DC-Optimal Power Flow and LMP Determination in the AMES Test Bed

Leigh Tesfatsion

Professor of Economics

Courtesy Professor of Mathematics and

Electrical & Computer Engineering

Iowa State University

Ames, IA 50011-1070

https://www2.econ.iastate.edu/tesfatsi/

tesfatsi@iastate.edu

Latest Revision: 13 June 2010

Presentation Outline

- Introduction
- Double auction basics for energy markets
  - Supply, demand, & market equilibrium
  - Net surplus extraction
- Market efficiency vs. social welfare: Implications for independent system operators in energy markets
- Illustrative AMES Test Bed experiments for a 5-bus test case with learning generators

Introduction

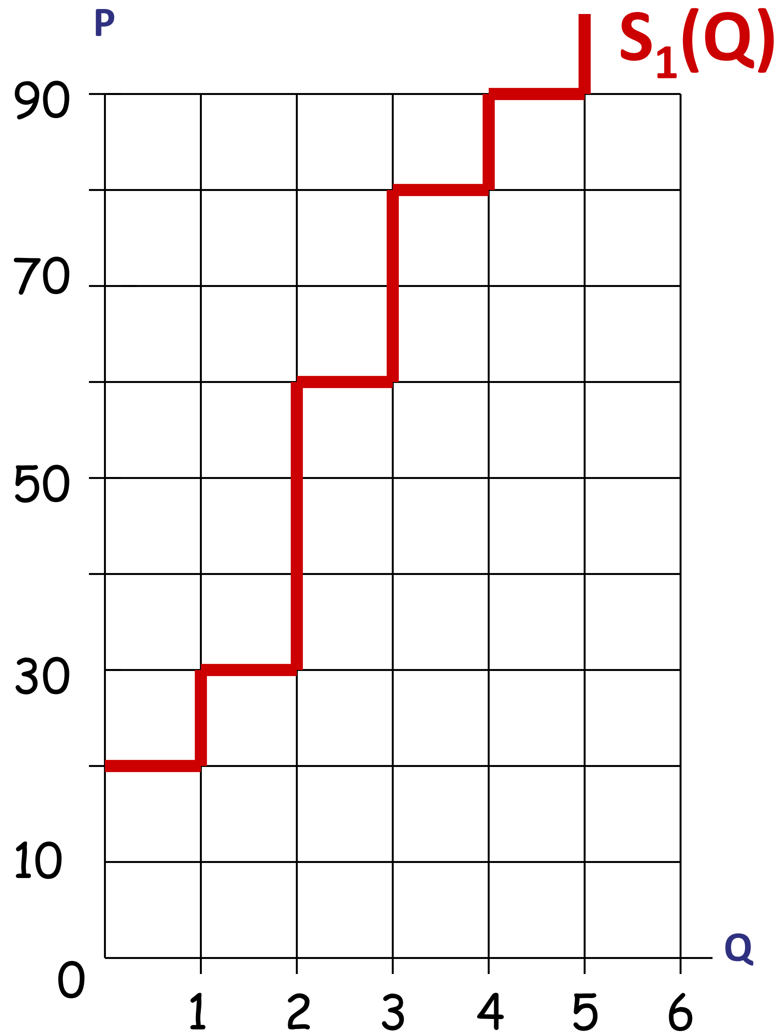
- In many regions of U.S., wholesale electric energy -- measured in megawatt-hours (MWh) -- is transacted in "day-ahead" markets designed as double auctions.

- ***Double Auction*** = A centrally-cleared market in which sellers make supply offers & buyers make demand bids.

- After review of basic double auction concepts, efficiency & welfare issues arising from use of double auctions for centrally-managed day-ahead markets for energy will be discussed.

DOUBLE-AUCTION BASICS: EXAMPLE

Seller 1's Supply Offer: $P = S_1(Q)$, where P = Price and Q = Quantity



Q = Quantity of specialty apples (in bushels)
P = Price of specialty apples ($ per bushel)

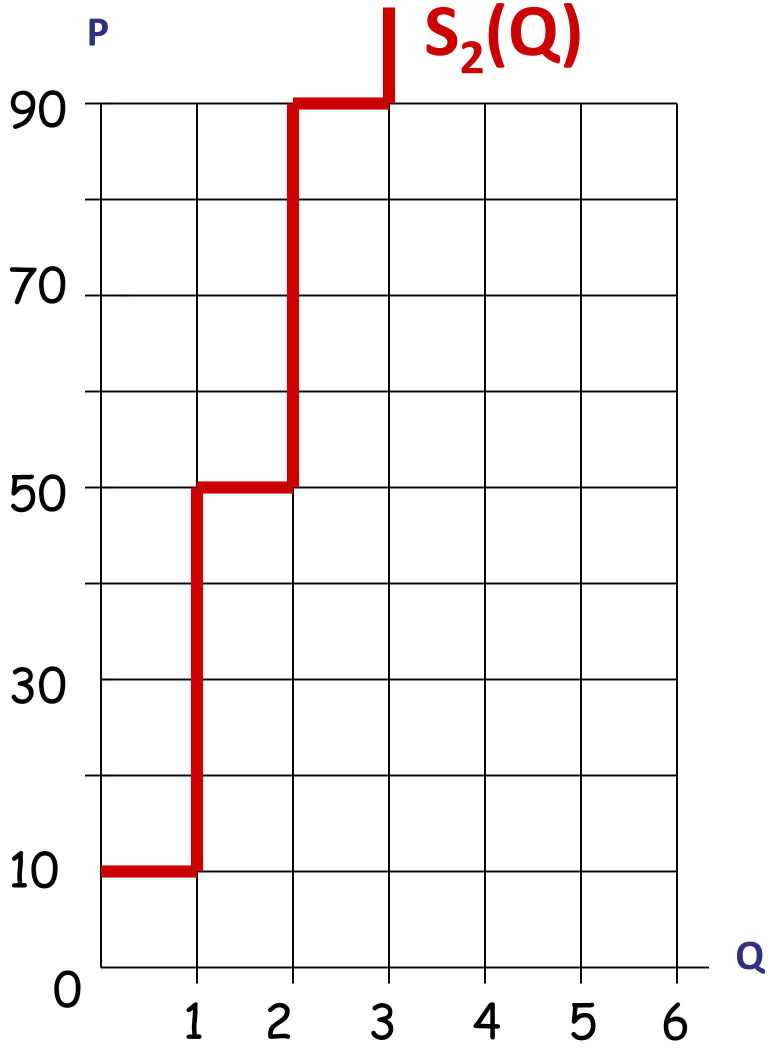
For each Q: $P=S_1(Q)$ is Seller 1's ***minimum acceptable sale price*** for the "last" bushel it supplies at Q.

Bushels Q	Price $P = S_1(Q)$
1	$20
2	$30
3	$60
4	$80
5	$90
6	∞

5 bushels = Seller S_1's max possible supply.

Note: ***"Minimum acceptable sale price"*** is also called a ***"(sale) reservation value"***

Seller 2's Supply Offer: $P = S_2(Q)$, where P = Price and Q = Quantity

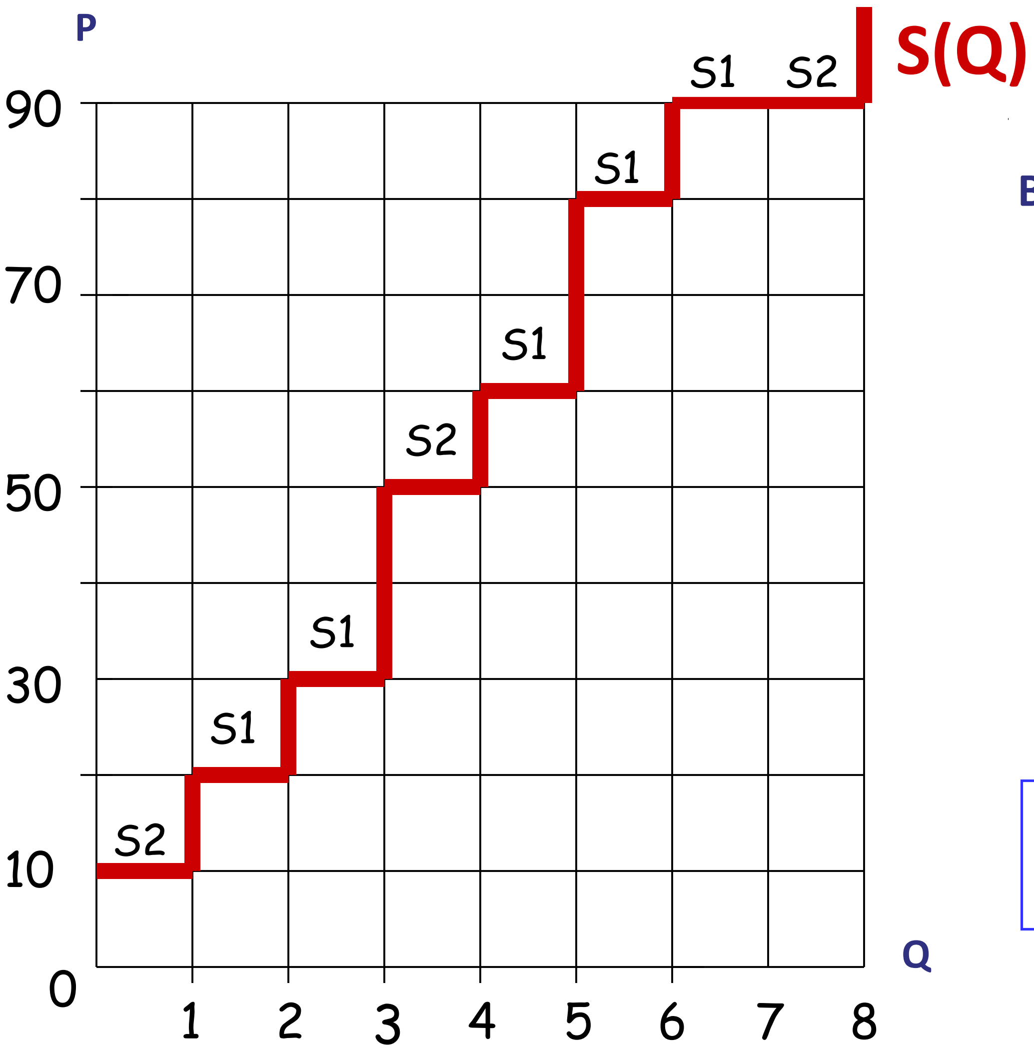


For each Q: $P = S_2(Q)$ is Seller 2's ***minimum acceptable sale price*** for the last bushel it supplies at Q.

Bushels Q	Price $P = S_2(Q)$
1	\$10
2	\$50
3	\$90
4	∞

3 bushels = Seller S_2's max possible supply.

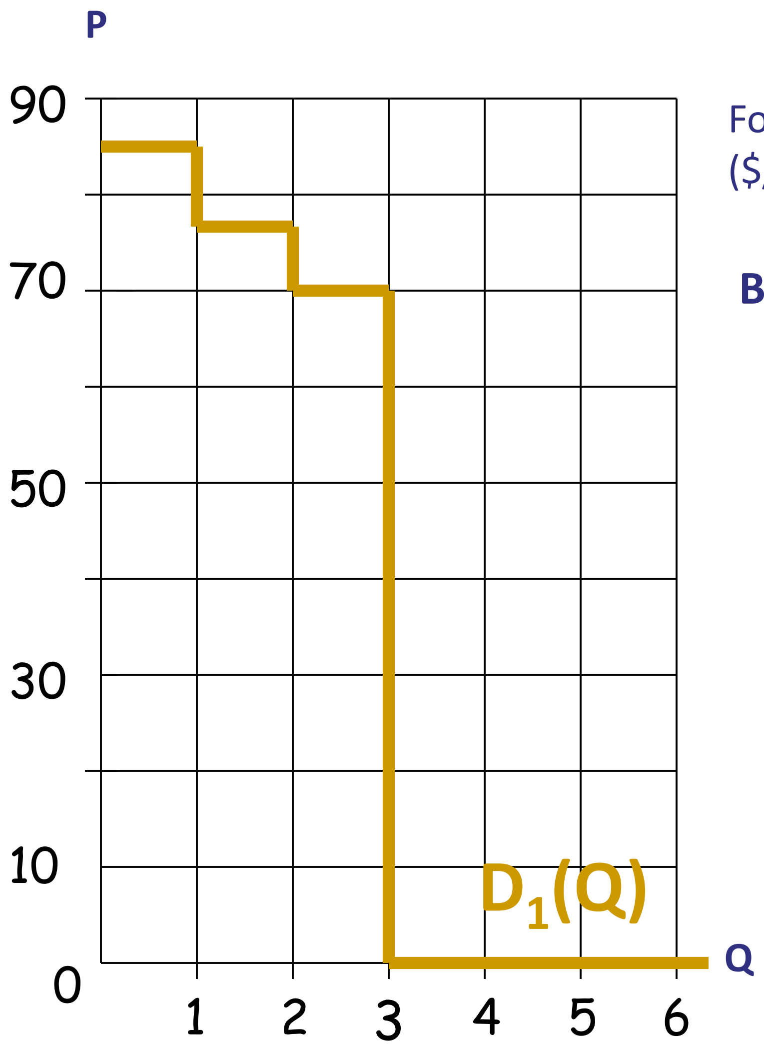
Total System (Inverse) Supply Function: P = S(Q)



Bushels Q	Price P = S(Q)
1	\$10 (S2)
2	\$20 (S1)
3	\$30 (S1)
4	\$50 (S2)
5	\$60 (S1)
6	\$80 (S1)
7	\$90 (S1/S2)
8	\$90 (S2/S1)
9	∞

Max possible total market supply = 8 bushels of apples.

Buyer 1's Demand Bid: $P = D_1(Q)$, where P = Price and Q = Quantity



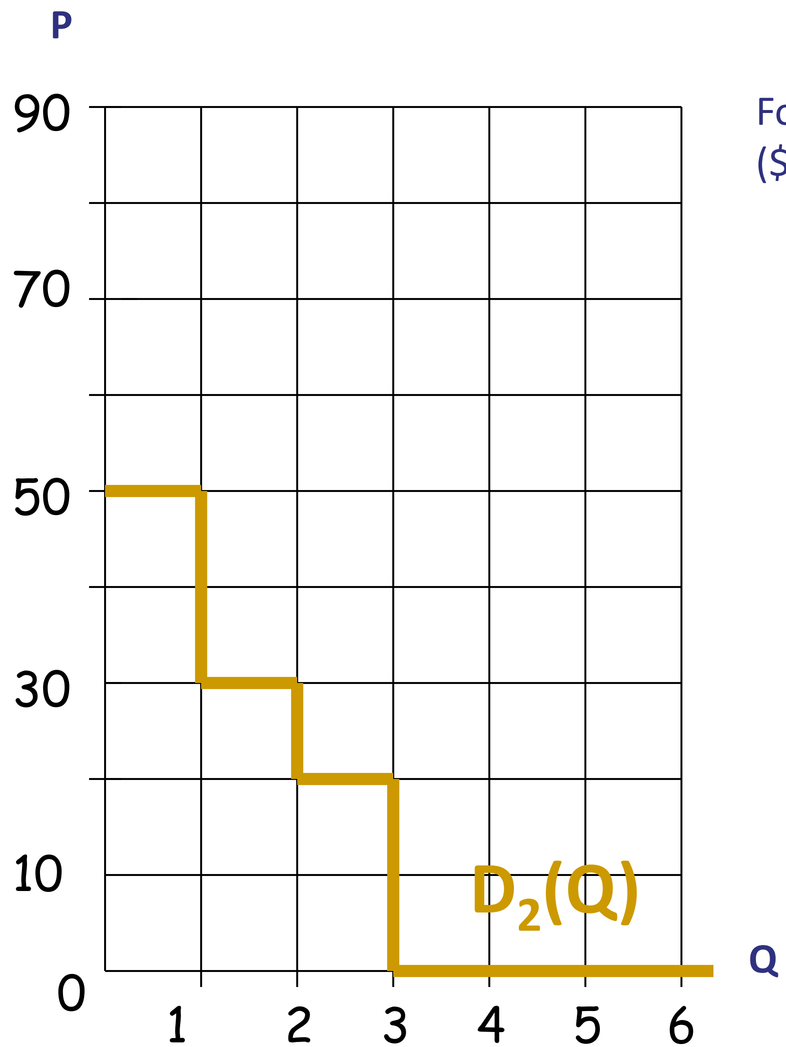
For each Q: $P = D_1(Q)$ is Buyer 1's ***max purchase price*** ($/bushel) for the last bushel it purchases at Q.

Bushels Q	Price $P = D_1(Q)$
1	$84
2	$76
3	$70
4	$ 0

Buyer 1's demand for apples is "satiated" at 3 bushels.

Note: ***"Maximum purchase price"*** ≡ ***"maximum willingness to pay"*** is also called a ***"(purchase) reservation value."***

Buyer 2's Demand Bid: $P = D_2(Q)$, where P = Price and Q = Quantity

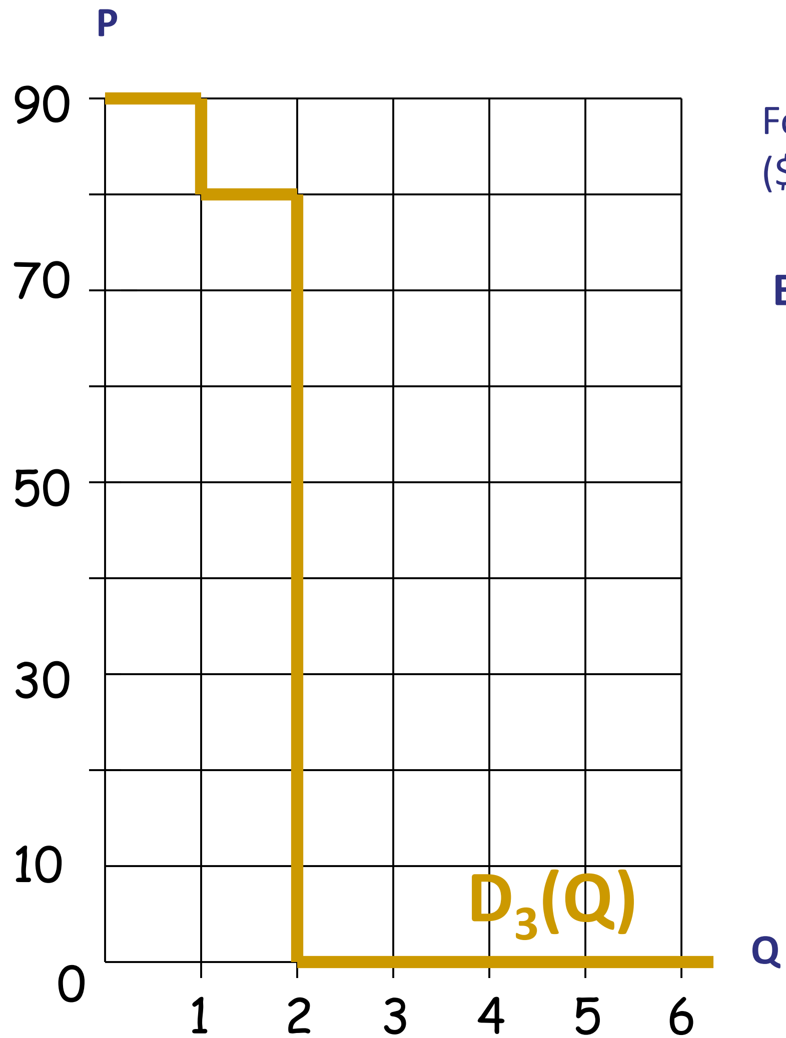


For each Q: $P = D_2(Q)$ is Buyer 2's ***max purchase price*** (\$/bushel) for the last bushel it purchases at Q.

Bushels Q	Price $P = D_2(Q)$
1	\$50
2	\$30
3	\$20
4	\$ 0

Buyer 2's demand for apples is "satiated" at 3 bushels.

Buyer 3's Demand Bid: $P = D_3(Q)$, where P=Price and Q = Quantity

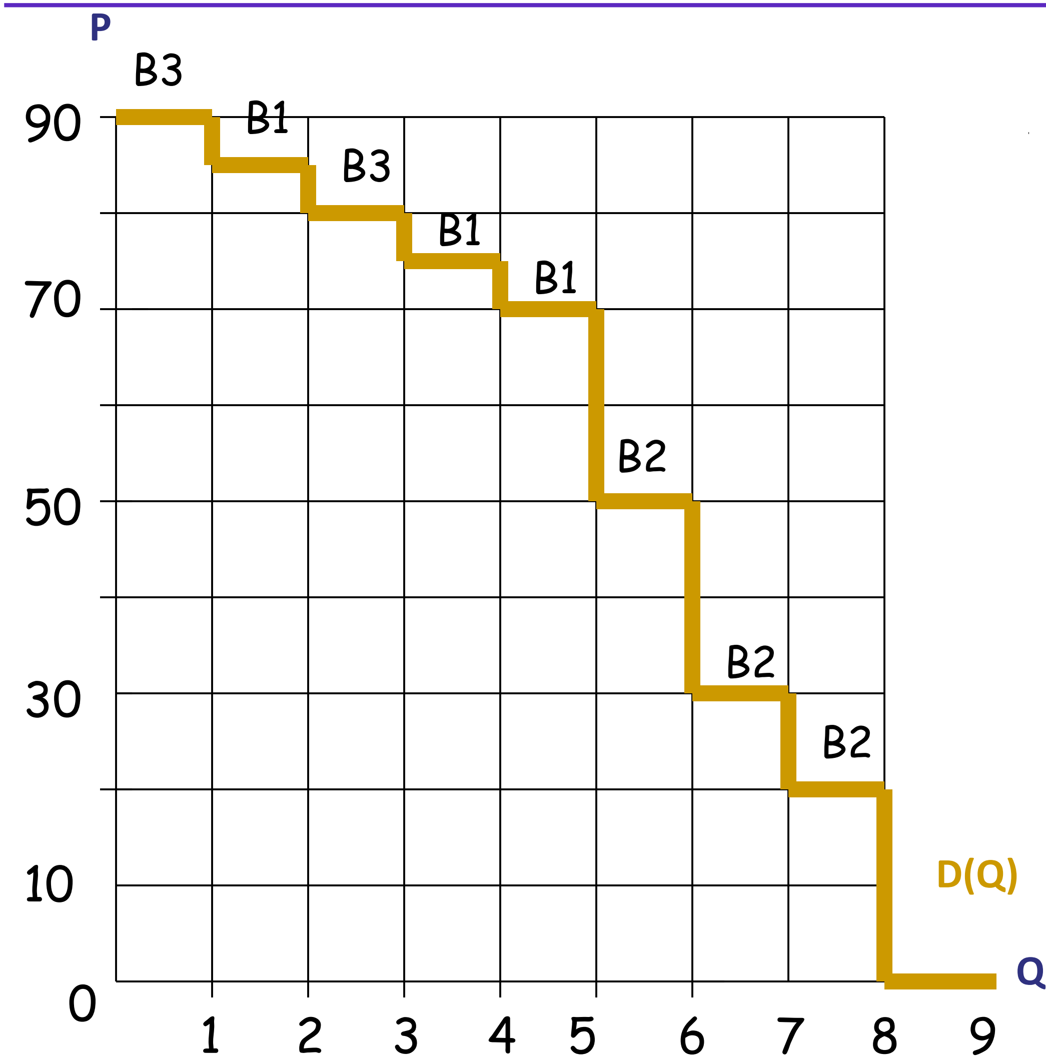


For each Q: $P = D_3(Q)$ is Buyer 3's ***max purchase price*** ($/bushel) for the last bushel it purchases at Q

Bushels Q	Price $P = D_3(Q)$
1	$90
2	$80
3	$ 0

Buyer 3's demand for apples is "satiated" at 2 bushels.

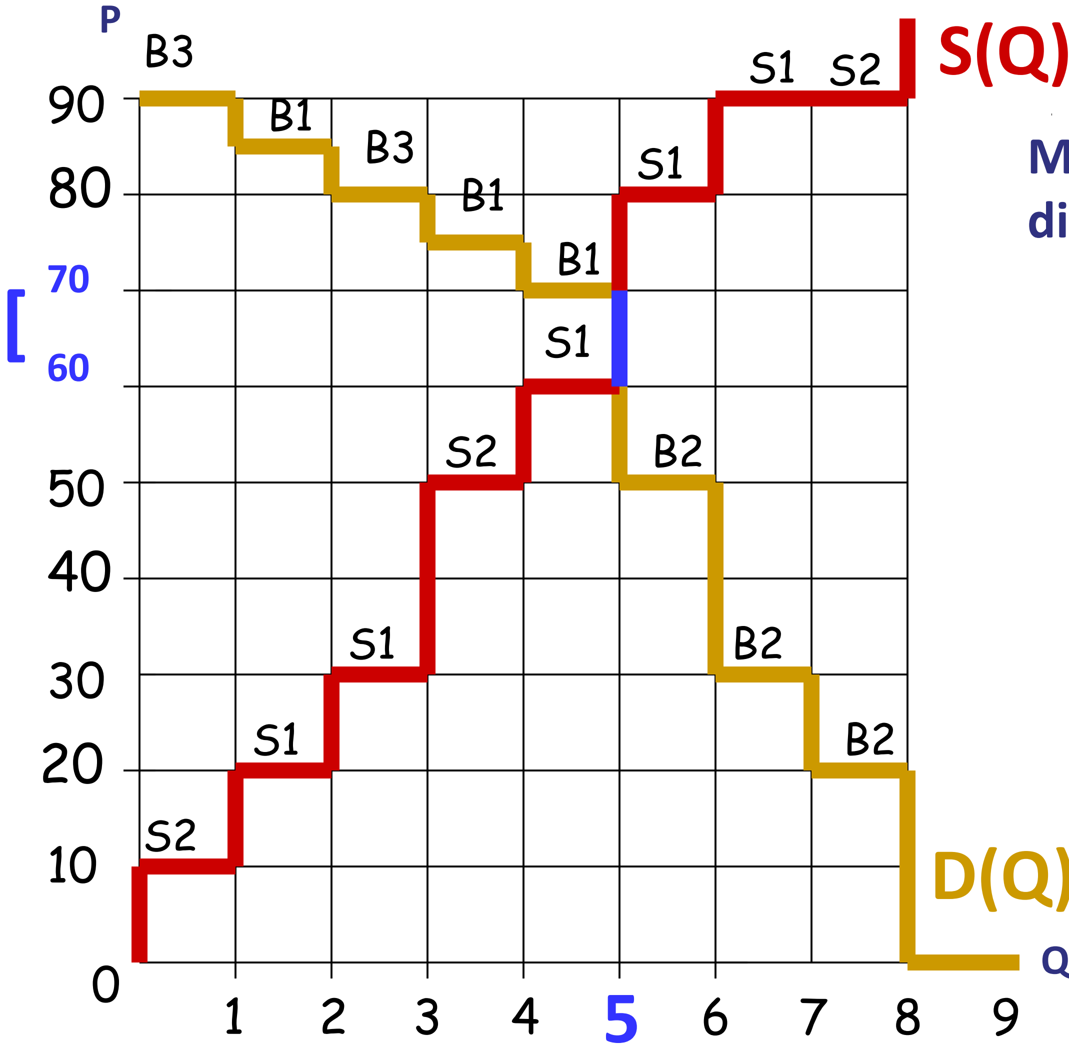
Total System (Inverse) Demand Function: P = D(Q)



Bushels Q	Price P = D(Q)
1	\$90 (B3)
2	\$84 (B1)
3	\$80 (B3)
4	\$76 (B1)
5	\$70 (B1)
6	\$50 (B2)
7	\$30 (B2)
8	\$20 (B2)
9	\$ 0

Competitive Market Clearing (CMC) Points

Points (Q,P) where the aggregate supply curve P = S(Q) intersects the aggregate demand curve P = D(Q): P = S(Q) = D(Q)



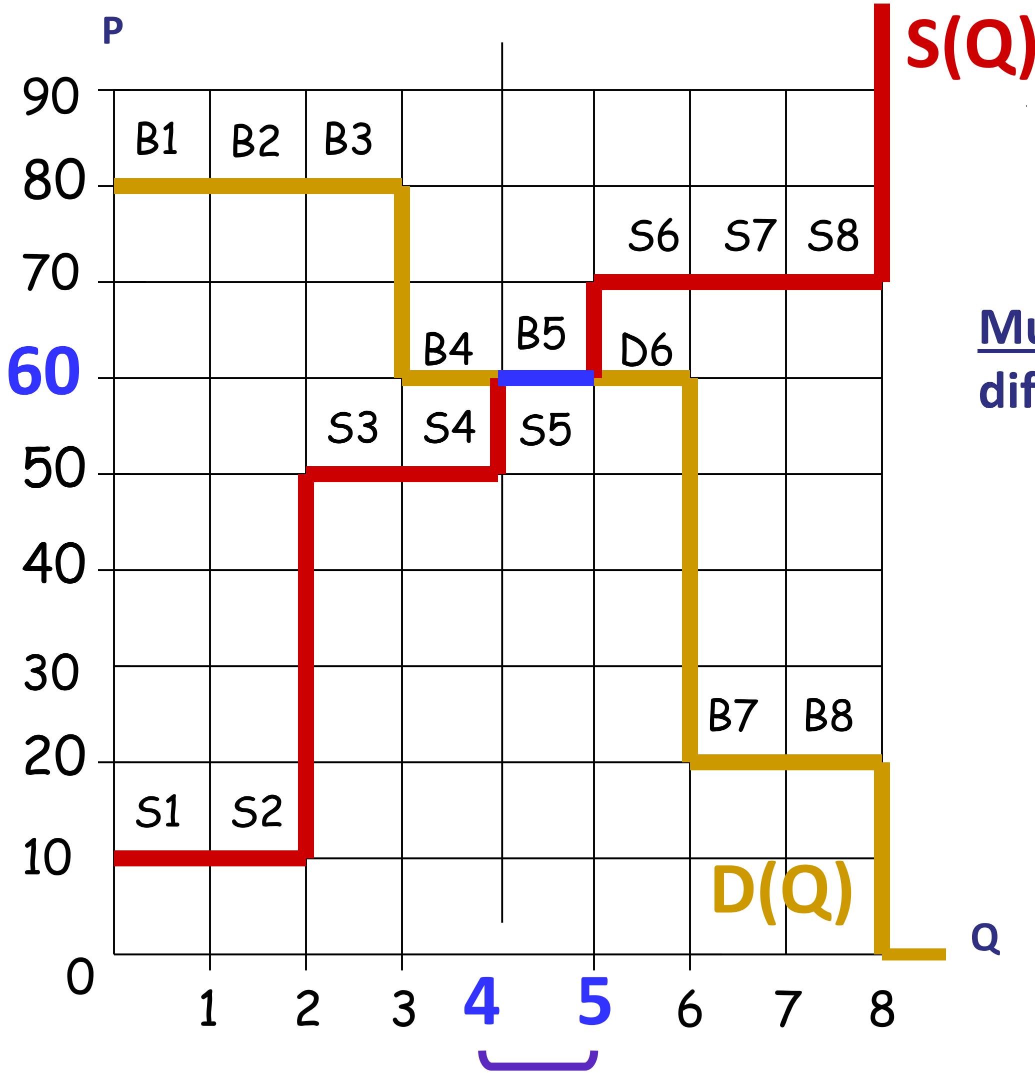
Multiple CMC points (Q*,P*) with different CMC prices P*:

Q*=5, \$60 ≤ P*≤ \$70

Bushels Q	Max Buy P	Min Sell P
1	\$90	\$10
2	\$84	\$20
3	\$80	\$30
4	\$76	\$50
5	\$70	\$60
6	\$50	\$80
7	\$30	\$90
8	\$20	\$90
9	0	∞

No bushel sales are possible beyond five bushels !

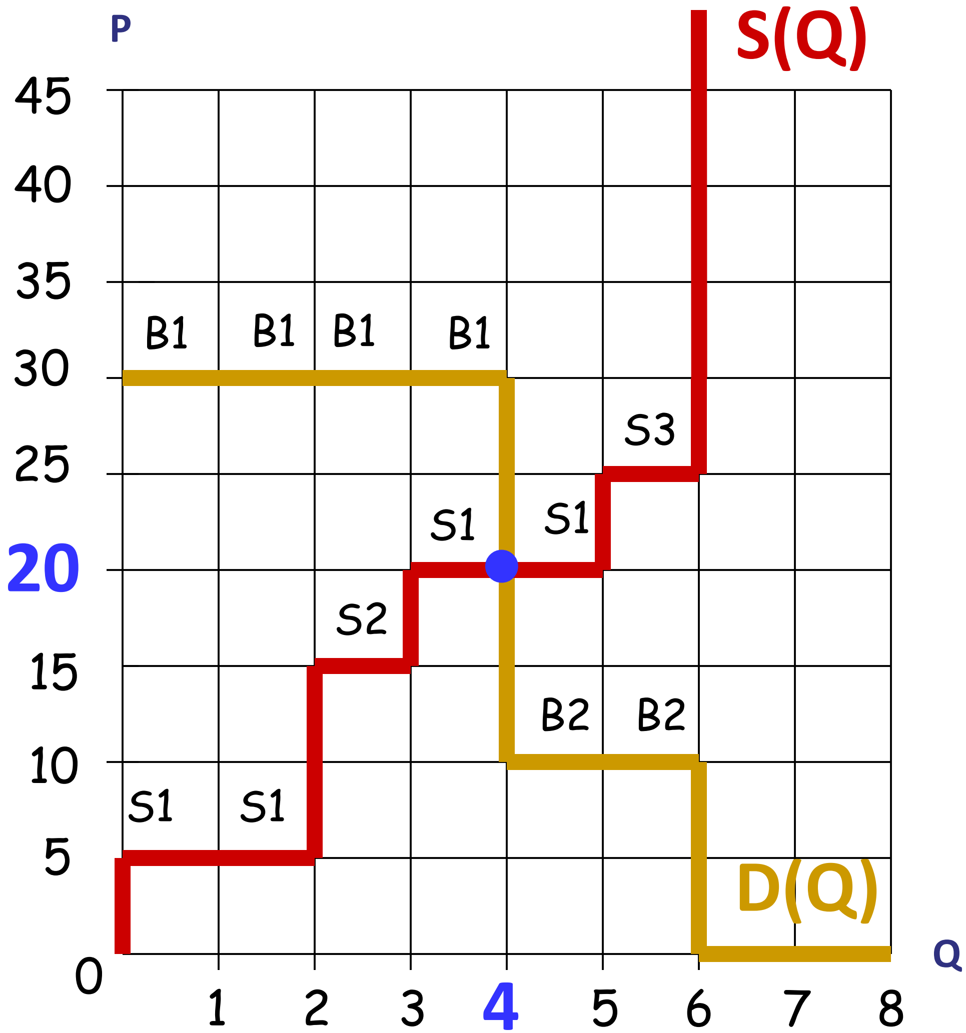
Can also possibly have <u>multiple</u> CMC points with a <u>range</u> of CMC quantities



<u>Multiple</u> CMC points (Q*,P*) with different CMC <u>quantities</u> Q*:

$4 \le Q^* \le 5$, P*=$60

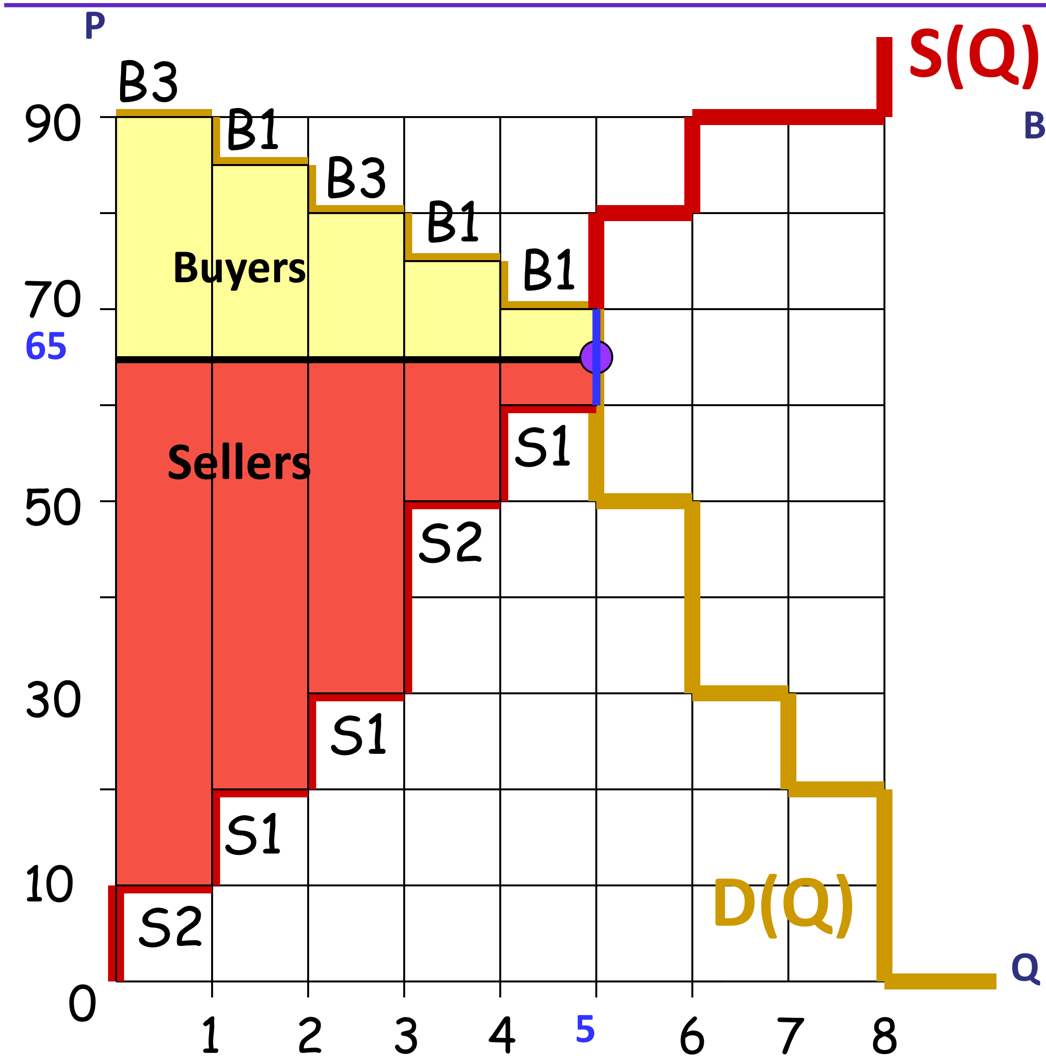
Can also possibly have a <u>unique</u> CMC point



<u>Unique</u> CMC Point:

Q*=4, P*= $20

Seller & Buyer Net Surplus Amounts at CMC Points



Ex 1: CMC Point Q*=5, P*=$65

Bushels Q	MaxBPrice		P*=65		BuyNetSur
1	$90	-	$65	=	$25
2	$84	-	$65	=	$19
3	$80	-	$65	=	$15
4	$76	-	$65	=	$11
5	$70	-	$65	=	$ 5

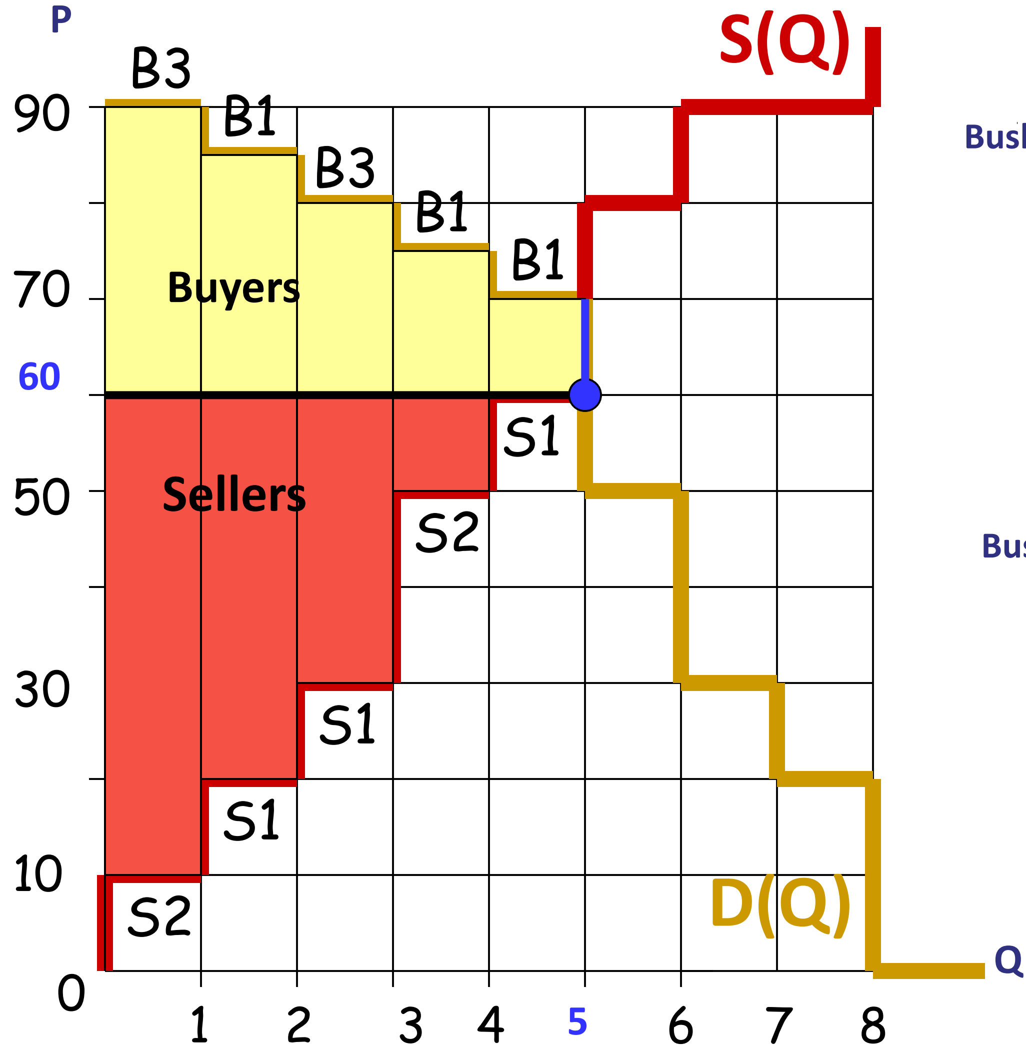
BUYER NET SURPLUS: $75

Bushels Q	P*=65		MinSPrice		SellNetSur
1	$65	-	$10	=	$55
2	$65	-	$20	=	$45
3	$65	-	$30	=	$35
4	$65	-	$50	=	$15
5	$65	-	$60	=	$ 5

SELLER NET SURPLUS: $155

Total Net Surplus: $230

A *different* selected CMC point
➔ *different* seller & buyer net surplus amounts



Ex 2: CMC Point Q*=5, P*=$60

Bushels Q	MaxBuyPrice		P*=60		BuyNetSurplus
1	$90	-	$60	=	$30
2	$84	-	$60	=	$24
3	$80	-	$60	=	$20
4	$76	-	$60	=	$16
5	$70	-	$60	=	$10

BUYER NET SURPLUS: $100

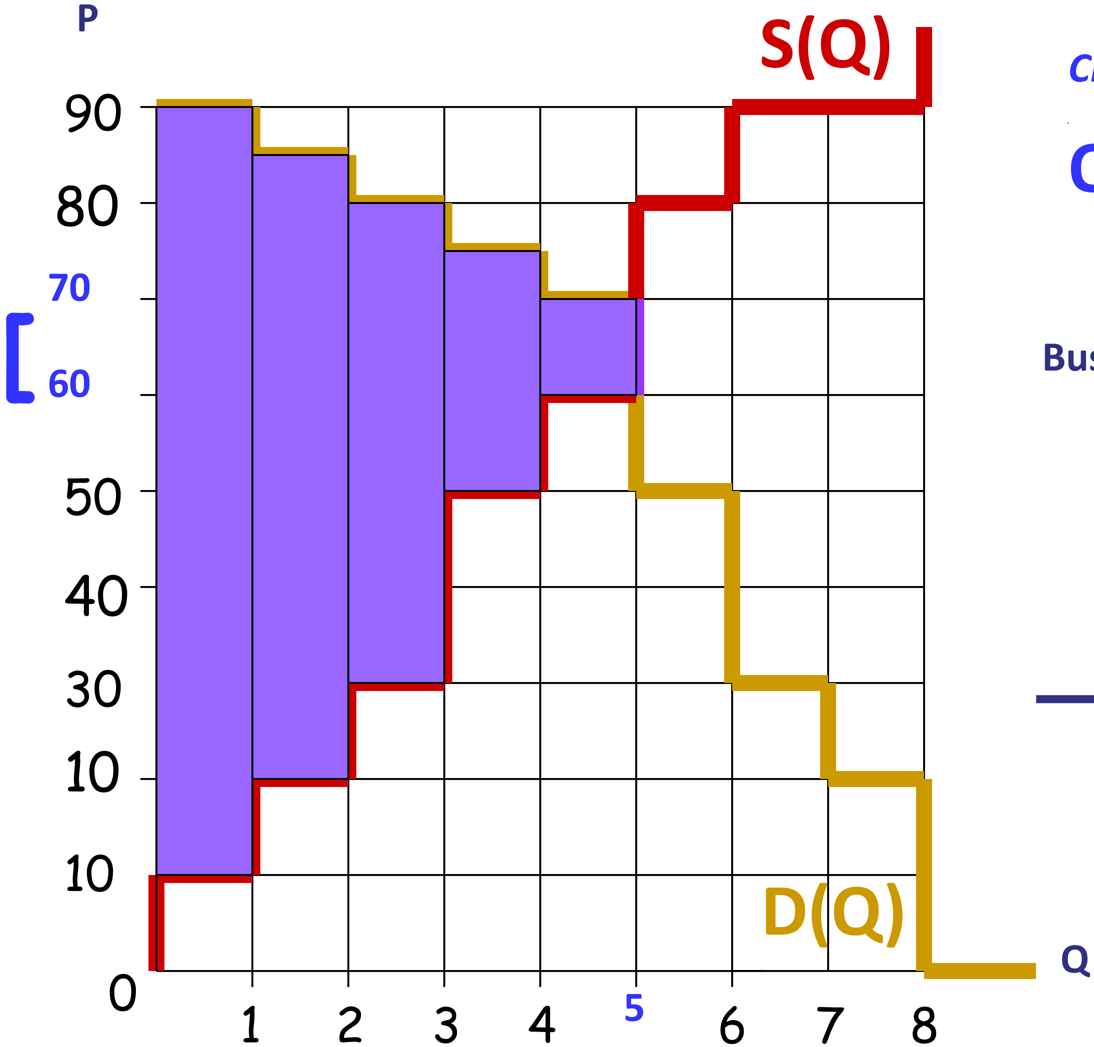
Bushels Q	P*=65		MinSellPrice		SellNetSurplus
1	$60	-	$10	=	$50
2	$60	-	$20	=	$40
3	$60	-	$30	=	$30
4	$60	-	$50	=	$10
5	$60	-	$60	=	$ 0

SELLER NET SURPLUS: $130

Total Net Surplus: $230

Total Net Surplus at a CMC Point

(*If multiple CMC points exist, TNS = same for each point.*)



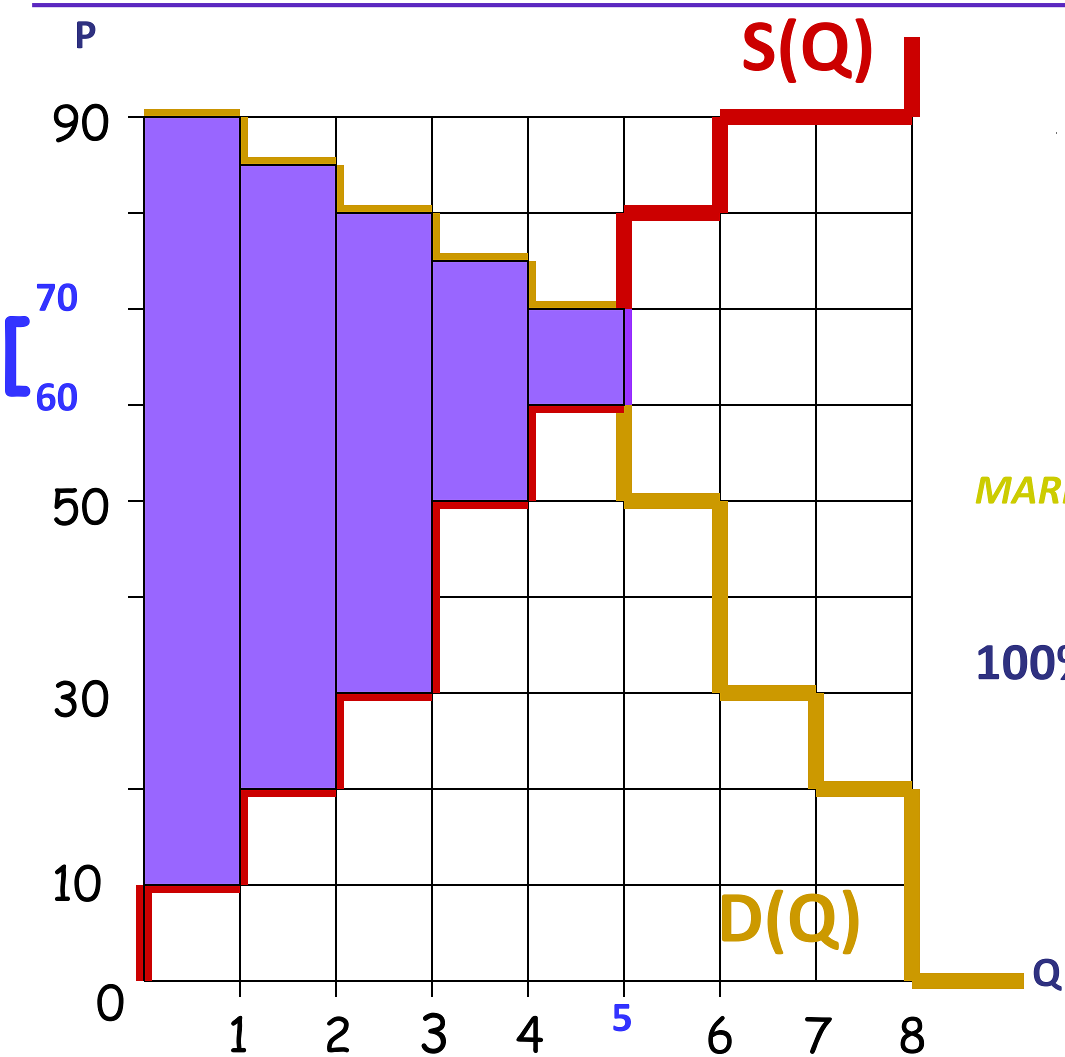
CMC Points:

Q*=5, $60 ≤ P*≤ $70

Bushels Q	MaxBuyP		MinSellP		Net Surplus
1	$90	-	$10	=	$80
2	$84	-	$20	=	$64
3	$80	-	$30	=	$50
4	$76	-	$50	=	$26
5	$70	-	$60	=	$10

TOTAL NET SURPLUS: $230

Standard Measure of Market Efficiency
(Non-Wastage of Resources)



CMC Points:

Q*=5, $60 ≤ P*≤ $70

CMC Total Net Surplus

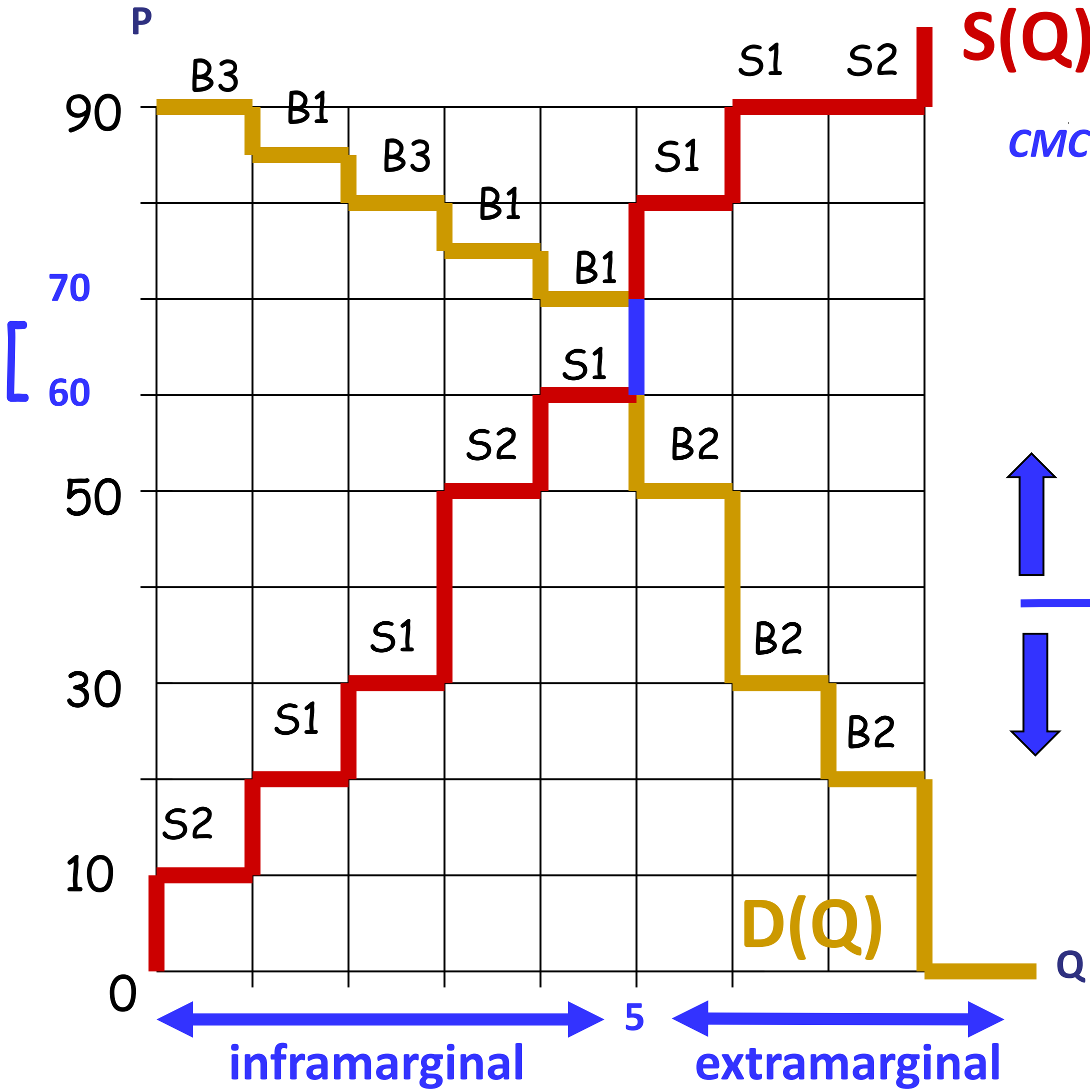
= $230 (Maximum Possible)

MARKET EFFICIENCY (ME):

$$100\% \times \frac{\text{Extracted Total Net Surplus}}{\text{Max Possible Total Net Surplus}}$$

How can ME be less than 100% ?

Inframarginal vs. Extramarginal Quantity Units at CMC Points



CMC Pts: Q=5, $60 ≤ P*≤ $70*

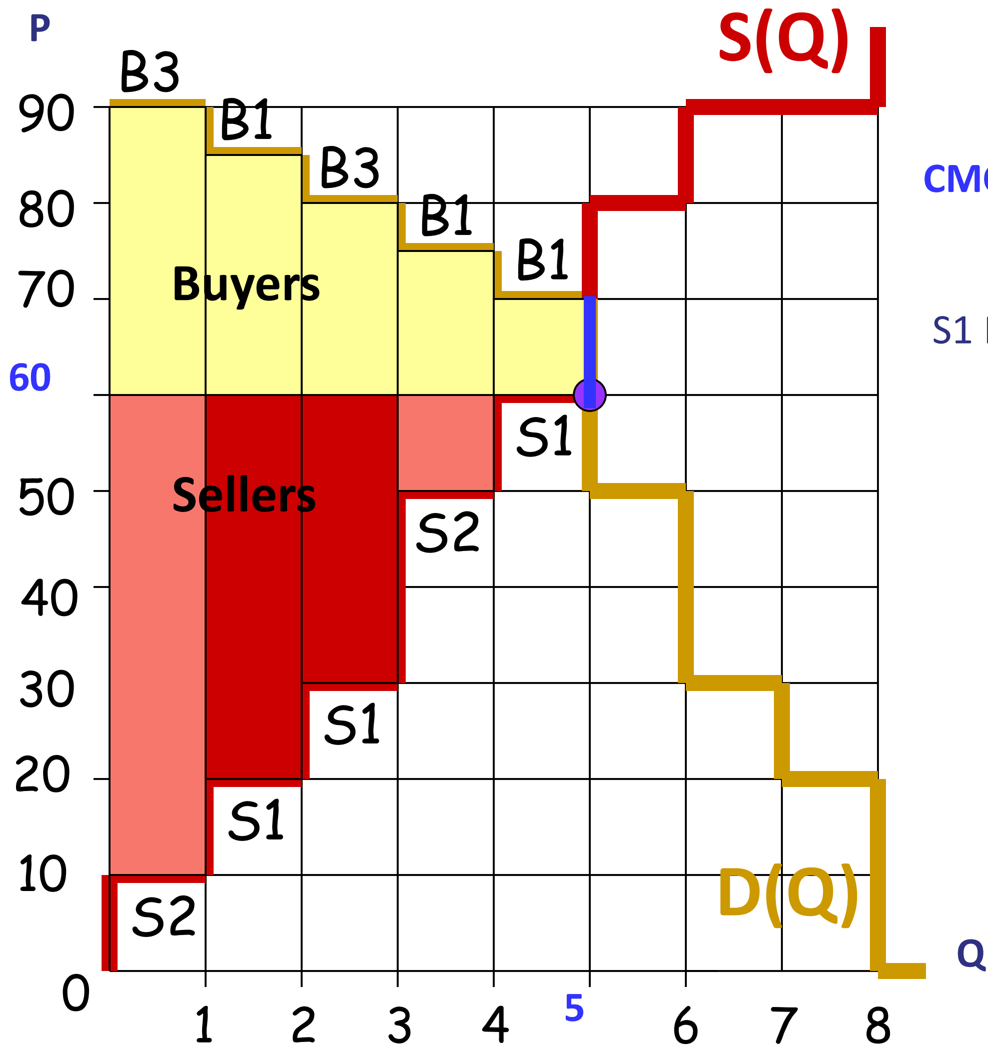
Bushels Q	MaxBuyPrice		MinSellPrice
1	$90	>	$10
2	$84	>	$20
3	$80	>	$30
4	$76	>	$50
5	$70	>	$60
6	$50	<	$80
7	$30	<	$90
8	$20	<	$90
9	$0	<	∞

Market Efficiency < 100% can arise if …

- some **inframarginal** quantity unit **fails to trade**
  - E.g., physical capacity withholding (**"market power"***)
- some **extramarginal** quantity unit **is traded**
  - a more costly unit is sold in place of a less costly unit ("out-of-merit-order dispatch")
  - and/or a less valued unit is purchased in place of a more valued unit ("out-of-merit-order purchase")

* ***Market Power:*** Ability of a seller or buyer to extract more net surplus from a market than they would achieve at a CMC point.

Example: Exercise of market power by Seller S1 that results in ME < 100%



CMC Point: $Q^*=5$, $P^*=\$60$

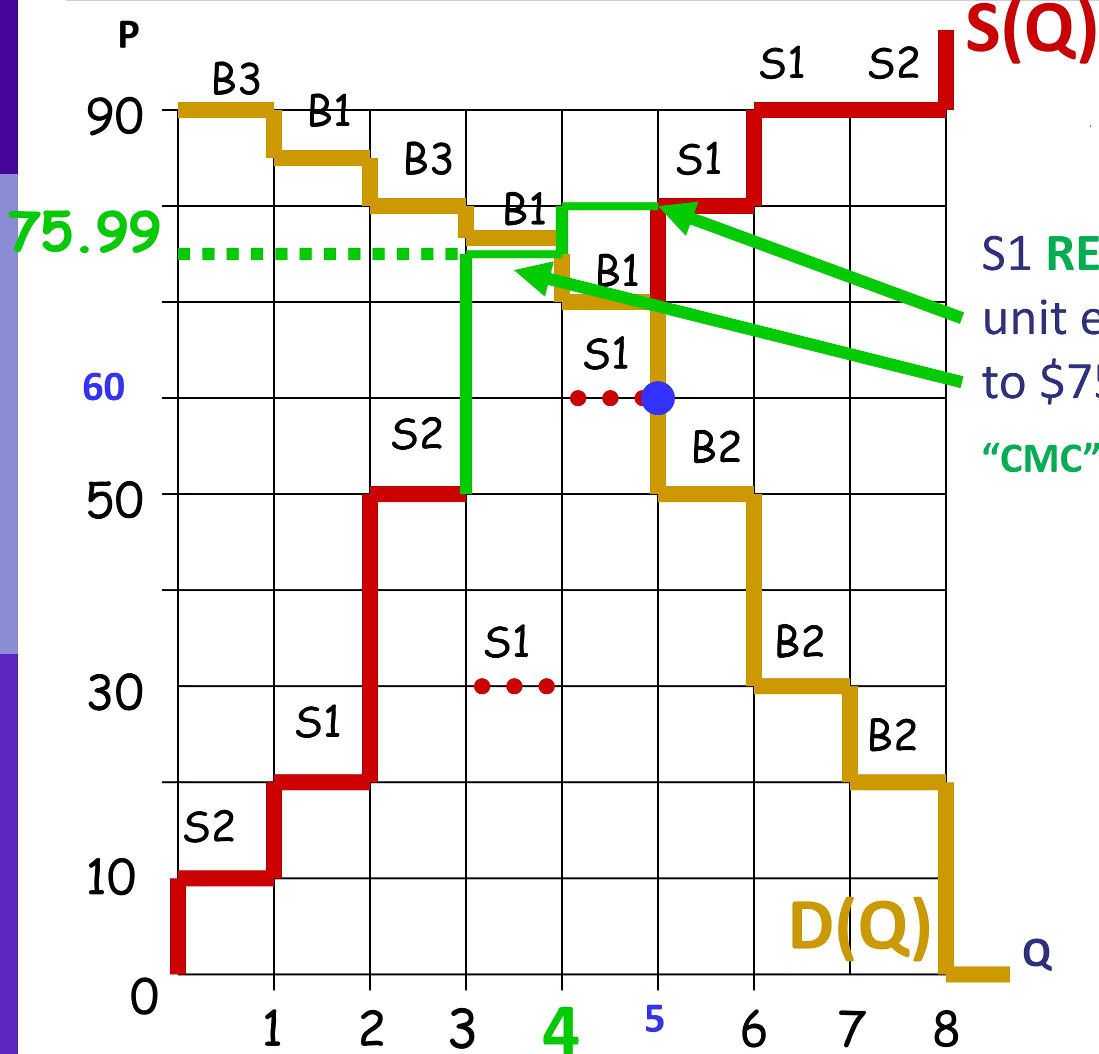
S1 Net Surplus at CMC Point:

\$60-\$20 = \$40
\$60-\$30 = \$30
\$60-\$60 = \$ 0

S1 Net Surplus = \$70

Total Net Surplus: \$230

Example: ME < 100% … Continued



CMC Point: Q*=5, P*=$60

S1's CMC Net Surplus = $70

S1 **REPORTS** a max sale price on his 3rd unit equal to $80 & on his 2nd unit equal to $75.99.

"CMC" Point: Q'=4, P'≅$76

At new "CMC" point, S1 only sells its first 2 units, but **S1's net surplus increases to ≅ $102 = [$56+$46]**

Extracted total net surplus **DECREASES FROM 230 TO 220** because inframarginal 5th unit now fails to sell.

Market Efficiency vs. Social Welfare

- ***Efficiency*** for one market at one time point is a very narrow measure of resource non-wastage.
- Ideally, ***social*** efficiency should be measured by resource non-wastage across ***all*** markets and across ***all*** current and future time periods.
- Moreover, economists measure ***social welfare*** in terms of the ***"utility" (well-being) of people*** in their roles as consumers/users of final goods and services.
- **Social efficiency** is ***necessary but not sufficient*** for the optimization of **social welfare**.

Market Efficiency, Social Welfare, and the Extraction of Net Surplus by "Third Parties"

- Suppose [price P_S paid to a seller] < [price P_B charged to a buyer] for some quantity unit sold in a market

➔ **Net surplus [P_B-P_S] is extracted by some type of "third party"**

Examples: Gov't tax revenues; **<u>ISO net surplus extractions</u>** that result from grid congestion in **D**ay-**A**head **M**arkets (**DAM**s) for grid-delivered energy (MWh) settled by means of **L**ocational **M**arginal **P**rices **LMP(**b,H**)** ($/MWh) conditional on grid delivery location b and operating hour H.

- "First order effect" of this **third-party extracted net surplus** is a decrease in the net surplus going to sellers & buyers.

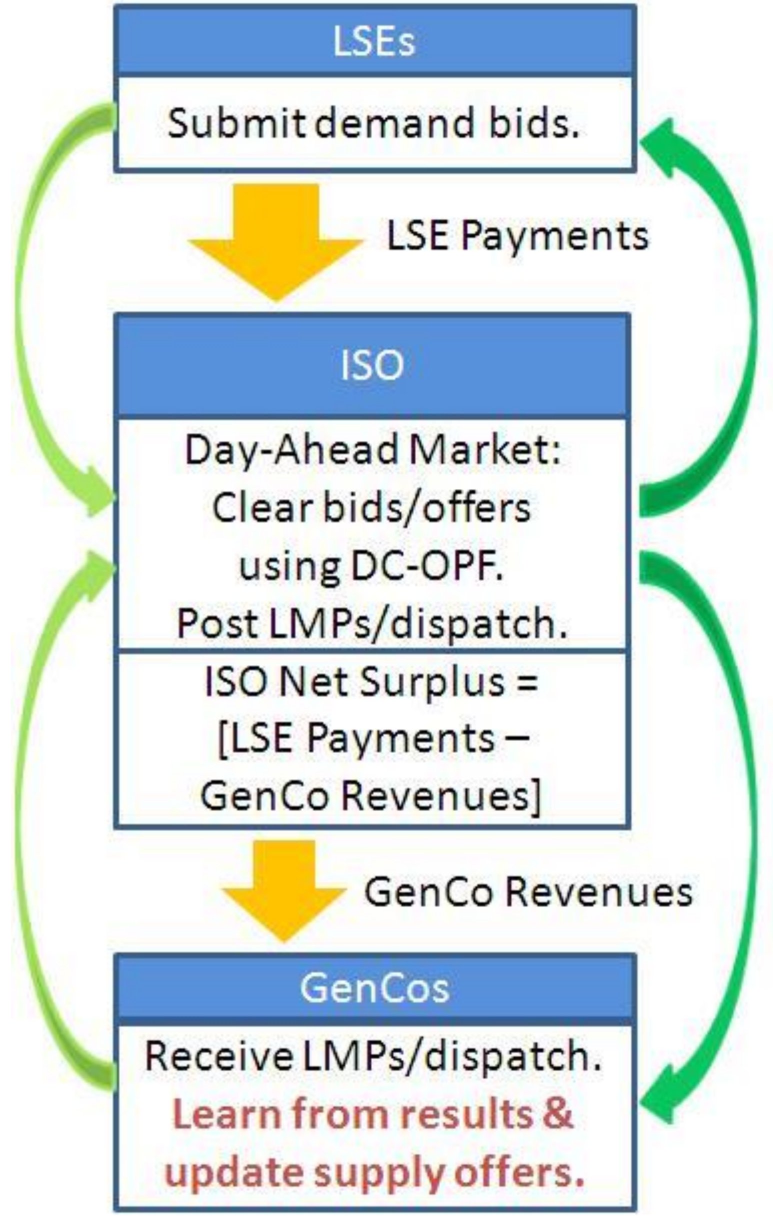
- **Social efficiency/welfare implications** of this third-part extracted net surplus depend on precisely <u>how</u> it is extracted and <u>to what uses it is subsequently put.</u>

AMES DC-OPF Formulation

Caution: Notation Switch

- P (in MWs) now denotes *amounts of power*
- $LMP_{k,T}$ (\$/MWh) = *Locational Marginal Price* at bus k for operating period T, roughly defined as the least cost of maintaining one additional MW of generated power at bus k during operating period T.

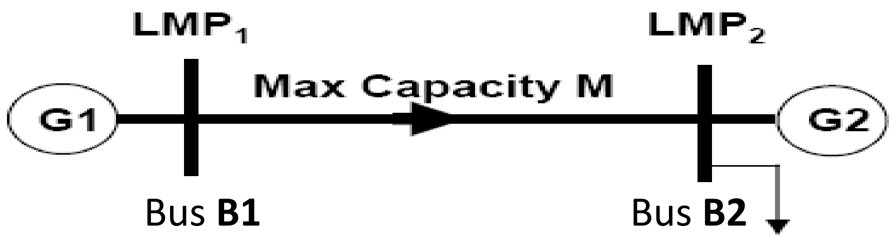
Discussion of double auctions, market efficiency, & social welfare specialized to an ISO managed ***D****ay-****A****head* ***M****arket* (***DAM***) for grid-delivered energy (MWh) with LMP settlements ($/MWh):



Day-ahead market activities on a typical operating day D

ISO Net Surplus Extraction: DAM Example

(adapted from Harold Salazar, MS Thesis, Nov 2008)



$$p_L = p^F_L + p^S_L \quad \text{(MW)}$$

-- A **D**ay-**A**head **M**arket (**DAM**) is held on day **D** for an operating hour **H** on day **D+1**

-- The transacted good is grid-delivered energy (MWh), expressed in terms of power levels p (MW) to be maintained during hour **H** (1h)

-- **G1, G2** = Generation Companies (GenCos) located at grid buses **B1** and **B2**

-- p_L = Total demand (MW) of a **L**oad-**S**erving **E**ntity (**LSE**) at bus **B2** for hour **H**

-- p^F_L = Fixed (i.e., not price sensitive) demand (MW) of **LSE** at bus **B2** for hour **H**

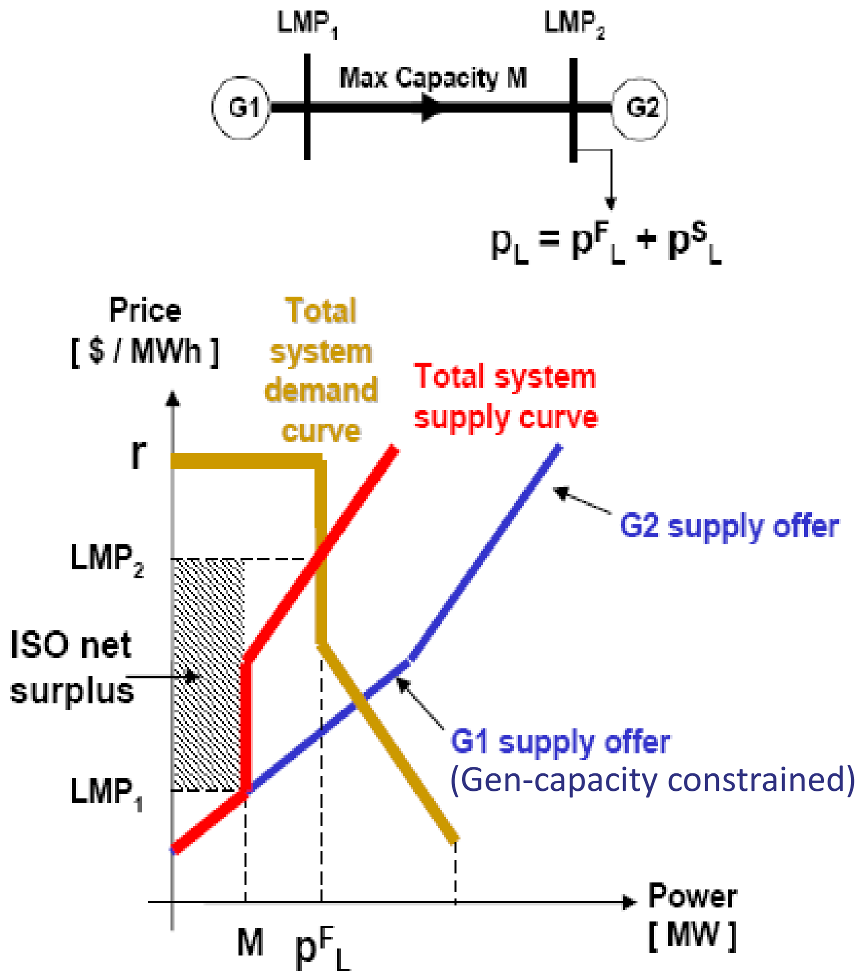
-- p^S_L = Price-sensitive demand (MW) of **LSE** at bus **B2** for hour **H**

-- LMP_1 = Locational Marginal Price ($/MWh) at bus **B1** for hour **H**

-- LMP_2 = Locational Marginal Price ($/MWh) at bus **B2** for hour **H**

-- r = Regulated rate ($/MWh) for **LSE** retail resale of fixed demand for hour **H**

ISO Net Surplus Example … Continued



Cleared load = p^F_L. LSE at bus 2 pays $LMP_2 > LMP_1$ for each unit of p^F_L. M units of p^F_L are supplied by cheaper G1 at bus 1 who receives only LMP_1 per unit.

ISO collects payment difference:

ISO Net Surplus

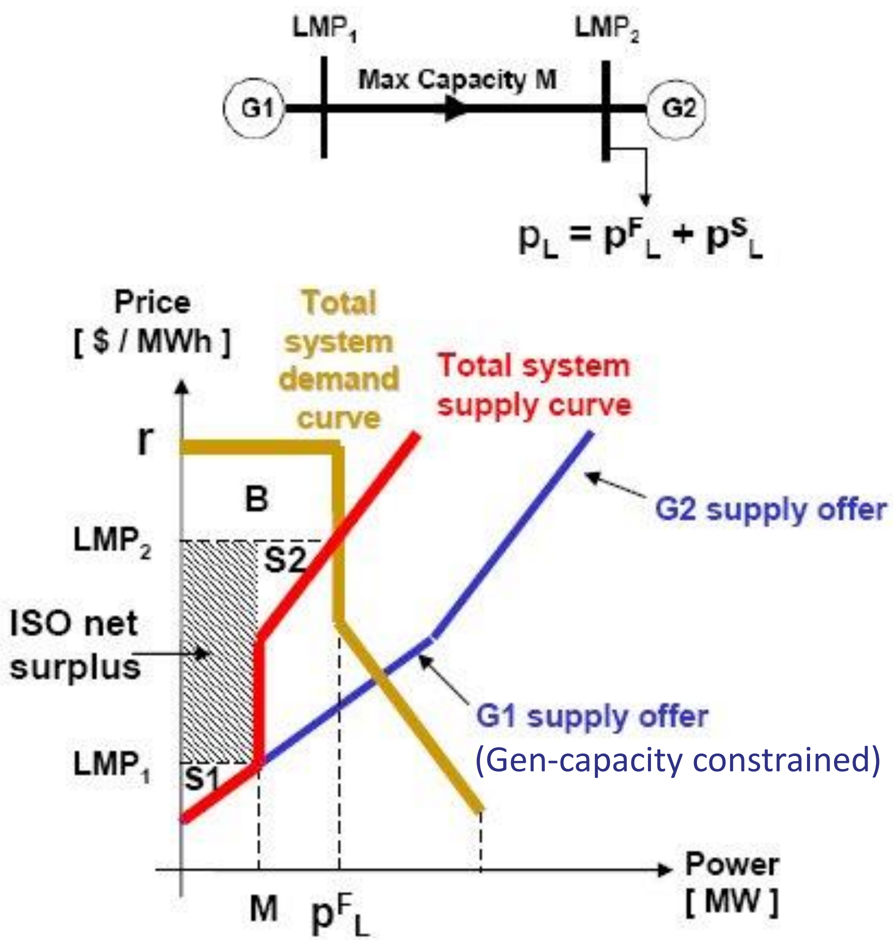
= [LSE Payments

– GenCo Revenues]

= $M \times [LMP_2 – LMP_1]$

Note: The transmission line capacity limit **M** is <u>binding</u> in this example.
Otherwise, the market-clearing price outcome is $LMP_1 = LMP_2$ = CMC Point!

ISO Net Surplus Example … Continued



ISO Net Surplus:

INS = M x [LMP_2 – LMP_1]

GenCo Net Surplus:

GNS = Area S1 + Area S2

LSE Net Surplus:

LNS = Area B

Total Net Surplus:

TNS = [**INS** + **GNS** + **LNS**]

ISO Objective (Optimal Power Flow):

Maximize **TNS** subject to transmission & generation constraints.

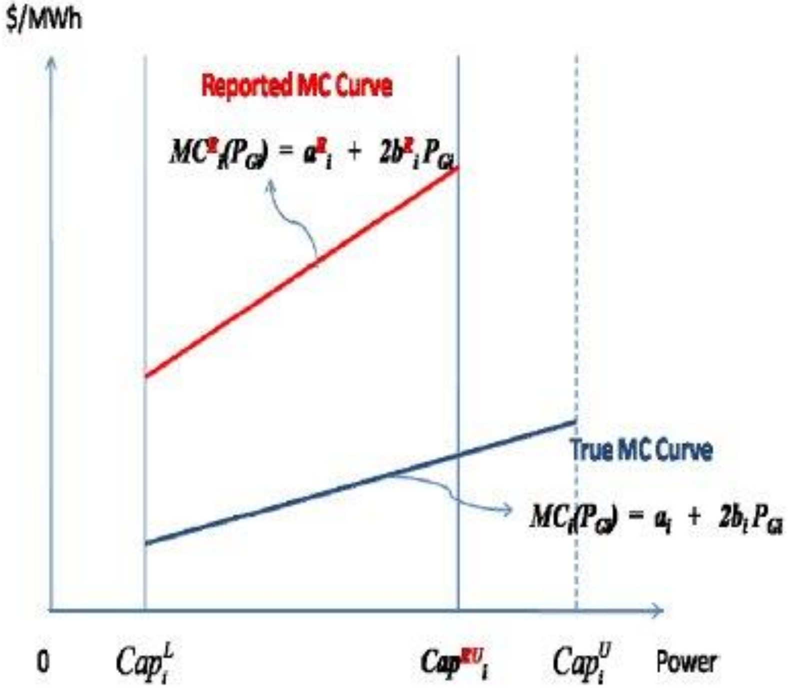
AMES GenCo Supply Offers

Hourly supply offer for each GenCo i = ***Reported*** linear marginal cost function over a ***reported*** operating capacity interval for real power p_{Gi} (in MWs):

$$MC^R_i(p_{Gi}) = a^R_i + 2b^R_i \, p_{Gi}$$

$$Cap_i^L \leq p_{Gi} \leq Cap_i^{RU}$$

GenCos can learn to report ***higher-than-true*** marginal costs and/or to report ***lower-than-true*** maximum capacity.



AMES LSE Demand Bids

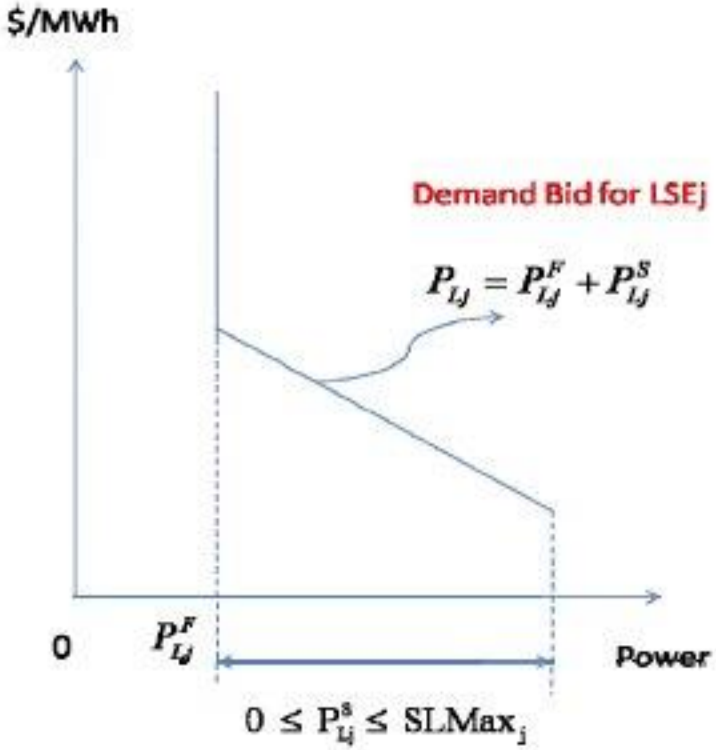
Hourly demand bid for each LSE j = ***Fixed demand bid*** + ***Price-sensitive demand bid***

- Fixed demand bid = p^F_{Lj} (MWs)
- Price-sensitive demand bid

  = Inverse demand function for real power p^S_{Lj} (MWs) over a purchase capacity interval:

$$F_j(p^S_{Lj}) = c_j - 2d_j\, p^S_{Lj}$$

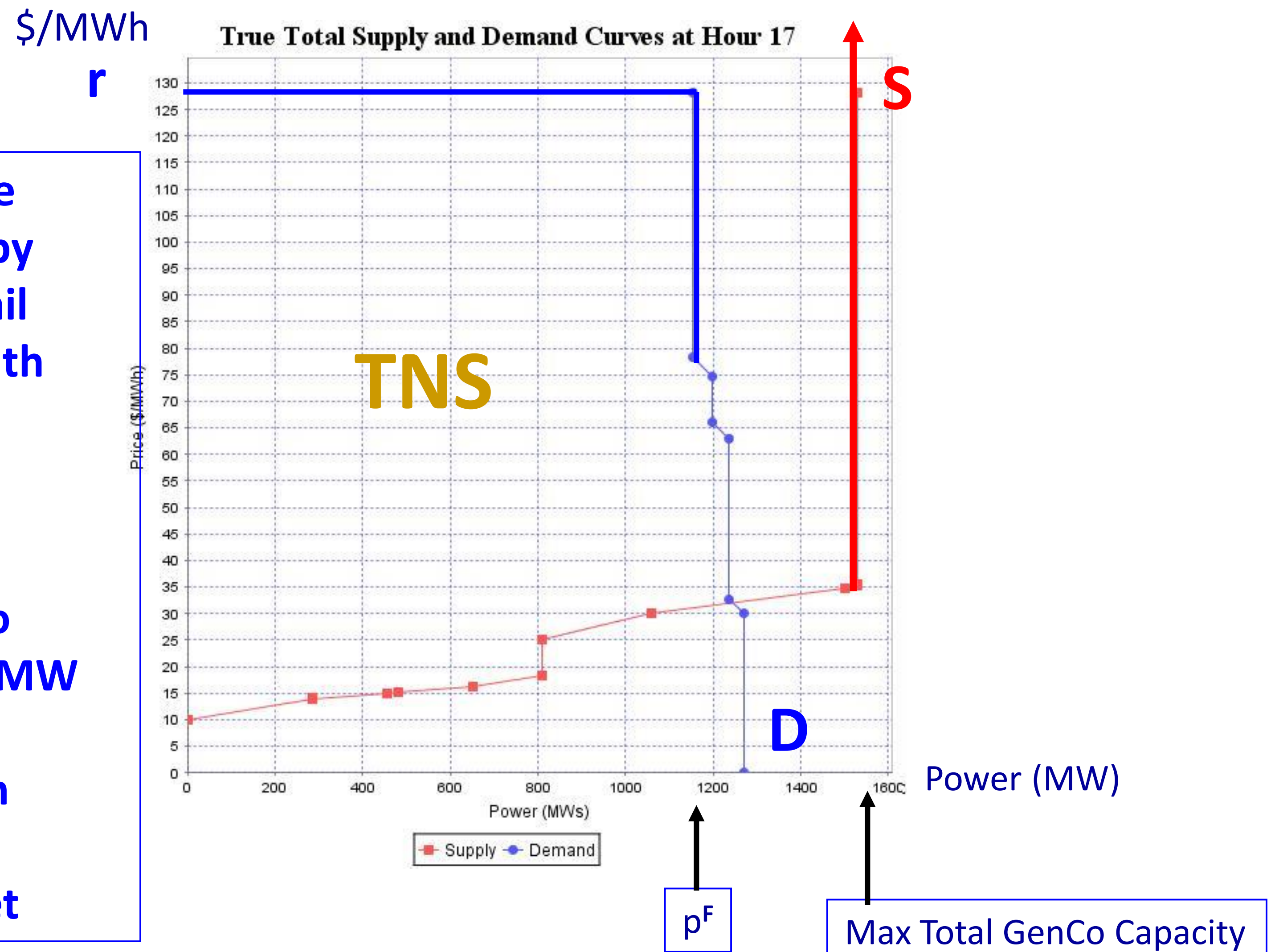
$$0 \leq p^S_{Lj} \leq SLMax_j$$



AMES Illustration: **Total Net Surplus (TNS)** in Hour 17 for 5-Bus Test Case with 5 GenCos and 3 LSEs

r = Fixed price paid to LSEs by the LSEs' retail customers with flat-price contracts

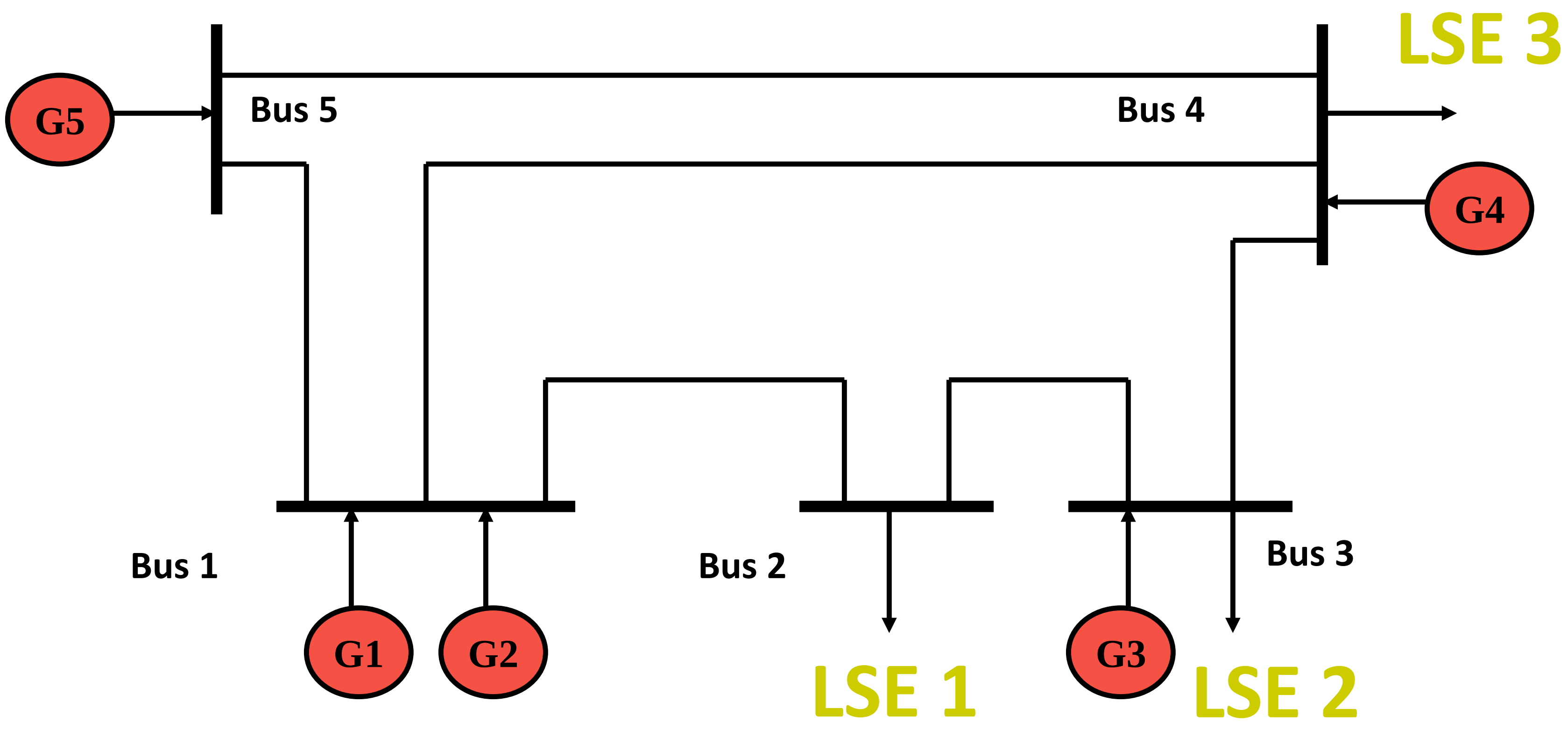
= LSEs' max willingness to pay for each MW of their fixed demand pF in wholesale power market



ISO Net Surplus Experiments (Li/Tesfatsion, 2009)

(Experiments run with AMES Wholesale Power Market Test Bed)

Five GenCo sellers G1,…,G5 and three LSE buyers LSE 1, LSE 2, LSE 3



R Measure for Demand-Bid Price Sensitivity

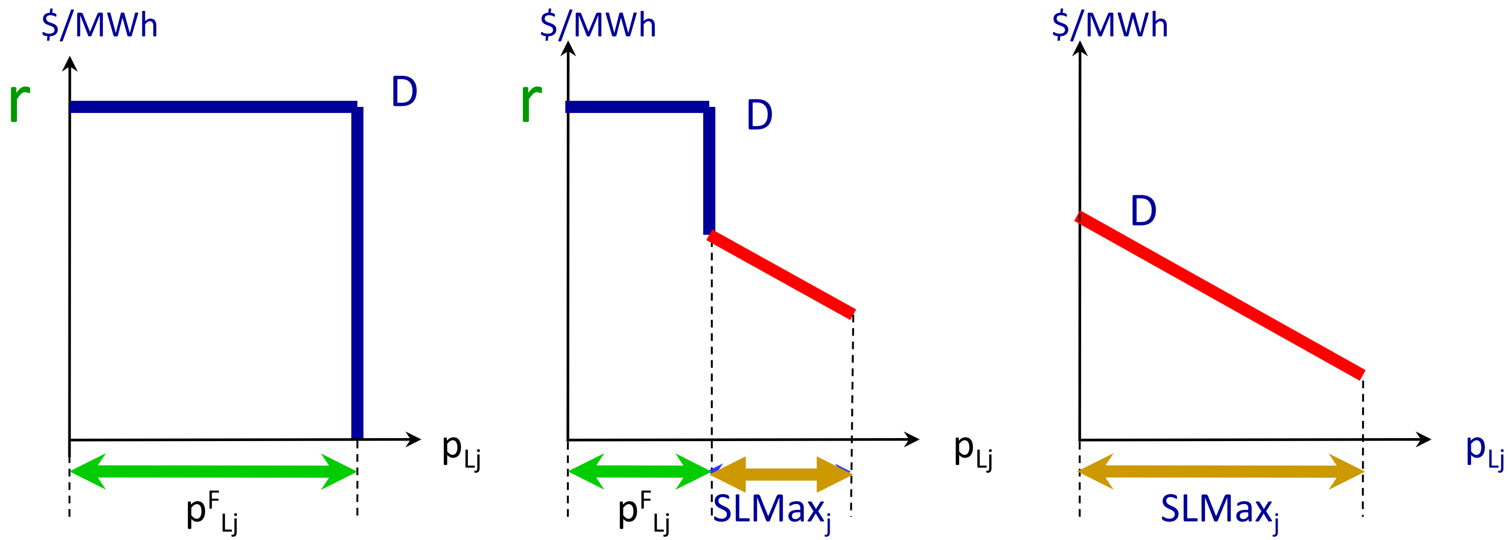
Note: **In actual U.S. ISO energy regions, price-sensitivity R ≅ 0.01**

For LSE j in Hour H:

pF_{Lj} = Fixed demand for real power (MWs)

$SLMax_j$ = Maximum potential price-sensitive demand (MWs)

$$R = SLMax_j / [\, p^F_{Lj} + SLMax_j \,]$$



R=0.0

(100% Fixed Demand)

R=0.5

R=1.0

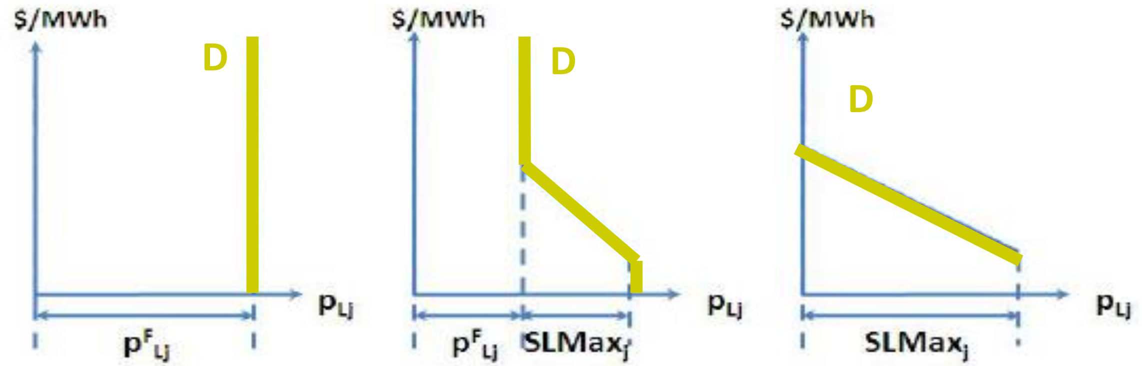
(100% Price-Sensitive Demand)

Experimental Outcomes:
Varied Price-Sensitivity for Demand Bids

Demand bid for LSE j (MW):

Fixed demand bid p^F_{Lj} + Price-sensitive demand bid p^S_{Lj},

where $0 \leq p^S_{Lj} \leq SLMax_j$



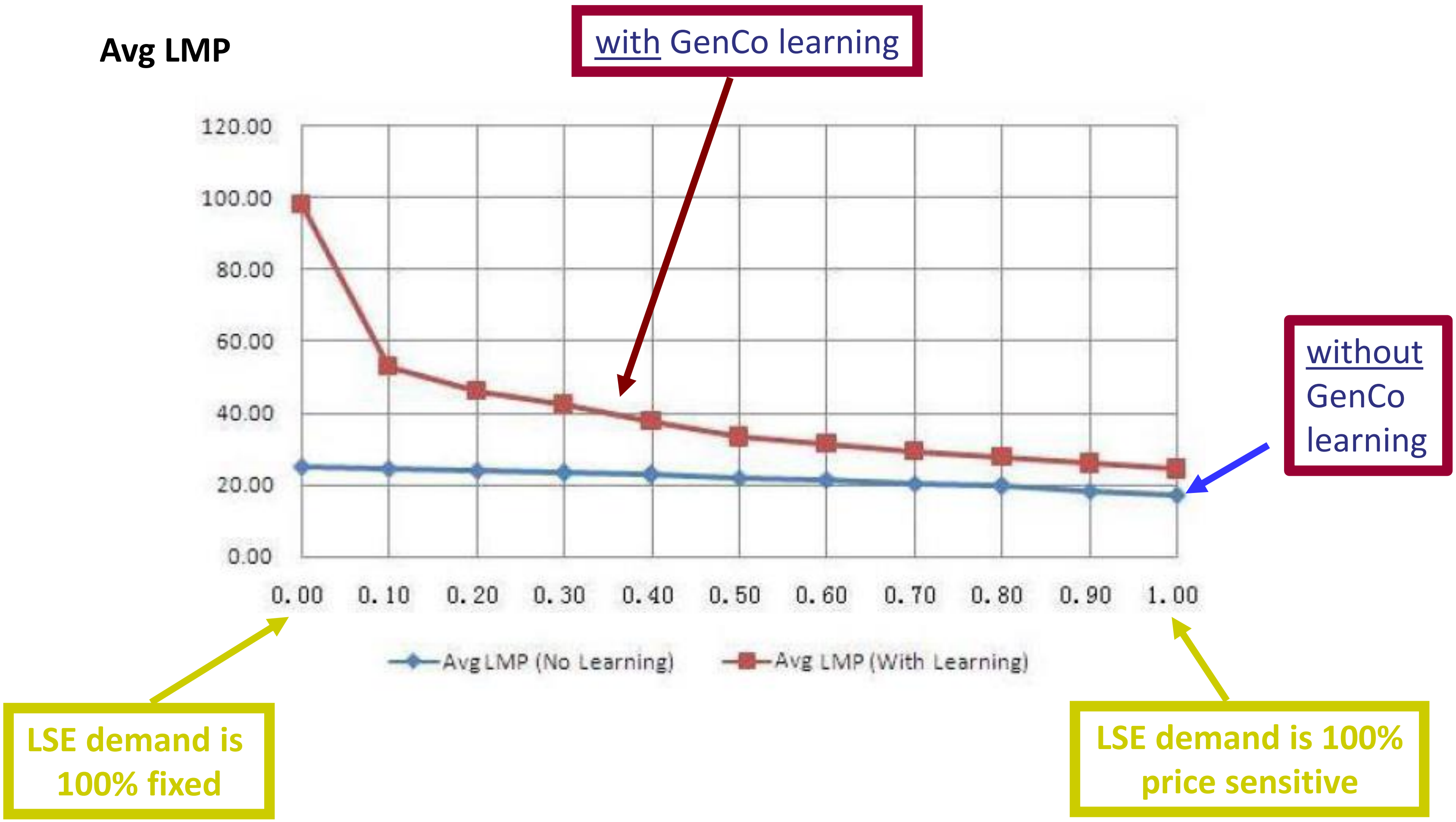
(1) R=0.0 (2) R=0.5 (3) R=1.0

100% fixed demand

100% price-sensitive demand

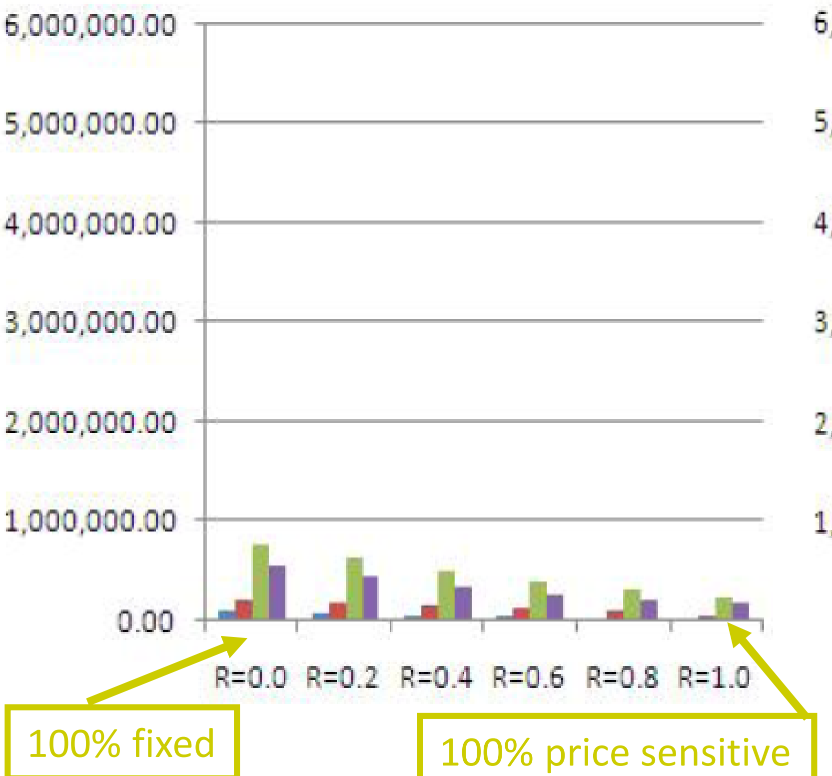
Average LMP Outcomes on Day 1000

(under varied **GenCo learning** & **LSE demand price-sensitivity** treatments)

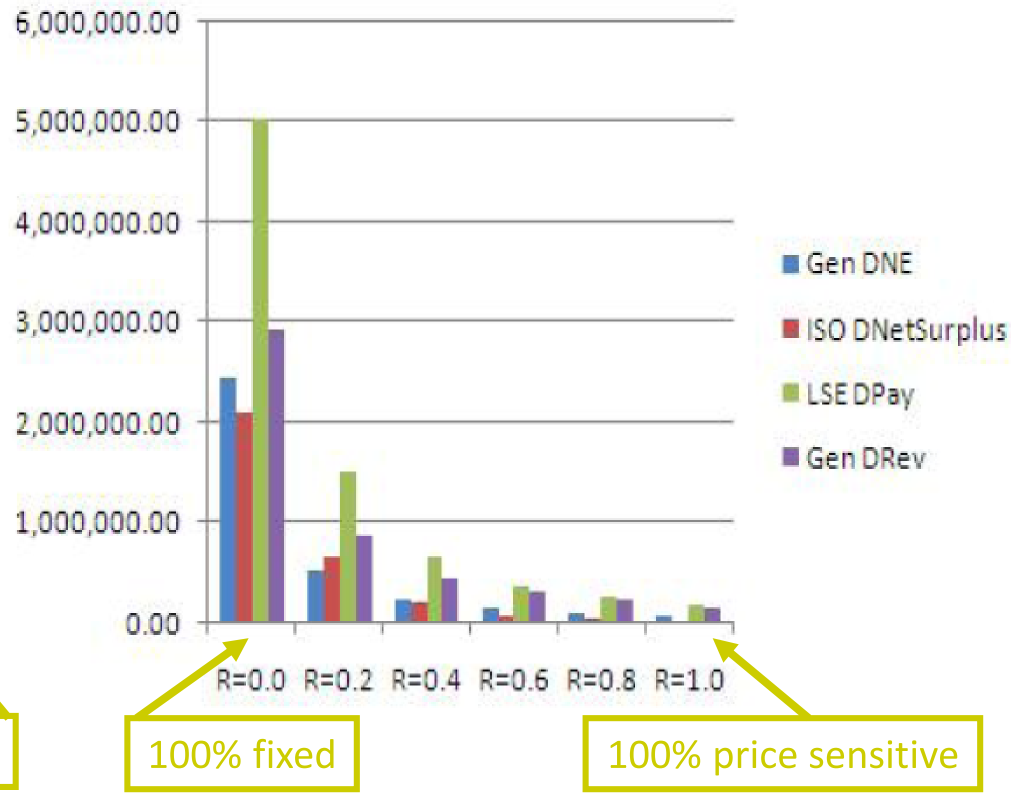


Average ISO Net Surplus Outcomes on Day 1000
for varied learning & demand treatments

Without Learning



With Learning GenCos



= GenCo Net Earnings

= ISO Net Surplus

ISO Net Surplus, Market Efficiency, and Social Welfare

- Two-bus example and experimental findings suggest ISO net surplus extractions can be **substantial**, and can **dramatically increase** with:
  - ***decreases*** in price sensitivity of demand
  - ***increases*** in GenCo learning ability resulting in the reporting of supply offers at higher-than-true costs (especially profitable in presence of fixed demand)

- **Important Issue:** How to ensure ISO financial incentives are properly aligned with goal of ensuring market efficiency/soc welfare?

AMES Calculation of TNS: General Form
(Note LMPs cancel out of TNS expression!)

Total Net Surplus for Hour H of Day D+1, based on Day D Supply Offers and Demand Bids:

$$TNS(H,D)$$

$$= \text{LSENetSur}(H,D) + \text{GenNetSur}(H,D) + \text{ISONetSur}(H,D)$$

$$= \sum_{j=1}^{J} GS_j(H,D) - \sum_{i=1}^{I} [\mathrm{C}_i^a(H,D)]$$

where

$$GS_j(H,D) = [r \cdot p_{Lj}^F(H,D) + \int_0^{p_{Lj}^S(H,D)} \mathrm{F}_{jHD}(p)dp]$$

$$\mathrm{C}_i^a(H,D) = \int_0^{p_{Gi}(H,D)} \mathrm{MC}_i(p)dp$$

LSE j's gross surplus from its retail fixed demand sales

LSE j's gross surplus from its retail price- sensitive demand sales

GenCo i's total avoidable costs of production

AMES Basic DC-OPF Formulation:

SI unit representation for AMES ISO's DC-OPF problem for hour H of the day-ahead market on day D+1, solved on day D.

DC-OPF formulation is derived from AC-OPF under three assumptions:

(a) Resistance on each branch km = 0

(b) Voltage magnitude at each bus k= base voltage V_o

(c)Voltage angle difference d_{km} = [delta_k - delta_m] across each branch km is small so that $\cos(d_{km}) \cong 1$ and $\sin(d_{km}) \cong d_{km}$

$$\max \quad \text{TNS}^R \tag{15}$$

with respect to LSE real-power price-sensitive demands, GenCo real-power generation levels, and voltage angles

$$p^S_{Lj},\ j = 1, ..., J;\ p_{Gi},\ i = 1, ..., I;\ \delta_k,\ k = 1, ..., K \tag{16}$$

subject to

(i) a real-power balance constraint for each bus k=1,...,K:

$$\sum_{i \in I_k} p_{Gi} - \sum_{j \in J_k} p^S_{Lj} - \sum_{km} P_{km} = \sum_{j \in J_k} p^F_{Lj} \tag{17}$$

where, letting x_{km} (ohms) denote reactance for branch km, and V_o denote the base voltage (in line-to-line kV),

$$P_{km} = [V_o]^2 \cdot [1/x_{km}] \cdot [\delta_k - \delta_m]$$

(ii) a limit on real-power flow for each branch km:

$$|P_{km}| \leq P^U_{km} \tag{18}$$

(iii) a real-power operating capacity interval for each GenCo i = 1,...,I:

$$\text{Cap}^L_i \leq p_{Gi} \leq \text{Cap}^U_i \tag{19}$$

(iv) a real-power purchase capacity interval for price-sensitive demand for each LSE j = 1,...,J:

$$0 \leq p^S_{Lj} \leq \text{SLMax}_j \tag{20}$$

(v) and a voltage angle setting at angle reference bus 1:

$$\delta_1 = 0 \tag{21}$$

TNS^R = Total Net Surplus based on *reported* GenCo marginal cost functions rather than *true* GenCo marginal cost functions.

Lagrange multiplier (or "shadow price") solution for the bus-k balance constraint (17) gives the LMP_k at bus k

AMES DC-OPF problem is a special type of GNPP, and LMPs are Lagrange Multiplier Solutions for this GNPP

General Nonlinear Programming Problem (GNPP):

- $\mathbf{x}$ = n_x1 choice vector;
- $\mathbf{c}$ = m_x1 vector & $\mathbf{d}$ = S_x1 vector (constraint constants)
- $f(\mathbf{x})$ maps $\mathbf{x}$ into R (all real numbers)
- $\mathbf{h}(\mathbf{x})$ maps $\mathbf{x}$ into R^m (all m-dimensional vectors)
- $\mathbf{z}(\mathbf{x})$ maps $\mathbf{x}$ into R^s (all s-dimensional vectors)

GNPP: Minimize f(x) with respect to x subject to

$\mathbf{h}(\mathbf{x}) = \mathbf{c}$ (e.g., DC-OPF bus balance constraints)

$\mathbf{z}(\mathbf{x}) \geq \mathbf{d}$ (e.g., DC-OPF branch constraints & GenCo capacity constraints)

AME DC-OPF as a GNPP … Continued

- Define the ***Lagrangean Function*** as

$$L(\mathbf{x},\boldsymbol{\lambda},\boldsymbol{\mu},\mathbf{c},\mathbf{d}) = f(\mathbf{x}) + \boldsymbol{\lambda}^T[\mathbf{c} - h(\mathbf{x})] + \boldsymbol{\mu}^T[\mathbf{d} - \mathbf{z}(\mathbf{x})]$$

- Assume ***Kuhn-Tucker Constraint Qualification (KTCQ)*** holds at **x***, roughly stated as follows:

The true set of feasible directions at **x***

= Set of feasible directions at **x*** assuming a linearized set of constraints in place of original set of constraints.

AMES DC-OPF as a GNPP … Continued

- Given KTCQ, the ***First-Order Necessary Conditions (FONC)*** for $\mathbf{x}^*$ to solve (GNPP) are: There exist vectors $\boldsymbol{\lambda}^*$ and $\boldsymbol{\mu}^*$ of ***Lagrange multipliers (or "shadow prices")*** such that $(\mathbf{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$ satisfies:

$$0 = \nabla_x L(\mathbf{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*, \mathbf{c}, \mathbf{d})$$

$$= [\ \nabla_x f(\mathbf{x}^*) - \boldsymbol{\lambda}^{*T} \bullet \nabla_x h(\mathbf{x}^*) - \boldsymbol{\mu}^{*T} \bullet \nabla_x z(\mathbf{x}^*)\ ]\ ;$$

$$\mathbf{h}(\mathbf{x}^*) = \mathbf{c}\ ;$$

$$\mathbf{z}(\mathbf{x}^*) \ge \mathbf{d};\ \ \boldsymbol{\mu}^{*T} \cdot [\mathbf{d} - \mathbf{z}(\mathbf{x}^*)] = 0;\ \boldsymbol{\mu}^* \ge \mathbf{0}$$

These FONC are often referred to as the *Karush-Kuhn-Tucker (KKT) conditions*.

Solution as a Function of (c,d)

By construction, the components of the solution vector ($\mathbf{x}^*$, $\boldsymbol{\lambda}^*$, μ^*) are <u>*functions*</u> of the constraint constant vectors c and d

- $\mathbf{x}^* = \mathbf{x}(\mathbf{c},\mathbf{d})$
- $\boldsymbol{\lambda}^* = \boldsymbol{\lambda}(\mathbf{c},\mathbf{d})$
- $\boldsymbol{\mu}^* = \boldsymbol{\mu}(\mathbf{c},\mathbf{d})$

GNPP Lagrange Multipliers as Shadow Prices

Given certain additional regularity conditions…

- The solution $\boldsymbol{\lambda}^*$ for the m x 1 multiplier vector $\boldsymbol{\lambda}$ is the derivative of the minimized value f($\mathbf{x}$*) of the objective function f($\mathbf{x}$) with respect to the constraint vector $\mathbf{c}$, all other problem data remaining the same.

$$\partial f(\mathbf{x}^*)/\partial \mathbf{c} = \partial f(\mathbf{x}(\mathbf{c},\mathbf{d}))/\partial \mathbf{c} = \boldsymbol{\lambda}^{*T}$$

GNPP Lagrange Multipliers as Shadow Prices …

Given certain additional regularity conditions…

- The solution $\boldsymbol{\mu}^*$ for the s x 1 multiplier vector $\boldsymbol{\mu}$ is the derivative of the minimized value f($\mathbf{x}^*$) of the objective function f($\mathbf{x}$) with respect to the constraint vector **d**, all other problem data remaining the same.

$$0 \le \partial f(\mathbf{x}^*)/\partial \mathbf{d} = \partial f(\mathbf{x}(\mathbf{c},\mathbf{d}))/\partial \mathbf{d} = \boldsymbol{\mu}^{*T}$$

GNPP Lagrange Multipliers as Shadow Prices …

Consequently…

- The solution $\boldsymbol{\lambda}^*$ for the multiplier vector $\boldsymbol{\lambda}$ thus essentially gives the ***prices (values)*** associated with unit changes in the components of the constraint vector $\mathbf{c}$, all other problem data remaining the same.

- The solution $\boldsymbol{\mu}^*$ for the multiplier vector $\boldsymbol{\mu}$ thus essentially gives the ***prices (values)*** associated with unit changes in the components of the constraint vector $\mathbf{d}$, all other problem data remaining the same.

- Each component of $\boldsymbol{\lambda}^*$ can take on ***any sign***

- Each component of $\boldsymbol{\mu}^*$ must be ***nonnegative***

Counterpart to Constraint Vector c for AMES DC-OPF?

AMES DC-OPF Has K Equality Constraints:

(i) a real-power balance constraint for each bus k=1,...,K:

$$\sum_{i \in I_k} p_{Gi} - \sum_{j \in J_k} p^S_{Lj} - \sum_{km} P_{km} = \sum_{j \in J_k} p^F_{Lj} \qquad (17)$$

Fixed demand of LSE j

Index set for LSEs located at bus k

Below is the kth Component of Kx1 Constraint Vector c :

$$\sum_{j \in J_k} p^F_{Lj} = FD_k$$ = **Total Fixed Demand at Bus k**

LMP as Lagrange Multiplier

- **TNS*(H,D)** = Maximized Value of TNS(H,D) from the ISO's DC-OPF solution on Day D for hour H of the day-ahead market on Day D+1

- **$\mathbf{LMP_k}$ (H,D)** = Least cost of servicing one additional MW of fixed demand at bus k during hour H of day-ahead market on day D+1

$$\mathbf{LMP_k(H,D)} = \frac{\partial TNS^*(H,D)}{\partial FD_k}$$

Online Resources

- Notes on DC-OPF Formulation in AMES

  https://www2.econ.iastate.edu/tesfatsi/DCOPFInAMES.LT.pdf

- AMES Wholesale Power Market Testbed

  https://www2.econ.iastate.edu/tesfatsi/AMESMarketHome.htm

- Market Basics for Price-Setting Agents

  https://www2.econ.iastate.edu/classes/econ458/tesfatsion/MBasics.SlidesIncluded.pdf

- Optimization Basics for Electric Power Markets

  https://www2.econ.iastate.edu/classes/econ458/tesfatsion/OptimizationBasics.LT458.pdf

- Power Market Trading with Transmission Constraints

  https://www2.econ.iastate.edu/classes/econ458/tesfatsion/OPFTransConstraintsLMP.KS6.1-6.3.2.9.pdf

Online Resources … Continued

- L. Tesfatsion (2009), **"Auction Basics for Wholesale Power Markets: Objectives & Pricing Rules,"** *IEEE PES General Meeting Proceedings*, July.
  https://www2.econ.iastate.edu/tesfatsi/AuctionTalk.LT.pdf (Slide-Set)
  https://www2.econ.iastate.edu/tesfatsi/AuctionBasics.IEEEPES2009.LT.pdf (Paper)

- H. Li & L. Tesfatsion (2011), "**ISO Net Surplus Collection and Allocation in Wholesale Power Markets Under Locational Marginal Pricing,"** *IEEE Transactions on Power Systems*, Vol. 26, No. 2, pp 627-641.
  https://www2.econ.iastate.edu/tesfatsi/ISONetSurplus.WP09015.pdf