DISCOVERING ARTIFICIAL ECONOMICS

How Agents Learn and Economies Evolve

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Preface

We live in an astonishingly complex world. Yet what we do in our everyday lives seems simple enough. Most of us conform to society's rules, pursue familiar strategies and achieve reasonably predictable outcomes. In our role as economic agents, we simply peddle our wares and earn our daily bread as best we can.

So where on earth does this astonishing complexity come from? Much of it's ubiquitous in nature, to be sure, but part of it lies within and between us. Part of it comes from those games of interaction that humans play - games against nature, games against each other, games of competition, games of cooperation. In bygone eras, people simply hunted and gathered to come up with dinner. Today you can find theoretical economists scratching mysterious equations on whiteboards (not even blackboards), and getting paid to do this. In the modern economy, most of us make our living in a niche created for us by what others do. Because we've become more dependent on each other, our economy as a whole has become more strongly interactive.

A strongly-interactive economy can behave in weird and wonderful ways, even when we think we understand all its individual parts. The resulting path of economic development is packed with unexpected twists and turns, reflecting the diversity of decisions taken by different economic agents. But an understanding of aggregate economic outcomes requires an understanding of each agents' beliefs and expectations and the precise way in which the agents interact. In a strongly interactive economy, the cumulative pattern of interactions can produce unexpected phenomena, emergent behaviour which can be lawful in its own right. Yet this is far from obvious if we study economics.

Most of twentieth-century economics has been reductionist in character. Reductionism tries to break down complex economies into simpler parts, like industries and households, and those parts, in turn, into even simpler ones, like jobs and persons. While this approach has enjoyed some success, it's also left us with a major void. Reductionism can never tell us how our economy really works? To find this out, we must combine our knowledge of the smallest parts, the individual agents, with our knowledge of their interactions, to build up a behavioural picture of the whole economy? To date, macroeconomics has not devised a convincing way of doing this.

Almost thirty years of research have convinced me that the conventional wisdom in economics fails to explain how economies behave collectively and develop over time. There are several reasons for this. First, the key elements of our economy, human agents, are not homogeneous. They're amazingly diverse. Second, human reasoning is not just deductive, it's often inductive, intuitive, adaptive. Third, geographical and economic patterns that we take for granted have not been forged by economic necessity alone. They're the outcome of a highly evolutionary interplay between two different architects: the expected and the unexpected. Yet it's the world of the expected, where necessity rules, that dominates our classical views about social and economic behaviour. This classical economic world is a fully deterministic one, a world of stasis resting at a stable equilibrium.

A world at rest is a world that isn't going anywhere. Static determinism has been bought at the expense of structural change. Our world is not static, but incredibly dynamic. And it's this dynamic world, where chance reigns supreme, that's triggered most of our economy's significant developments. To learn how to live with the unexpected, we must look into this dynamic world more deeply. And that's precisely what this book does. What we find is a world that's often far from equilibrium, a world that's teeming with complex interactions between coevolving agents, a world that literally begs us to be more adaptive. These are the real games that agents play. In short, we live in a world of morphogenesis, working to shape our future just as it has carved out our past.

What follows is a search for the laws of complexity that govern how human agents interactively alter the state of economies. Economies don't merely evolve over time, they coevolve. What people believe affects what happens to the economy and what happens to the economy affects what people believe. This positive feedback loop is the signature of coevolutionary learning. Some investment gurus call it "reflexivity." In a nutshell, success or failure for various agents depends on which other agents are present, because their own state depends on the states of these other agents. Agents learn and adapt in response to their unique experiences, such that the aggregate economy evolves in a manner determined by the pattern of their interactions. If this pattern catalyzes unforeseen chain reactions of change, the collective outcome can surprise everyone. Economies self-organize. Sometimes something unexpected emerges.

Some of this emergent behaviour is discussed and illustrated in the pages of this book, which takes a look at a handful of unexpected socio-economic changes during the last millenium. We find ourselves poised on the threshold of a new kind of social science: the science of surprise. Oddly enough, we seem to be dancing to a particular tune, conjured up by an invisible choreographer. The score designed by this choreographer suggests an implicit faith in two things: adaptive learning and self-organization. If this is true, then the social sciences are entering a new era, one in which more and more economists will conduct experiments inside their own computers. Instead of traditional, closed-form models, the new scientific tool for these lab experiments will be agent-based simulations. Welcome to the Age of Artificial Economics!

Acknowledgments

Shortly after I moved to Sweden in 1986, Åke E. Andersson suggested that a book be written on knowledge, networks and economic development. He envisaged that the two of us would join forces with our creative colleague in Kyoto, Kiyoshi Kobayashi. That book remains unwritten. In the meantime, Åke has written at least five books on this subject in Swedish and Kiyoshi has probably written the equivalent of five in Japanese. Despite my natural command of the English language, this is my very first. Some of us are living proof of the pervasiveness of slow processes!

Immense thanks are due to Åke for his inspiring insights into slow and fast processes, the C-society, and the catalytic role of networks. While Director of the Institute for Futures Studies (IFS) in Stockholm, he provided generous grants to transform my thoughts into written words. Given the Institute's stimulating atmosphere, perhaps it's not surprising that I hastened slowly! Timely reminders and pragmatic suggestions came from a scientific ringmaster at the IFS, Folke Snickars. Gradually, a draft manuscript began to take shape, aided by creative IFS residents and visitors. Helpful in many ways at this early stage were Martin Beckmann, John Casti, Börje Johansson, T.R. Lakshmanan, Don Saari, Peter Sylvan, and Wei-Bin Zhang.

Returning to Australia, an unexpected phase transition occurred: I lost my enthusiasm for the manuscript. A critical review by Kevin O'Connor identified the need for a major rewrite. Fortunately, stays at Monash and Curtin Universities revived my flagging morale. The rewrite was duly completed. Special thanks go to Kevin and a close friend, Barry Graham, for organizing these opportunities. Further suggestions by Bertil Marksjö and two anonymous reviewers have generated valuable refinements to the final manuscript.

Though it may appear to be the work of one author, this book is precisely the opposite. It's packed with the creative ideas of many gifted scholars. Two scientists who inspired me in the early days were the joint pioneers of self-organization: Hermann Haken and Ilya Prigogine. More recently, the work of the Sante Fe Institute, notably that of Brian Arthur and Stuart Kauffman, has left an indelible impression. In addition to the IFS scholars mentioned above, many others have helped to shape various parts of the manuscript. Among these, I want to thank Chris Barrett, Sergio Bertuglia, Dimitrios Dendrinos, Britton Harris, Jeff Johnson, Dino Martellato, and Aura Reggiani. Organizational support from CERUM in Umeå (where it all started), the Regional Planning Group at the Royal Institute of Technology in Stockholm (who provided a second office) and the Swedish Council for Building Research (who funded my research chair in Sweden) is also gratefully acknowledged.

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List of Illustrations

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Chance and Necessity

"Everything existing in the universe is
the fruit of chance and of necessity."
DEMOCRITUS

{A}Wetting the Appetite{/A}

According to the MIT economist, Paul Krugman, we're caught up in the "Age of Diminished Expectations."[1] Sluggish growth, and the persistent stagnation of living standards since 1973, have triggered a great deal of critical debate about economics. In many parts of the Western world, it's been the age of the *policy entrepreneur*: that economist who tells politicians precisely what they want to hear. Thankfully, the nonsense preached by some of these opportunists has been condemned by most serious economists.[2] But the fallout still lingers. In the eyes of an unforgiving public, misguided policy entrepreneurship has undermined the credibility of economics as a trustworthy discipline.

Oddly enough, the problem with economics is much more challenging than most policy entrepreneurs and many economists would have us believe. The truth is that we know very little about how people, societies and economies are likely to change as time goes on. But admission of ignorance is hardly a suitable trait for a policy entrepreneur or an academic, so it's difficult to get this message of uncertainty across to the the public.

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[1] Krugman presents some enjoyable anecdotes about economic sense and nonsense in the political economy of the last few decades in the United States. For a closer look at why our expectations have diminished, see Krugman (1994b,c).

[2] A typical example of policy entrepreneurship was the spurious "supply-side economics", which proliferated during Ronald Reagan's term of Presidency.

Krugman tells a cute story of an Indian-born economist, who tried to explain his personal theory of reincarnation to his graduate economics class: "If you are a good economist, a virtuous economist," he said, "you are reborn as a physicist. But if you are an evil, wicked economist, you are reborn as a sociologist."[3] If you happened to be a sociologist, you'd have every right to be upset by this. How could a subject that's fundamentally about human beings, with all their idiosyncrasies, possibly hope to solve its problems with the mathematical certainty of the hard sciences? You're probably thinking that there's too much mathematics in the economics journals. Economics is not just mathematics. Fondly enough, the Indian-born economist was making a different point. His real message was that the more we learn about the economy, the more complicated it seems to get. Economics is a *hard* subject. Economists like Krugman believe that it's harder than physics.[4]

Is economics harder than physics? Before we try to answer this question, let's hear what another well-known economist has to say. Paul Samuelson feels that we can't be sure whether the traditional methods of the physical sciences – observation, quantitative measurements, and mathematical model building – will ever succeed in the study of human affairs.[5] Part of his reasoning is that physics relies on controlled experiments, whereas in the socio-economic fields it's generally impossible to perform such experiments. Nevertheless, experiments in the form of computer simulation have begun in earnest in the social sciences. In the short space of twenty years, a small group of evolutionary economists have embarked on a fascinating journey towards wider use of experimental methods. As we'll see shortly, agent-based simulation is at the forefront of this new world of economic theorizing.

Samuelson also claims that physics is not necessarily as lawful as it appears, because the so-called laws of physics depend subjectively on one's point of view. How

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[3] See Krugman (1994c), page xi.

[4] Krugman is certainly not alone in this belief. Another of like mind is Herbert Simon, who has argued that the seemingly "soft" social sciences are really "hard" (i.e. difficult); see Simon (1987).

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[5] See Samuelson (1976), page 10.

we perceive or interpret the observed facts depends on the theoretical spectacles we wear. Part of his argument is based on an ambiguity drawn from the visual perception of art. Take a close look at Figure 1.1. Do you see birds gazing to the left or antelopes staring to the right? Perhaps you see rabbits instead of antelopes? All answers are admissible, but someone who has no knowledge of living creatures might say that each shape is simply a continuous line between two points plus a closed curve that, unlike a bird or an antelope or a rabbit, is topologically equivalent to a straight line plus a circle. There's no universal truth in a picture like this. Multiple impressions prevail.

[Figure 1.1 near here]

Samuelson's point about the subjectivity of science is an important one. Various leading schools of scientific thought argue that physical reality is observer-created.[6]. If there's doubts about the existence of a unique, observer-independent reality in the physical world, what are our chances of coming up with universal laws that are mathematical in the fuzzy world of human decisionmaking? Rather slim, one would have thought. But before we launch into a deeper discussion of how law-abiding our socio-economic behaviour might be, let's take a closer look at the conventional view of what physics and economics are construed to be.

Physics is the science of matter and energy, and their interactions. As such, it does very well at explaining simple, contained systems – such as planets orbiting the sun. In classical physics (and chemistry, for that matter) the conceptual palette used to paint the big picture is thermodynamics. Of great significance in this field is the equilibrium state, that full stop at the end of all action.

To gain a mental picture of a state of equilibrium, consider what would happen if you released a marble near the top of a mixing bowl, pushing it sideways. There are no prizes for guessing where it will end up. After rolling around briefly, it falls to the bottom of the bowl under the influence of gravity. Eventually it settles in the centre

[6] For an entertaining summary of the arguments, both for and against, the contention that no objective reality exists independent of an observer, see Casti (1989, Chapter 7).

where its motion ceases. The convex shape of the bowl "attracts" the marble to its base. In mathematical jargon, this point of stability is even called an *attractor*. Once it reaches that safe haven, its pretty much like the equilibrium state of a chemical reaction. It's trapped in a minimum energy state. To simplify matters, we'll just say that it's trapped in the world of *stasis*.[7]

A system at a stable equilibrium *is* trapped. It's like a crystal, not doing anything or going anywhere. It becomes immortal, forever frozen into an ordered state. With the advent of Newtonian mechanics, much of physics found itself locked inside this world of stasis. And for very good reasons. Newton's laws of motion strengthened our faith in this immortal world, because his laws are a classical example of determinism. At the dawn of the twentieth century, most physicists agreed that the fundamental laws of the universe were deterministic and reversible. The future could be uniquely determined from the past. All that occurred had a definite cause and gave rise to a definite effect. Since predictability was the ruling paradigm, a mathematical approach worked perfectly.

But this kind of physics breaks down badly if called upon to explain nature and all its magic. Imagine trying to forecast weather patterns using the properties of a stable equilibrium. Faced with these stark realities, physics was forced to move on. And move on it has. The advent of quantum physics made sure of that. As we enter the new millenium, a large number of physicists will have agreed that many fundamental processes shaping our natural world are stochastic and irreversible. Physics is becoming more historical and generative. Of course, headaches like weather forecasting will remain. Despite massive expenditure on supercomputers and satellites, predicting the weather remains an inexact science. Why? Because it rarely settles down to a quasi-equilibrium for very long. On all time and distance scales, it goes through never-repeating changes. Our climatic system is a complex dynamic system.

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[7] The term "stasis" is an abbreviated form of the word "morphostasis", a group of negative feedback processes studied in cybernetics. Since its inception, cybernetics has been more-or-less regarded as the science of self-regulating and equilibrating systems. But its scope turns out to be broader, as we'll learn in Chapters 2 and 3.

Unlike physics, economics has hardly changed at all. Despite the rumblings of a handful of evolutionary economists, its central dogma still revolves around stable equilibrium principles. Goods and services are assumed to flow back and forward between agents in quantifiable amounts until a state is reached where no further exchange can benefit any trading partner. Any student of economics is taught to believe that prices will converge to a level where supply equates to demand.

Boiled down to its bare essentials, equilibrium economics is no more sophisticated than water flowing between two containers.[8] Suppose a farmer owns two water tanks, which we'll call "A" and "B." A contains eighty litres of rainwater, while B has twenty litres. One day the farmer decides to combine his water resources by linking the tanks. He lays a pipe from A to B, allowing water to flow between them until the levels in each are identical (see Figure 1.2).[9] For all intents and purposes, this balanced equilibrium outcome is imperturbable. Obviously, the water level in each tank will always match perfectly unless the pipe is blocked.

[Figure 1.2 near here]

Now substitute fruit for water. Suppose that farmer A has a case of eighty apples and farmer B a bag of twenty oranges. Because farmer A is fond of oranges and farmer B loves apples, they agree that an exchange would serve their joint interests. Apples being far more plentiful than oranges, farmer B sets the price: four apples for every orange. They agree to trade. Farmer A parts with forty apples in return for ten oranges. Both end up with fifty pieces of fruit. Being equally satisfied with the outcome, there's no point in trading further. Displaying perfect rationality, each farmer deduces the optimal strategy. The equilibrium outcome turns out to be predictable and perfectly stable. Just like the two tanks of water.

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[8] This simple analogy was suggested by the physicist, Per Bak. For an unconventional look at the boundary between the natural world and the social sciences, see Bak (1996, especially Chapter 11).

A stable equilibrium *is* the best possible state in a static world. There's simply nowhere better to go. Everything adds up nicely and linearly. The effect on the water level of adding additional litres of water is proportional to the number of litres added. Generalizing to many agents simply corresponds to connecting more tanks together. In physics, this kind of treatment is referred to as a "mean field approximation." A single macrovariable, such as the water level, is considered. Many traditional economic theories are mean field theories, to the extent that they focus on the macrovariables that are associated with an equilibrium state. Examples are GNP (gross national product), the interest rate or the unemployment rate.

Mean field theories work quite well for systems that are static and ordered. They also work well for systems that are full of disorder. However, they don't work well for systems that are subject to diversity and change. For example, they don't work well when differences in economic agents' behaviour become so significant that they can't be overlooked. Furthermore, they don't work well if our economy happens to be at or near a bifurcation point, such as a critical stage of decisionmaking. In short, they don't work well if we wish to understand all those weird and wonderful ways in which the economy really works.

The point of departure for this book, in fact, is that our economic world is *heterogeneous and dynamic*, not homogeneous and static. It's full of pattern and process. Development unfolds along a trajectory which passes through a much richer phase space, one in which multiple possibilities abound. While this creates spectacular diversity, it also poses a major problem. How do we predict likely outcomes, least of all the whole development process, if we don't know what the system's trajectory looks like along the way? It's mostly impossible to predict details of this trajectory unless we know exactly what the system's initial state was. As well as this, many other questions arise. Does the system reach any equilibrium state at all? If it does and such equilibria are temporary, when will it move on? What happens when it's far from equilibrium?

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[9] Tank A can be thought of as "selling" thirty litres of water to Tank B. In this abstract case, the "selling" price would need to cover the price of the pipe.

In a dynamic economy, traditional equilibrium models only provide a reasonable description of the state of an economic system under very limited circumstances: namely if the system just happens to evolve towards a fixed-point attractor. We can think of a fixed-point attractor as a point along the way, with a signpost saying: "Endpoint: all motion stops here!" Under different conditions, however, an economic system may never reach such a point. There's growing evidence that certain economic processes may never come to such a deadend.[10] Instead, some may converge towards a periodic attractor set, or to a chaotic attractor.[11] Because periodic attractor sets are unstable, one imagines that their signposts might say: "Resting place: stop here briefly!" A suitable sign for a chaotic attractor will be left to the avid reader's imagination.

What, then, is the best possible state in a dynamic world? This is a very thorny question to answer. Consider the following statement in a recent book exploring facets of the new science of complexity: "In the place of a construction in which the present implies the future, we have a world in which the future is open, in which time is a construction in which we may all participate."[12] These are the words of the Belgian chemist, Ilya Prigogine, 1977 Nobel laureate in chemistry for his novel contributions to non-equilibrium thermodynamics and the process of self-organization. They remind us that, in an open, dynamic world, we find evolution, heterogeneity and instabilities; we find stochastic as well as deterministic phenomena; we find unexpected regularities as well as equally unexpected large-scale fluctuations. Furthermore, we find that a very special kind of transformation can occur. Many systems *self-organize* if they're far-from-equilibrium. Obviously, we must postpone our discussion of what constitutes the best

[10] In 1972, Hugo Sonnenschein surprised many mathematical economists by showing that the rule of price adjustment arising from a given set of agent preferences and endowments can literally be *any* rule you like. More importantly, it need not be the kind of rule that leads to one of Adam Smith's invisible-hand equilibria. In view of this result, a static equilibrium becomes a very unlikely state of economic affairs. For a discussion of Sonnenschein's result, as well as some other paradoxical aspects of economic processes, see Saari (1995).

[11] For the technically-minded, a *fixed-point* attractor contains only one state; a *periodic* attractor set is a sequence of states periodically occupied by the system at each iteration; a *chaotic* attractor doesn't show any simple geometrical structure, but is often fractal, and is such that the sequence of states depends sensitively on the initial state.

possible state in such a world until we know much more about it. We'll look at the nitty-gritty of self-organization in the next section.

One thing is certain: we live in a pluralistic economy. Pluralism stems from the fact that trajectories of economic development depend on the deterministic and the stochastic. Moreover, some processes are reversible while others are irreversible. Since there's a privileged direction in time, what we're beginning to realize is that many economic phenomena appear to be stochastic and irreversible. For example, an economy that started as a primitive, agrarian one may eventually develop a more sophisticated, multisectoral structure. By evolving towards a more complex state, an economy gives the impression that it can never return to its original, primitive state. But the more sophisticated it becomes, the more difficult it is to predict what it will do next.[13] To understand the multitude of ways in which economies can change, we must acknowledge the existence of stochastic processes - those whose dynamics is nondeterministic, probabilistic, possibly even random and unpredictable. A high degree of unpredictability of the future may well be the hallmark of human endeavour, be it at the individual level of learning or at the collective level of history making.

Another Nobel laureate in the natural sciences, the biologist Jacques Monod, puts the argument for pluralism concisely: "Drawn out of the realm of pure chance, the accident enters into that of necessity, of the most implacable certainties."[14] Our world is pluralistic because two "strange bedfellows" are at work together: chance and necessity. Chance events, or accidents of history, play a vital role whenever an economy's trajectory of development is confronted with alternative choice possibilities. We can think of them as key moments of decision. Technically, they're points of instability or bifurcation.

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[12] See Nicolis and Prigogine (1989), page 3.

[13] Archaeologists think in terms of millenia instead of merely generations or centuries. Thus they know that economies can collapse rapidly and revert to a more primitive regime. In relatively isolated cultures, the socioeconomic process can also become trapped in a more-or-less stationary or fluctuating state for a very long time. A simulated example of this kind of socio-economic dynamics is discussed in Chapter 8. Another unusual model describing socioeconomic evolution in the very long run can be found in Day and Walter (1995).

[14] See Monod (1971), page 118.

Alternative pathways into the future introduce an element of uncertainty which, in turn, invalidates simple extrapolations. Under these conditions, prediction of future economic outcomes becomes impossible.

[Figure 1.3 near here]

This book will argue that we live under just such conditions. More exactly, we're both spectators and participants in a dynamic, pluralistic economy. Patterns of economic evolution change by way of fluctuations in time and space. The interesting thing is that seemingly simple interactions between individual agents can accumulate to a critical level, precipitating unexpected change. What's even more surprising is that some of this change can produce patterns displaying impressive order. Order through fluctuations, if you like. We're left wondering whether the sole source of this order is "chance caught on the wing", as Monod suggests. On the surface at least, there seems to be more to it than that. The rest of this book attempts to find out.

{A}Sandpiles, Self-Organization and Segregation{/A}

Perhaps you're beginning to wonder whether a dynamic economy ever reaches any equilibrium state? Surprisingly enough, the answer to this question may have more in common with piles of sand than with tanks of water, according to the physicist, Per Bak.[15] Decisions made by human agents tend to be discrete, like grains of sand; not continuous, like levels of tank water. Many decisions are sticky. So are many prices. We buy or sell many capital goods only when the need arises or the opportunity of a bargain presents itself, remaining passive in between. We buy or sell stocks and shares only when some threshold price is reached, remaining passive in between. Very few of us *continually* adjust our own holdings in response to fluctuations in the market. In other

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[15] See Bak (1996).

words, there's plenty of friction in real economies; just like in sandpiles.[16] It might just be the friction of distance that binds villages, towns and cities together in special patterns to form a stable, yet dynamic, economy. Oddly enough, it's also friction that prevents a sandpile from collapsing completely to a flat state. It may even be responsible for a special kind of dynamic equilibrium.

No doubt you're thinking to yourself: 'Economic agents can think but grains of sand can't think! Surely economics must be more sophisticated than sandpiles!' Perhaps you're right. But before we start to delve more deeply into the quirks and foibles of economic agents, let's explore a few of the surprising features of "unthinking" sandpiles. Try the following experiment in your backyard sandpit. Starting from scratch on a flat base, build up a pile by randomly adding sand at the centre; slowly and carefully, a few grains at a time. Notice how the grains tend to stick together. The peaked landscape formed by the sand doesn't revert automatically to the flat state when you stop adding sand. Static friction keeps the pile together. Gradually it becomes steeper. Then a few small sand slides start to occur. One grain lands on top of others and topples to a lower level, causing a few other grains to topple after it. In other words, that single grain of sand can cause a local disturbance, but nothing dramatic happens to the pile as a whole.

At this formative stage, events in one part of the pile have no effect on other grains in more distant parts of the pile. We might say that the pile is only weakly-interactive, featuring local disturbances between individual grains of sand. As you add more grains and the slope increases, however, a single grain is more likely to cause a larger number of others to topple. If you've created it properly, eventually the slope of your pile will reach a stationary state - where the amount of sand you add is balanced on average by the amount falling off.

There's something very special about this stationary state. Remember that you're adding sand to the pile in the centre, but the sand that's falling off is at the edges. For this to happen, there must be communication between grains at the centre and grains at the edge. How on earth could grains of sand communicate with each other? What transforms this collection of grains from a weakly-interactive to a strongly-interactive pile? Perhaps there's communication throughout the entire pile. In the words of its discoverer, Per Bak, the sandpile has *self-organized*. It has attained a self-organized critical state.

The marvelous thing about self-organization is that it can transform a seemingly incoherent system into an ordered, coherent whole. Weakly-related grains of sand suddenly become a strongly-interactive sandpile. Adding a few grains of sand at a crucial stage transforms the system from a state in which the individual grains follow their own local dynamics to a critical state where the emergent dynamics are global. This is a transition of an unusual kind: a *non-equilibrium phase transition*. Space scales are no longer microscopic, suddenly they're macroscopic. A new organizing mechanism, not restricted to local interactions, has taken over. Occasional sandslides or *avalanches* will span the whole pile, because the sandpile has become a complex system with its own emergent dynamics. What's most important is that the emergence of the sandpile, with its full range of avalanche sizes, could not have been anticipated from the properties of the individual grains.

Now go back to your own sandpit again. Once you've reached this critical state, try adding more sand. See how it slides off. Try adding wet sand instead. Wet sand has greater friction, so the avalanches will be smaller and local for a while. Your pile becomes steeper. But eventually it will return to the critical state with system-wide avalanches again. Admittedly it's not an easy experiment to conduct successfully. So you may need to try the whole thing again if you're not convinced. The pile always bounces back whenever you try to force it away from this critical state. Formally speaking, it exhibits homeostasis. In other words, it's resistant to small perturbations.

Another fascinating thing is that the whole sandpile evolves to this critical state independently of any intentions on your part. You can't force it to do something else. In

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[16] The effects of friction in economics have been at the core of distribution and welfare issues for more than a century. For a review of such frictional effects, see Griffin (1998). Other chapters in the same book also highlight the importance of friction; see Åkerman (1998).

fact, you can't control it at all. All you can do is add sand, a few grains at a time. Nobody knows the sandpile's initial conditions. Whatever they happen to be is of no significance anyway. Repeated experiments produce the same result. In the words of Stuart Kauffman, Santa Fe Institute scientist and devout advocate of self-organization, this kind of emergent order seems to be the work of an "invisible choreographer."[17] An ordered pattern has sprung up from nowhere. Order through fluctuations, if you like. Technically speaking, the critical state is an attractor for the dynamics. It's a dynamic equilibrium.

We can now return to that challenging question posed earlier. What's the best possible state in a dynamic world? With all its fluctuations, perhaps the self-organized critical state doesn't strike you as being the very best possible state. But it might just be the best of all those states that are dynamically feasible and more-or-less efficient from a collective viewpoint.

So what, you might say! This still has nothing to do with economics. Yes, I remember. People can think, but grains of sand can't think. So it's time to take a look at some of those quirks and foibles of human nature. To introduce the human element, we turn to work done a generation ago by Harvard's Thomas Schelling.[18] His ideas on complexity and self-organization were summed up in a deceptively simple account of how people in a city could get segregated. In this section, we'll simply describe the model and its results. In later chapters, we'll elaborate on the implicit features of Schelling's important work. In particular, we'll look at other collective outcomes that were neither expected nor intended by the agents who engineered them. Such outcomes turn out to be instances of self-organization, i.e. emergent order through fluctuations.

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[17] See Kauffman (1995), page 209.

[18] Schelling's ideas on complexity and self-organization can be found in a book entitled *Micromotives and Macrobehaviour*. Krugman suggests that the first chapter of this book is "surely the best essay on what economic analysis is about, on the nature of economic reasoning, that has ever been written." (Krugman, 1996, page 16). The two chapters on "sorting and mixing" provide an excellent, non-mathematical introduction to the idea of self-organization in economics. See Schelling (1978) for the original material and Krugman's book for a modern interpretation.

In Schelling's model, there are two classes of agents. He thought of them as blacks and whites, but they could be any two classes of individuals that have some cultural difficulty in getting along together - e.g. boys and girls, smokers and non-smokers, butchers and vegetarian restaurants. Instead of a sandpit, a chessboard can play the role of our "simplified city". Think of the sixty-four squares as a symmetrical grid of house locations, although the principles hold just as convincingly over much larger (and irregularly-shaped) domains.

The key thing is that each agent cares about the class of his immediate neighbors, defined as the occupants of the abutting squares of the chessboard. Preferences are honed more by a fear of being isolated rather than from a liking for neighbors of the same class. It's pretty obvious that such preferences will lead to a segregated city if each agent demands that a majority of his neighbors be the same class as himself. But the novelty of Schelling's work was that he showed that much milder preferences, preferences that seem to be compatible with an integrated structure, typically lead to a high degree of segregation - once the interdependent ramifications of any changes are considered.

Consider the following simple rule: an individual who has one neighbor will only try to move if that neighbor is a different class; one with two neighbors wants at least one of them to be of the same class; one with three to five neighbors wants at least two to be his or her class; and one with six to eight neighbors wants at least three of them to be like him or her.[19] At the level of each individual, this rule of neighborhood formation is only mildly class-conscious. For example, with these preferences it's possible to form an integrated residential pattern that satisfies everybody. The familiar checkerboard layout, where most individuals have four neighbors of each class, does the trick as long as we leave the corners vacant.

Nobody can move in such a layout, except to a corner. There are no other vacant cells. But nobody wants to move anyway. Because it's an integrated equilibrium structure, there's no incentive to change it. But what if a few people are forced to move.

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[19] An equivalent way of stating this rule is that each individual is satisfied as long as at least 37 percent of his or her neighbors are of his or her type.

What if three neighbors, who happen to work together, are relocated interstate by their company. They must sell up and move to another city. Will the integrated equilibrium remain? Let's try to find out.

After they move out, the neighborhood layout looks like the chessboard shown on the left hand side of Figure 1.4. The departing workmates vacate the squares located at coordinates C4, D3 and E2. Once they move out, other nearby neighbors of the same type suddenly feel too isolated. For example, residents at D1 and F1 discover that only one of their four neighbors is the same type as them. Thus they decide to move to locations where the neighborhood rule is satisfied again, say A1 and H8.

[Figure 1.4 near here]

A self-reinforcing pattern of interdependency quickly becomes evident. Another resident can become unhappy because the departing resident tips the balance in his neighborhood too far against his own class, or because his arrival in a new location tips the balance there too far against agents of the other class. Surprisingly, our integrated equilibrium begins to unravel. An unsatisfied individual at C2 moves to C4, leaving another at G2 with nowhere to go. G2 has no alternative but to move out of the city completely, precipitating a chain reaction of moves in response to his decision. Residents at F3, H3, G4, H5, E4, F5 and G6 all follow suit. Despite the fact that agents only have mild preferences against being too much in the minority, some of them are forced to move out and pockets of segregation begin to appear on our chessboard city (see Figure 1.4b).

There are now 49 agents residing in the city. Let's trigger some more change by removing another nine of them using a random number generator, then picking five empty squares at random and filling them with a new class of agent on a 50/50 basis. In a similar manner, Schelling showed that an equilibrium like that in Figure 1.4(a) was unstable with respect to some random shuffling, and tends to unravel even further. Figure 1.5(a) shows the result after my random number generator has done the job.

[Figure 1.5 near here]

It's clear that some other residents will now be unhappy with their locations and will move (or move again). Seemingly simple moves provoke responses. Thus a new chain reaction of moves and countermoves is set in motion. To simulate this chain reaction on a computer, the order in which people move, and the way they choose their new location, would need to be specified. As we're doing this by hand on a chessboard, we can watch the structure evolve. When it finally settles down, my series of moves leads to the layout shown in Figure 1.5(b).[20]

What a surprise! Even though the individuals in our city are tolerant enough to accept an integrated pattern, they end up highly segregated. Even though their concerns are local - they only care about the type of their immediate neighbors - the whole chessboard gets reorganized into homogeneous residential zones. How remarkable that short-range interactions can produce large-scale structure. Like the sandpile we discussed earlier, our chessboard city has been self-organized. Large-scale order has emerged from a disordered initial state. Segregation may not be our favorite form of order, but it's order nevertheless. All of our city-dwellers in Figure 1.5(b) are now content.

This large-scale order emerges because the original state - that integrated pattern shown in Figure 1.4(a) - is *unstable*. Scramble it a little and you trigger a chain reaction of moves that eventually produces a strongly segregated city. We could say that you get *order from instability*. This is one of the hallmarks of self-organization.

The interesting thing is that such a chain reaction of moves never would have happened if class consciousness had been slightly weaker. Schelling fine tuned his rules very carefully. His resident could only be satisfied if at least 37.5% of his or her neighbors were of the same class. If that figure had been 33.3%, then only one resident in

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[20] This layout is one of a number of possibilities, since the order in which individuals move remains unspecified. The final outcome will also be sensitive to the initial conditions (as depicted in Figure 1.4b). As Schelling noted, repeating the experiment several times will produce slightly different configurations, but an emergent pattern of segregation will be obvious each time.

Figure 1.4(a) - the one located at position F1 - would have wanted to move. Once he had moved - say to A1 - then everyone else in the city would have been satisfied. In other words, the original integrated equilibrium would have remained stable. This makes it clear that a very small change in the sensitivity of neighbors (i.e. the migration rule) can result in a very large change in the number of moves. Conversely, if the figure had been 50% instead of 37.5%, then a highly segregated residential pattern would have appeared immediately.

The sudden and unexpected appearance of highly segregated areas, when the migration rule is increased from 33.3% to 37.5%, is indicative of a qualitative change in the aggregate pattern of behaviour. We might say that the location pattern has "flipped" into an entirely different state. In fact this nonlinear change is indicative of something like a *phase transition*. Alternatively, it's the kind of nonlinear jump portrayed in percolation theory. Both of these abrupt transitions are shown in Figure 1.6. At first the integrated equilibrium remains rather stable to slight increases in class consciousness. Then, rather suddenly, the number of moves skyrockets dramatically. Once the whole city has reached this state of self-organized criticality, various avalanches of change (in the form of clusters of migration of different sizes) can occur. Just like those sandslides we referred to a little earlier. Global order emerges from the expanding reach of local interactions.

[Figure 1.6 near here]

The idea that local interactions can produce global structure - via non-equilibrium phase transitions - came from the pioneering work of some physicists and chemists studying self-organization in physical systems.[21] Yet Schelling's model permits us to see

[21] The notion of phase transitions has its roots in the physical sciences, but it's relevance to economic evolution has been recognized recently. In the social sciences, phase transitions are difficult to grasp because the *qualitative* changes are hard to see. Far more transparent is the effect of temperature changes on water. As a liquid, water is a state of matter in which the molecules move in all possible directions, mostly without recognizing each other. When we lower its temperature below freezing point, however, it changes to a crystal lattice - a new solid phase of matter. Suddenly, its properties are no longer identical in all directions. The translational symmetry characterizing the liquid has been broken. This type of change is

exactly how the process works in a socio-economic context. To some extent, of course, the model oversimplifies urban realities. The tendency is to divide the whole city into vast # and O areas. What typically happens in a real city is that the chain reaction of moving households dies out at some point, leaving the city locked into various # and O domains of different sizes. And the resulting classes of individuals are not simply two-dimensional. They're n-dimensional, so much so that it's sometimes difficult to discern the true class or "colors" of all your neighbors. Despite these drawbacks, Schelling's insights were well ahead of their time, and the rich dynamics contained therein are extraordinary.

{A}Power Laws and Punctuated Equilibria{/A}

Odd as it may seem, Bak's sandpile experiment and Schelling's segregation model have plenty in common. First and foremost, both are examples of self-organizing systems. They develop macroscopic order without interference from any outside agent. Nothing more than the local, dynamic interactions among the individual elements are needed to produce this global order. Each system gets transformed from a state where individual elements follow their own local rules, to one displaying an emergent, global pattern. Space scales that were once microscopic, suddenly become macroscopic. Even more mysteriously, an unexpected and unpredictable chain reaction of events produces this coherent, stationary state.

What an incredible discovery! A mysterious process called self-organization can transform disordered, incoherent systems into ordered, coherent wholes. What's even more amazing is that each emergent whole could not have been anticipated from the properties of the individual elements. Order from incoherence. Who would have thought that a coherent sandpile could result from so many weakly-interactive grains of sand.

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known as an equilibrium phase transition. Recent advances in systems theory, especially studies led by Ilya Prigogine and the Brussels school of thermodynamacists, have discovered a new class of phase transitions - one in which the lowering of temperature is replaced by the progressively intensifying application of nonequilibrium constraints. It's nonequilibrium phase transitions that are associated with the process of self-organization. See, for example, Nicolis and Prigogine (1977, 1989).

Who would have thought that a strongly segregated city could result from such weakly sensitive rules about local neighborhood structure?

But that's not all. Once these systems reach a state of spontaneous order, their holistic behaviour seems to follow a dynamic pattern which is lawful in its own right. Take sandpiles first. Minor disturbances to a self-organized sandpile can trigger avalanches of all different sizes. Most of these avalanches are small, toppling only a few grains at a time. Some are much larger. Now and then, an avalanche collapses the entire pile. If we were clever and patient enough, we could measure how many avalanches there are of each size, just like earthquake scientists measure how many earthquakes there are of each magnitude. Let's skip this step and assume that we already have the data. An interesting thing might happen if we could plot the size distribution of avalanches on double logarithmic paper. The likely outcome is shown in Figure 1.7.

[Figure 1.7 near here]

Surprisingly, the result is a straight line. The x-axis shows the size class, c, to which each avalanche belongs, whereas the y-axis shows how many avalanches, N(c), occurred in that size class. Linearity on a log-log plot confirms that the number of avalanches is given by the simple power law:

$$N(c) = c^{-s} .$$

Taking logarithms of both sides of this equation, we find that

$$\log N(c) = -s \log c$$

Thus the exponent s is nothing more than the slope of the straight line formed when log N(c) is plotted against log c.

Now reconsider Schelling's segregated city. Chain reactions of relocation – like the sequences of household moves that were triggered by small disturbances to the original, integrated equilibrium – bear a striking resemblance to the avalanches of change depicted in Figure 1.7. For starters, the majority of such chain reactions in a city tend to be small in terms of spatial scale. Most of them die out locally. But the few larger ones affect a bigger catchment area of residents. Very occasionally, a modest disturbance in a city can trigger a huge chain reaction of responses across the city. Such a skewed size distribution of chain reactions has much in common with the distribution of avalanches underpinning the sandpile model. If we were to collect the data or compute the possibilities exhaustively, the size distribution of chain reactions in our chessboard city would surely obey a power law distribution. Once again, the aggregate pattern of potential moves may be lawful in its own right.

There's another reason for suspecting that the size distribution of chain reactions leading to segregation may conform to a power law. Schelling's chessboard city, together with his rules determining moves to other locations, correspond to a two-dimensional cellular automaton. Cellular automata were originally put into practice by John von Neumann to mimic the behaviour of complex, spatially extended structures.[22] Because they're really cellular computers, today they're being put to use as simulators, designed to help with time-consuming calculations by taking advantage of fast parallel processing.[23] Since cellular automata employ repetitive application of fixed rules, we should expect them to generate self-similar patterns. Indeed, many do produce such patterns. If Schelling had used computer simulation to explore a much larger chessboard city, self-similar patterns of segregation may have even been visible in his results. Being akin to periodicity on a logarithmic scale, such self-similar patterns would conform to power laws.

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[22] See von Neumann (1966).

[23] In later chapters, we'll discuss various examples of cellular automata that have been used to sharpen our intuition about socio-economic behaviour via computer simulation. Schelling's model is not strictly a cellular automaton, since it allows agents to migrate from one cell to another. The interested reader can find a cellular automaton defined and applied to urban dynamics in Chapter 5.

Although it's too early to say for sure, it's likely that many dynamic phenomena discussed in this book obey power laws.[24] Power laws mean scale invariance, and scale invariance means that no kinks appear anywhere. Economic change may be rife with scale invariance. Nearly 100 years ago, the Italian economist, Vilfredo Pareto, found that the number of people whose personal incomes exceed a large value follows a simple power law.[25] In some socio-economic contexts, of course, linearity may break down at the smaller and larger scales. The fact that scaling usually has limits does no harm to the usefulness of thinking "self-similar." In the next section, we'll look more closely at scale invariance in economics. We'll take a further look at power laws when we discuss urban evolution in Chapter 5.

Yet another observation links sandpiles to economies. A great many unexpected socio-economic changes may be nothing more sinister than large avalanches which "punctuate" the quiescent state of affairs. Once it reaches a self-organized critical state, for example, a sandpile exhibits *punctuated equilibrium* behaviour. In 1972, the paleontologists, Nils Eldredge and Stephen Jay Gould, argued that evolutionary change is not gradual, but proceeds in "fits and starts".[26] Long periods of stasis are interupted, or punctuated by, bursts of dramatic change. Perhaps the most spectacular examples of such punctuations are the Cambrian explosion (500 million years ago) and the extinction of dinosaurs (about 60 million years ago). Out of the Cambrian explosion came a sustainable network of species, believed to be the collective result of a self-organized, learning process. The evolution of single species are thought to follow a similar pattern.

The theory of punctuated equilibria melds together stasis and adaptive change associated with speciation. Stasis recognizes that most species hardly change at all once they show up in the fossil record. But these quiet periods are interrupted occasionally by shorter periods, or punctuations, during which their attributes change dramatically.

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[24] In a delightful book about fractals, chaos and power laws, Manfred Schroeder reviews the abundance and significance of power laws in nature and human life; see Schroeder (1991).

[25] See Pareto (1896).

[26] See Eldredge and Gould (1972).

Speciation recognizes that major evolutionary change comes from new species, mutants which tend to show up unexpectedly. Again we find two worlds at work - speciation and stasis, punctuation and equilibria, chance and necessity.

Oddly enough, punctuated equilibria have turned up in many other places. For example, Kauffman and his colleagues at the Santa Fe Institute have produced computer algorithms that exhibit this kind of behaviour: relatively long periods of stasis interrupted by brief periods of rapid change. The dramatic changes are not coded into the programs in advance. They appear spontaneously and unexpectedly from within the programs themselves. Tom Ray, a naturalist from the University of Delaware, created an experimental world inside his computer. The digital life he created is capable of replication and open-ended evolution.[27] Part of the open-ended repertoire displayed by Ray's digital world includes "periods of stasis punctuated by periods of evolutionary change, which appears to parallel the pattern of punctuated equilibrium described by Eldredge and Gould."

Another scene of punctuated calm is the scientific world. Remember that book "The Structure of Scientific Revolutions," a best-seller in the sixties written by Thomas Kuhn.[28] Kuhn's central observation was that science proceeds for long periods as status quo paradigms, interrupted occasionally by creative spurts that finally force out the old paradigm in favour of a new one. The new arrival handles the anomalies swept under the table by its predecessor. Kuhn also argued that the historian constantly encounters many smaller, but structurally similar, revolutionary episodes that are central to scientific advance. Because the old must be revalued and reordered when assimilating the new, discovery and invention in the sciences are intrinsically revolutionary.

Economies also evolve in fits and starts. The Austrian economist, Joseph Schumpeter, coined the term industrial mutation for the process of creative destruction

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[27] In Ray's experimental world, which he calls *Tierra* (the Spanish word for Earth), self-reproducing programs compete for CPU time and memory. These programs show all of the evolutionary splendour that we have come to admire in the natural world. For further details of this digital life, which takes place inside a virtual computer, see Ray (1992).

[28] See Kuhn (1962).

that incessantly revolutionizes the economic structure from within, destroying the old one and creating a new one. He further states: "Those revolutions are not strictly incessant; they occur in discrete rushes which are separated from each other by spans of comparative quiet. The process as a whole works incessantly however, in the sense that there always is either revolution or absorption of the results of revolution."[29]

In one form or another, the idea of punctuated equilibria looks to be at the heart of the dynamics of complex systems. In fact, the footprints of power laws and punctuated equilibria can be found everywhere. They turn up in the frequency distribution of many catastrophic events - like floods, forest fires and earthquakes. They're also thought to be responsible for pink noise, and the music most listeners like best – a succession of notes that's neither too predictable nor too surprising. In each case, the activity going on is relatively predictable for quite long periods. Suddenly this quiescent state is interrupted by brief and tumultuous periods of major activity, roaming and changing everything along the way. Such punctuations are another hallmark of self-organized criticality.

Large, intermittent bursts of activity lie beyond the world of stasis. They can change the very nature of the system itself. Their effects can be self-reinforcing. Self-organization affects form and structure in a fundamental way. A world ripe with punctuations is a world of *morphogenesis*. The process of morphogenesis is ubiquitous in history, biology and economics. We can think of morphogenesis as a topological conflict, a struggle between two or more attractors. In the next section, we'll look for further footprints, direct evidence of self-organizing tendencies in the economic marketplace. Hopefully, this may dispel any lingering doubts in the reader's mind about the potential for self-organization in the economic realm!

{A}Bulls, Bears and Fractals{/A}

One of the most baffling puzzles in financial markets is the fact that academic theorists, by and large, see markets quite differently from the way that actual traders see them.

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[29] See Schumpeter (1934).

Academics see investors as being perfectly rational, thus ensuring that markets are efficient in the sense that all available information is discounted into current prices. The sole driving force behind price changes for any stock or commodity is assumed to be new information coming into the market from the outside world. Traders process this information so efficiently that prices adjust instantaneously to the news. Because the news itself is assumed to appear randomly, so the argument goes, prices must move in a random fashion as well.

Known as the efficient markets hypothesis, this notion was first put forward by the little-known French mathematician, Louis Bachelier.[30] It's a long-standing equilibrium theory which suggests that prices are unpredictable and therefore technical trading using price charts is a waste of time and money. Why is it, then, that newspapers and financial tabloids still feature graphs and advertisements by self-styled "chartists" claiming to be able to predict future price movements? Technical traders feel the geometry of price histories is important. As a result, they view markets quite differently from academics. Not only do they believe that technical trading can be profitable. Some of them have demonstrated that it *can* be consistently profitable. They also believe that factors such as market "psychology" and "herd" effects influence price changes.

Which group should we believe? It's a difficult question to settle empirically. Markets do seem to be reasonably efficient. Despite this, statistical tests and real results have shown that technical trading can produce modest profits over time.[31] Other tests have shown that trading volume and price volatility are more volatile in real markets than the standard theory predicts.[32] Temporary bubbles and slumps, like the major crash in 1987, seem well beyond the scope of rational adjustments to market news. Although a spate of economists have looked for signs that prices are being generated by chaotic mechanisms, we shall not dwell on these tests here. It suffices to say that the evidence

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[30] This hypothesis was not widely appreciated at the time, since it appeared in his doctoral dissertation on price fluctuations in the Paris bond market; see Bachelier (1900).

[31] See, for example, Brock et al. (1991).

[32] See, for example, Shiller (1981, 1989).

implicating chaos as a factor influencing price fluctuations in financial markets is mixed.[33] But there's growing evidence that markets do undergo phase transitions between two different regimes of behaviour: the simple and the complex. Could it be that chance and necessity are at play again?

What interests us most is that price histories do exhibit geometrical regularities. Charles Dow, one of America's earliest students of stock market movements, noted a certain repetition in various price gyrations. Dow observed that the market in its primary uptrend was characterized by three upward swings. But at some point in every upswing, there was a reverse movement cancelling three-eighths or more of that swing. Dow's principles motivated Ralph Elliott to develop his Wave Principle which suggests that market behaviour trends and reverses in recognizable patterns. The ever-changing path of prices reflects a basic harmony found in nature. Elliott isolated thirteen patterns, or "waves", which recur in markets and are repetitive in form, but not necessarily in time or amplitude.[34] He also described how these patterns link together to form larger versions of the same patterns. Without realizing it, he had discovered patterns of self-similarity on different timescales.

Remarkably, Elliott reached his conclusions fifty years before the advent of the science of fractals. Yet his findings showed that historical price patterns bear a striking resemblance to the fractal character of the natural world. Benoit Mandelbrot's studies of fractals and multifractals have confirmed that nature and markets abound with a special symmetry. He analysed daily and monthly data for the variation of cotton prices over different periods, drawing on statistics spanning more than a century. Then he counted how often the monthly variation was between 10 and 20 percent, how often it lay between 5 and 10 percent, and so on. After plotting the results on a double logarithmic plot, he found that the resulting distributions of price changes in different periods were horizontal

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[33] Two admirable summaries of recent findings in the ongoing search for chaos in financial markets can be found in Brock, Hsieh and LeBaron (1991) and Benhabib (1992).

[34] For an overview of Elliott's Wave Principle, see Frost and Prechter (1985).

translates of each other (see Figure 1.8). Furthermore, their shape conformed to a familiar pattern: the ubiquitous power law.[35]

[Figure 1.8 near here]

Mandelbrot was the first to interpret such power laws in terms of scaling. The unifying concept underlying fractals and power laws is self-similarity: invariance against changes in scale or size. Finding the power law distribution in financial data was a major discovery. It showed that small scale patterns combine to form similar patterns at larger scales.[36] Mandelbrot looked at price variations for other commodities, finding similar patterns which matched across different timescales. His scaling principles echo Elliott's observation that the market traces out characteristic patterns at all levels or trend sizes.

However, price charts themselves are not self-similar. A more exact term for the resemblance between the parts and the whole in financial markets is *self-affinity*. Mandelbrot concluded that much in economics is self-affine. Two renormalized price charts will never be identical, of course, but their resemblance over different timescales can be striking and worthy of our attention. Such price variations are "scale-free" with no typical size of variations, just like the sandpile avalanches that we discussed in the previous section. As remarkable as it may seem, markets and sandpiles have something in common after all. In Chapter 7, we'll return to the issue of price fluctuations, fractals and self-affinity in financial data.

Until very recently, most economists (and all policy entrepreneurs) have ignored Mandelbrot's important work, presumably because it doesn't fit into the traditional picture. Classical economists have a tendency to discard large events, attributing them to specific abnormal circumstances - such as program trading in the case of the crash in

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[35] Mandelbrot suggests that most scientists did not expect to encounter power-law distributions, and thus were unwilling to acknowledge their existence. An account of his work on cotton prices appears in Mandelbrot (1963). For a full account of all his work on fractals and scaling in finance, see Mandelbrot (1997).

[36] Mandelbrot refers to the power-law distribution as the scaling distribution.

October 1987. If you happen to believe in the world of stasis, it would be difficult to believe in a general theory of events that occur just once! Yet history is riddled with such events. The paleontologist, Stephen Jay Gould, co-inventor of the theory of punctuated equilibria, argues that, in many sciences, we're compelled to engage in "storytelling" because particular outcomes are contingent on many single and unpredictable events.

Despite their potentially devastating consequences, the fact that rare, large events might follow the same law as a host of small events suggests that there's nothing very special about large events. Furthermore, they may not even be so rare after all. Although the magnitude of price movements may remain roughly constant for more than a year, suddenly the variability may increase for an extended period. Big price jumps become more common as the turbulence of the market grows. Then one observes such spikes on a regular basis - sometimes as often as once a month. Should such occurrences be regarded as abnormal? Not if one believes in fractal geometry and the scaling properties of such markets.

The important thing to learn from phenomena like self-organized criticality, punctuated equilibria and fractal geometry is that complex patterns of behaviour are created by a long period of evolution. A universal law of economics, for example, cannot be understood by studying economic change within a timeframe which is short compared with the economy's overall evolution. Mandelbrot's work spanned five human generations. Suddenly that familiar phrase "you cannot understand the present without understanding the past" takes on a deeper and more exact meaning.

Hopefully you're beginning to sense how and why disciplines like geophysics, biology and economics differ from physics. Modern physicists are accustomed to dealing with probabilistic theories in which the specific outcome of an experiment cannot be predicted. Only certain statistical features can be determined with any accuracy. Statistical mechanics, quantum mechanics and chaos theory are important theories in physics which are of a statistical nature. What makes geophysics, biology and economics different is that their outcomes impinge on our everyday lives as human beings. The fact that we may understand the statistical properties of earthquakes is of little consolation to those who have suffered from one large, devastating earthquake. A similar statement can be made about biological and economic catastrophes. Many affect us personally.

It's quite correct to attribute the variability of things, and thus their complexity, to contingency. History depends on freak accidents, so if the tape of history is replayed many times with slightly different initial conditions, the outcome will be different each time. The discovery of each new nation, for example, usually involved a long series of events, each of crucial importance for the eventual outcome. Because of this, even the pioneers involved in such discoveries had little idea of what the likely outcome would be. Wherever contingency is pervasive, detailed long-term prediction becomes impossible. For example, many kinds of economic changes are unpredictable. But that very fact doesn't mean that they're also unexplainable. The main problem with understanding our economic world is that we have no reliable benchmarks with which to compare it.

Fortunately, a few economists have recognized the important role of history, and chance events, in economic development. We'll sample some of their ideas in the next section, before moving on to a more detailed discussion of some of them in the chapters that follow.

{A}Stasis and Morphogenesis{/A}

In case you're still wondering if equilibrium economics is really like tank water, here's another way of testing the analogy. It comes from the youthful field of cybernetics, which deals mostly with self-regulating and equilibrating systems. Thermostats, physiological regulation of body temperature, and automatic steering devices are examples of self-regulating systems. So are equilibrium economies and our tank water example. They're all systems in which negative feedback processes tend to counteract, or cancel out, deviations from the equilibrium state. In other words, they all possess *negative feedback loops*. Such loops promote stability in a system, because they tend to negate change.

[Figure 1.9 near here]

Negative feedback is *assumed* to occur in economics. The belief is that economic actions will force the economy back to a stable equilibrium point, because of the respective shapes of the supply and demand curves. You can see the logic behind this self-regulating process in Figure 1.9. Suppose the apple farmer (who we met earlier) sets his price initially at p_A. Before long, he realizes that he's not selling as many apples as he would like. Supply exceeds demand. He's building up an unwanted surplus, some of which will soon turn bad. So he drops his price to p_C. A little later, he sells out of apples. Demand has outstripped supply. Thinking that he must have set his price too low, he increases it again. As if guided by an "invisible hand," he finally converges on the equilibrium price, p_E.

Negative feedback loops like this are fine in principle. They seem to provide a stabilizing influence in an otherwise volatile marketplace. But does our economy really work this way in practice? Many believe that it has done so in the past and still does to some extent. It's certainly true that the price of a specific brandname product, like a McDonald's cheeseburger or a Diet Coke, may not vary greatly from place to place. Although primary products do vary in price from season to season, Tasmanian apples and Californian oranges may not vary greatly when their prices are measured from place to place at the same point in time. Any difference might simply be due to differences in transportation costs to the marketplace. The prices of various manufactured goods, like clothing or sports equipment, never seem to vary greatly when we shop around at different stores.

But how can we be sure that all the producers of the same product will behave in this way? The truthful answer is that we can't. In addition to all the sales, discounts, consumer loyalty privileges, and neverending suite of devious tactics that firms introduce to attract buyers away from their competitors, there's another reason why it's hard to believe in the broad existence of equilibrium prices. Take a look at Figure 1.10. It shows a typical average cost curve faced by an "efficient" manufacturing firm over the long run. Cost per unit of output is plotted against output. The firm is efficient in the sense that it adopts a least-cost method of production for the level of output involved. Efficient production possibilities lie on the thick black line. For example, producing output OB at a cost of Ob can be done using the least-cost technique. Cost levels above Ob are inefficient whereas cost levels below Ob are impossible at that level of output.

[Figure 1.10 near here]

Strangely enough, two different kinds of economic worlds are implicit in this one curve. The one which we've been discussing, the negative feedback world, lies to the right of the point C. At smaller output levels than OC, a very different regime prevails. In this region, positive feedback mechanisms prevail. An expansion in production results in a decrease in costs per unit of output. On average, each unit of output becomes cheaper to produce. Under these conditions, a firm has every incentive to expand production as much and as quickly as possible, because the firm can then enjoy scale economies, i.e. *increasing returns to scale*. Beyond OC, however, the curve begins to rise, signifying that unit costs have changed direction. Now they're increasing rather than decreasing. At these higher output levels, negative feedback loops prevail and the firm faces *diminishing returns to scale*.

Conventional economic theory tends to frown upon the left hand part of this curve. Yet this is a realistic and most profitable cost structure for a firm. Why would any serious analyst want to overlook part of it? One answer is that Zone 2 is much simpler to model and understand than Zone 1. Negative feedback loops serve to stabilize the economy; any major changes will be offset by the reactions they generate. A stable, closed economy is a predictable economy -- easily identified and interpreted.[37] This classical world of diminishing returns is epitomized by the agricultural sector. Suppose a

[37] Critics of economic theory see this "stable, closed-world" model as a vast abstraction from reality. For example, Daniel Bell regards it as a convenient Utopia dreamt up by John Locke and Adam Smith. He points to the need for studies of human behaviour, the codification of theoretical knowledge, and the influence of time and history. Clearly, many economists have begun to realize that they cannot afford to ignore the nature and relentless pace of social and technological change. For a blunt view of what's been wrong with economics for some time, see Bell (1981).

wheat farmer wants to expand production. Because of the scarcity of arable land, eventually he'll have to pay more for suitable land or put up with land that's less suitable for wheat. This pushes unit costs up, reducing unit profits. Hence the label "diminishing returns."

Primary producers are plagued by an additional problem. If there are too many wheat producers competing for scarce parcels of land, for example, wheat prices may come under downward pressure. As prices fall towards average production costs, some farmers will struggle to earn any profits. In this so-called world of perfect competition, profits are marginal at best. As we've said earlier, perfect competition belongs to the world of stasis; a world at equilibrium, stable, predictable and resistant to change. What's rarely said, however, is that firms in such a world are actually flirting with extinction. Once we picture it as part of a dynamic economy, the true identity of a competitive equilibrium reveals itself. Basically, it's a deadend. Instead of engendering a perfect marketplace, negative feedback breeds extinction!

Feedback processes can also be positive. In many sectors of our economy, the stabilizing forces needed to maintain an equilibrium state are absent. Instead, radically different forces prevail. Positive feedback loops amplify the effects of small initial changes.[38] Hi-tech monopolies and oligopolies are a good example. They belong to a vastly different world, a world of increasing returns. Whereas diminishing returns imply a single equilibrium point for the economy, increasing returns imply many possible states. Such open-ended pluralism presents two problems. First, there is no certainty that the particular outcome selected from among the many alternatives will be optimal. Nor can it be predicted in advance. Chance dominates over necessity. Second, once a particular economic outcome is selected, that choice may become securely "locked-in", thereafter tending to prevail regardless of its advantages or disadvantages.

Classical theories of industrial location have tended to resist the idea that historical chance plays a role. For example, the Sante Fe Institute economist, Brian

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[38] See Maruyama (1963).

Arthur, has noted two different views in the literature on spatial economics.[39] The first, associated with the writings of von Thünen, the early Weber, Predöhl, Christaller, Lösch, and Isard, saw the spatial evolution of industry as *preordained* - by geographical endowments, transport possibilities, and economic needs. In this view, locational history does not matter. The key factors are geographical differences, shipment costs, market interactions, and the spatial distribution of prices and rents. The outcome is determinate and easily predictable: a unique equilibrium pattern. Because this is a static and unique view of the locational world, Arthur calls it *stasis*.

The second group saw industry location as *path-dependent* - more like an organic process with new industry influenced by, and thus reinforcing, the locational landscape already in place. Included among this group were the later Weber, Englander, Ritschl and Palander. Although there's still a role for geographical endowments and economic factors (such as transportation costs) in this view, the dominant driving forces are agglomeration economies. Englander and Palander were severe critics of Weber's theory on this point, claiming that he grossly underemphasized the actual development process and the historical advantages of existing production points as self-reinforcing centres of agglomeration. In a path-dependent world, chance events in history play a crucial role. We'll refer to this view as *morphogenesis*.

Here's a modern example. Japan Railways East, believed to be the largest carrier in the world, ran into some water problems when it was building a train line through the mountains of Tokyo. As engineers made plans to drain the water out of the tunnel, the company learned that the workers were drinking it because it tasted good. So JR East decided to bottle and sell it as a premium mineral water. It became so popular that vending machines were installed on JR East's platforms and a home-delivery service was launched. A new $75 million-a-year beverage industry had been triggered by nothing more than an accidental discovery. Once again, such an outcome could not be foreseen in advance. Chance ruled out determinism. Morphogenesis reigned supreme.

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[39] See Arthur (1994b).

Whether small events in history matter in determining the pattern of settlement, growth and change in an economy reduces, strangely enough, to a question of topology. In the matter of industrial location, it hinges on whether the underlying structure of locational forces guiding the location pattern is convex or nonconvex.[40] History does matter when the these forces are nonconvex, and nonconvexity stems from some form of agglomeration or increasing returns in space. Path dependence can be illustrated by by a firm's decision to locate its headquarters in one of several alternative cities (or regions). We'll discuss agglommerative forces more fully in Chapter 5, where Chicago's development is portrayed as a path-dependent, coevolutionary process.

Agglomeration is a powerful force. Firms that are not heavily reliant on raw material locations, but are more sensitive to their industry's learning curve, are often attracted by the presence of other like-minded firms in a region. Some densely settled regions can offer better infrastructure, more diverse labour markets, more specialized services, and more opportunity to do business face-to-face. They may also provide an active forum for the continuous exchange of ideas. This is a vital part of Arrow's "learning-by-doing."[41] Under these conditions, the world of morphogenesis dominates.

Brian Arthur has suggested the following example.[42] Stasis would see today's electronics industry in the United States distributed across the country, but with a substantial part of it in California (e.g. Silicon Valley) - because that location is close to Pacific sources of supplies, and because it has better access to skilled labour and to advances in academic engineering research. By way of contrast, morphogenesis would see concentrations of high-tech industry, like Silicon Valley, as largely the outcome of chance events - such as the vision of the Vice-President of Stanford University, Frederick Terman, who just happened to support a few key entrepreneurs - the Hewletts, the Varians, the Shockleys - who then decided to set up shop near Stanford in the 1940s and 1950s. The attractive work environment that they helped to create made subsequent

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[40] See Allen and Sanglier (1981) and Arthur (1994b).

[41] See Arrow (1962).

location there very attractive for the thousand or so firms that followed them. If Terman or those key entrepreneurs had thought or acted differently, Silicon Valley might have happened somewhere else.

Stasis or morphogenesis? Which explanation is correct? It's likely that most of the spatial patterns we observe today have been forged by a mixture of chance and necessity, rather than by either element alone.[43] Whenever industry and people are attracted to places where resources of interest are already gathered, those small concentrations established initially by chance will have sown the seeds of the resulting urban configurations. This is a world of morphogenesis. To the extent that the locational choices of the pioneering agents were preordained by geographical or economic needs, however, the resulting configurations will reflect pure necessity. This is a world of stasis.

The important point to note is that positive feedback loops never let the economy return to its original state. Even an accidental (or seemingly insignificant) kickstart will cause divergence from the initial condition. This has troubled conventional economic theorists for decades. Most have refused to tackle the complexities of increasing returns economics, or even to acknowledge their importance. Given the lack of attention devoted to them, it's surprising to find that positive feedback processes are so ubiquitous in societies: the evolution of living organisms, the accumulation of knowledge and physical capital, the rise of specific cultures, for example. Because the term *morphogenesis* is used in cybernetics to cover this category of feedback processes, for ease of exposition we'll regard all economic systems which are governed by positive feedback loops as belonging to the world of morphogenesis.

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[42] See Arthur (1994b), page 50.

[43] This conjecture seems to be turning into a serious hypothesis. To gain an accurate picture of urban development, for example, Peter Allen and Michele Sanglier have demonstrated that a dynamic model of a central place system must consider the self-organizing aspects of urban evolution; see Allen and Sanglier (1979, 1981). Although the monopolistically competitive, general equilibrium model formulated by Paul Krugman demonstrated that the process of city formation is one of cumulative causation (i.e. positive feedbacks), he found that the eventual locations of cities tend to have a roughly central-place pattern; see Krugman (1993). The Kyoto scholar, Kiyoshi Kobayashi, has shown that Japanese industrial R&D laboratories tend to cluster in one dominant location, which depends on geographical attractiveness as well as the historical choices of others; see Kobayashi et al. (2000). These modelling and simulation

Once we start to think of development as a lifecycle process of evolution, the respective roles of positive and negative feedback loops – or increasing and diminishing returns – fall into place. Consider the forces behind the typical S-shaped growth curve in population dynamics. The binding constraint is carrying capacity. When the population is well below this upper limit, it's being driven by a positive feedback loop. Additions to population increase in proportion to population itself. Thus it expands exponentially. This self-reinforcing process produces the initial upward sweeping part of the curve. As population nears carrying capacity, however, a dormant negative feedback loop becomes active, interacting nonlinearly with the positive feedback loop, neutralizing its influence and converting the system to a search for an equilibrium at the population limit. As Jay Forrester suggests, S-shaped growth curves depict shifting loop dominance at different times.[44]

Such S-shaped curves also form part of the trajectory traced out by the product lifecycle of a firm. Like humans, products pass through a familiar sequence of recognizable stages. Self-reinforcing stages of the human lifecycle include incubation, infancy, adolescence, and young adulthood. Here, positive feedback loops underpin the growth process. By the time we reach middle age, however, negative feedback loops have taken over. Their growing influence eventually leads to senility. Death -- that ultimate equilibrium state of human existence – follows thereafter.

[Figure 1.11 near here]

Stages of the product lifecycle follow a similar pattern. Invention, innovation or imitation, and rapid growth correspond to self-reinforcing stages of market growth. They're the hallmark of an increasing returns economy. Once competitive turbulence sets in, however, market share stabilizes and begins to decline. A mature, stable, saturated

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experiments confirm the importance of chance and determinism in the evolution of urban systems. In other words, pluralism prevails.

[44] See Forrester (1987).

market confirms that diminishing returns have taken over. Even products cannot avoid senility, with rapidly declining market share signalling the ultimate death knoll.

Unless a firm is unusually innovative, it can only expect to enjoy increasing returns in the early stages of the cycle. Before final choices are locked in, the development process is characterized by a high degree of risk and uncertainty. Learning processes are rapid but haphazard. Many different solutions are possible and frequent major changes are necessary. Chance invariably takes a hand. A host of ideas and products are triggered by accidental discovery or even by mistake. Any accidental kickstart via the invention process triggers the divergence mentioned earlier. But the high initial costs of research and testing usually become a distant memory once production expands and the cost per unit of output begins to fall. Increasing returns take over, bestowing on the firm a temporary period of competitive advantage over its rivals. Chance can breed windfall profits.

Economists can now explore the challenging terrain of increasing returns with much better equipment than they could a few decades ago. The early chapters of Adam Smith's *Wealth of Nations* placed considerable emphasis on increasing returns to explain both specialization and economic growth. Since then, many others have taken up the challenge. What they're discovering is a world of growing complexity. Among the early pioneers were A.A. Cournot, Alfred Marshall, Allyn Young, Edward Chamberlin, Joan Robinson, Gunnar Myrdal and Nicholas Kaldor. Today's champions of increasing returns are led by Brian Arthur, Paul Krugman and Paul Romer. Some of their work will be discussed in more detail as our story unfolds in the ensuing chapters.

In the words of Brian Arthur, an increasing returns world is a world of evolution rather than equilibrium; a world full of instability and chance events.[45] It's also a world of process and pattern change, placing it in the world of morphogenesis. If one firm gets ahead by historical accident or innovation, increasing returns serve to magnify this advantage. Regardless of its ultimate efficiency, a product can "lock in" considerable advantages by being first. Chance events in the past may have set the wheels in motion.

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[45] See Arthur (1994b).

But once they're turning, increasing returns turn them even more quickly - breeding uncertainty and instability. In brief, increasing returns are the tendency for that which gets ahead to get further ahead.

In stark contrast, diminishing returns are the tendency for that which falls behind to fall further behind. Such conditions tend to dominate during the later stages of the product lifecycle. Once a product has become standardized, further innovation becomes marginal at best. Improvements are only incremental. Low cost imitation takes over. The emphasis switches to productivity, marginal improvements, and cost control. Saturated markets breed numerous competitors and unit profits are thin. The classical zero-profit equilibrium of economic theory is a reasonable approximation of the ultimate deadend state of this frozen world. For the firms involved, death is imminent. Without fresh innovation, diminishing returns signal that the market and its customer base have matured, and that the risk of extinction is growing.

Our analysis reveals two economic worlds: the seemingly static one (stasis) is heavier on resources, lighter on know-how and subject to diminishing returns; the dynamic one (morphogenesis) is lighter on resources, heavier on know-how, and subject to increasing returns. These two worlds are readily visible in the economies of the past and the present. Our traditional mainstays of economic life - agriculture and manufacturing - have been surrendering market share on a global basis to dynamic newcomers built around newer technology. Instead of processing resources, these pioneers of hi-tech products process knowledge and information. Instead of applying raw energy, they apply new ideas. The relentless pace of change in this hi-tech world is nothing short of remarkable. Chance is setting such a cracking pace that necessity has trouble simply staying in touch.

[Table 1.1 near here]

{A}On Learning Curves{/A}

To reiterate, two contrasting views of our economic world proliferate today: chance and necessity, punctuation and equilibria, morphogenesis and stasis. Our primary focus in the rest of book will be on morphogenesis - those chance events that punctuate the calm, deterministic landscape of an economic system, propelling it into an uncertain future. Real economies evolve in fits and starts. Calm is nothing more than the precursor of storm. Morphogenesis and disequilibrium are more influential states in an evolving economy than stasis and equilibrium.

Recent simulation work in economics has also shown that rational expectations equilibria cannot be seen as stationary states of adaptive processes.[46] Instead of equilibrating, evolving economies adapt and select continuously. The work of nonequilibrium scientists like Ilya Prigogine and Peter Allen has revealed that self-organizing human systems possess an evolutionary drive that selects for populations with an ability to learn, rather than for populations exhibiting optimal behaviour. Schumpeter was an early champion of the innovative entrepreneur. Creatively destructive entrepreneurs have been stoking the engine of economic change for centuries. The rest of this book attempts to unravel facets of their adaptive behaviour.

Learning takes place individually and collectively. The collective learning process can be illustrated in the following way. Fundamental inventions spawn an early explosion of diverse forms as many tinkerers try out new variants on the basic invention. Tinkering occurs with very little real understanding of the likely consequences. After the early frenzy dies away, we settle down to finer, more incremental tinkering among a mere handful of designs that dominate. Once these better designs have been found, it becomes progressively more difficult to do much better. Variations become more modest. Such qualitative features are reminiscent of the Cambrian explosion: branching radiation to create diverse forms is bushy at the base; then the rate of branching dwindles, extinction sets in, and a few final, major alternative forms persist.[47]

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[46] Some of this simulation work is discussed in later chapters.

[47] See Kauffman (1995), page 202.

The more copies of an item produced by a firm, the more efficient production tends to become. Learning curves are a means of tracking such efficiency improvements by relating the unit costs of the firm to its accumulated output. According to empirical economists, the cost per unit for high-tech products entering the marketplace can fall by as much as half at each doubling of the number of units produced. Being heavy on know-how and light on resources, high-tech products typically have high R&D costs when compared with their unit production costs. As the technology matures, however, this rate of improvement slows considerably to a few percentage points. It may even start to rise if marketing costs become excessive. Being closely related to a product's lifecycle (Figure 1.11), the learning curve reveals a rapid improvement in performance at first, followed by an eventual slowdown and deterioration.

Formally speaking, then, learning curves relate unit costs to accumulated output. Let's plot such a curve for Microsoft's Windows software. Being a high-tech product in the early phase of its lifecycle, it enjoys increasing returns to scale. In fact, the learning curve can be thought of as the result of economies of scale which just happen to be defined temporally. [48] The first disk of Windows entering the market cost Microsoft $50 million; the second and subsequent disks cost $3. For such high-tech products, the Nth unit typically costs about 1/Nth of the cost of the first unit produced. Once again, the special character of this property shows up when the logarithm of the cost per unit is plotted against the logarithm of the total number of units produced. The resulting straight line confirms an already familiar shape for this pattern of learning. Yes, it's another power law (see Figure 1.12).

[Figure 1.12 near here]

How fascinating! Mathematically speaking, a learning curve appears to follow a power law. We're back to sandpiles again! Note how closely the linear plot resembles

[48] If we think of the market as being segmented over time, then the learning curve can also be regarded as embodying economies of scope. For a discussion of this idea, see Spence (1981).

the one shown in Figure 1.7. In the early stages of a new product's lifecycle, the power-law exponent may be near -1. This exponent approaches zero as the cycle runs its course. But the actual number, N(c), could just as well be the number of sandpile avalanches of size c, or the number of fjords of length c or the number of earthquakes with energy c. As Mandelbrot has shown, it could also be the number of months during which stock price variations exceeded a given fraction c. The message we're getting is that power laws may be rather ubiquitous in nature and in human endeavour.

Like sandpiles, fractals and earthquakes, learning is a coupled dissipative process. Thus it can't be fully understood by limiting our study to a single human lifetime. Even an evolutionary approach is insufficient. Because it takes place individually **and** collectively, learning isn't just evolutionary; it's coevolutionary. Agents react to the moves of other agents. Each agent's decision affects the collective outcome and, in turn, this collective outcome influences the agents' future beliefs and decisions. Such outcomes may be quite different to what each agent expected or intended. Unexpected outcomes can trigger avalanches of anxiety and uncertainty, causing each agent to react and modify his view of the world. Because such avalanches of economic change vary greatly in magnitude, perhaps they also conform to a power law.[49] If the system of interest self-organizes, a new regime may take over. Future expectations and decision strategies change dramatically. So do future collective outcomes.

This seems to be the way of the world, the way we respond to the unexpected and accumulate experience. Experience is cumulative skill or judgment acquired through practice. They say that "practice makes perfect." But practice involves making mistakes, learning from them, and adapting future strategies accordingly. Experience can't be gained in isolation and is suboptimal. It's accrued through an interactive, coevolutionary process. But now we've moved ahead of ourselves, skipping over some of our story. We

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[49] In a simplified network of producers and consumers, like those represented in the classical input-output model, intermediate producers play the dual role of purchasers and vendors. It's possible to demonstrate that small initial shocks to parts of such an economy can sometimes trigger large avalanches of orders and back-orders. The collaborative work between Per Bak and two economists at the University of Chicago – Jose Scheinkman and Michael Woodford – goes even further. It suggests that some large fluctuations observed in economics are indicative of an economy operating at the self-organized critical state, in which

know so little about the nature of knowledge and the mechanics of learning. How do creative entrepreneurs acquire the know-what and know-how to make innovative decisions? What does it mean to learn *adaptively*? Can adaptive learning cause an economy to self-organize? We'll begin to tackle these intriguing questions in the next chapter, where we look at the behaviour of adaptive economic agents as they journey along the road to "know-ware."

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minor shocks can lead to avalanches of all sizes. For further details on economic avalanches of this kind, see Bak et al (1993) or Scheinkman and Woodford (1994).

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