer[3]:	ZP (0.00)	Buyer[3]:	ZP (0.00)	
	Seller[1]:	ZP (0.00)	Seller[1]:	ZP (0.00)	Seller[1]:	ZP (0.00)	
	Seller[2]:	ZP (0.00)	Seller[2]:	-0.37 (1.89)	Seller[2]:	-0.10 (0.34)	
	Seller[3]:	0.76* (0.63)	Seller[3]:	0.34 (0.45)	Seller[3]:	-0.11 (0.24)	
	Efficiency:	92.13 (0.09)	Efficiency:	94.59 (0.07)	Efficiency:	100.00 (0.00)	
	1/2	MP	StdDev	MP	StdDev	MP	StdDev
		All Buyers:	-0.21* (0.12)	All Buyers:	-0.14* (0.08)	All Buyers:	0.09 (0.24)
All Sellers:		0.67* (0.46)	All Sellers:	0.30 (0.31)	All Sellers:	-0.07 (0.19)	
Buyer[1]:		-0.18* (0.12)	Buyer[1]:	-0.14* (0.10)	Buyer[1]:	0.09 (0.27)	
Buyer[2]:		-0.37 (0.47)	Buyer[2]:	-0.77* (0.44)	Buyer[2]:	ZP (0.00)	
Buyer[3]:		ZP (0.00)	Buyer[3]:	ZP (0.00)	Buyer[3]:	ZP (0.00)	
Buyer[4]:		-0.20* (0.11)	Buyer[4]:	-0.11 (0.11)	Buyer[4]:	0.10 (0.25)	
Buyer[5]:		-0.38 (0.47)	Buyer[5]:	-0.73* (0.46)	Buyer[5]:	ZP (0.00)	
Buyer[6]:		ZP (0.00)	Buyer[6]:	ZP (0.00)	Buyer[6]:	ZP (0.00)	
Seller[1]:		ZP (0.00)	Seller[1]:	ZP (0.00)	Seller[1]:	ZP (0.00)	
Seller[2]:		ZP (0.00)	Seller[2]:	0.14 (2.69)	Seller[2]:	-0.08 (0.27)	
Seller[3]:		0.63* (0.55)	Seller[3]:	0.32 (0.48)	Seller[3]:	-0.07 (0.17)	
Efficiency:	91.84 (0.09)	Efficiency:	94.24 (0.07)	Efficiency:	100.00 (0.00)		

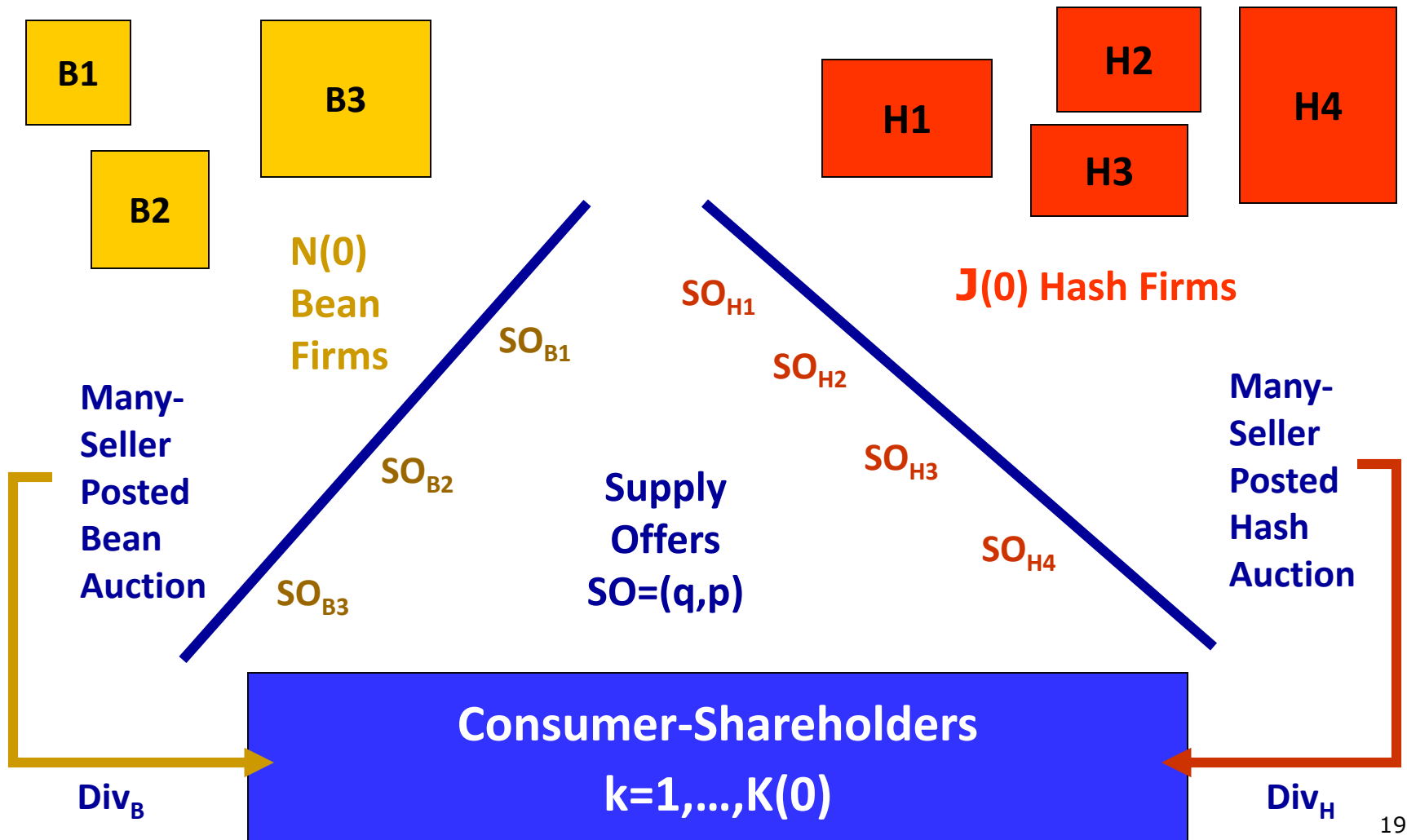
ZP indicates that zero profits were earned both in the auction and in competitive equilibrium.

# Summary of Policy-Relevant Findings for Example 1: A Double-Auction Market Economy

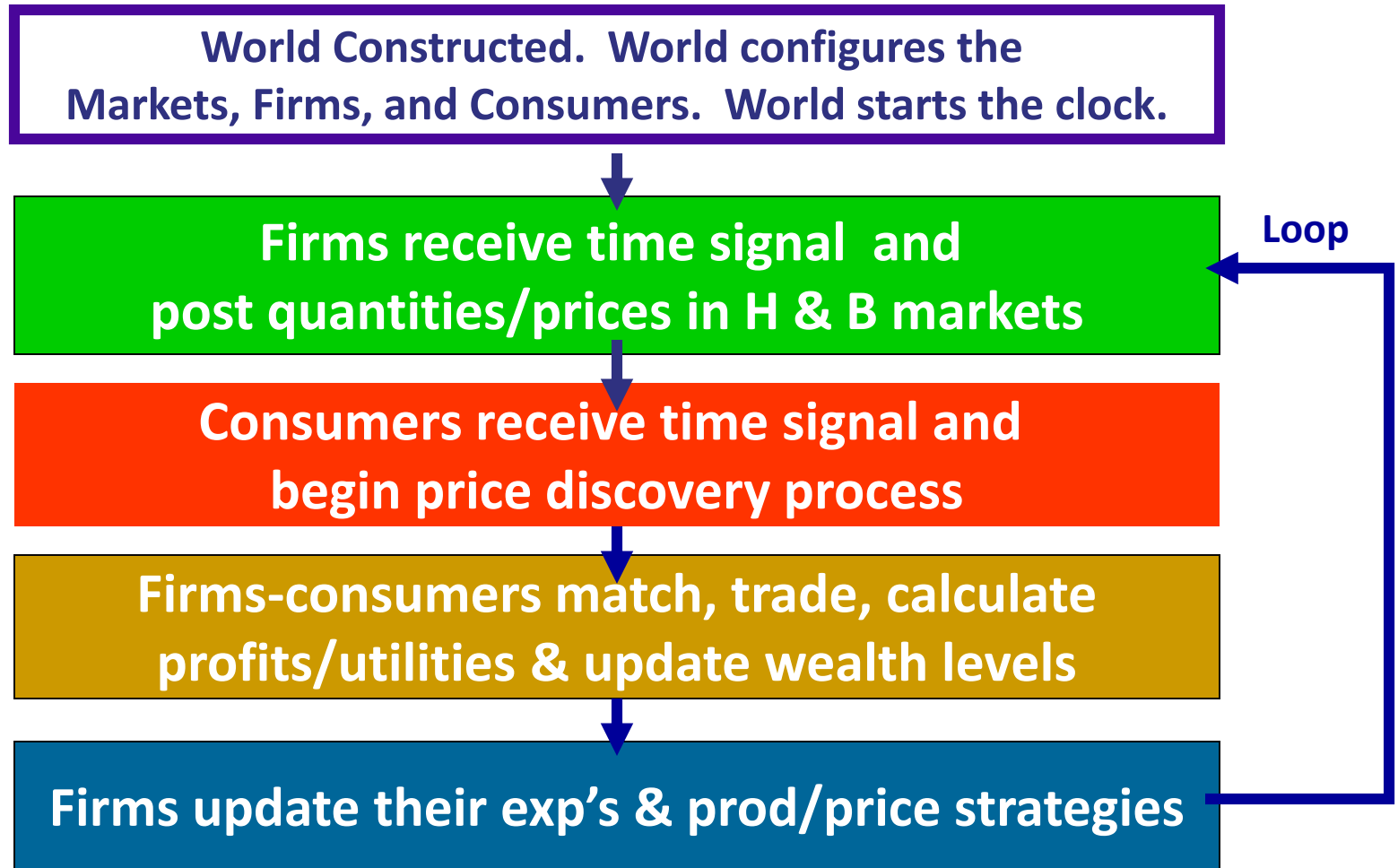
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- **Market Efficiency:** Generally high when traders use MRE-RRL (Modified Roth-Erev Reactive Reinforcement Learning) **but not** when traders use GA (Genetic Algorithm) social mimicry (*type of learning matters*).
- **Structural Market Advantage:** Market microstructure is strongly predictive for the relative market advantage of the seller and buyer traders (*structural aspects matter*).
- **Strategic Market Advantage:** Traders are **not** able to change their relative market advantage through learning alone (*the importance of built-in structural market advantage*).

# Example 2: An ACE Posted-Auction Hash-and-Beans Economy



# Dynamic Flow of ACE H&B Economy



# Dynamic Flow of Activity for H & B Firms

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- ◆ Each firm  $f$  starts out ( $T=0$ ) with *money*  $M_f(0)$  and a *production capacity*  $Cap_f(0)$
- ◆ Firm  $f$ 's *fixed cost*  $FC_f(T)$  in each  $T \geq 0$  is proportional to its *current capacity*  $Cap_f(T)$
- ◆ At beginning of each  $T \geq 0$ , firm  $f$  selects a *supply offer* =: *(production level, unit price)*
- ◆ At end of  $T \geq 0$ , firm  $f$  is **solvent** if it has a *NetWorth*( $T$ ) =:  $[Profit(T)+M_f(T)+ValCap_f(T)] \geq 0$
- ◆ If solvent, firm  $f$  allocates its *profits (+ or -)* between  $M_f$ ,  $CAP_f$ , and dividend payments.

# Dynamic Flow of Activity for H&B Consumers

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- ◆ Each consumer  $k$  starts out ( $T=0$ ) with a *lifetime money endowment profile*

$$(Mk_{youth}, Mk_{middle}, Mk_{old})$$

- ◆ In each  $T \geq 0$ , consumer  $k$ 's **utility** is measured by

$$U_k(T) = (\text{hash}(T) - h_k^*)^{\alpha_k} \cdot (\text{beans}(T) - b_k^*)^{[1-\alpha_k]}$$

- ◆ In each  $T \geq 0$ , consumer  $k$  seeks to secure maximum utility by **searching** for hash and beans to buy at **lowest possible prices**.
- ◆ At end of each  $T \geq 0$ , consumer  $k$  **dies** unless consumption meets *subsistence needs for hash and beans*:

$$(h_k^*, b_k^*).$$

# Experimental Design Treatment Factors

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- ◆ **Initial size of consumer sector** [  $K(0)$  ]
- ◆ **Initial concentration** [  $N(0), J(0), \text{Cap}(0)$  values ]
- ◆ **Firm learning** (supply offers & profit allocations)
- ◆ **Firm cost functions**
- ◆ **Firm initial money holdings** [  $M_f(0)$  ]
- ◆ **Firm rationing protocols** (for excess demand)
- ◆ **Consumer price discovery processes**
- ◆ **Consumer money endowment profiles**  
(rich, poor,  $\nearrow$ ,  $\searrow$ , life cycle u-shape)
- ◆ **Consumer preferences** ( $\theta$  values)
- ◆ **Consumer subsistence needs** ( $b^*, h^*$ )

# The ACE H&B World Agent

## Public Access:

### // **Public Methods**

The *World Event Schedule*, i.e., a system clock that permits inhabitants to time and synchronize activities (e.g., opening/closing of H & B markets);

Protocols governing firm collusion;

Protocols governing firm insolvency;

Methods for receiving data;

Methods for retrieving World data.

## Private Access:

### // **Private Methods**

Methods for gathering, storing, and sending data;

### // **Private Data**

World attributes (e.g., spatial configuration);

World inhabitants (H & B markets, firms, consumers);

World inhabitants' methods and data.



# An ACE H&B Market Agent

## Public Access:

### // **Public Methods**

getWorldEventSchedule(clock time);

Protocols governing the public posting of supply offers;

Protocols governing matching, trades, and settlements;

Methods for receiving data;

Methods for retrieving Market data.

## Private Access:

### // **Private Methods**

Methods for gathering, storing, and sending data.

### // **Private Data**

Data recorded about firms (e.g., sales);

Data recorded about consumers (e.g., purchases);

Address book (communication links).

# An ACE H&B Consumer Agent

## Public Access:

### // **Public Methods**

getWorldEventSchedule(clock time);  
getWorldProtocols (stock share ownership);  
getMarketProtocols (price discovery process, trade process);  
Methods for receiving data;  
Methods for retrieving stored Consumer data.

## Private Access:

### // **Private Methods**

Methods for gathering, storing, and sending data;  
Method for determining my budget constraint;  
Method for searching for lowest prices.

### // **Private Data**

Data about me (history, utility function, current wealth,...);  
Data about external world (posted supply offers, ...);  
Address book (communication links).

# An ACE H&B Firm Agent

## Public Access:

### // **Public Methods**

getWorldEventSchedule(clock time);  
getWorldProtocols (collusion, insolvency);  
getMarketProtocols (posting, matching, trade, settlement);  
Methods for receiving data;  
Methods for retrieving Firm data.

## Private Access:

### // **Private Methods**

Methods for gathering, storing, and sending data;  
Methods for calculating expected & actual profit outcomes;  
Method for allocating my profits to my shareholders;  
Method for updating my supply offers (**LEARNING**).

### // **Private Data**

Data about me (history, profit function, current wealth,...);  
Data about external world (rivals' supply offers, ...);  
Address book (communication links).

# Interesting Issues for Exploration

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- ◆ Initial conditions → **carrying capacity ?**  
(Survival of firms/consumers in long run)
- ◆ Initial conditions → **market clearing ?**  
(Walrasian equilibrium benchmark)
- ◆ Initial conditions → **market efficiency ?**  
(Walrasian equilibrium benchmark)
- ◆ Standard concentration measures at  $T=0$  →  
**good predictors of long-run market advantage ?**
- ◆ Importance for market performance of **trader learning abilities vs. market structure ?** (*Gode/Sunder, JPE, 1993*)

# ACE Hash-and-Beans Economy: Computational Laboratory Implementation

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Christopher Cook and Leigh Tesfatsion, **“An Agent-Based Computational Laboratory for the Experimental Study of Complex Economic Systems”**

- Computational laboratory for the ACE Hash-and-Beans Economy
- Programming language C#/.Net (all WinDesktops)
- Development initiated for Econ 308 (ACE course)  
<https://www2.econ.iastate.edu/classes/econ308/tesfatsion/>
- Superseded by later ACE macroeconomic model developments  
<https://www2.econ.iastate.edu/tesfatsi/amulmark.htm>

# ACE Hash & Beans Economy: Computational Laboratory Main Screen

Form1  
File Tools Window Help

Untitled 1 (Empty Lab)

## Hash & Bean Multi-Market Economy Model

**CONSUMERS**

Group	Count
Cons Type 1	100
Cons Type 2	100
<b>Total:</b>	<b>200</b>

**Consumer Details**

Group Name:  Consumption Needs: Hash:  Beans:  Endowment Schedule: Lifecycle  [\[edit\]](#)

Count:  Initial:

Preference: [\[edit\]](#)  
 $\alpha = 0.505$  Slightly Prefers Hash

**FIRMS**

Group	Count	Count
Large	1	1
Small	20	20
<b>Total:</b>	<b>21</b>	<b>21</b>

**Firm Details**

Group Name:  Initial Assets: Money:  Capacity:  Cost Function: Default  [\[edit\]](#)

Hash Firms:  Bean Firms:

Profit Distribution: Money:  Dividends:  Learning Strategy: Random P & Q (Det)  [\[edit\]](#)

Experiment Number:

Trial Count:

Trial Length ( TMax ):