C^3 Modeling with Symmetrical Rationality

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ABSTRACT

In the absence of contrary information, it would seem prudent for competitors to attribute to their opposition the same level of rationality they attribute to themselves. In the context of a simple but interesting C^3 (command, control, and communication) model presented in Ref. [3], a method is proposed for incorporating symmetrical rationality without resorting to the general multistage game framework which has proved difficult to apply in practice. A technique is suggested for the approximate solution of the resulting C^3 model which does not require integration operations, and which appears to be especially well suited for C^3 problems with finite admissible control sets.

1. INTRODUCTION

The mathematical C^3 (command, control, and communication) model developed in Ref. [3] incorporates many of the esssential elements inherent in tactical campaigns, i.e., strategic nonstationary interaction between two opposing forces, each subject to imperfect information. One important use envisioned for the model is the cost evaluation of alternate information infrastructures for one of the forces (blue), given a fixed known mode of opposition from the second force (red).

A linear-quadratic version of the C^3 model with unconstrained controls is solved in Ref. [1] by applying the Kagiwada-Kalaba solution technique for

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integral equations [4] to the first-order minimization conditions, viewed as a system of Fredholm integral equations. One interpretation suggested for the linear-quadratic C^3 model is as a perturbation model, for which the controls derived under various comparative static changes in information infrastructure are to be viewed as deviations from previously assigned values.

In attempting to apply the results developed in Refs. [1] and [3] to general C^3 problems, several potential difficulties arise. If controls are constrained, or if the C^3 problem cannot be modeled in a linear-quadratic framework, the Kagiwada-Kalaba solution technique is no longer directly applicable. A second difficulty, of a more conceptual nature, is the asymmetrical modeling of the blue and red opposing forces in terms of their rational decision-making capabilities. Specifically, in Refs. [1] and [3] the red-force activity is modeled as a nonstationary unconditioned probability distribution, whereas the blue-force optimal-control law is derived as a nonstationary function of past state and control realizations.

For short-term decision processes, asymmetrical rationality may be a justifiable approximation. However, for long-term decision processes, especially those for which it is desired to evaluate alternative control objectives and information infrastructures, the presumption of asymmetrical rationality can result in significantly inaccurate scenario projections. For example, after observing sufficiently many state and control realizations, the red force could presumably form an accurate estimate of the blue control objective and information infrastructure and vary its own actions accordingly. Thus a shift in objective or infrastructure by the blue force could eventually lead, in reality, to a radically different red response than envisioned by blue in its original scenario projections. Even if the blue control objective and information infrastructure were fixed and known, the red force could gain an unanticipated informational advantage by taking into account, however crudely, the rational decision-making processes of the blue force as reflected in past state and control realizations.

In the absence of contrary information, it thus seems a prudent rule of thumb for competitors to attribute to their opposition the same level of rationality they attribute to themselves. Ideally, the analytical planning framework should permit each opposing force to optimally exploit all the information to which it has access.

In the following two sections a method is proposed for incorporating symmetrical rationality in general C^3 models without resorting to the multistage differential game framework, which has proved difficult to apply in actual decision contexts. In addition, an approximate solution technique is suggested for the resulting C^3 model which does not require integration operations, and which appears to be especially well suited for C^3 problems with finite admissible control sets. Although the suggested solution technique is not competitive with the Kagiwada-Kalaba solution technique for linear-quadratic C^3 models with unconstrained controls, it does appear to be applicable to a wide range of C^3 problems for which no practical solution techniques currently exist.

2. A SIMPLE C^3 MODEL WITH SYMMETRICAL RATIONALITY

As in Ref. [3], Sec. II, consider an amphibious N-day campaign in which a blue naval force lands ground troops and then provides close air support. At the beginning of each day n, the blue air and ground commanders must decide on the air and ground force strengths to be used that day.[1] The objective of the blue commanders is to reach a ground position s^0 at minimum total cost within the specified time limit N. Simultaneously, red naval commanders daily deploy air support and ground troops in an attempt to prevent the inland penetration.

Suppose the blue air and ground commanders have no intelligence concerning red air and ground activity for each coming day. In the discussion of this case in Ref. [3], p. 8, the nth-period red air and ground activity is assumed to be a random vector $\omega_n \equiv (p_n, q_n)$ with density function $w(\cdot)$ known to both blue commanders. The nth-period blue air and ground activity $v_n \equiv (\alpha_n, \beta_n)$ is then optimally derived as a function of the current time n and the current front-line position s_n. The red force is thus modeled as less rational than the blue force in the sense that, in contrast to blue, red makes no use of the information (n, s_n) to which it has access on day n.

Symmetrical rationality would require that blue commanders attribute to red commanders the same level of rationality they attribute to themselves. If blue derives an optimal-control law functionally dependent on time and position, then, in the absence of contrary information, red activity should also be viewed by blue as the result of an optimal-control law functionally dependent on time and position. Other known factors, e.g., current weather, might be presumed to affect red activity; but then the question arises: why is blue activity not also derived as a function of these additional factors? If certain known factors affect red activity, then *a priori* they must also be important for blue.

On the other hand, it is entirely possible that additional factors unknown to blue are affecting red activity—e.g., red objectives, or red modeling of blue activity. For this reason, blue cannot in general derive the red optimal-control law. However, blue could presumably account for these unknown factors by modeling nth-period red activity as a random drawing from a

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[1]The time-to-go variable K used in Ref. [3] has been replaced by the time-past variable $n \equiv N - K$.

probability distribution

$$p_n(\cdot\,|s_n) \tag{1}$$

conditioned on the current time n and the current front-line position s_n. If blue knew the red control law exactly, then (1) would be a degenerate distribution.

The blue specification (1) for red activity is consistent with symmetrical rationality for red, in the following sense. If red likewise models blue activity as a random drawing from a probability distribution conditioned on time and position, then its optimal-control law is still a function only of time and position. No new variables are introduced.

The modeling of red activity by means of a conditional probability distribution (1) resolves in principle the problem of second guessing. How might (1) be assessed in practice? One possibility would be by means of successive min-min approximation. Consider first the case in which blue knows red's objective. For the initial iteration, blue models red activity via an unconditioned prior distribution, and derives the resulting blue optimal-control law. Substituting this blue control law into the red optimization problem, blue then derives an optimal-control law for red. For the second iteration, blue uses the red optimal-control law resulting from the first iteration to rederive a blue optimal-control law, which in turn is then used to rederive an optimal-control law for red; and similarly for successive iterations. Assuming this process converges, the limiting blue and red control laws represent a Nash equilibrium in control-law space. If blue does not know red's objective, but has a discrete prior distribution for this objective, then separate consideration of possible red objectives at each iteration results in a probability distribution over possible red control laws, i.e., a conditional distribution having the form (1).

The modifications needed in order to symmetrically model red activity by the conditional distribution (1) in place of an unconditioned density, as in Refs. [1] and [3], will now be outlined in the context of a C^3 model which generalizes the C^3 model presented in Ref. [3].

Suppose the progression of the blue ground troops is determined in accordance with the system of equations

$$s_1 = \bar{s} \qquad \text{(initial landing position)}, \tag{2a}$$

$$s_{n+1} = f_n(\omega_n, v_n, s_n), \qquad 1 \leqslant n \leqslant N, \tag{2b}$$

where the *n th-period state (front-line position)* s_n is an element of a set $S \subset R^q$, the *n th-period blue control* v_n is constrained to lie in an admissible control set $V(n,s_n)\subset V\subset R^r$, the *nth-period blue observation* ω_n on red activity is an element of a set $\Omega\subset R^m$, and $f_n:\Omega\times V\times S\to S$ is a continuous[2] *state function*. Letting $\mathcal{F}$ denote the σ-algebra generated by the open sets of Ω, assume the blue force believes that red activity ω_n is governed by a transition probability[3] $p_n(\cdot|s_n):\mathcal{F}\to R$ conditioned on the current time n and the current state s_n. In addition, assume that the loss associated by the blue force with each possible observation, control, and state configuration (ω_n,v_n,s_n) for period n is measured by a continuous *cost function* $C_n:\Omega\times V\times S\to R$.

An *admissible feedback control law* for the problem at hand is any vector

$$\mathrm{v}=(v_1(\cdot),\ldots,v_N(\cdot)) \tag{3}$$

of measurable functions $v_n:S\to V$ satisfying $v_n(s)\in V(n,s)$ for each $s\in S$. The symbol $\mathfrak{L}$ will be used to denote the set of all admissible feedback control laws v. The objective assumed for the blue force is the minimization of expected total cost

$$E\left[\sum_{n=1}^{N} C_n(\omega_n,v_n(s_n),s_n)\middle|\mathrm{v},\bar{s}\right] \tag{4}$$

via selection of a feedback control law $\mathrm{v}\in\mathfrak{L}$.[4]

[2]It is assumed throughout the paper that S, V, and Ω have the relative topology with respect to Euclidean q-space, r-space, and m-space, respectively, and that products of S, V, and Ω have the corresponding product topology. Each of the spaces S, V, and Ω will also be regarded as a measurable space, with σ-algebra generated by its open sets.

[3]More precisely, it is assumed that $p_n(\cdot|s):\mathcal{F}\to R$ is a probability measure for each $s\in S$, and $p_n(A|\cdot):S\to R$ is a measurable function for each $A\in\mathcal{F}$.

[4]The expectation operator $E[\cdot|\mathrm{v},s]$ is more precisely defined as follows. Let Ω^N denote the set of all observation sequences $\omega^N\equiv(\omega_1,\ldots,\omega_N)$ satisfying $\omega_n\in\Omega$, $1\leqslant n\leqslant N$, and let $\mathcal{F}^N$ denote the product σ-algebra generated by all cylinder sets of the form

$$\prod_{n=1}^{N} A_n=\{\omega^N\in\Omega^N|\omega_1\in A_1,\ldots,\omega_N\in A_N\},$$

where $A_n\in\mathcal{F}$, $1\leqslant n\leqslant N$. Finally, for each $\mathrm{v}\in\mathfrak{L}$ and $s\in S$, let $p^N(\cdot|\mathrm{v},s)$ denote the unique probability measure on $(\Omega^N,\mathcal{F}^N)$ satisfying

$$p^N\left(\prod_{n=1}^{N} A_n|\mathrm{v},s\right)=\int_{A_1}\int_{A_2}\cdots\int_{A_N} p_N(d\omega_N|s_N)\cdots p_2(d\omega_2|s_2)p_1(d\omega_1|s)$$

for each cylinder set $\Pi N_{n=1}A_n\in\mathcal{F}^N$, where $s_{n+1}=f_n(\omega_n,v_n(s_n),s_n)$, $1\leqslant n\leqslant N-1$. (See Ref. [2], Theorem A5, p. 148.) Expectation with respect to $\langle\Omega^N,\mathcal{F}^N,p^N(\cdot|\mathrm{v},s)\rangle$ is then denoted by $E[\cdot|\mathrm{v},s]$.

For brevity, any C^3 problem meeting the above specifications will be said to have the *basic format*.[5]

Let $E_n[\cdot|s]$ denote expectation with respect to the nth-period transition probability $\langle\Omega,\mathfrak{F},p_n(\cdot|s)\rangle$; and, for each $n\in\{1,\ldots,N\}$ and $s\in S$, let $H_n(s)$ denote the minimum attainable expected total cost beginning in period n with initial state s, and using feedback control. Then [2, Theorem 14.4, p. 101; pp. 104–105; Theorem 17.6, p. 111]

$$H_N(s)=\inf_v E_N[\,C_N(\omega,v,s)|s\,],\qquad v\in V(N,s); \tag{5a}$$

$$H_n(s)=\inf_v E_n[\,C_n(\omega,v,s)+H_{n+1}\circ f_n(\omega,v,s)|s\,],\qquad v\in V(n,s), \tag{5b}$$

$s\in S$, $n\in\{1,\ldots,N-1\}$; and a feedback control law $\mathrm{v}^{\mathrm{opt}}\in\mathfrak{L}$ minimizes total expected cost (4) if and only if it satisfies the dynamic-programming optimality equations

$$H_N(s_N)=E_N[\,C_N(\omega_N,v_N^{\mathrm{opt}}(s_N),s_N)|s_N\,], \tag{6a}$$

$$H_n(s_n)=E_n[\,C_n(\omega_n,v_n^{\mathrm{opt}}(s_n),s_n)+H_{n+1}\circ f_n(\omega_n,v_n^{\mathrm{opt}}(s_n),s_n)|s_n\,], \tag{6b}$$

$1\leqslant n\leqslant N-1$, for almost every[6] observation sequence $(\omega_1,\ldots,\omega_N)$.

In Ref. [1] an exact solution procedure is suggested for the optimality equations corresponding to a linear-quadratic C^3 problem with unconstrained blue controls, based on the Kagiwada-Kalaba representation of the first-order conditions for minimization as a system of Fredholm integral equations. An alternate approach is suggested here for the approximate solution of the optimality equations (6) which does not require integration operations, and which appears to be especially well suited for basic format C^3 problems with finite admissible control sets $V(n,s_n)$.[7]

[5] An axiomatization for a one-period version of the basic format model with discrete control-dependent probability distributions is provided in Ref. [6], where it is also shown that the basic format model with control-dependent distributions generalizes the Savage expected-utility model, the Marschak-Radner team model, and the standard Bayesian statistical decision model. The basic format model has proved to be useful in the development of a new approach to adaptive control, direct criterion-function updating. See Ref. [7].

[6] More precisely, using the definitions presented in footnote 4, the optimality equations must hold for p^N-almost every observation sequence $\omega^N\in\Omega^N$.

[7] Considering the crude approximations which generally must be incorporated into the basic structuring of state equations for socio-economic problems, the specification of control sets with infinitely many elements appears to be a computation-increasing luxury which planners can ill afford. As usual, one exception would be linear-quadratic control problems, for which solutions can be obtained in closed form.

Period 1. For each $n \in \{1,\ldots,N\}$ and $s \in S$, generate[8] a random sample $\{\omega_{-1}(n,s),\ldots,\omega_{-M}(n,s)\}$ from the probability distribution $\langle \Omega, \mathcal{F}, p_n(\cdot|s) \rangle$, and compute the cost-to-go estimates

$$H_N^0(s) \equiv \inf_{v \in V(N,s)} \left[\frac{\sum_{i=1}^{M} C_N(\omega_{-i}(N,s),v,s)}{M} \right] \tag{7a}$$

$$\equiv \inf_{v \in V(N,s)} C_N^0(v,s), \tag{7b}$$

$$H_n(s) \equiv \inf_{v \in V(n,s)} \left[\frac{\sum_{i=1}^{M} \left[C_n(\omega_{-i}(n,s),v,s) + H_{n+1}^0 \circ f_n(\omega_{-i}(n,s),v,s) \right]}{M} \right], \tag{7c}$$

$$\equiv \inf_{v \in V(n,s)} C_n^0(v,s), \tag{7d}$$

$s \in S$, $n \in \{1,\ldots,N-1\}$. Select $v_1^0(s_1) \in V(1,s_1)$ which satisfies $H_1^0(s_1) = C_1^0(v_1^0(s_1),s_1)$. Record an observation ω_1 for period 1 and the new state $s_2 = f_1(\omega_1, v_1^0(s_1), s_1)$ for period 2.

Period n $(2 \leqslant n \leqslant N)$. Select $v_n^0(s_n) \in V(n,s_n)$ which satisfies $H_n^0(s_n) = C_n^0(v_n^0(s_n),s_n)$. Record an observation ω_n for period n and the new state $s_{n+1} = f_n(\omega_n, v_n^0(s_n), s_n)$ for period $n+1$.

For each $n \in \{1,\ldots,N\}$, $s \in S$, and $v \in V(n,s)$, the cost-to-go approximation $C_n^0(v,s)$ defined by (7) is an averaging of M independent and identically distributed random variables. Thus, by a strong-law argument,

$$C_n^0(v,s) \xrightarrow{M} E_n\left[C_n(\omega,v,s) + H_{n+1}^0 \circ f_n(\omega,v,s) | s \right] \qquad \text{a.s.} \tag{8}$$

Given suitable additional regularity conditions (e.g., finiteness of the admissible control sets), it follows that

$$H_n^0(s) \xrightarrow{M} H_n(s) \qquad \text{a.s.} \tag{9}$$

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[8]More realistically, one would presumably generate random samples for a suitably selected finite grid. Although Monte Carlo integration methods using random sample generation are not as efficient as classical methods for well-behaved real functions on the real line, they often compare favorably with classical methods for multivariate integration or for integration involving integrand functions with discontinuities or kinks. See Ref. [8], Chapter 6.

One would therefore expect the outlined approximation procedure to perform satisfactorily for sufficiently large M.

3. EXTENSION: THE DEGRADED-COMMUNICATIONS CASE

Consider now the case in which the blue naval air commander receives exact intelligence concerning red ground strength for each coming day, but he is unable to communicate this intelligence to the blue ground commander. In the discussion of a similar[9] case in Ref. [3], p. 3, nth-period red air and ground activity is assumed to be a random vector $\omega_n \equiv (p_n, q_n)$ with known density function $w(\cdot)$. However, to be optimal, nth period blue air activity α_n must now be derived as a function $\alpha(q_n, s_n)$ of the current time n, the current front line position s_n, and the current red ground strength q_n.

In the absence of contrary information, it would seem prudent for blue commanders to assume that red intelligence is as effective as their own. This presumption of symmetrical rationality would require red activity $\omega_n \equiv (p_n, q_n)$ to be modeled as a random drawing from a probability distribution

$$p_n(\cdot \mid \beta_n, s_n) \tag{10}$$

conditioned on the current time n, the current position s_n, and the current blue ground activity β_n. (Cf. the discussion in Sec. 2.)

Ignoring for the moment the asymmetry in blue naval intelligence, the modifications needed to replace the state-conditioned distribution (1) in the basic-format C^3 model by the control and state-conditioned distribution (10) are conceptually straightforward. The blue objective is still the minimization of expected total cost (4); but now the expectation operator $E[\cdot \mid \mathrm{v}, s]$ is generated, as in footnote 4, by the transitional densities $p_n(\cdot \mid \beta_n(s_n), s_n)$ in place of the densities $p_n(\cdot \mid s_n)$. Letting $E_n[\cdot \mid \beta, s]$ denote expectation with respect to $p_n(\cdot \mid \beta, s)$, and $G_n(s)$ denote the minimum attainable expected total cost beginning in period n with initial state s, and using feedback control, one again obtains [2, *loc. cit.*] that

$$G_N(s) = \inf_{(\alpha,\beta) \in V(N,s)} E_N[C_N(\omega, \alpha, \beta, s) \mid \beta, s]; \tag{11a}$$

$$G_n(s) = \inf_{(\alpha,\beta) \in V(n,s)} E_n[C(\omega, \alpha, \beta, s) + G_{n+1} \circ f_n(\omega, \alpha, \beta, s) \mid \beta, s], \tag{11b}$$

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[9]In Ref. [3] it is hypothesized that blue air and ground commanders know the red air and ground strength, respectively, for each coming day. Symmetrical rationality is therefore ruled out *a priori*; i.e., it would be logically impossible for the blue and red air commanders to each know the air strength of the other for the coming day, and to base their own air strength upon this intelligence.

$s \in S$, $n \in \{1,\ldots,N-1\}$; and a feedback control law $\mathbf{v}^{\mathrm{opt}} \equiv (\boldsymbol{\alpha}^{\mathrm{opt}}, \boldsymbol{\beta}^{\mathrm{opt}}) \in \mathfrak{L}$ minimizes expected total cost (4) if and only if it satisfies the dynamic-programming optimality equations

$$G_N(s_N) = E_N\big[\, C_N(\omega_N, v_N^{\mathrm{opt}}(s_N), s_N) \,|\, \beta_N^{\mathrm{opt}}(s_N), s_N \big]; \tag{12a}$$

$$G_n(s_n) = E_n\Big[\, C_n(\omega_n, v_n^{\mathrm{opt}}(s_n), s_n) + G_{n+1} \circ f_n(\omega_n, v_n^{\mathrm{opt}}(s_n), s_n) \,\Big|\, \beta_n^{\mathrm{opt}}(s_n), s_n \Big], \tag{12b}$$

$1 \leqslant n \leqslant N-1$, for almost every observation sequence $(\omega_1, \ldots, \omega_N)$.

However, the optimality equations (12) make no explicit use of the additional intelligence available to the blue air commander concerning red ground activity for each coming day. Using this additional information, the Nth period minimization problems relevant for the blue air and ground commanders, respectively, are

$$\min_{\alpha} E_N\big[\, C_N(p_N, q_N, \alpha, \beta, s_N) | q_N, \beta, s_N \big], \qquad (\alpha, \beta) \in V(N, s_N), \tag{13a}$$

$$\min_{\beta} E_N\big[\, C_N(p_N, q_N, \alpha, \beta, s_N) | \beta, s_N \big], \qquad (\alpha, \beta) \in V(N, s_N), \tag{13b}$$

where $E_N[\cdot \,|\, q_N, \beta, s]$ denotes expectation with respect to the distribution $p_N(\cdot \,|\, \beta, s)$ restricted to the subspace $\{\omega = (p,q) | q = q_N\}$. A solution to (13) has the form of a pair of functions, $\alpha_N(q_N, s_N)$ and $\beta_N(s_N)$. The nth period minimization problems facing the two blue commanders are then

$$\min_{\alpha} E_n\big[\, B_n(p_n, q_n, \alpha, \beta, s_n) | q_n, \beta, s_n \big], \qquad (\alpha, \beta) \in V(n, s_n), \tag{14a}$$

$$\min_{\beta} E_n\big[\, B_n(p_n, q_n, \alpha, \beta, s_n) | \beta, s_n \big], \qquad (\alpha, \beta) \in V(n, s_n), \tag{14b}$$

where

$$B_n(p,q,\alpha,\beta,s) \equiv C_n(p,q,\alpha,\beta,s) + G_{n+1}^{*} \circ f_n(p,q,\alpha,\beta,s), \tag{15}$$

and G_{n+1}^{*} is the nth-period cost-to-go generated recursively by use of the solution functions $\alpha_k(q_k, s_k)$ and $\beta_k(s_k)$, $k \in \{n+1, \ldots, N\}$.

An exact solution technique is suggested in Ref. [1] for a linear-quadratic C^3 problem with unconstrained blue controls and degraded communications, based on the Kagiwada-Kalaba representation of the first-order conditions for minimization as a system of Fredholm integral equations. An approximate solution technique for (14) is suggested below, based on Monte Carlo integration.

Briefly, it is first suggested that G^*_{n+1} in (15) be replaced by the cost-to-go function G_{n+1} generated by (11), and β and α in (14a) and (14b), respectively, be replaced by the feedback controls $\beta(s_n)$ and $\alpha(s_n)$ generated as solutions to (12). These approximations reduce to the following simple maxim for each commander: In each period n, act as if your own intelligence concerning current red activity is the only intelligence available now and for the future. This maxim is analogous to the principle governing open-loop feedback control techniques commonly used in adaptive-control theory: In each period n, ignore the fact that future observations will be made. (See Ref. [5].)

Secondly, it is suggested that estimates for the cost-to-go functions $\{G_n\}$ in (11) be generated as follows: For each $n\in\{1,\ldots,N\}$, $s\in S$, and $(\alpha,\beta)\in V(n,s_n)$ generate[8] a random sample $\{\omega_{-1}(n,\beta,s),\ldots,\omega_{-M}(n,\beta,s)\}$ from the probability distribution $\langle\Omega,\mathcal{F},p_n(\cdot\,|\,\beta,s)\rangle$, and compute the estimates

$$G_N^0(s)\equiv\inf_{v\in V(N,s)}\left[\frac{\sum_{i=1}^{M}C_N(\omega_{-i}(n,\beta,s),v,s)}{M}\right]; \tag{16a}$$

$$G_n^0(s)\equiv\inf_{v\in V(n,s)}\left[\frac{\sum_{i=1}^{M}C_n(\omega_{-i}(n,\beta,s),v,s)+G_{n+1}^0\circ f_n(\omega_{-i}(n,\beta,s),v,s)}{M}\right], \tag{16b}$$

$s\in S$, $n\in\{1,\ldots,N-1\}$. Given the first set of approximations, the estimates (16) can be justified by strong-law arguments similar to those presented in Sec. 2 for the estimates $H_n^0(s)$.

Finally, it is suggested that the expectation operator $E_n[\cdot\,|q,\beta,s]$ in (14) be replaced by a finite average using a random sample generated from the appropriate distribution.

4. CONCLUSION

A C^3 model has been presented which generalizes the C^3 model developed in Ref. [1]. Within the context of the general model, a method has been proposed for incorporating symmetrical rationality by use of appropriately conditioned probability distributions.

Not surprisingly, the incorporation of symmetrical rationality results in significantly more complex C^3 models. The Kagiwada-Kalaba solution technique [4] can still be applied if the model is linear-quadratic and controls are unconstrained. For more general C^3 problems, an alternate approximate solution technique is suggested which does not require integration operations, and which appears especially well suited for C^3 models with finite admissible control sets.

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