DISCOVERING ARTIFICIAL ECONOMICS

How Agents Learn and Economies Evolve

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Artificial Economics

"Truth is much too complicated to allow anything but approximations."
JOHN VON NEUMANN

{A}Limits to Knowledge{/A}

It's hard to let old beliefs go. They're so familiar and comforting that we depend heavily on them for peace-of-mind. Most of us forget Kant's message, that the way the world looks is nothing more than the way we happen to see it through our own particular set of lenses. So it has been in economics. That old set of lenses, still the most popular pair on the block, remains stubbornly homogeneous, static and linear. But a new set have arrived on the scene. These new lenses are dauntingly heterogeneous, dynamic and nonlinear.

A pressing need for new lenses has prompted a focus in this book on the less predictable elements underpinning economic change, those chance events that punctuate the calm, deterministic landscape of the classical economic system, propelling it into an uncertain future. Hopefully, we've convinced you that real economies evolve in fits and starts. Calm is merely the precursor of the next storm. There's also structure and recurrent pattern to these fits and starts. In an evolving economy, morphogenesis and disequilibrium are more often the norm than stasis and equilibrium.

But this is only symptomatic of a more complicated problem. As we've stressed throughout, the real difficulty is that each of us is part of the very economy that we're desperately trying to understand. This has the hallmark of a systems problem. But it's not a classical systems problem, like how a clock "tells the time" or how a car "moves".[1] Clocks and cars are structurally complex, but they're behaviourally simple. Their behavioural simplicity transcends the structural complexity of their intricate parts. An

[1] As Cohen and Stewart have noted, "You can dissect axles and gears out of a car but you will never dissect out a tiny piece of motion;" see Cohen and Stewart (1994, page 169).

economy, however, is behaviourally complex. Because the “parts” are human agents, they’re observers as well as participants, learning from their experiences while contributing to the collective outcome. Playing these dual roles really puts the cat among the pigeons! What people believe affects what happens to the economy and what happens to the economy affects what people believe. Because each agent’s beliefs are affected differently, nobody knows exactly what will happen!

Whenever agents learn from, and react to, the moves of other agents, building a simple predictive model to forecast the collective future is fraught with danger unless the economy is linear. A linear economy obeys the principle of superposition. It’s easy to analyse because we can extrapolate our understanding of the agents in isolation. Learning is only weakly-interactive, so the economy’s behaviour is just the sum of the behaviour of its constituent parts.

But weakly-interactive learning, like that which is associated with repetition of much the same problem, is subject to diminishing returns. We’re trapped in the frozen world of stasis. For learning to be truly adaptive, the stimulus situations must themselves be steadily evolving rather than merely repeating conditions. The existence of a recursive, nonlinear feedback loop is the familiar signature of coevolutionary learning. People learn and adapt in response to their *interactive* experiences. In turn, the whole economy reacts and adapts collectively based on the choices which people make. In other words, the behaviour of the whole is more than the sum of its parts.

Under strongly-interactive conditions, we’ve seen that collective outcomes can differ from what each agent expected or intended. Unexpected outcomes trigger avalanches of uncertainty, causing each agent to modify his view of the world. As Kant has suggested, nobody can have certain knowledge of things "in themselves." Each of us only knows how things seem to us. If we’re only privy to part of the information about the economy, then there are clear limits to what we can know. Each agent’s mind sets these limits. When we ask questions about the economy, we’re asking about a totality of which we’re but a small part. We can never know an economy completely; nor can we see into the minds of all its agents and their idiosyncracies.

From the above, it's pretty clear that knowledge becomes a much fuzzier concept in a coevolutionary economy. If there are definite limits to what we can know, then our ability to reach identical conclusions under similar conditions should not be taken for granted. We're unique products of our uniquely individual experiences. Our personal knowledge is honed by the constructs, models and predictors which we choose to use to represent it. All of this has to be created, put together over time by us as well as by others in society as a whole. Despite the fact that learning can be strongly-interactive, it can also be frustratingly slow, partly because some knowledge stocks are surprisingly resilient to change. They're also surprisingly complex.

In Chapter 2, we mentioned that deductive rationality fails us when we're forced to deal with complicated decision problems. Beyond a certain degree of complicatedness, our rationality is bounded. Even more ominous was the fact that, in strongly-interactive decision situations, each agent may be forced to guess the behaviour of other agents. Suddenly we're all plunged into a world of subjective beliefs, and subjective beliefs about subjective beliefs. Complete, consistent, well-defined premises are impossible under these trying conditions. Deductive reasoning breaks down because the problem has become ill-defined.

Whenever deductive reasoning breaks down, we argued that human agents tend to resort to inductive reasoning. In other words, we search for patterns.[2] The right hand side of the brain handles pattern recognition, intuition, sensitivity and creative insights. By putting a combination of these processes to work, we use perceived patterns to fashion temporary constructs in our mind. These simple constructs fill the gaps in our understanding. They "localize" our decision-making, in the sense that we can do no better than act on the best construct at our disposal. When feedback changes our perceptions, thereby strengthening or weakening our confidence in our current set of constructs, we may decide to discard some and retain others.

We looked at the importance of inductive reasoning in three economic contexts: (1) estimating the periodic demand for a public facility; (2) estimating travel times and

[2] Induction as a search for patterns should be distinguished from mathematical induction, the latter being a technique for proving theorems.

costs on a congested highway; and (3) estimating price movements in financial markets. Each of these examples typifies a broader class of problems which arise in economics.[3] Yet all three possess common features. If there was an obvious model that all agents could use to forecast the outcome, then a deductive solution would be possible. But no such model has been found to date. Irrespective of recent history, a wide range of plausible hypotheses could be adopted to predict future behaviour. This multiplicity of possibilities means that nobody can choose their own strategy in a well-defined manner. Each problem is ill-defined and the agents involved are catapulted into an uncertain world. Thus they're forced to resort to intuition and other inductive modes of reasoning.

There's an even more diabolical dimension to each of these problems: any shared expectations will tend to be broken up. For example, if all of our bar lovers believe most will go to the El Farol next Thursday night, then nobody will go. But by all staying home, that common belief will be destroyed immediately. If all of our peak-hour commuters believe that most drivers will choose to commute at peak hour, then most explorers will search for ways to avoid peak hour congestion. On the other hand, if all believe few will do this, then all will commute at peak hour, thereby undermining that belief. Not only do expectations differ, but they're also changing incessantly. Adaptive agents, who persistently alter their mental models of other agents' behaviour, will decide and behave differently.[4] They're forever changing their mental images of each others' likely behaviour. Beliefs about beliefs are mostly volatile. There's no evidence to suggest that adaptive behaviour ever settles down into a steady, predictable pattern.

{A}Adaptive Agents and the Science of Surprise{/A}

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[3] For example, the need to estimate the periodic demand for a facility applies to crowding problems at annual meetings, at monthly luncheons, at weekly sporting events, at weekend markets or at daily shopping centers.

[4] There is a lengthy literature on mental models, although the term has been used in many different ways. For a good review, see Rouse and Morris (1986). For an interesting discussion about their use in the scientific field, see Gorman (1992).

The key to understanding adaptive behaviour lies with explanation rather than prediction. When economic agents interact, when they must think about what other agents may or may not be thinking, their coevolving behaviour can take a variety of forms. Sometimes it may look chaotic, sometimes it may appear to be ordered, but more often than not it will lie somewhere in between. At one end of the spectrum, chaotic behaviour would correspond to *rapidly* changing models of other agents' beliefs. If beliefs change too quickly, however, there may be no clear pattern at all. Such a volatile state could simply appear to be random. At the other end of the spectrum, ordered behaviour could emerge, but only if the ocean of beliefs happens to converge onto a mutually consistent set of models of one another. One familiar example is that stalwart of the economic theorist's world -- a state of equilibrium among a set of deductively rational agents.

For most of the time, however, we'd expect that mental models of each other's beliefs would lie somewhere in between these two extremes, poised ready to unleash avalanches of many small and a few large changes throughout the whole population of interacting agents. Why should we expect this? The plentiful evidence supporting the ubiquitous applicability of power laws is one reason. Given more data, we would expect each agent to improve his ability to generalize about the other agents' behaviour by constructing *more complex* models of their behaviour. These more complex models would also be more sensitive to small alterations in the other agents' behaviour. Thus as agents develop more complex models to predict better, the coevolving system of agents tends to be driven away from the ordered regime toward the chaotic regime. Near the chaotic regime, however, such complexity and changeability would leave each agent with very little reliable data about the other agents' behaviour. Thus they would be forced to simplify, to build *less complex* models of the other agents' behaviour. Such simplified models can succeed in calmer times.

Economic enigmas -- like the periodic demand for public facilities, for road space, or for financial instruments -- have several key features in common. Each contains the essential elements of a *complex adaptive system* (CAS). CAS possess three important attributes. First, they involve a large (but not infinite) number of agents. Second, these agents are adaptive and intelligent, making decisions on the basis of mental models (like travel time predictors or financial models), which they modify in the light of their experiences and replace with new ones if necessary. Finally, no single agent knows what all the other agents are (thinking of) doing, because each has access to a limited amount of information only.

The upshot of all this is that there's no *optimal* predictor in a CAS. The best each agent can do is apply the predictor that has worked best so far, be willing to revealuate the effectiveness of his favorite predictor, and adopt more convincing ones as new information becomes available. An agent's *active* predictor may be the most plausible or most profitable one at the time. But the total population of active predictors coevolves incessantly. As we've stressed repeatedly, coevolutionary learning means that the total population of active predictors determines the outcome, but the outcome history also shapes the total population of active predictors.

One of the difficulties with a CAS is that nobody really knows the total population of active predictors at any point in time. Because it's impossible to formulate a closed-form model to deduce future outcomes, traditional economic models fail in this environment. In John Holland's jargon, the population of predictors forms an *ecology*. If we want to understand how this ecology might evolve over time, we're forced to resort to simulation experiments. Simulation doesn't simplify the economy, but incorporates as much detail as necessary to produce emergent behaviour. There's simply no other way of accommodating such a large, ever-changing population of active predictors.

The defining characteristic of a CAS is that some of its global behaviours cannot be predicted simply from knowledge of the underlying interactions.[5] We spoke earlier about emergence. An emergent phenomenon was defined as collective behaviour which doesn't seem to have any clear explanation in terms of its microscopic parts. What does this kind of emergent simplicity tell us? It tells us that an economic system of interacting agents (like urban residents, bar attendees, traffic commuters or traders in a financial market) can spontaneously develop collective properties that are not at all obvious from our knowledge of the agents themselves. These statistical regularities are large-scale

[5] See Darley (1995, page 411).

features that emerge purely from the microdynamics. They signify order despite change. Sometimes, they display self-similarity at different scales.

Furthermore, the laws governing economic change can't be understood by limiting our study to a single human lifetime or a few generations. A deeper understanding of how the economy coevolves can only be gained by adopting a long term perspective. Only then can we see that the best thing to do - to move or not to move, to go or not to go, to commute or not to commute, to buy or to sell - really depends on what everyone else is doing. But since no individual agent knows what everyone else will do, all he can do is apply the set of predictors that has worked best for him so far. An individual agent has no option but to "suck it and see," so to speak.

Since the study of CAS, with changing patterns of interactions between adaptive agents, often gets too difficult for a mathematical solution, finding a new way of doing social science has become imperative. We might call the new boy on the block the science of "surprise." This kind of science is gradually gaining ground inside the computers of various social scientists. The computer itself represents a promising laboratory for social science experiments, and the primary research tool in this new field is simulation.

The simulation of agents and their interactions goes by different names: agent-based modeling, bottom-up modeling, and artificial social systems, to name a few. What's crystal clear, however, is its purpose. Agent-based simulation attempts to gain a deeper understanding of CAS through the analysis of simulations. As Robert Axelrod suggests, this new method of doing social science can be contrasted with the two standard methods discussed throughout this book: deduction and induction. Like deduction, it starts with a set of explicit assumptions. But unlike deduction, it does not prove theorems. Instead, an agent-based model generates simulated data that can be analyzed inductively. The simulated data come from a rigorously specified set of rules rather than from direct measurement of the real world.[6] Whereas the aim of induction is to discern patterns in data, and that of deduction is to discover consequences of assumptions, the aim of agent-based modeling is to enhance our intuition.

Numerous experiments throughout this book have shown how locally interacting agents can produce surprising, large-scale effects. Agent-based simulation is a rigorous way of conducting such "thought experiments." The assumptions are often simple, but the full consequences are rarely obvious. We have referred to the large-scale effects of locally interacting agents as emergent properties. Emergent properties seem surprising because it can be difficult to anticipate the consequences of even very simple forms of interaction. In an economy, for example, emergent properties arise from seemingly simple interactions between agents engaged in the business of exchange. Congestion and market volatility are typical emergent properties of socio-economic interactions.

The Harvard systems scientist, Vince Darley, has argued that emergence is purely the result of a phase change in the amount of computation necessary for the optimal prediction of certain phenomena.[7] Imagine that $s(n)$ denotes the amount of computation required to simulate a system of size n and arrive at a prediction of the given phenomenon. Further imagine that $u(n)$ is the amount of computation needed to arrive at the same result by way of a creative analysis -- founded, for example, on a deeper understanding of the system. Darley suggests that if $u(n) < s(n)$, the system is non-emergent, but if $u(n) > s(n)$, the system is emergent.

We can visualize Darley's phase change in the context of traffic congestion. In Chapter 6, we discussed dynamic jamming transitions. Remember how they transform the traffic from free-flow to stop-start waves at a critical flow density. As long as the density of vehicles remains below this critical threshold, it's rather easy to estimate individual travel times. They're roughly the same for each driver, and variations between vehicles are small. Experience engenders reliable predictions. We can deduce the outcome readily from our understanding of the system's performance as a whole. We can do this because the behaviour of this simple system is easily understandable. There's no need to carry out a detailed simulation of it to arrive at a travel time prediction. In light traffic, obviously $u(n) << s(n)$ and the traffic system is non-emergent.

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[6] However, the rules themselves are usually based on direct observations of the real world. For a more comprehensive discussion of agent-based simulation, see Axelrod (1997).

[7] See Darley (1995, page 413).

Once the critical density is exceeded, however, travel time predictability quickly starts to fade. As we mentioned earlier, the traffic can change from a regime where the travel time is predictable with an accuracy of about 3% to a regime where the error climbs to 65% or higher. There's a critical region around maximal capacity where the traffic as a whole is very sensitive to small perturbations. This emergent phase transition in the traffic's collective behaviour results in a much greater spread of individual travel times. The business of predicting your own travel time suddenly becomes much more challenging. Under heavily congested conditions, perfect understanding of the system is replaced by a bemuddled picture of what's happening. In this emergent situation, $u(n) > s(n)$. Under these conditions, we must resort to simulation if we wish to improve our understanding of the way the traffic behaves collectively.

The surprising thing about self-organization is that it can transform a seemingly simple, incoherent system (e.g. light traffic) into an ordered, coherent whole (a strongly-interactive traffic jam). Adding a few more vehicles at a crucial stage transforms the system from a state in which the individual vehicles follow their own local dynamics to a critical state where the emergent dynamics are global. This involves a phase transition of an unusual kind: a non-equilibrium phase transition. Space scales change suddenly from microscopic to macroscopic. A new organizing mechanism, not restricted to local interactions, has taken over. Occasional jamming transitions will even span the whole vehicle population, because the traffic has become a complex system with its own emergent dynamics. What's most important is that the emergence of stop-start waves and jams, with widely varying populations of affected vehicles, could not have been anticipated from the properties of the individual drivers or their vehicles.

[Figure 8.1 near here]

As the size and rule complexity of many classes of socio-economic system changes, various phase changes can occur when the curves u(n) and s(n) cross. Darley argues that there's no discontinuity separating non-emergent and emergent systems, just a phase change in the optimal means of prediction. Beyond this, perfect understanding of the system does no better than a simulation. Our astonishment at the fact that we seem unable to predict emergent properties doesn't stem from any inability to understand, but from the inherent properties of the system attributable to the accumulation of interactions. As systems become more emergent, the propagation of information through accumulated interaction will blur the boundaries of any analysis which we try to perform. All useful predictive knowledge is contained in the accumulation of interactions.

The advantage of an agent-based approach to any CAS is that the system's dynamics is generated by way of the simulation. Interactions can accumulate, multiple pathways can be recognized, and emergent properties can be revealed, all without making any ad hoc assumptions or aggregated models for these properties. The major disadvantages of simulation are the extremely high computational demands and the fact that it may not always lead to a better understanding of the basic mechanisms that caused the dynamics. Although the inherent dynamics is revealed, it's not always explained.

The fact that a given system lies far beyond the realms of deductive reasoning does not necessarily mean that we should lose all faith in our traditional means of comprehension, explanation and prediction. Consider the game of chess, first discussed in Chapter 2. Modern computer chess programs use extremely sophisticated, brute force approaches to simulate the game and decide on moves. By way of contrast, human grand-masters use a subtle combination of pattern-recognition, generalization and analogy-making with foresight to "understand" the game and make their decisions. In this instance, the phase change where the curves u(n) and s(n) cross is at such a value that humans can still boast superiority at determining such elusive concepts as positional advantage.[8] But the computer is rapidly bridging such gaps.

Many scientists now believe that chess lies on the emergent side of the phase boundary, so much so that solution by simulation is ultimately the best approach. However, human experience and understanding can often do surprisingly well, despite all the limits of knowledge and reasoning. Perhaps a sophisticated combination of both approaches may be the best bet for predicting the behaviour of a CAS. The brain itself is

[8] Especially if the usual time constraints on moves are removed.

an extremely complex system, one whose functioning would appear to lie far beyond the phase change. Our state of mind seems to be an emergent property of our brains, more mysterious than the motion of a car because we can't see the mental wheels going around. A human mind is a process, not a thing, emerging from the collective interactions of appropriately organized bits of ordinary matter. This has very important ramifications for the youthful field of Artificial Life.[9] We'll look at some of the economic ramifications of these exciting developments in the next section.

{A}The New Age of Artificial Economics{/A}

There's no universally agreed definition of economic activity. Classical economics texts place most of their emphasis on exchange processes, such as trade between producers and consumers. In other words, they emphasize transactions between agents. In this book, our emphasis has not been on the transactional character of economic interactions, but on the dynamic, learning aspects. Why? Because the adaptive behaviour of human agents makes a dynamic approach obligatory. Human learning means that economies are not just transactional systems to be analyzed as if they're simply part of a giant accounting system. Instead, economies should be treated as something very much "alive".

Traditionally, the scientific study of life has been restricted to biology. But some economists have recognized the nexus between biology and economics. It was Alfred Marshall who contended that biology, not mechanics, is the true Mecca of economics.[10] Economics should be a branch of biology concerned with the study of socio-economic life. That makes it cultural and dynamic, not purely financial and transactional. Economics must embrace a cluster of properties associated with life in general: self-

[9] Traditional knowledge-based approaches to AI, based on conceptual ideas and understanding, are less likely to succeed than approaches relying on agent-based, interactive phenomena.

[10] "But economics has no clear kinship with any physical science. It is a branch of biology broadly interpreted." (Marshall, 1920, page 772).

organization, emergence, growth, development, reproduction, evolution, coevolution, adaptation, and morphogenesis.[11]

Above all, economic development depends crucially on path-dependent principles of self-organization and coevolution, unfamiliar processes that have remained largely untouched by traditional analytical tools. The challenge is to create a bottom-up approach, a *synthetic* methodology in which the behaviour of agents is examined in each other's presence. Its pursuit lies at the heart of agent-based simulation. The collective behaviour that results from such an approach can be radically different to that posited from studies of agents in isolation.

The time seems ripe for a radically new approach to economics. We might call it *Artificial Economics*! Like its predecessor, Artificial Life, Artificial Economics (AE) would adopt a synthetic approach. Instead of taking economies apart, piece-by-piece, AE would attempt to put economies together in a coevolutionary environment. Its primary aim would be to link economic macrostructure to agents' microeconomic behaviour in a consistent, path-dependent manner. We might even find that such a synthetic approach may lead us beyond known economic phenomena: beyond *economic-life-as-we-know-it* and into the less familiar world of *economic-life-as-it-could-be*.[12]

Instead of those stubbornly homogeneous agents who dominate the classical economist's world, AE would concern itself with a rich diversity of agents generating *lifelike* economic behaviour. To produce lifelike economic outcomes, it would create diverse *behaviour generators*. This problem is partly psychological and partly computational. We've discussed behaviour generators in earlier chapters, under the guise of constructs, predictors and mental models. Many of the mechanisms by which economic reasoning and behaviour arises are known. There are still some gaps in our knowledge, but the general picture is falling into place. Like nature, an economy is

[11] Some early attempts to make progress in these directions can be found in Allen and Sanglier (1979, 1981) and Batten (1982). Interest in coevolution and self-organization has grown in the nineties, especially within the field of economic geograhy; see Fujita (1996) and Krugman (1996).

[12] Some recent work by David Lane, in which he describes a class of models called artificial worlds, designed to provide insights into a process called emergent hierarchical organization, could be said to typify the kinds of experiments that fall under the heading of Artificial Economics. For details of these artificial worlds, see Lane (1993).

fundamentally parallel. Thus AE can start by recapturing economic life as if it's *fundamentally and massively parallel*.[13] If our models are to be true to economic life, they must also be highly distributed and massively parallel.

AE would be concerned with the application of computers to the study of complex, economic phenomena. This doesn't mean that the computational paradigm would be the underlying methodology of behaviour generation.[14] Nor would AE seek to explain economic life as a kind of computer program. Instead, for example, it might use insights from evolutionary biology and human psychology to explore the dynamics of interacting agents and the resulting collective economic outcomes. This was the synthetic approach in the El Farol problem. Artificial music lovers were assigned different sets of predictors to aid in their decision-making. If the ecology of active predictors is suitably diverse, it's likely that it would mimic the diverse approaches of an assortment of real music lovers. The same may be said of artificial commuters and artificial investors.

In the days before computers, economists worked primarily with systems whose defining equations could be solved analytically. For obvious reasons, they politely ignored those whose defining equations could *not* be solved. This led to gross approximations, sometimes even to gross misrepresentations! With the advent of computers, however, mundane calculations can be handled routinely. Agent-based simulation allows one to explore an economic system's behaviour under a wide range of parameter settings and conditions. The heuristic value of this kind of experimentation cannot be overestimated. One gains much richer insights into the potential dynamics of an economy by observing the behaviour of its agents under many different conditions. Let's look briefly at a product of this new age of Artificial Economics: the evolution of an artificial society of agents, initially engaged in some relatively primitive economic activity.

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[13] Massively parallel "architecture" means that living systems consist of many millions of parts, each one of which has its own behavioural repertoire.

[14] This has been the approach taken in Artificial Intelligence. Methodologies to be explored in Artificial Economics have much more in common with the embryonic field of Artificial Life. For an introduction to Artificial Life, see Langton (1995).

{A}Growing a Silicon Society{/A}

It has been said that the ultimate goal of the social sciences is to discover laws of cultural dynamics. Economic development in the very long run must play a part in this scientific exercise. The problem is that most economic analyses focus on the short to medium run. Furthermore, what economists typically regard as the long run is only a relatively short-run movement from the perspective of archaeologists. For the latter, a generation or even a century is a relatively short period. Archaeologists tend to think in terms of millenia when considering changes in human culture.

Archaeology can help economics because the field has gleaned a reasonably clear picture of socio-economic development in the very long run. In terms of broad epochs, that development has taken us from the epoch of hunting and gathering to horticultural settlements and complex, nonliterate societies, then to the historical epoch of urban civilization, household agriculture, and trading empires, and more recently to the industrialized economy.[15] There's plenty of evidence suggesting that we're currently in the midst of another major transition to a new epoch of postindustrialization, largely associated with the advent of a knowledge-based economy and increasingly sophisticated information-processing devices.

Sometimes the transition between epochs seems to have been smooth, but on other occasions there's evidence of crises and abrupt upheavals prior to the successful adoption of a new regime. Much of the archaeological evidence supports the notion of a series of punctuated equilibria, as discussed in earlier chapters. Examples even exist where a rapid collapse and reversion to a previous regime has occurred. In some well-known cases of relatively isolated cultures, the socio-economic process seems to have stuck in a more-or-less stationary or fluctuating state for a very long time. All these documented examples provide convincing evidence in support of the multiplicity of outcomes that are possible as the economic agents in a society coevolve.

One of the more baffling cultures to have challenged the minds of archaeologists is that of the Anasazi Indians. The earliest settlement built by this Native American

[15] For an interesting discussion of economic change in the very long run, see Day and Walter (1995).

culture dates from AD 100, but nobody seems to know where they lived before they set up house in north-eastern Arizona. What is known is that over a period of 1200 years, the Anasazi established a flourishing culture of villages, shrines and farms. They enjoyed a golden age of over a hundred years towards the end of the thirteenth century. Then, quite suddenly, they abandoned their elaborate dwellings and fertile farmland and travelled south-east to the Rio Grande and Arizona's White Mountains. No one seems to know why the Indians abandoned the place that had been their fertile home for over a thousand years. But after the exodus, the number of Anasazi dwindled to a third of what it was in its heyday. Only a few remnants of their culture have survived today, such as their pottery and agriculture. Most of their ancient history has been lost.

But now the Indians have been brought back to "life." This ancient tribe has been resurrected and their native landscape is once again dotted with Anasazi settlements. The big difference on this occasion is that each Anasazi community is actually a colored zone on a grid which sits inside the memory of a Macintosh computer. In a broader sense, this artificial society consists of two main elements: a population of "agents" (like the Anasazi) and an environment on which these agents "live." This two-level, silicon world is the joint brainchild of Joshua Epstein and Robert Axtell, two researchers at the Brookings Institution in Washington, D.C.

Epstein and Axtell set out to "grow" a social order from scratch, by creating an ever-changing environment and a set of agents who interact with each other and their environment according to a set of behavioural rules. History is said to be an experience that's only run once. Clearly Epstein and Axtell don't hold with that view. Their idea is that an entire society like the Anasazi – complete with its own production, trade and culture – could be "recreated" or evolved from the interactions among the agents. As Epstein suggests: "You don't solve it, you evolve it." They call the laboratory in which they conduct their simulation experiments a *CompuTerrarium*, and the landscape which the interacting agents inhabit a *Sugarscape*.[16] Let's take a closer look at how socio-economic life develops in this artificial world.

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[16] For a full account of life on the Sugarscape, see Epstein and Axtell (1996). Readable summaries of this metaphoric world of artificial life can be found in Casti (1997, Chapter 4) and Ward (1999, Chapter 2).

The action takes place on a small grid of fifty-by-fifty cells. But Sugarscape is not a pure CA (cellular automaton). The landscape denoted by this grid is not blank, as it is on a typical CA. On it is scattered this silicon world's only resource: sugar. In order to survive, the entities that inhabit this sweetened landscape must find and eat the sugar. The entities themselves are not just cells that are turned on or off, mimicking life or death. Each is an agent that's imbued with a variety of attributes and abilities. Epstein and Axtell call these internal states and behavioural rules. Some states are fixed for the agent's life, while others change through interaction with other agents or with the environment. For example, an agent's sex, metabolic rate, and vision are hard-wired for life. But individual preferences, wealth, cultural identity and health can all change as agents move around and interact.

Although every interacting agent appears on the grid as a colored dot, each may be quite different. Some are far-sighted, spotting sugar from afar. Others are thrifty, burning the sugar they eat so slowly that each meal lasts an eternity. Still others are short-sighted or wasteful. Rapacious consumers eat their sugar too quickly. The obvious advantage of this heterogeneity is that it's capable of mimicking (albeit simplistically) the rich diversity of human populations in terms of their preferences and physiological needs. Any agent that can't find enough sugar to sustain its search must face that ultimate equilibrium state: it simply dies!

Sugarscape resembles a traditional CA in its retention of rules. There are rules of behaviour for the agents and for the environmental sites (i.e. the cells) which they occupy. Rules are kept simple, and may be no more than the commonsense ones for survival and reproduction. For example, a simple movement rule might be: *Look around as far as you can -- find the nearest location containing sugar – go there – eat as much as you need to maintain your metabolism – save the rest*. Epstein and Axtell speak of this as an agent-environment rule. A rule for reproduction might be: *Breed only if you've accumulated sufficient energy and sugar*. Also, there are rules governing socio-economic behaviour, such as: *Retain your current cultural identity (e.g. consumer preferences) unless you see that you're surrounded by many agents of a different kind – if you are, change your identity to fit in with your neighbors or try to find a culture like your own.*

This rule smacks of Schelling's segregation model, because it highlights coevolutionary possibilities among nearby neighbors.

The *CompuTerrarium* leaps into action when hundreds of agents are unleashed randomly onto the grid. Colored dots distinguish agents who can spy sugar easily from more myopic agents. Naturally, all the agents rush towards the sugar. The latter may be piled into two or more huge heaps or scattered more evenly throughout the landscape. Strikingly, many agents tend to "stick" to their own terrace, adjacent to their "birthplace." Because natural selection tends to favour those agents with good eyesight and a low metabolic rate, they survive and prosper at the expense of the short-sighted, rapacious consumers. In short, the ecological principle of carrying capacity quickly becomes evident. Soon the landscape is covered entirely with red dots (high-vision agents).

Even with relatively simple rules, fascinating things start to happen as soon as the agents begin to interact on the Sugarscape. For example, when seasons are introduced and sugar concentrations change periodically over time, high-vision agents migrate. But low vision, low metabolism agents prefer to hibernate. Agents with low vision and high-metabolism usually die, because they're selected against.

All of the time, the surviving artificial agents are accumulating wealth (i.e. sugar). Thus there's an emergent wealth distribution on the Sugarscape. Herein lies the first topic of particular interest to economists. Will the overall wealth be distributed equally, or will agents self-organize into a Pareto distribution? In other words, will equity prevail or will the ubiquity of power laws prevail again? No doubt you've guessed already. Although quite symmetrical at the start, the wealth histogram on the Sugarscape ends up highly skewed. Because such skewed distributions turn up under a wide range of agent and environment conditions, they resemble an emergent structure -- a stable macroscopic pattern induced by the local interaction of agents. Self-organization is on the job as usual, and the power law wins out again!

Although these few examples are a useful way of illustrating the variety of artificial life evolvable inside the *CompuTerrarium*, they hardly herald an impending revolution in our understanding of how the economy works. For that we must expand the behavioural repertoire of our agents, allowing us to study more complex socio-economic phenomena. Epstein and Axtell have made a start on this expansion. When a second commodity, spice, is added to the landscape, a primitive trading economy emerges. By portraying trade as welfare-improving barter between agents, reminiscent of those Merchants of Venice that we met in Chapter 4, they implement a trading rule of the form: *Look around for a neighbor with a commodity you desire, bargain with that neighbor until you agree on a mutually acceptable price, then make an exchange if both of you will be better off*.

Surprisingly, this primitive exchange economy allows us to test the credentials of that classical theory of market behaviour: the efficient market hypothesis. The first stage of the test involves imbuing agents with attributes consistent with neoclassical economic wisdom – homogeneous preferences and infinite lifespans for processing information. Under these conditions, an equilibrium price is approached. But this equilibrium is not the general equilibrium price of neoclassical theory. It's *statistical* in nature. Furthermore, the resulting resource allocations, though locally optimal, don't deliver the expected global optimum. There remain additional gains from trade that the agents can't extract. What we find is that two competing processes – exchange and production – yield an economy that's perpetually out of equilibrium.

Once we imbue agents with human qualities – like finite lives, the ability to reproduce sexually, and the ability to change preferences, the trading price never settles down to a single level. It keeps swinging between highs and lows, very much like price oscillations in real markets (as discussed in the previous chapter). Basically, it appears to be a random distribution. But it turns out that there's structure after all! Although the seemingly-random price fluctuations continue indefinitely, the fluctuations appear to be variations from an identifiable price level. This particular price just happens to be the same equilibrium level as the one attained under those all-too-unrealistic assumptions underpinning the efficient market hypothesis. Thus we gain the distinct impression that any equilibrium state associated with the efficient market hypothesis is nothing more than a limiting case among a rich panorama of possible states that may arise in the marketplace. As Epstein suggests: "If the agents aren't textbook agents – if they look a little bit human – there is no reason to assume markets will perform the way economic textbooks tell us they should."

How is the distribution of wealth affected by trade? It turns out that the overall effect of trade is to further skew the Sugarscape's distribution of wealth. By increasing the carrying capacity, and allowing more agents to survive, it also magnifies differences in wealth. Trade increases the interactions between agents, thereby strengthening the power law fit even further. Experiments with a wider set of choice possibilities endorsed the view that it's devilishly difficult to find conditions under which a society's wealth ends up being evenly distributed. We may conclude that there's a definite tradeoff between economic equality and economic performance. This bears a striking qualitative similarity to findings in various economies around the world.

There's so much more to say about the socio-economic laboratory constructed by Epstein and Axtell. When agents can enter into credit relationships, for example, some turn out to be borrowers and lenders simultaneously. This is of fundamental importance for economic evolution. Many other issues – such as the emergence of cultural groups, webs of economic intercourse, social clusters, institutional structures, and disease – can all be scrutinized under the Sugarscape microscope. The pair are now working to extend the Sugarscape in order to capture the way of human life in the late twentieth century. Thus far, the agents have sex but there are no families, no cities, no firms and no government. Over the next few years, they hope to produce conditions under which all of these things emerge spontaneously. As life on the Sugarscape is in its infancy at present, who's to say what might happen in time?

Sugarscape is an important example of agent-based simulation for several reasons. First, although economists and other social scientists study society, they do so in isolation. Economists, geographers, psychologists and sociologists rarely interact meaningfully or pool the knowledge they've accumulated. The organization of university departments further endorses this regrettable divide. Yet life on the Sugarscape brings all these narrow views together, broadening our understanding in a meaningful way. Second, Sugarscape activities are interactive and dynamic. Thus it's far more process-dependent than classical models. Third, Sugarscape recognizes and preserves differences in culture and skills that human populations exhibit. Finally, for the first time in history, the social sciences have the opportunity to conduct and repeat experiments and test hypotheses to do with socio-economic behaviour. Sugarscape typifies this new way of doing social science as we enter the unprecedented era of Artificial Economics. So if your business is modelling economic behaviour, it's an excellent starting point for rule-based experiments.

{A}Some Final Words{/A}

Like it or not, computers have handed scientists a new paradigm for modelling the world. With the incredible drop in the cost of computing power, computers are now capable of simulating many physical systems from first principles. For example, it's now possible to model turbulent flow in a fluid by simulating the motions of its constituent particles - not just approximating changes in concentrations of particles at particular points, but actually computing their motions exactly. But such advances may not be unique to the physical world. Perhaps it may not be so long before we can model the turbulence observed in financial markets.in much the same way.

Agent-based simulation models like Sugarscape and TRANSIMS have shown us that complex behaviour need not have complex origins. Some of the complex behaviour exhibited collectively by economic agents, for example, may come from relatively simple predictors. Other emergent behaviour may be attributable to predictors which differ in terms of the time horizons over which they're applied. Since it's hard to work backwards from a complex outcome to its generator(s), but far simpler to create many different generators and thus synthesize complex behaviour, a promising approach to the study of complex economic systems is to undertake a general study of the kinds of collective outcomes that can emerge from different sets of predictors as behaviour generators.[17] As we've stressed already, most work of this kind must be done by simulation experiments.

There are many exciting new efforts underway which attempt to replicate the rich diversity of socio-economic life inside the computer. We've discussed a few of these in this book. Sadly, space has precluded discussion of them all. The common feature of these experiments is that the main behaviours of interest are properties of the *interactions between agents*, rather than the agents themselves. Accumulations of interactions constitute the fundamental parts of nonlinear economic systems. They're the *virtual parts* of an economy, which depend on nonlinear interactions between human agents for their very existence. If we choose to isolate the agents, then the virtual parts disappear. If we choose to aggregate the agents, then the virtual parts disappear. It's the virtual parts of an economy that Artificial Economics is after. The goal is the *synthesis*, rather than *analysis*. In this quest, synthesis by simulation is the primary methodological tool and the computer is the scientific laboratory.

Like nature, economies coevolve incessantly. They add and subtract mechanisms, components, and interactions over time. They're just as alive as any biological organism. Their unique quality is an evolutionary drive that selects for human agents with an ability to learn and adapt, rather than for those choosing optimal behaviour. Because economic diversity springs from the heterogeneity of human learning and creativity, economic evolution may be subject to "The Baldwin Effect."[18] The importance of adaptive learning shows up more clearly when the economy is viewed in a long-run perspective. Learning and adaptation should not be addenda to the central theory of economics. They should be right at its core in strongly-interactive environments of high complexity.

As interactions grow, the natural trend of human progress is forever towards the more complex. But the path of progress is not a smooth curve, and never will be; unless human nature is somehow repealed. It's a very haphazard path, straight enough for much of the time, but boasting tortuous twists and turns at unexpected times. Economic progress looks like a series of punctuated equilibria. So does the path of environmental quality. Because the interactions between agents in an economy can produce robust, self-organized dynamic equilibria, the frequency of disturbances from this critical state may obey a power-law distribution with respect to their size. Power law distributions seem to

[17] For a deeper discussion of behaviour generators and the theory of simulation, see Rasmussen and Barrett (1995) or Barrett et al. (1998).

[18] First proposed in the late nineteenth century, the Baldwin Effect suggests that the course of evolutionary change can be influenced by individually learned behaviour. The existence of this effect is still a hotly debated topic in biology and related fields. For evidence of how the Baldwin Effect may alter the course of evolution, see Hinton and Nowlan (1987) or French and Messinger (1995).

be ubiquitous in nature and human societies. Thus we shouldn't be surprised by occasional large fluctuations. Archaeologists never are. Very big changes are part of a frequency distribution that reflects many more smaller changes alongside fewer larger changes.

In the preceding chapters, we've looked at how economies (and their parts) can self-organize. Self-organization in a complex economy results from a set of agents, driven by their own behavioural biases, interacting to produce unexpected collective outcomes. The goals, strategies, ethics and understanding of these agents fashion the collective behaviour that emerges which, in turn, forces each agent to react and adapt differently. Sometimes something new emerges. Sometimes a different regime takes over. Future expectations and decision strategies change dramatically. So do future collective outcomes.

If nothing else, an appreciation of power laws, adaptive learning and self-organization teaches us humility. Perhaps science is revealing our own limitations. Understanding that we can't know everything is a crucial step in the quest for wisdom. Our inability to predict can be soothed by a growing ability to adapt and coevolve harmoniously - just like we find in nature. We live in a world full of remarkable emergence and diversity. History matters. We can't be sure where we're going next, but thankfully we're getting to know some of the rules by which the game is being played. What's most reassuring is that sometimes we seem to get something for nothing; emergent order on this neverending road to know-ware.

TABLE 1.1:
Two Economic Worlds - The Simple and the Complex

NECESSITY	CHANCE
Stasis	Morphogenesis
Resource-Based	Knowledge-Based
Unique Outcome	Multiple Outcomes
Equilibrium	Path-Dependent
Mechanistic	Organic
Predictable	Unpredictable
Diminishing Returns	Increasing Returns
Convex	Nonconvex
Easy to Model	Difficult to Model
A SIMPLE WORLD	A COMPLEX WORLD

TABLE 2.1:
Information and Knowledge

Characteristic	Information	Knowledge
Source	External	Internal
Nature	Weakly-interactive	Strongly-interactive
Primary exchange mode	Interface	Face-to-face
Learning rate	Fast	Slow
Usefulness	Temporary	Longlasting
Exchange process	Simple	Complex
Unit of measurement	Quantitative (e.g. bits)	Qualitative (e.g. deep)

TABLE 4.1
Population Growth in Europe

Date	European Population	Margin of Error (%)
200	48	35
500	36	30
800	32	30
1000	39	20
1300	75	20
1500	76	10
1700	102	8

TABLE 4.2:
The Ten Largest Cities in Europe by Population, 1000-1400

1000	1100	1200	1300	1400
Cordova	Constantinople	Constantinople	Paris	Paris
Constantinople	Fez	Palermo	Granada	Bruges
Seville	Seville	Seville	Constantinople	Milan
Palermo	Palermo	Paris	Venice	Venice
Kiev	Cordova	Venice	Genoa	Genoa
Venice	Granada	Cordova	Milan	Granada
Thessalonika	Venice	Granada	Sarai	Prague
Ratisbon	Kiev	Milan	Seville	Constantinople
Amalfi	Salerno	Cologne	Florence	Rouen
Rome	Milan	London	Cologne	Seville

Table 5.1: Changes in Rank of Selected American Cities, 1810-1910

City	Rank in 1810	Rank in 1860	Rank in 1910
New York	1	1	1
Philadelphia	2	2	3
Baltimore	3	3	7
Boston	4	4	5
New Orleans	6	5	14
Cincinatti	42	6	13
St. Louis	-	7	4
Chicago	-	8	2
Buffalo	-	9	10
Louisville	-	10	22
Albany	17	11	44
Washington	12	12	16
San Francisco	-	13	11
Providence	8	14	21
Pittsburgh	28	15	8
Rochester	-	16	23
Detroit	-	17	9
Milwaukee	-	18	12
Cleveland	-	19	6
Charleston	4	20	77

Table 5.2: Similarities Between CAs and Socio-Economic Dynamics

	Cellular Automata	Socio-Economic Dynamics
Basic elements	Cells are the basic units or "atoms" of a CA	Individual agents are the basic units of an economy
Possible states	Cells assume one of a set of alternative states	Agents form mental models which enable them to make choices from alternatives
Interdependence	The state of a cell affects the state of its closest neighbors	The choices made by agents affect the choices made by other agents
Applications and tasks	Modeling the emergence of order, macro outcomes explained by micro rules, and the path dependence of dynamic processes	Important tasks include: understanding the emergence of order, macro to micro relationships, and economic dynamics

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