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A Consensus-Based Transactive Energy Design for Unbalanced Distribution Networks

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***Abstract*—This study develops a consensus-based transactive energy design managed by an Independent Distribution System Operator (IDSO) for an unbalanced distribution network. The network is populated by welfare-maximizing customers with price-sensitive and fixed loads who make multiple successive power decisions during each Operating Period (OP). The IDSO and customers engage in a negotiation process in advance of each OP to determine retail prices for OP that align customer power decisions with network constraints in a manner that preserves customer privacy. Convergence and optimality properties of this proposed design are established for an analytically formulated illustration: an unbalanced radial distribution network, populated by households, that is electrically connected to a relatively large RTO/ISO-managed transmission network. Numerical test cases are reported for a 123-bus unbalanced radial distribution network that demonstrate these properties.**

***Index Terms*—Transactive energy, unbalanced distribution network, IDSO-managed negotiation process, network reliability, IDSO-customer alignment, customer privacy, FERC Order 2222.**

I. Introduction

THE growing reliance of centrally-managed wholesale power markets on non-dispatchable power poses new challenges for their operation. For example, wind power not fully firmed by storage increases the volatility and uncertainty of net load, hence the difficulty of ensuring continual power balance across the transmission network. These challenges have led to efforts by the U.S. Federal Energy Regulatory Commission, most recently FERC Order 2222 [2], to encourage the increased participation of dispatchable distributed power resources in these markets in various aggregated forms.

Transactive Energy System (TES) design is a relatively new approach to electric power management that could provide important support for FERC Order 2222 objectives. As defined in [3, Sec. 3.1], a TES design is a collection of economic and control mechanisms that allows the dynamic balance of power supply and demand across an entire electrical infrastructure, using value as the key operational parameter.

This study proposes a TES design managed by an *Independent*[1] *Distribution System Operator (IDSO)* within an *Integrated Transmission and Distribution (ITD)* system. As discussed more carefully in subsequent sections, this proposed TES design has four important advantages relative to many previously developed TES designs.

First, the general form of the proposed TES design is applicable for distribution networks that are either *unbalanced or balanced*, and in either *meshed or radial* form. The distribution network can consist of an arbitrary mix of 1-phase, 2-phase, and 3-phase lines.

Second, the proposed TES design is *consensus-based*. Retail prices for each operating period are determined by an iterative negotiation process between the IDSO and its customers that aligns customer goals/constraints with distribution network constraints in a manner that preserves customer privacy.

Third, the proposed TES design supports *multiperiod* decision-making, thus allowing correlations among successive decisions to be taken into account. More precisely, each operating period, of arbitrary duration, is partitioned into finitely many sub-periods; and a negotiation process between the IDSO and its customers held in advance of this operating period determines retail price *profiles* and corresponding planned power *profiles* for these sub-periods.

Fourth, the negotiated retail prices determined by the proposed TES design have an *informative structure*. Each customer's negotiated retail price profile for an operating period OP is the sum of an initial IDSO-set retail price profile plus customer-specific price deviations entailed by the IDSO's fiduciary responsibility to maintain distribution network reliability. Thus, for example, customers at different distribution network locations with otherwise identical attributes might be charged different negotiated retail power prices because the same power withdrawn at different locations has different effects on voltage reliability constraints.

Remaining sections are organized as follows. The relationship of this study to previous electric power management studies is discussed in Section II. The general features of the proposed IDSO-managed consensus-based TES design are described in Section III. Convergence and optimality properties of this TES design are established in Sections IV – VIII for an analytically-formulated ITD system. Section IX reports numerical test cases that demonstrate these properties in more concrete form. The concluding Section X discusses ongoing and planned future studies. A comprehensive quick-reference Nomenclature Table is provided in an appendix.

II. Relationship to Existing Literature

As extensively surveyed in [4]–[7], current management strategies for electric power systems can be roughly divided into four categories: top-down switching; centralized optimization; price reaction; and TES design. In contrast to the first

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[1] The qualifier *independent* means the IDSO has no financial or ownership stake either in distribution system participants or in the operations of the distribution network itself.

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three categories, *TES design* management methods use participant benefit and cost valuations to maintain balance between power withdrawals (usage and/or losses) and power injections across an entire supporting electric power network [3, Sec. 3.1]. Thus, TES designs permit careful consideration of *economic efficiency*[2] for an electric power system as well as reliability and resiliency.

Demonstration projects have been conducted for various TES designs; see, for example, [8]–[11]. These designs range from peer-to-peer designs based on bilateral customer transactions (e.g., [12], [13]) to designs for which customer power requirements are centrally managed, either by direct two-way communications[3] with customers (e.g., [15]–[18]) or by distribution locational marginal prices (e.g., [19, pp. 50-85]).

Centrally-managed TES designs have several advantages relative to peer-to-peer TES designs. A central manager can take timely actions to maintain the overall reliability of distribution system operations, based on continually updated information about the state of the system as a whole. In addition, a central manager can cluster its managed customers into distinct aggregated groups based on their particular power requirements and capabilities. This clustering could facilitate the participation of these central managers in transmission system operations as providers of various types of ancillary services harnessed from customers in return for suitable compensation, in accordance with the objectives of FERC Order 2222 [2].

However, previously proposed centrally-managed TES designs leave open three critical issues. First, many of these TES designs do not handle network constraints for the empirically relevant case of *unbalanced distribution networks*. Thus, they cannot ensure the reliable operation of these networks.

Second, many of these TES designs do not align customer goals/constraints with network constraints in a manner that ensures *voluntary customer participation*. Ensuring voluntary customer participation has two crucial implications for TES design: (i) customer constraints (e.g., budget limits) and benefit/cost valuations should be expressed from the local vantage point of the customer, in a locally measurable manner; and (ii) the central manager should respect customer privacy, implying the information the central manager has about local customer goals and constraints will typically be very limited. Given (i) and (ii), alignment of customer goals/constraints with distribution network constraints in a computationally tractable manner becomes an extremely challenging problem.

Third, these TES designs typically focus on the sequential determination of decisions with *single-period look-ahead horizons*. This myopic single-period focus prevents decision makers from taking into account the intertemporal correlations among their successive decisions.

As carefully established in subsequent sections, the IDSO-managed consensus-based TES design proposed in the current study addresses all three of these critical issues. The design permits the IDSO to ensure distribution network constraints are satisfied, whether the network is balanced or unbalanced. The design aligns customer goals/constraints with distribution network constraints in a computationally tractable manner that respects customer privacy. Finally, the design permits the IDSO and customers to make successive decisions based on multi-period look-ahead horizons.

The previous TES design study closest to this study is Hu et al. [17]. The authors develop a DSO-managed multiperiod TES design based on a negotiation process between the DSO and a collection of aggregators managing the charging schedules for *Electric Vehicle (EV)* owners. However, the authors address a different type of coordination problem than the current study: namely, a coordination problem between a DSO and *aggregators*. The authors do not consider whether the resulting negotiated EV charging schedules are the best possible schedules *from the vantage point of the EV owners*. In the current study an IDSO is attempting to align network constraints directly with the goals and constraints of a collection of retail end-use customers, where customer benefits, costs, and constraints are formulated locally by the customers themselves.

III. The Proposed IDSO-Managed Consensus-Based TES Design: General Features

A. Design Context

The proposed consensus-based TES design is assumed to be implemented within an ITD system. The transmission system, managed by an *Independent System Operator (ISO)* or *Regional Transmission Organization (RTO)*, operates over a high-voltage transmission network. The distribution system, managed by an IDSO, operates over a lower-voltage distribution network. The transmission network electrically connects to the distribution network at a unique *T-D linkage bus* b^*.

The IDSO uses the proposed consensus-based TES design to manage the power needs for all customers electrically connected to the distribution network. The IDSO has a fiduciary responsibility to ensure the welfare of these customers, subject to the maintenance of distribution network reliability.

Each customer has a mix of price-sensitive and conventional loads. Customer load that exceeds distributed generation must be balanced by the IDSO by procuring bulk power from the transmission system at the T-D linkage bus b^*.

Each operating period OP is partitioned into a finite number of customer-decision sub-periods. Prior to each OP, the IDSO engages its customers in a multi-round negotiation process N(OP). The purpose of N(OP) is to determine customer-specific retail prices for the sub-periods comprising OP that ensure subsequent customer power transactions during these sub-periods satisfy all distribution network constraints.

B. Design Timing Relative to Real-Time Market Processes

The RTO/ISO conducts a *real-time market* shortly in advance of each operating period OP, denoted by RTM(OP). The market clearing process for RTM(OP) determines a locational

[2]The *economic efficiency* of a transaction-based system refers to non-wastage in two senses: (i) non-wastage of *resources*, such as services, intermediate goods, and consumption goods; and (ii) *Pareto-efficiency*, i.e., non-wastage of *resource reallocation opportunities* that would result in increased net benefit (i.e., benefit minus cost) for some system participants without reducing the net benefit of any other system participants. Property (i) is a necessary condition for property (ii) unless all system participants are satiated with respect to some resource.

[3]The study of institutions mapping private activities into social outcomes by means of communication processes is referred to as *mechanism design* in the economics literature; see [14].

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marginal price LMP(b^*,OP) for power transactions at the T-D linkage bus b^* during OP.[4] The RTO/ISO then publicly posts LMP(b^*,OP) along with all other RTM LMPs for OP.

Figure 1 depicts the timing of the consensus-based TES design in relation to RTM(OP). The *Look-Ahead Horizon* for RTM(OP), denoted by LAH(OP), is the time interval between the close of RTM(OP) and the start of OP. Let $\mathcal{K} = (1, \ldots, NK)$ denote the sequence of NK customer-decision sub-periods t that comprise OP. During LAH(OP), the IDSO conducts a multi-round negotiation process N(OP) with its managed customers to determine customer-specific retail price profiles $\boldsymbol{\pi}(\mathcal{K})$ for power transactions during $\mathcal{K}$. During OP, the customers engage in power transactions based on their negotiated retail price profiles $\boldsymbol{\pi}(\mathcal{K})$.

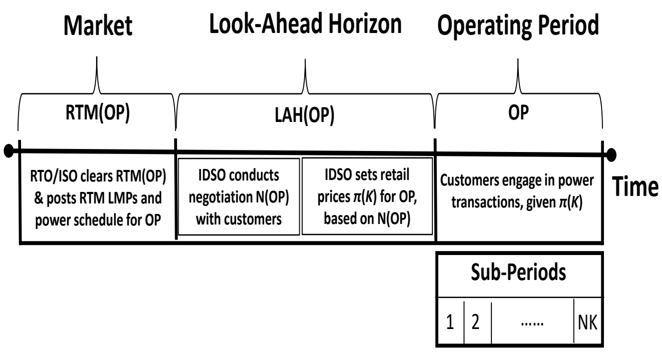

Fig. 1. Timing of the IDSO-managed consensus-based TES design in relation to the timing of a real-time market RTM(OP) for an operating period OP.

C. Design Negotiation Process: Three-Stage Structure

Let OP denote any given operating period. The IDSO understands that LMP(b^*,OP) is the price the IDSO must pay during OP for any procurement of bulk power from the transmission system at the T-D linkage bus b^*. Hence, the IDSO records this price at the close of RTM(OP).

As depicted in Fig. 1, the close of RTM(OP) occurs prior to the start of the negotiating process N(OP) for OP. This negotiation process consists of three general stages:

N(OP) Initialization. At the start of N(OP), the IDSO knows LMP(b^*,OP) as well as the distribution network point-of-connection for each customer. The IDSO receives from each customer a slider-knob control setting between 0 and 1 for the customer's smart (price-sensitive) devices indicating the customer's preferred emphasis on power benefit ("0") relative to power cost ("1") during OP. Based on this information, the IDSO communicates to each customer a customer-specific *initial retail price profile* for OP.

N(OP) Adjustment Step. Upon receipt from the IDSO of a customer-specific retail price profile for OP, each customer communicates back to the IDSO its optimal power profile for OP. Each customer determines its optimal power profile subject to its local physical and financial constraints, taking its received retail price profile as given. The IDSO then checks whether these customer-determined optimal power profiles for OP would result in any violation of distribution network constraints during OP. If so, and if the N(OP) stopping rule has not been activated, the IDSO determines adjusted customer-specific retail price profiles for OP and communicates these adjusted profiles back to its customers to commence another negotiation round. Otherwise, the IDSO halts N(OP).

N(OP) Stopping Rule. If the negotiation process has not terminated by a publicly-designated time prior to the start of OP, the IDSO uses a publicly-designated rule to stop N(OP) and set final retail price profiles for OP that ensure reliable distribution network operations during OP.

As seen from the above general description, the negotiation process N(OP) is a *Stackelberg game in multi-round form*. At the start of each N(OP) round, the IDSO – as Leader – offers customer-specific retail price profiles for operating period OP. Each customer – as a Follower – then responds to its received price-profile offer by communicating back to the IDSO its optimal power profile for OP conditional on this offer.

In consequence, viewed over the course of *successive* operating periods OP, the consensus-based TES design proposed in this study is structured as an *open-ended sequential Stackelberg game* between an IDSO and its managed customers.

IV. Analytical Illustration: Overview

The next five sections develop a complete analytical modeling of the IDSO-managed consensus-based TES design implemented for an ITD system. A comprehensive quick-reference Nomenclature Table for this modeling is given in an appendix.

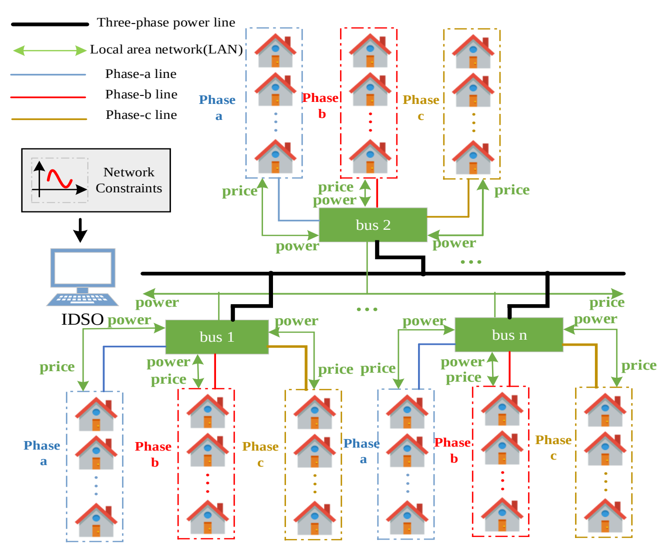

Fig. 2. Depiction of key features for the analytical illustration of the proposed IDSO-managed consensus-based TES design.

As depicted in Fig. 2, the (primary) distribution network for the analytical illustration is an unbalanced radial network consisting of multiple buses connected by multi-phase line segments. The network is populated by a set Ψ of finitely many

[4] U.S. RTMs are typically cleared by means of *Security-Constrained Economic Dispatch (SCED)*. The SCED constraints implicitly or explicitly impose a power balance balance constraint (Kirchhoff's Current Law) at each transmission bus. The RTM LMP at each transmission bus is calculated from the SCED solution as the dual variable for the power balance constraint imposed at this bus. See [20] for a detailed discussion of RTM LMP determination.

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households ψ. Each household ψ is electrically connected to a single distribution network bus by a secondary 1-phase line; this bus is referred to as ψ's distribution network *location*.

The distribution network is electrically connected to a relatively large RTO/ISO-managed transmission network at a unique T-D linkage bus b^*, assumed to be the head bus of the radial distribution network. Given the difference in network sizes, the effects of distribution system operations on transmission system outcomes are negligible.

Each household ψ has a smartly-controlled (price-sensitive) *Heating, Ventilation, and Air-Conditioning (HVAC)* system plus conventional (non-price sensitive) appliances. Hereafter, household HVAC load is referred to as *Thermostatically-Controlled Load (TCL)* and household conventional load is referred to as *non-TCL*. In addition:

- Households do not have power generation capabilities.
- Households are not charged or paid for reactive power.
- At the start of each operating period OP, each household ψ sets a slider-knob control γ_ψ(OP) $\in (0,1)$ for its smart HVAC system that indicates $\psi's$ preferred emphasis on power benefit ("0") relative to power cost ("1") for OP.
- Each operating period OP consists of a sequence $\mathcal{K}$ = $(1,\ldots,NK)$ of NK household-decision sub-periods t with common duration $\triangle\tau$ measured in hourly units.[5]
- During each sub-period $t \in \mathcal{K}$, the HVAC system for each household ψ operates at a fixed power factor PF$_\psi(t) \in (0,1]$; hence, ψ's TCL reactive power usage is a function of ψ's TCL active power usage during t.
- Total household TCL active power usage is zero for a sub-period $t \in \mathcal{K}$ if the retail price for TCL active power during t is at or above $\pi^{\max}(t)$ (cents/kWh), a level known to the IDSO from historical experience.

Since households cannot generate power, household power usage for each operating period OP must be serviced by power withdrawn from the transmission network at the unique T-D linkage bus b^*. The IDSO manages this servicing by implementing a consensus-based TES design in coordination with the operations of an RTO/ISO-managed real-time market.

This servicing proceeds as follows. In advance of OP, the RTO/ISO conducts a real-time market RTM(OP) for power generated at the transmission level. At the close of RTM(OP) the RTO/ISO publicly posts RTM locational marginal prices for OP, including a price LMP(b^*,OP) (cents/kWh)[6] for active power withdrawal from the transmission network at the T-D linkage bus b^* during OP. The IDSO must pay LMP(b^*,OP) for any *actual* withdrawal of active power at bus b^* during OP to service household power needs.

The IDSO recoups power procurement costs for OP by charging households appropriately-set retail prices, determined by means of the negotiation process N(OP) for the consensus-based TES design. For the analytical illustration, N(OP) takes the following concrete three-stage form:

N(OP) Initialization. At the start of N(OP), the IDSO knows the location of each household and observes LMP(b^*,OP). The IDSO receives from each household ψ a slider-knob control setting γ_ψ(OP) and a fixed power-factor PF$_\psi(t)$ for each sub-period t of OP. The IDSO then determines its forecast for total household non-TCL during OP and communicates to each household a commonly-set *initial retail price profile* $\boldsymbol{\pi}^o(\mathcal{K}) = [\pi^o(1),...,\pi^o(NK)]$ for TCL active power during OP, where $\pi^o(t)$ = LMP(b^*,OP) for each sub-period t of OP.

N(OP) Adjustment Step. Upon receipt from the IDSO of a retail price profile for TCL active power during OP, each household ψ communicates back to the IDSO its optimal TCL active power profile for OP. The IDSO then checks whether these household TCL active power profiles, together with their corresponding (power-factor derived) TCL reactive power profiles, would result in any violation of distribution network constraints during OP, given the IDSO's forecast for total household non-TCL during OP. If so, and if the N(OP) stopping rule has not been activated, the IDSO determines adjusted household-specific retail price profiles for TCL active power during OP and communicates these adjusted profiles back to households to commence another negotiation round. Otherwise, the IDSO halts the negotiation process.

N(OP) Stopping Rule. If the negotiation process N(OP) has not terminated at least one minute prior to the start of OP, the IDSO stops N(OP) and sets the final retail price for TCL active power during each sub-period t of OP equal to $\pi^{\max}(t)$.

V. Analytical Illustration: Network Model

A. The Distribution Network

The distribution network for the analytical illustration is an unbalanced radial network with N+1 buses and unbalanced phases $\{a,b,c\}$. Let $\{0\}\bigcup\mathcal{N}$ denote the bus index set, where 0 is the index for the head bus and $\mathcal{N} = \{1,2,...,N\}$ is the index set for all non-head buses.

The distribution network has N distinct *line segments* connecting pairs of adjacent buses, where each line segment can be a 1-phase, 2-phase, or 3-phase circuit. For each $j \in \mathcal{N}$, let $b^p(j) \in \{0\}\bigcup\mathcal{N}$ denote the bus immediately *preceding* bus j along the radial network. Also, let $\mathcal{N}_j$ denote the set of all buses located *strictly after* bus j along the radial network. Then the set consisting of all distinct line segments for the distribution network can be expressed in the following compact form: $\mathcal{L} = \{\ell_j = (i,j) \mid i = b^p(j), j \in \mathcal{N}\}$.

As shown in [1, App. B], each line segment for a radial network can equivalently be represented as a 3-phase line segment by an appropriate introduction of virtual circuits with virtual phases whose self-impedance and mutual impedance are set to 0. This virtual extension to a 3-phase form does not affect any resulting power flow solutions. Let this equivalent virtual extension be called the *3-phase distribution network*.

Hereafter, the distribution network for the analytical illustration is assumed to be in its equivalent 3-phase form.

[5]More precisely, each sub-period $t \in \mathcal{K} = (1,\ldots,NK)$ is a half-open interval of time points along the real line, defined as follows: $t = [s(t),e(t))$ with *start-time* $s(t) = \tau^{\text{op}} + (t-1)\triangle\tau$ and *end-time* $e(t) = \tau^{\text{op}} + t\triangle\tau$ for some fixed time point $\tau^{\text{op}} \geq 0$ and some fixed time duration $\triangle\tau > 0$. Thus, $\mathcal{K}$ is a partition of the operating period OP, where OP is the half-open time interval $[\tau^{\text{op}}, \tau^{\text{op}} + NK\triangle\tau)$ along the real line. The start-time for the next operating period is then given by $\tau^{op} + NK\triangle\tau$.

[6]RTM LMPs are assumed to be measured in (cents/kWh) to simplify analytical expressions. In actuality, U.S. RTM LMPs are measured in \$/MWh.

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B. Power Flow Model for the 3-Phase Distribution Network

Let OP denote an operating period, partitioned into NK household-decision sub-periods $t \in \mathcal{K} = (1, \ldots, NK)$. Making use of [21], which assumes 3-phase bus voltages are approximately balanced, the following extended version of the LinDistFlow model [22] is used to represent power flow relations for the 3-phase distribution network during OP. For each sub-period $t \in \mathcal{K}$ and each line segment $\ell_j = (i,j) \in \mathcal{L}$:

$$\boldsymbol{P}_{ij}(t) = \sum_{k\in\mathcal{N}_j} \boldsymbol{P}_{jk}(t) + \boldsymbol{p}_j(t) \tag{1}$$

$$\boldsymbol{Q}_{ij}(t) = \sum_{k\in\mathcal{N}_j} \boldsymbol{Q}_{jk}(t) + \boldsymbol{q}_j(t)$$

$$\boldsymbol{v}_i(t) = \boldsymbol{v}_j(t) + 2\big[\bar{\boldsymbol{R}}_{ij}\boldsymbol{P}_{ij}(t) + \bar{\boldsymbol{X}}_{ij}\boldsymbol{Q}_{ij}(t)\big]$$

$\boldsymbol{R}_{ij}$ = 3-phase resistance matrix (p.u.) for $\ell_j = (i,j)$

$\boldsymbol{X}_{ij}$ = 3-phase reactance matrix (p.u.) for $\ell_j = (i,j)$

$\boldsymbol{a} = [1, e^{-j2\pi/3}, e^{j2\pi/3}]^T$, $\boldsymbol{a}^H$ = conjugate transpose of $\boldsymbol{a}$

$\bar{\boldsymbol{R}}_{ij} = Re(\boldsymbol{a}\boldsymbol{a}^H) \odot \boldsymbol{R}_{ij} + Im(\boldsymbol{a}\boldsymbol{a}^H) \odot \boldsymbol{X}_{ij}$

$\bar{\boldsymbol{X}}_{ij} = Re(\boldsymbol{a}\boldsymbol{a}^H) \odot \boldsymbol{X}_{ij} - Im(\boldsymbol{a}\boldsymbol{a}^H) \odot \boldsymbol{R}_{ij}$

$\odot$ = element-wise multiplication operator

In (1), the 3×1 column vectors $\boldsymbol{P}_{ij}(t) = [P^{\phi}_{ij}(t)]_{\phi\in\Phi}$, $\boldsymbol{Q}_{ij}(t) = [Q^{\phi}_{ij}(t)]_{\phi\in\Phi}$, $\boldsymbol{v}_j(t) = [v^{\phi}_j(t)]_{\phi\in\Phi}$, $\boldsymbol{p}_j(t) = [p^{\phi}_j(t)]_{\phi\in\Phi}$, and $\boldsymbol{q}_j(t) = [q^{\phi}_j(t)]_{\phi\in\Phi}$, with $\Phi = \{a,b,c\}$, respectively depict the 3-phase active and reactive power flows for line segment ℓ_j, the squared 3-phase voltage magnitudes at bus j, and the 3-phase active and reactive loads at bus j. All terms are measured per unit (p.u.) and ordered using the phase ordering (a,b,c).

To greatly simplify subsequent derivations, a compact matrix representation will next be developed for the power flow relations (1). Let $\bar{\boldsymbol{M}} = [\boldsymbol{m}_0, \boldsymbol{M}^T]^T$ denote the standard $(N+1) \times N$ incidence matrix for a radial distribution network with $N+1$ buses connected entirely by 1-phase line segments [23]. As shown in [1, App. C], if all 1-phase lines for this radial network are replaced by 3-phase lines, the standard incidence matrix for the resulting 3-phase radial network is a $3[N+1] \times 3N$ matrix expressible in the following form:

$$\bar{\boldsymbol{A}} \;=\; [\boldsymbol{A}_0, \boldsymbol{A}^T]^T \;=\; \bar{\boldsymbol{M}} \otimes \boldsymbol{I}_3 \tag{2}$$

where the $3\times 3N$ submatrix $\boldsymbol{A}_0^T$ constitutes the first three rows of $\bar{\boldsymbol{A}}$, the symbol $\otimes$ denotes the Kronecker product operation, and $\boldsymbol{I}_3$ denotes the 3×3 identity matrix.

Let the active/reactive power flows over line segments, squared bus voltage magnitudes, and active/reactive bus loads for the 3-phase distribution network be denoted by the following column vectors:[7] $\boldsymbol{P}(t) = [\boldsymbol{P}_{b^p(j)j}(t)]_{(b^p(j),j)\in\mathcal{L}}$, $\boldsymbol{Q}(t) = [\boldsymbol{Q}_{b^p(j)j}(t)]_{(b^p(j),j)\in\mathcal{L}}$, $\boldsymbol{v}(t) = [\boldsymbol{v}_j(t)]_{j\in\mathcal{N}}$, $\boldsymbol{p}(t) = [\boldsymbol{p}_j(t)]_{j\in\mathcal{N}}$, and $\boldsymbol{q}(t) = [\boldsymbol{q}_j(t)]_{j\in\mathcal{N}}$. Also, let resistances and reactances for the line segments in $\mathcal{L}$ be denoted by the $3N\times 3N$ block diagonal matrices $\boldsymbol{D}_r$ and $\boldsymbol{D}_x$ such that the main-diagonal blocks are 3×3 square matrices and all off-diagonal blocks are zero matrices, as follows: $\boldsymbol{D}_r = \mathrm{diag}(\bar{\boldsymbol{R}}_{b^p(1)1}, \ldots, \bar{\boldsymbol{R}}_{b^p(N)N})$ and $\boldsymbol{D}_x = \mathrm{diag}(\bar{\boldsymbol{X}}_{b^p(1)1}, \ldots, \bar{\boldsymbol{X}}_{b^p(N)N})$. Finally, let the squared bus voltage magnitudes for the head bus 0 be denoted by the column vector $\boldsymbol{v}_0(t) = [v^a_0(t), v^b_0(t), v^c_0(t)]^T$.

Given these notational conventions, the power flow relations (1) can be expressed in the following matrix form:

$$\boldsymbol{A}\boldsymbol{P}(t) = -\boldsymbol{p}(t); \quad \boldsymbol{A}\boldsymbol{Q}(t) = -\boldsymbol{q}(t); \tag{3a}$$

$$\big[\boldsymbol{A}_0 \;\; \boldsymbol{A}^T\big]\begin{bmatrix}\boldsymbol{v}_0(t)\\ \boldsymbol{v}(t)\end{bmatrix} = 2\Big[\boldsymbol{D}_r\boldsymbol{P}(t) + \boldsymbol{D}_x\boldsymbol{Q}(t)\Big] \tag{3b}$$

Since $\boldsymbol{M}^T$ is invertible [23], the matrix $\boldsymbol{A}^T$ is also invertible. Thus, (3) can equivalently be expressed as

$$\boldsymbol{v}(t) \;=\; -[\boldsymbol{A}^T]^{-1}\boldsymbol{A}_0\boldsymbol{v}_0(t) - 2\boldsymbol{R}_D\boldsymbol{p}(t) - 2\boldsymbol{X}_D\boldsymbol{q}(t) \tag{4a}$$

$$\boldsymbol{R}_D \;=\; [\boldsymbol{A}^T]^{-1}\boldsymbol{D}_r\boldsymbol{A}^{-1} \tag{4b}$$

$$\boldsymbol{X}_D \;=\; [\boldsymbol{A}^T]^{-1}\boldsymbol{D}_x\boldsymbol{A}^{-1} \tag{4c}$$

VI. Analytical Illustration: Household Model

To engage in the negotiation process N(OP) for an operating period OP, each household ψ must be able to determine its optimal TCL active power profile for OP in response to any IDSO-offered retail price profile for OP. This section develops the specific model used in the analytical illustration to express this price-conditional household optimization problem for any given OP. For ease of notation, dependence of terms on the given OP will generally be suppressed.

Let $\psi = (u, \phi, j)$ be the generic designation for a household with preference and structural attributes u that is connected by a secondary 1-phase line with phase $\phi \in \Phi = \{a,b,c\}$ to a distribution bus $j \in \mathcal{N}$, referred to as ψ's *location*; see Fig. 2. As noted in Section IV, the TCL for each household ψ consists of smartly controlled (price-sensitive) HVAC power usage.

The goal of household ψ is to attain maximum possible net benefit during OP through its choice of a TCL active power profile for OP, where net benefit takes the general form:

$$\mathrm{NetBen}_{\psi} \;=\; \mathrm{Comfort}_{\psi} \;-\; \mu_{\psi}\mathrm{Cost}_{\psi} \tag{5}$$

$\mathrm{Comfort}_{\psi}$ (utils) measures the benefit (thermal comfort) attained by household ψ from its TCL active power usage during OP, and Cost_{ψ} (cents) measures the cost incurred by household ψ for its TCL active power usage during OP.[8] Household ψ's *marginal utility of money* μ_{ψ} (utils/cent) is a commonly used transformation factor in economics; any money amount (cents) that is multiplied by μ_{ψ} is transformed into a benefit amount (utils). Here, μ_{ψ} is approximated by

$$\mu_{\psi} \;=\; \frac{\gamma_{\psi}}{1-\gamma_{\psi}} \times (\text{utils/cent}) \tag{6}$$

where $\gamma_{\psi} \in (0,1)$ denotes household ψ's slider-knob control setting for its smart HVAC system during OP, communicated to the IDSO during the initialization stage of N(OP).[9]

[7] The active/reactive power flows over line segments ℓ_j are sorted in accordance with the ordering of these line segments from small to large j. The bus voltage magnitudes and active/reactive loads at buses j are sorted in accordance with the ordering of these buses from small to large j.

[8] Recall from Section III that the *non*-TCL power usage of each household ψ in the analytical illustration is assumed to be fixed (non-price-sensitive). Thus, benefits and costs arising from *non*-TCL household power usage are omitted from (5) since their inclusion would not affect household optimal (net benefit maximizing) choices of TCL power profiles for OP, conditional on IDSO-offered retail price profiles for OP.

[9] See [1, App. D] for a careful constructive definition of γ_{ψ}.

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A complete analytical formulation will next be developed for household ψ's price-conditional optimization problem for an operating period OP, where OP is partitioned into household-decision sub-periods $t \in \mathcal{K} = (1, \ldots, NK)$.

Let $p_\psi(t)$ (p.u.) and $q_\psi(t)$ (p.u.) denote the TCL active and reactive power-usage levels that household ψ selects at the start-time $s(t)$ for sub-period $t \in \mathcal{K}$ and maintains during t. Let the $NK \times 1$ column vectors $\mathcal{P}_\psi(\mathcal{K}) = [p_\psi(1), ..., p_\psi(NK)]^T$ and $\mathcal{Q}_\psi(\mathcal{K}) = [q_\psi(1), ..., q_\psi(NK)]^T$ denote ψ's *TCL active and reactive power profiles* for $\mathcal{K}$.

Also, let TB^{a}_ψ (^oF) denote household ψ's *bliss inside air temperature* for OP, i.e., the inside air temperature at which ψ would attain maximum thermal comfort $u^{\max}_\psi$ (utils) during OP. The *discomfort* (utils) experienced by ψ for each sub-period $t \in \mathcal{K}$ is measured by the discrepancy between TB^{a}_ψ and ψ's realized inside air temperature $T^{\mathsf{a}}_\psi(p_\psi(t), t)$ (^oF) at the end-time $e(t)$ for t, multiplied by a conversion factor c_ψ $\big(\text{utils}/(^oF)^2\big)$. The analytical form of Comfort_ψ (utils) in (5), expressing the total comfort attained by ψ for any choice $\mathcal{P}_\psi(\mathcal{K})$ of its TCL active power profile for $\mathcal{K}$, is then

$$\mathrm{U}_\psi(\mathcal{P}_\psi(\mathcal{K})) = \sum_{t \in \mathcal{K}} \big(u^{\max}_\psi - c_\psi [T^{\mathsf{a}}_\psi(p_\psi(t), t) - TB^{\mathsf{a}}_\psi]^2\big) \quad (7)$$

The common duration $\triangle\tau$ of each sub-period t is measured in hourly units (e.g., 0.25h, 1.0h, 1.5h). Let S_{base} (kVA) denote the base-power level used to transform active power (kW) into per unit (p.u.) form by simple division. Also, let $\boldsymbol{\pi}_\psi(\mathcal{K}) = [\pi_\psi(1), \ldots, \pi_\psi(NK)]$ denote household ψ's $1 \times NK$ *retail price profile* for OP. The analytical form of Cost_ψ (cents) in (5), expressing the total cost incurred by household ψ for any choice $\mathcal{P}_\psi(\mathcal{K})$ of its TCL active power profile for $\mathcal{K}$, is then

$$\text{Cost}_\psi(\mathcal{P}_\psi(\mathcal{K}) \mid \boldsymbol{\pi}_\psi(\mathcal{K})) \;=\; \boldsymbol{\pi}_\psi(\mathcal{K})\mathcal{P}_\psi(\mathcal{K}) S_{\mathsf{base}} \triangle\tau \quad (8)$$

Household ψ's participation in the negotiation process N(OP) will typically require ψ to solve, repeatedly, for a TCL active power profile $\mathcal{P}_\psi(\mathcal{K})$ to maximize its net benefit (5) during OP in response to an IDSO-offered retail price profile $\boldsymbol{\pi}_\psi(\mathcal{K})$ for OP. These optimizations are conditional on the following forecasted temperature conditions for OP, determined by household ψ prior to the start of N(OP):

- $\hat{T}^{\mathsf{a}}_\psi(0)$ = Forecast (^oF) for household ψ's *inside* air temp at the *start-time* s(1) for sub-period 1 in $\mathcal{K}$;
- $\hat{T}^o(0)$ = Forecast (^oF) for common network-wide *outside* air temp at the *start-time* s(1) for sub-period $1 \in \mathcal{K}$;
- $\hat{T}^o(t)$ = Forecast (^oF) for common network-wide *outside* air temp at the *end-time* e(t) for sub-period $t \in \mathcal{K}$.

The complete analytical formulation for household ψ's net benefit maximization problem is then as follows:

$$\max_{\mathcal{P}_\psi(\mathcal{K})} \Big[\mathrm{U}_\psi(\mathcal{P}_\psi(\mathcal{K})) - \mu_\psi \text{Cost}_\psi\big(\mathcal{P}_\psi(\mathcal{K}) \mid \boldsymbol{\pi}_\psi(\mathcal{K})\big)\Big] \quad (9)$$

subject to the following constraints:

$$\begin{aligned} T^{\mathsf{a}}_\psi(p_\psi(1), 1) \;&= \alpha^H_\psi \hat{T}^{\mathsf{a}}_\psi(0) \pm \alpha^P_\psi p_\psi(1) S_{\mathsf{base}} \triangle\tau \qquad (10a)\\ &\quad + (1 - \alpha^H_\psi)\hat{T}^o(0) \;; \\ T^{\mathsf{a}}_\psi(p_\psi(t+1), t+1) \;&= \alpha^H_\psi T^{\mathsf{a}}_\psi(p_\psi(t), t) \qquad (10b)\\ &\quad \pm \alpha^P_\psi p_\psi(t+1) S_{\mathsf{base}} \triangle\tau \\ &\quad + (1 - \alpha^H_\psi)\hat{T}^o(t), \; t = 1, \ldots, NK-1; \end{aligned}$$

$$0 \;\le\; p_\psi(t) \;\le\; p^{\max}_\psi, \; t = 1, \ldots, NK. \quad (10c)$$

The thermal dynamic constraints (10a)-(10b), based on the discrete-time linearized thermal dynamic model developed in ([24],[25]), model the forecasted fluctuation in household ψ's inside air temperature $T^{\mathsf{a}}_\psi(p_\psi(t), t)$ during $\mathcal{K}$, from the start-time $s(1)$ for sub-period 1 to the end-time $e(NK)$ for sub-period NK.[10] The parameters α^H_ψ (unit-free) and α^P_ψ (^oF/kWh) are positively valued. Constraint (10c) imposes an upper limit $p^{\max}_\psi$ (p.u.) on ψ's TCL active power usage during each sub-period $t \in \mathcal{K}$, assumed to represent the rated active power (p.u.) of household ψ's HVAC system.

Finally, since the retail price profile $\boldsymbol{\pi}_\psi(\mathcal{K})$ for household ψ appears in the objective function for the net benefit maximization problem (9), any optimal solution for (9) will typically depend on $\boldsymbol{\pi}_\psi(\mathcal{K})$. Let $\mathcal{P}_\psi(\boldsymbol{\pi}_\psi(\mathcal{K}))$ denote an optimal solution for (9), given $\boldsymbol{\pi}_\psi(\mathcal{K})$. Also, define

$$\mathcal{X}_\psi(\mathcal{K}) = \{\mathcal{P}_\psi(\mathcal{K}) \in \mathbb{R}^{NK} | \mathcal{P}_\psi(\mathcal{K}) \text{ satisfies } (10)\} \quad (11)$$

Then the (possibly empty) set of all optimal solutions for (9) can be characterized as follows:

$$\mathcal{P}_\psi(\boldsymbol{\pi}_\psi(\mathcal{K})) \in \operatorname*{argmax}_{\mathcal{P}_\psi(\mathcal{K}) \in \mathcal{X}_\psi(\mathcal{K})} \Big[\mathrm{U}_\psi(\mathcal{P}_\psi(\mathcal{K})) - \mu_\psi \text{Cost}_\psi\big(\mathcal{P}_\psi(\mathcal{K}) \mid \boldsymbol{\pi}_\psi(\mathcal{K})\big)\Big] \quad (12)$$

VII. Analytical Illustration: Benchmark Complete-Information IDSO Optimization

A. Overview

This section develops a *benchmark complete-information IDSO optimization* for the analytical illustration. For any given operating period OP, the IDSO maximizes total household net benefit subject to all household constraints *and* all distribution network constraints under the presumption the IDSO has all information needed to perform this optimization. This benchmark optimization is used in Section VIII to establish, analytically, the convergence and optimality properties of a dual decomposition algorithm newly developed to implement the negotiation process N(OP) for each OP. This benchmark optimization is also used in Section IX to demonstrate these convergence and optimality properties for numerical test cases.

B. Benchmark IDSO Optimization: Analytical Derivation

Let $p^{\mathsf{non}}_\psi(t)$ (p.u.) and $q^{\mathsf{non}}_\psi(t)$ (p.u.) denote household ψ's estimates at the start-time of sub-period $t \in \mathcal{K} = (1, \ldots, NK)$ for its *non*-TCL active and reactive power-usage levels during sub-period t. Also, let $\mathcal{P}^{\mathsf{non}}_\psi(\mathcal{K}) = [p^{\mathsf{non}}_\psi(1), ..., p^{\mathsf{non}}_\psi(NK)]^T$ and $\mathcal{Q}^{\mathsf{non}}_\psi(\mathcal{K}) = [q^{\mathsf{non}}_\psi(1), ..., q^{\mathsf{non}}_\psi(NK)]^T$ denote ψ's estimates for its *non-TCL active and reactive power profiles* for $\mathcal{K}$.

Recall from Section IV that the TCL device (HVAC system) for each household ψ operates at a unit-free constant power factor $\mathrm{PF}_\psi(t) \in (0, 1]$ for each sub-period $t \in \mathcal{K}$. Thus:

$$q_\psi(t) = \eta_\psi(t) p_\psi(t), \;\text{ where } \eta_\psi(t) = \sqrt{\frac{1}{[\mathrm{PF}_\psi(t)]^2} - 1} \quad (13)$$

[10]Temperature fluctuation, given by the terms preceded by the symbol $\pm$ in (10a) and (10b), takes a '+' sign for heating and a '-' sign for cooling.

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Let $\mathcal{U}_{\phi,j}$ denote the set of all household attributes u such that (u,ϕ,j) denotes a household $\psi \in \Psi$. For each $\phi \in \Phi$, $j \in \mathcal{N}$, and $t \in \mathcal{K}$, let $p_j^{\phi}(t)$ and $q_j^{\phi}(t)$ denote the active and reactive load for phase ϕ at bus $j \in \mathcal{N}$ during sub-period t, as follows:

$$p_j^{\phi}(t) = \sum_{u \in \mathcal{U}_{\phi,j}} [p_\psi(t) + p_\psi^{\mathsf{non}}(t)],\ \forall \phi \in \Phi,\ \forall j \in \mathcal{N} \quad (14a)$$

$$q_j^{\phi}(t) = \sum_{u \in \mathcal{U}_{\phi,j}} [q_\psi(t) + q_\psi^{\mathsf{non}}(t)],\ \forall \phi \in \Phi,\ \forall j \in \mathcal{N} \quad (14b)$$

Using the matrix representation for the 3-phase distribution network developed in Section V-B, together with (13) and (14), the power flow relations (4) can equivalently be expressed as follows: For any sub-period $t \in \mathcal{K}$,

$$\boldsymbol{v}(t, \boldsymbol{p}_\Psi(t)) = \boldsymbol{v}^{\mathsf{non}}(t) - 2\boldsymbol{s}(t, \boldsymbol{p}_\Psi(t)) \quad (15)$$

where:

$$\boldsymbol{p}_\Psi(t) = \{p_\psi(t) \mid \psi \in \Psi\};\ \boldsymbol{s}(t, \boldsymbol{p}_\Psi(t)) = \sum_{\psi \in \Psi} \left[\boldsymbol{h}_\psi(t, p_\psi(t))\right]$$

$$\boldsymbol{h}_\psi(t, p_\psi(t)) = \boldsymbol{r}_D(j, N_\psi^{\mathsf{ph}}) p_\psi(t) + \boldsymbol{x}_D(j, N_\psi^{\mathsf{ph}}) \eta_\psi(t) p_\psi(t)$$

$N_\psi^{\mathsf{ph}} = 1$, 2, or 3 if household ψ connects to phase a, b, or c

$$\boldsymbol{v}^{\mathsf{non}}(t) = -[\boldsymbol{A}^T]^{-1} \boldsymbol{A}_0 \boldsymbol{v}_0(t) - 2\boldsymbol{s}^{\mathsf{non}}(t)$$

$$\boldsymbol{s}^{\mathsf{non}}(t) = \sum_{\psi \in \Psi} \left[\boldsymbol{r}_D(j, N_\psi^{\mathsf{ph}}) p_\psi^{\mathsf{non}}(t) + \boldsymbol{x}_D(j, N_\psi^{\mathsf{ph}}) q_\psi^{\mathsf{non}}(t)\right]$$

In (15), the $3N \times 1$ column vector $\boldsymbol{v}^{\mathsf{non}}(t)$ gives the 3-phase squared voltage magnitudes for t at all non-head buses, assuming zero TCL; and the 3×1 column vector $\boldsymbol{v}_0(t)$ gives the 3-phase squared voltage magnitudes for t at head bus 0. Also, $\psi = (u,\phi,j)$ is the generic term for a household in the household set Ψ, and the $3N \times 1$ column vectors $\boldsymbol{r}_D(j, N_\psi^{\mathsf{ph}})$ and $\boldsymbol{x}_D(j, N_\psi^{\mathsf{ph}})$ are the $\{3(j-1) + N_\psi^{\mathsf{ph}}\}$-th columns of the $3N \times 3N$ matrices $\boldsymbol{R}_D$ and $\boldsymbol{X}_D$ defined as in (4b) and (4c).

Given the above notation and derivations, and the household model developed in Section VI, the *benchmark complete-information IDSO optimization* for a given operating period OP consisting of sub-periods $t \in \mathcal{K}$ is expressed as follows:

$$\max_{\mathcal{P}(\mathcal{K}) \in \mathcal{X}(\mathcal{K})} \sum_{\psi \in \Psi} \left[U_\psi(\mathcal{P}_\psi(\mathcal{K})) - \mu_\psi \mathbf{LMP}(\mathcal{K}) \mathcal{P}_\psi(\mathcal{K}) S_{\mathsf{base}} \triangle\tau\right] \quad (16a)$$

$$\text{s.t.} \quad \sum_{\psi \in \Psi} [p_\psi(t) + p_\psi^{\mathsf{non}}(t)] \le \bar{P},\ \forall t \in \mathcal{K} \quad (16b)$$

$$\boldsymbol{v}_{\mathsf{min}}(t) \le \boldsymbol{v}(t, \boldsymbol{p}_\Psi(t)) \le \boldsymbol{v}_{\mathsf{max}}(t),\ \forall t \in \mathcal{K} \quad (16c)$$

In (16): $\mathbf{LMP}(\mathcal{K})$ = $[\text{LMP}(b^*, \text{OP}), ..., \text{LMP}(b^*, \text{OP})]_{1 \times NK}$; LMP($b^*$,OP) = RTM LMP at the T-D linkage bus b^* for OP; $\bar{P}$ (p.u.) is the *peak demand upper limit* imposed by the IDSO on total household active power usage for each t; the $3N \times 1$ column vectors $\boldsymbol{v}_{\mathsf{min}}(t)$ and $\boldsymbol{v}_{\mathsf{max}}(t)$ give the *min and max voltage limits* (p.u.) imposed by the IDSO on the 3-phase squared voltage magnitudes at each distribution bus during t; and

$$\mathcal{P}(\mathcal{K}) = \{\mathcal{P}_\psi(\mathcal{K}) \mid \psi \in \Psi\} = \{\boldsymbol{p}_\Psi(t)) \mid t \in \mathcal{K}\}$$

$$\mathcal{X}(\mathcal{K}) = \prod_{\psi \in \Psi} \mathcal{X}_\psi(\mathcal{K})$$

Finally, let the $(3N \cdot NK) \times 1$ column vectors $\boldsymbol{v}(\mathcal{P}(\mathcal{K}))$, $\boldsymbol{v}_{\mathsf{max}}(\mathcal{K})$, and $\boldsymbol{v}_{\mathsf{min}}(\mathcal{K})$ be defined as follows:

$$\boldsymbol{v}(\mathcal{P}(\mathcal{K})) = [\boldsymbol{v}(1, \boldsymbol{p}_\Psi(1))^T, \ldots, \boldsymbol{v}(NK, \boldsymbol{p}_\Psi(NK))^T]^T$$

$$\boldsymbol{v}_{\mathsf{max}}(\mathcal{K}) = [\boldsymbol{v}_{\mathsf{max}}(1)^T, \ldots, \boldsymbol{v}_{\mathsf{max}}(NK)^T]^T$$

$$\boldsymbol{v}_{\mathsf{min}}(\mathcal{K}) = [\boldsymbol{v}_{\mathsf{min}}(1)^T, \ldots, \boldsymbol{v}_{\mathsf{min}}(NK)^T]^T$$

C. Benchmark IDSO Optimization: Primal Problem Form

The benchmark complete-information IDSO optimization (16) for operating period OP can be expressed in standard *Nonlinear Programming (NP)* form, as follows:

$$\max_{\boldsymbol{x} \in \mathcal{X}} F(\boldsymbol{x}) \text{ subject to } \boldsymbol{g}(\boldsymbol{x}) \le \boldsymbol{c} \quad (17)$$

where:

$$\mathcal{X} = \mathcal{X}(\mathcal{K}) = \prod_{\psi \in \Psi} \mathcal{X}_\psi(\mathcal{K}) \subseteq \mathbb{R}^d$$

$$x_\psi(t) = p_\psi(t) \in \mathbb{R};\ \boldsymbol{x}_\psi = \{x_\psi(t) \mid t \in \mathcal{K}\} = \mathcal{P}_\psi(\mathcal{K}) \in \mathbb{R}^{NK}$$

$$\boldsymbol{x} = \{\boldsymbol{x}_\psi \mid \psi \in \Psi\} = \mathcal{P}(\mathcal{K}) \in \mathbb{R}^d;\ F(\boldsymbol{x}) = \sum_{\psi \in \Psi} F_\psi(\boldsymbol{x}_\psi)$$

$$F_\psi(\boldsymbol{x}_\psi) = \left[U_\psi(\boldsymbol{x}_\psi) - \mu_\psi \mathbf{LMP}(\mathcal{K}) \boldsymbol{x}_\psi S_{\mathsf{base}} \triangle\tau\right]$$

$$\boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} \sum_{\psi \in \Psi} [\boldsymbol{x}_\psi + \mathcal{P}_\psi^{\mathsf{non}}(\mathcal{K})] \\ \boldsymbol{v}(\boldsymbol{x}) \\ -\boldsymbol{v}(\boldsymbol{x}) \end{bmatrix}_{m \times 1} \quad \boldsymbol{c} = \begin{bmatrix} \bar{\boldsymbol{P}}(\mathcal{K}) \\ \boldsymbol{v}_{\mathsf{max}}(\mathcal{K}) \\ -\boldsymbol{v}_{\mathsf{min}}(\mathcal{K}) \end{bmatrix}_{m \times 1}$$

and: NH = number of households $\psi \in \Psi$; NK = number of sub-periods $t \in \mathcal{K}$; $d = NK \cdot NH$; N = number of non-head buses; and $m = ([1 + 6N] \cdot NK)$.

Definition: Benchmark Primal Problem. Problem (17) will hereafter be called the *benchmark primal problem*. Any solution $\boldsymbol{x}^*$ for (17) can equivalently be expressed as $\boldsymbol{x}^*$ = $\{\boldsymbol{x}_\psi^* \mid \psi \in \Psi\}$ = $\{\boldsymbol{\mathcal{P}}_\psi^*(\mathcal{K}) \mid \psi \in \Psi\}$ = $\boldsymbol{\mathcal{P}}^*(\mathcal{K})$. Note, also, the following identities hold for each sub-period $t \in \mathcal{K}$: $\boldsymbol{x}_\Psi(t) = \{x_\psi(t) \mid \psi \in \Psi\}$ = $\{p_\psi(t) \mid \psi \in \Psi\} = \boldsymbol{p}_\Psi(t)$.

VIII. Analytical Illustration: Implementation of the IDSO-Managed Negotiation Process

A. Overview

Let OP denote any operating period for the analytical illustration, partitioned into *NK* household-decision sub-periods $t \in \mathcal{K} = (1, \ldots, NK)$. This section develops a new form of *Dual Decomposition Algorithm (DDA)* [26, Sec.2] to implement the negotiation process N(OP) between the IDSO and the households for OP. Convergence and optimality properties of this DDA are established by means of five propositions.[11]

B. TES Equilibrium: Definition and Properties

Let $\{\boldsymbol{\pi}_\psi(\mathcal{K}) \mid \psi \in \Psi\}$ = $\boldsymbol{\pi}(\mathcal{K})$ denote the set of household-specific retail price profiles communicated by the IDSO to households during some round of the negotiation process N(OP) for OP. Also, let $\{\mathcal{P}_\psi(\boldsymbol{\pi}_\psi(\mathcal{K})) \mid \psi \in \Psi\}$ = $\boldsymbol{\mathcal{P}}(\boldsymbol{\pi}(\mathcal{K}))$ denote the set of optimal TCL active power profiles that households communicate back to the IDSO, conditional on these retail price profiles.

[11]Complete proofs for these propositions are provided in [1, Apps. G-J].

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Definition: TES Equilibrium for OP. Suppose an optimal solution $\boldsymbol{x}^* = \boldsymbol{\mathcal{P}}^*(\mathcal{K})$ for the benchmark complete-information IDSO optimization (16) in benchmark primal problem form (17) coincides with $\boldsymbol{\mathcal{P}}(\boldsymbol{\pi}^*(\mathcal{K}))$ for some set $\boldsymbol{\pi}^*(\mathcal{K})$ of retail price profiles for OP. Then the quantity-price pairing $(\boldsymbol{\mathcal{P}}^*(\mathcal{K}), \boldsymbol{\pi}^*(\mathcal{K}))$ will be called a *TES equilibrium for OP*.

For each sub-period $t \in \mathcal{K}$, let $\lambda_{\bar{P}}(t)$ denote the non-negative dual variable (utils/p.u.) associated with the peak demand constraint (16b). Also, let the $1 \times 3N$ row vectors $\boldsymbol{\lambda}_{v_{\max}}(t)$ and $\boldsymbol{\lambda}_{v_{\min}}(t)$ denote the non-negative dual variables (utils/p.u.) associated with the upper and lower 3-phase voltage magnitude inequality constraints (16c). The $1 \times m$ row vector $\boldsymbol{\lambda}$ whose components consist of all of these non-negative dual variables is then denoted by

$$\boldsymbol{\lambda} = [\boldsymbol{\lambda}_{\bar{P}}(\mathcal{K}), \boldsymbol{\lambda}_{v_{\max}}(\mathcal{K}), \boldsymbol{\lambda}_{v_{\max}}(\mathcal{K})] \tag{18}$$

where the component row vectors for $\boldsymbol{\lambda}$ are given by:

$$\begin{aligned}
\boldsymbol{\lambda}_{\bar{P}}(\mathcal{K}) &= [\lambda_{\bar{P}}(1), \ldots, \lambda_{\bar{P}}(NK)]_{1\times NK} \\
\boldsymbol{\lambda}_{v_{\max}}(\mathcal{K}) &= [\boldsymbol{\lambda}_{v_{\max}}(1), \ldots, \boldsymbol{\lambda}_{v_{\max}}(NK)]_{1\times(3N\cdot NK)} \\
\boldsymbol{\lambda}_{v_{\min}}(\mathcal{K}) &= [\boldsymbol{\lambda}_{v_{\min}}(1), \ldots, \boldsymbol{\lambda}_{v_{\min}}(NK)]_{1\times(3N\cdot NK)}
\end{aligned}$$

Finally, let the dual variables corresponding to the upper and lower 3-phase voltage magnitude inequality constraints (16c) be expressed in the following $NK \times 3N$ matrix forms:

$$\boldsymbol{\Lambda}_{v_{\max}}(\mathcal{K}) = \begin{bmatrix} \boldsymbol{\lambda}_{v_{\max}}(1) \\ \vdots \\ \boldsymbol{\lambda}_{v_{\max}}(NK) \end{bmatrix};\ \boldsymbol{\Lambda}_{v_{\min}}(\mathcal{K}) = \begin{bmatrix} \boldsymbol{\lambda}_{v_{\min}}(1) \\ \vdots \\ \boldsymbol{\lambda}_{v_{\min}}(NK) \end{bmatrix}$$

Definition: Benchmark Lagrangian Function. The *benchmark Lagrangian function* L: $\mathcal{X} \times \mathbb{R}^m_+ \rightarrow \mathbb{R}$ for the benchmark primal problem (17) is given by

$$L(\boldsymbol{x}, \boldsymbol{\lambda}) = F(\boldsymbol{x}) + \boldsymbol{\lambda}[\boldsymbol{c} - \boldsymbol{g}(\boldsymbol{x})] \tag{19}$$

where $\boldsymbol{x} = \{\boldsymbol{x}_\psi \mid \psi \in \Psi\} = \mathcal{P}(\mathcal{K})$.

Definition: Benchmark Dual Problem. The *benchmark dual function* D:$\mathbb{M} \rightarrow \mathbb{R}$ for (17) is given by:

$$D(\boldsymbol{\lambda}) = \max_{\boldsymbol{x}\in\mathcal{X}} L(\boldsymbol{x}, \boldsymbol{\lambda})\,; \tag{20}$$

$$\mathbb{M} = \{\boldsymbol{\lambda} \in \mathbb{R}^m_+ \mid D(\boldsymbol{\lambda}) \text{ is well-defined and finite}\} \tag{21}$$

The *benchmark dual problem* for (17) is then

$$\min_{\boldsymbol{\lambda}\in\mathbb{M}} D(\boldsymbol{\lambda}) \tag{22}$$

Proposition 1 (Classical): *A point $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)$ in $\mathcal{X} \times \mathbb{R}^m_+$ is a saddle point for the benchmark Lagrangian function $L(\boldsymbol{x}, \boldsymbol{\lambda})$ given by (19) if and only if:*

- **[P1.A]** $\boldsymbol{x}^*$ *solves the benchmark primal problem (17) ;*
- **[P1.B]** $\boldsymbol{\lambda}^*$ *solves the benchmark dual problem (22) ;*
- **[P1.C]** $D(\boldsymbol{\lambda}^*) = F(\boldsymbol{x}^*)$ *(strong duality).*

Recall from Section VII-B that the TCL active and reactive power usage levels $(p_\psi(t), q_\psi(t))$ for each household $\psi \in \Psi$ in each subperiod $t \in \mathcal{K}$ satisfy $q_\psi(t) = \eta_\psi(t) p_\psi(t)$, where $\eta_\psi(t)$ is defined in (13). Let $\boldsymbol{H}_\psi(\mathcal{K})$ denote ψ's $NK \times NK$ *TCL power-ratio matrix* for operating period OP, defined as:

$$\boldsymbol{H}_\psi(\mathcal{K}) = \mathrm{diag}\big(\eta_\psi(1), \eta_\psi(2), ..., \eta_\psi(NK)\big) \tag{23}$$

Proposition 2: *Suppose $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)$ in $\mathcal{X} \times \mathbb{R}^m_+$ is a saddle point for the benchmark Lagrangian function $L(\boldsymbol{x}, \boldsymbol{\lambda})$ given by (19), where $\boldsymbol{x}^* = \boldsymbol{\mathcal{P}}^*(\mathcal{K})$. Suppose, also, that $\boldsymbol{x}^*$ uniquely maximizes $L(\boldsymbol{x}, \boldsymbol{\lambda}^*)$ over $\boldsymbol{x} \in \boldsymbol{\mathcal{X}}$. Define $\boldsymbol{\pi}^*(\mathcal{K}) = \{\boldsymbol{\pi}^*_\psi(\mathcal{K}) \mid \psi \in \Psi\}$, where the retail price profile $\boldsymbol{\pi}^*_\psi(\mathcal{K})$ for each household $\psi = (u, \phi, j) \in \Psi$ takes the following form:*

$$\begin{aligned}
\boldsymbol{\pi}^*_\psi(\mathcal{K}) \;=\; & \boldsymbol{LMP}(\mathcal{K}) + \frac{1}{\mu_\psi S_{\mathsf{base}} \triangle\tau}\Big[\boldsymbol{\lambda}^*_{\bar{P}}(\mathcal{K}) \\
& - 2 \cdot \boldsymbol{r}_D(j, N^{\mathsf{ph}}_\psi)^T \big[\boldsymbol{\Lambda}^*_{v_{\max}}(\mathcal{K}) - \boldsymbol{\Lambda}^*_{v_{\min}}(\mathcal{K})\big]^T \\
& - 2 \cdot \boldsymbol{x}_D(j, N^{\mathsf{ph}}_\psi)^T \big[\boldsymbol{\Lambda}^*_{v_{\max}}(\mathcal{K}) - \boldsymbol{\Lambda}^*_{v_{\min}}(\mathcal{K})\big]^T \boldsymbol{H}_\psi(\mathcal{K})\Big]
\end{aligned} \tag{24}$$

Then $(\boldsymbol{\mathcal{P}}^*(\mathcal{K}), \boldsymbol{\pi}^*(\mathcal{K}))$ is a TES equilibrium for OP.

As seen from (24), in order for the profile $\boldsymbol{\pi}^*_\psi(\mathcal{K})$ of TES equilibrium retail prices charged to a household $\psi = (u, \phi, j)$ during OP to deviate from the profile LMP($\mathcal{K}$) of RTM LMPs determined for OP, at least one of the non-negative dual variables (18) associated with the reliability (peak demand and voltage) inequality constraints for the benchmark primal problem (17) must be strictly positive. Depending on which of these dual variables are positive (if any), the magnitude and sign of any resulting price deviations can depend on: ψ's preference and structural attributes $u = (\ \mu_\psi,\ \boldsymbol{H}_\psi(\mathcal{K})\ )$; ψ's phase attribute ϕ; and/or ψ's location attribute j.

Note, also, that some components of the price profile (24) could even be negative in value. In this case the IDSO is essentially paying household ψ for power usage as an ancillary service (power absorption) in order to ensure all distribution network reliability constraints are satisfied.

C. TES Equilibrium: Dual Decomposition Solution Method

This section presents a five-step DDA, called DDA-N(OP), that implements the negotiation process N(OP) for OP. A critical issue is whether any limit point for DDA-N(OP) determines a TES equilibrium for OP. Sufficient conditions ensuring this is the case are provided below in Propositions 3–5.

Proposition 3: *Suppose the following three assumptions hold for the benchmark primal problem (17) and DDA-N(OP):*

- **[P3.A]** $\mathcal{X}$ *is compact, and the objective function $F(\boldsymbol{x})$ and constraint function $\boldsymbol{g}(\boldsymbol{x})$ are continuous over $\mathcal{X}$.*
- **[P3.B]** *For every $\boldsymbol{\lambda} \in \mathbb{R}^m_+$, the benchmark Lagrangian function $L(\boldsymbol{x}, \boldsymbol{\lambda})$ given by (19) achieves a finite maximum at a unique point $\boldsymbol{x}(\boldsymbol{\lambda}) \in \mathcal{X}$; hence, the benchmark dual function domain $\mathbb{M}$ in (21) is given by $\mathbb{M} = \mathbb{R}^m_+$.*
- **[P3.C]** *The sequence $(\boldsymbol{x}^y, \boldsymbol{\lambda}^y)$ for DDA-N(OP) converges to a limit point $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)$ as the iteration time y approaches $+\infty$.*

Then $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)$ is a saddle point for the benchmark Lagrangian function (19) that determines a TES equilibrium for OP.

Proposition 4 establishes sufficient conditions for the critical convergence property [P3.C] in Proposition 3 to hold.

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Algorithm DDA-N(OP): Dual Decomposition Algorithm for Implementation of the Negotiation Process N(OP)

S1: Initialize. At the initial iteration time $y = 0$, the IDSO specifies positive scalar step-sizes β_1, β_2, and β_3. In addition, the IDSO sets the following initial dual variable values: $\boldsymbol{\lambda}^y_{\bar{P}}(\mathcal{K}) = \mathbf{0}$, $\boldsymbol{\lambda}^y_{v_{\mathsf{max}}}(\mathcal{K}) = \mathbf{0}$, and $\boldsymbol{\lambda}^y_{v_{\mathsf{min}}}(\mathcal{K}) = \mathbf{0}$.

S2: Set price profiles. The IDSO sets the price profile $\boldsymbol{\pi}^y_\psi(\mathcal{K})$ for each household $\psi = (u, \phi, j) \in \Psi$, as follows:

$$\begin{aligned}\boldsymbol{\pi}^y_\psi(\mathcal{K}) \;=\; & \mathbf{LMP}(\mathcal{K}) + \frac{1}{\mu_\psi S_{\mathsf{base}}\triangle\tau}\Big[\boldsymbol{\lambda}^y_{\bar{P}}(\mathcal{K}) \\ & - 2\cdot \boldsymbol{r}_D(j, N^{\mathsf{ph}}_\psi)^T\big[\boldsymbol{\Lambda}^y_{v_{\mathsf{max}}}(\mathcal{K}) - \boldsymbol{\Lambda}^y_{v_{\mathsf{min}}}(\mathcal{K})\big]^T \\ & - 2\cdot \boldsymbol{x}_D(j, N^{\mathsf{ph}}_\psi)^T\big[\boldsymbol{\Lambda}^y_{v_{\mathsf{max}}}(\mathcal{K}) - \boldsymbol{\Lambda}^y_{v_{\mathsf{min}}}(\mathcal{K})\big]^T \boldsymbol{H}_\psi(\mathcal{K})\Big]\end{aligned}$$

Note that $\boldsymbol{\pi}^y_\psi(\mathcal{K})$ reduces to $\mathbf{LMP}(\mathcal{K})$ if $y = 0$.

S3: Update primal variables. $\boldsymbol{x}^y = \mathrm{argmax}_{\boldsymbol{x}\in\mathcal{X}}\, L(\boldsymbol{x}, \boldsymbol{\lambda}^y)$, implemented as follows: The IDSO communicates to each household $\psi \in \Psi$ the price profile $\boldsymbol{\pi}^y_\psi(\mathcal{K})$. Each household $\psi \in \Psi$ then adjusts its TCL power profile according to

$$\boldsymbol{x}^y_\psi = \mathcal{P}_\psi(\boldsymbol{\pi}^y_\psi(\mathcal{K}))$$

and communicates $\boldsymbol{x}^y_\psi$ back to the IDSO. If this primal updating step triggers the *N(OP) Stopping Rule*, the negotiation process halts. Otherwise, the negotiation process proceeds to step **S4**.

S4: Update dual variables.

$$\boldsymbol{\lambda}^{y+1} = \big[\boldsymbol{\lambda}^y + [\boldsymbol{g}(\boldsymbol{x}^y) - \boldsymbol{c}]^T\boldsymbol{B}\big]^+$$

where $[\ \cdot\ ]^+$ denotes projection on R^m_+, and $\boldsymbol{B}$ is an $m \times m$ diagonal positive-definite matrix constructed as follows: The diagonal entries of $\boldsymbol{B}$ associated with $\boldsymbol{\lambda}_{\bar{P}}(\mathcal{K})$, $\boldsymbol{\lambda}_{v_{\mathsf{max}}}(\mathcal{K})$, $\boldsymbol{\lambda}_{v_{\mathsf{min}}}(\mathcal{K})$ are repeated entries of the **S1** step-sizes β_1, β_2, β_3, respectively.

S5: Update iteration time. The iteration time y is assigned the updated value $y + 1$ and the process loops back to step **S2**.

Proposition 4: *Suppose the following four assumptions hold for the benchmark primal and dual problems (17) and (22):*

- **[P4.A]** Conditions *[P3.A] and [P3.B] in Prop. 3 are true;*
- **[P4.B]** *The benchmark Lagrangian function (19) has a saddle point $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)$ in $\mathcal{X} \times \mathbb{R}^m_+$;*
- **[P4.C]** ***Extended Lipschitz Continuity Condition:*** *There exists a real symmetric positive-definite $m \times m$ matrix $\boldsymbol{J}$ such that, for all $\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2 \in \mathbb{R}^m_+$,*

$$\big\langle \nabla D_+(\boldsymbol{\lambda}_1) - \nabla D_+(\boldsymbol{\lambda}_2), \boldsymbol{\lambda}_1 - \boldsymbol{\lambda}_2\big\rangle \;\leq\; ||\boldsymbol{\lambda}_1 - \boldsymbol{\lambda}_2||^2_{\boldsymbol{J}}$$

  where: $\nabla D_+(\boldsymbol{\lambda})$ denotes the gradient of the benchmark dual function $D(\boldsymbol{\lambda})$ in (20) for $\boldsymbol{\lambda} \in \mathbb{R}^m_{++}$ and the right-hand gradient of $D(\boldsymbol{\lambda})$ at boundary points of $\mathbb{R}^m_+$; $\langle\ ,\ \rangle$ denotes vector inner product; and $||\cdot||^2_{\boldsymbol{J}} = (\cdot)\boldsymbol{J}(\cdot)^T$
- **[P4.D]** The matrix $[\boldsymbol{I} - \boldsymbol{J}\boldsymbol{B}]$ *is positive semi-definite, where $\boldsymbol{I}$ is the $m \times m$ identity matrix, and where $\boldsymbol{B}$ is the $m \times m$ diagonal positive-definite matrix defined in step **S4** of DDA-N(OP).*

Then the primal-dual point $(\boldsymbol{x}^y, \boldsymbol{\lambda}^y)$ for DDA-N(OP) at iteration time y converges to a saddle point $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)$ for the benchmark Lagrangian function (19) as $y \to +\infty$.

The Extended Lipschitz Continuity Condition [P4.C] in Proposition 4 is expressed in a relatively complicated form. Proposition 5 provides sufficient conditions for [P4.C] to hold that are easier to understand.

Proposition 5: *Suppose the benchmark primal problem (17) satisfies condition [P3.A] in Prop. 3 plus the following:*

- **[P5.A]** *$\mathcal{X}$ is a non-empty compact convex subset of $\mathbb{R}^d$.*
- **[P5.B]** *The objective function $F{:}\mathbb{R}^d \to \mathbb{R}$ restricted to $\mathcal{X} \subseteq \mathbb{R}^d$ has the quadratic form*

$$F(\boldsymbol{x}) \;=\; \frac{1}{2}\boldsymbol{x}^T\boldsymbol{W}\boldsymbol{x} + \boldsymbol{\rho}^T\boldsymbol{x} + \sigma \tag{25}$$

  where $\boldsymbol{W}$ is a real symmetric negative-definite $d \times d$ matrix, $\boldsymbol{\rho}$ is a real $d \times 1$ column vector, σ is a real scalar.
- **[P5.C]** *The constraint function $g{:}\mathbb{R}^d \to \mathbb{R}^m$ restricted to $\mathcal{X} \subseteq \mathbb{R}^d$ has the linear affine form*

$$g(\boldsymbol{x}) \;=\; \boldsymbol{C}\boldsymbol{x} + \boldsymbol{b} \tag{26}$$

  where $\boldsymbol{C}$ is a real $m \times d$ matrix, and $\boldsymbol{b}$ is a real $m \times 1$ column vector.

Then the Extended Lipschitz Continuity Condition [P4.C] in Prop. 4 holds with $\boldsymbol{J} = \boldsymbol{C}\boldsymbol{H}^{-1}\boldsymbol{C}^T$, where $\boldsymbol{H} = -\boldsymbol{W}$.

IX. Numerical Test Cases

A. Overview

The test cases[12] reported in this section are numerical implementations of the analytical illustration developed in Sections IV–VIII. An IDSO oversees the operations of a lower-voltage 123-bus unbalanced radial distribution network connected to a high-voltage transmission network at the distribution network's head bus. The distribution network is populated by 345 households, identical apart from their secondary connection-line phases and distribution network locations.

Each test case simulates a single day D partitioned into 24 operating hours OP. The goal of the IDSO for each OP is to maximize total household net benefit subject to distribution network constraints that include: an upper limit on peak demand; and lower and upper limits on bus voltage magnitudes.

Three key findings for the IDSO-managed consensus-based TES design were observed for each operating hour OP. First, all distribution network constraint violations occurring in the absence of customer management were eliminated under the TES design. Second, the negotiation process N(OP) for the TES design converged in less than 500s $\approx$ 8.4min. And third, the welfare and network outcomes resulting under the TES design closely approximated the welfare and network outcomes resulting under IDSO complete-information optimization.

B. Maintained Test-Case Specifications

(1) D, OP, NK, RTM(OP), LAH(OP), RTM LMPs: The maintained settings for these terms are based on ERCOT; see [27]. The simulated day D is partitioned into 24 one-hour operating periods OP. The number NK of sub-periods t for each OP is set to one, with duration $\triangle\tau$ = 1h. The duration of RTM(OP) and LAH(OP) are set to 1min and 59min; cf. Fig. 1. The day-D profile of hourly RTM LMPs is given in [1, Fig. 8].

(2) Distribution Network: The standard IEEE 123-bus unbalanced radial distribution network [28] is modified to

[12] All test-case simulations were conducted using MATLAB R2019b, which integrates the YALMIP Toolbox with the IBM ILOG CPLEX 12.9 solver. Additional technical test-case aspects are provided in [1, App. K].

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include 345 households located across the network, with S_{base} = 100 (kVA) and V_{base} = 4.16 (kV). The maintained p.u. settings for voltage parameters are: $\boldsymbol{v}_{\min}(t) = [0.95^2, 0.95^2, 0.95^2]^T$; $\boldsymbol{v}_{\max}(t) = [1.05^2, 1.05^2, 1.05^2]^T$; and $\boldsymbol{v}_0(t) = [1.04^2, 1.04^2, 1.04^2]^T$. The unique T-D linkage bus b^* is the head bus 0 for the radial distribution network.

(3) Households: All test-case households $\psi = (u, \phi, j)$ have identical preference and structural attributes $u = (\mu_\psi, \boldsymbol{H}_\psi(\mathcal{K}))$ but can differ with regard to their secondary connection-line phase ϕ and their bus-j distribution network location. The inside air temperature set for each household ψ at the start of day D is $\hat{T}^{a}_{\psi}(0) = 74$ (^oF). The day-D profiles for non-TCL power usage and outside air temperature commonly set for each household are depicted in [1, Figs. 7-8]. For each operating hour OP, the thermal dynamic parameter values set for each household ψ are $\alpha^H_\psi = 0.96$ (unit-free), $\alpha^P_\psi = 0.7$ $(^oF$/kWh) [24], $p^{\max}_\psi = 0.05$p.u., and $\text{PF}_\psi = 0.9$p.u.; and the preference parameter values set for each household ψ are $c_\psi = 6.12$ $\left(\text{utils}/(^oF)^2\right)$, $u^{\max}_\psi = 1.20 \times 10^4$ (utils), $TB^{a}_{\psi} = 72$ (^oF), and $\mu_\psi = 1$ (utils/cent).

(4) IDSO and N(OP): An RTM operates over the transmission network, and the IDSO purchases power at b^* from this RTM to meet household power-usage requirements. The parameter settings for the algorithm DDA-N(OP) used to implement the negotiation process N(OP) for each OP are: $\beta_1 = 15$; $\beta_2 = \beta_3 = 50{,}000$; and $I_{\max} = 200$.

(5) Benchmark Complete-Information IDSO Optimization: In form (17), this optimization is a concave programming problem with a strictly concave objective function $\boldsymbol{F}(\boldsymbol{x})$ and a linear-affine constraint function $\boldsymbol{g}(\boldsymbol{x})$, $\boldsymbol{x} \in \mathcal{X} \subseteq \mathbb{R}^d$, where the function domain $\mathcal{X}$ is non-empty, compact, and convex.

C. No Customer Management vs. TES Customer Management

Suppose the IDSO does *not* manage household power usage. Rather, the IDSO sets the retail prices for household non-TCL and TCL during each hour OP of day D equal to LMP(b^*,OP), the LMP determined in RTM(OP) for the linkage bus b^*. As seen in Fig. 3, the day-D peak demand for this Unmanaged System Case is 2962kW, realized for hour 17. Thus, as long as the day-D peak demand upper limit on total household active power usage, required for distribution network reliability, is at least 2962kW, no violation of this limit occurs. On the other hand, the bus voltage magnitude limits $[0.95, 1.05]$ (p.u.) are violated because the minimum phase-a voltage magnitude (p.u.) across the N buses for hour 17 is $0.9485 < 0.9500$.

Suppose the IDSO instead uses the consensus-based TES design to manage household power usage. The IDSO imposes an upper limit 3200kW on day-D peak demand as well as min/max limits $[0.95, 1.05]$ (p.u.) on day-D voltage magnitudes by phase. All network constraints are now satisfied. As seen in Fig. 4, the switch to the use of the consensus-based TES design enables the IDSO to eliminate the violation of the phase-a voltage magnitude lower limit 0.95p.u. without violating the peak demand upper limit 3200kW.

Finally, suppose the day-D peak demand upper limit is reduced from 3200kW to 2900kW. For the Unmanaged System Case shown in Fig. 3, this change has no effect on system operations. Consequently, the peak demand 2962kW that results

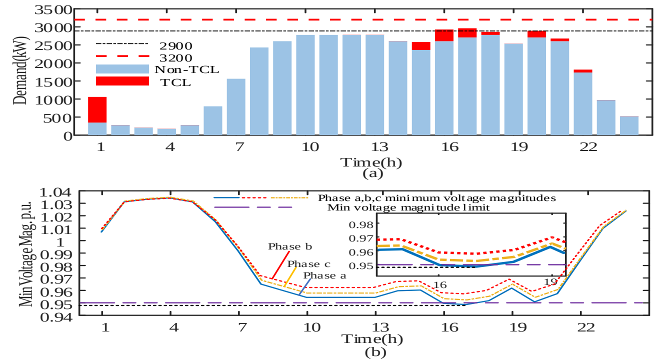

Fig. 3. **Unmanaged System Case (Peak Demand Upper Limit 3200kW):** (a) Total household power demand (kW), and (b) minimum bus voltage magnitude (p.u.) by phase across the N distribution buses, for each hour of day D. The peak demand upper limit 3200kW is satisfied; but the lower limit 0.95p.u. for the phase-a bus voltage magnitude is violated during hour 17.

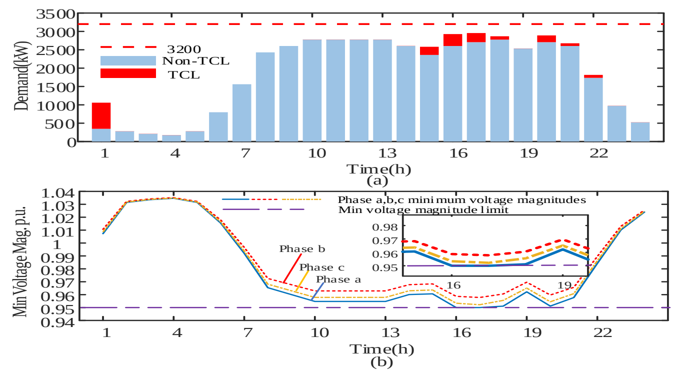

Fig. 4. **TES Management Case 1 (Peak Demand Upper Limit 3200kW):** (a) Total household power demand (kW), and (b) minimum bus voltage magnitude (p.u.) by phase across the N distribution buses, for each hour of day D. The consensus-based TES design ensures the day-D peak demand upper limit 3200kW and voltage magnitude limits [0.95, 1.05] (p.u.) are satisfied.

for this case during hour 17 violates the reduced upper limit 2900kW; and the phase-a voltage magnitude violation during hour 17 continues to occur.

In contrast, under TES management, the reduced day-D peak demand upper limit 2900kW changes the manner in which the IDSO conducts negotiations with its managed customers. As reported in Fig. 4, the day-D peak demand resulting for TES Management Case 1 with day-D peak demand upper limit 3200kW does *not* satisfy the reduced upper limit 2900kW during some hours. Thus, the IDSO must negotiate day-D retail prices *in a different manner* to ensure that day-D total household power usage satisfies this reduced upper limit as well as the min/max voltage magnitude constraints.

Fig. 5 reports the day-D demand outcomes resulting for TES Management Case 2 with reduced day-D peak demand upper limit 2900kW. Peak demand is now at or below 2900kW during each hour of day D. Also (not shown), all voltage magnitudes are within the required limits [0.95, 1.05] (p.u.) during each hour of day D. These results illustrate how the negotiation process supporting the consensus-based TES design permits the IDSO to pursue the goal of maximizing customer welfare conditional on the satisfaction of *all* distribution network constraints, whatever form these constraints take.

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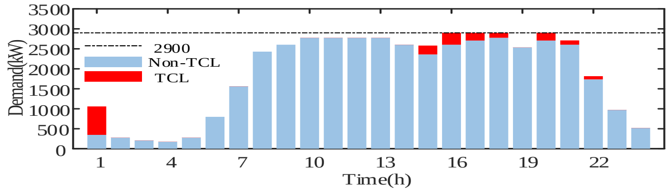

Fig. 5. **TES Management Case 2 (Peak Demand Upper Limit 2900kW):** Total household power demand (kW) during each hour of day D. The consensus-based TES design ensures the day-D peak demand upper limit 2900kW and voltage magnitude limits [0.95, 1.05] (p.u.) are satisfied.

D. Relationship Between Prices and Constraints

For the analytical illustration (hence for each test case), it follows from Propositions 1-5 that the final N(OP)-negotiated retail prices (24) for an operating period OP – determined by DDA-N(OP) – are given by (24). If the network inequality constraints (16b) and (16c) for the analytical illustration are strictly non-binding, then their corresponding dual variable solutions must all be zero[13] In this case it follows from (24) that the final N(OP)-negotiated retail price[14] for each household ψ must coincide with the retail price LMP(b^*,OP) the IDSO commonly sets for all households at the start of N(OP).

How do the final N(OP)-negotiated retail prices (24) deviate from LMP(b^*,OP) when at least one network inequality constraint is binding? For example, consider the retail prices (24) for OP = hour 17 reported in Fig. 6 for TES Management Case 1 with peak demand upper limit 3200kW. These prices vary across the 123 buses constituting the distribution network; and, at each bus, the prices also vary across the households located at this bus that have different secondary connection-line phases ϕ. What explains this retail price variation?

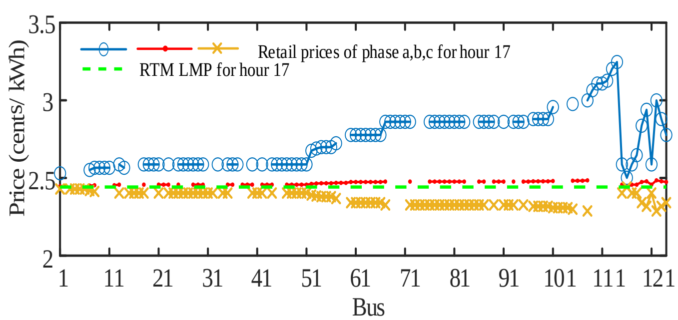

Fig. 6. **TES Management Case 1 (Peak Demand Upper Limit 3200kW):** Consensus-based TES design retail price outcomes across the 123-bus distribution network for OP = hour 17 of day D compared with LMP(b^*,OP), the RTM LMP at the T-D linkage bus b^* during OP = hour 17.

As reported in Fig. 4 for TES Management Case 1, the peak demand upper limit 3200kW is *strictly non-binding* for hour 17. In addition (not shown), the voltage magnitude upper limit 1.05p.u. (by phase) is *strictly non-binding* for hour 17. On the other hand, the *lower* limit 0.95p.u. for the phase-a voltage magnitude is *binding* for hour 17. For example, when the IDSO sets each household's retail price equal to LMP(b^*,OP) at the start of the negotiation process N(OP) for OP = hour 17, the violation of this lower limit can be inferred from Fig. 3.

Thus, for TES Management Case 1 with OP = $\mathcal{K}$ = hour 17, all components of the dual solution terms $\boldsymbol{\lambda}^*_{\bar{P}}(\mathcal{K})$ and $\boldsymbol{\Lambda}^*_{v_{\max}}(\mathcal{K})$ appearing in the final N(OP)-negotiated retail price (24) for each household ψ are necessarily zero. On the other hand, at some of the buses for which the phase-a voltage magnitude lower-limit 0.95p.u. is binding, the corresponding dual variable solution turns out to be strictly positive; hence, the non-negative dual solution term $\boldsymbol{\Lambda}^*_{v_{\min}}(\mathcal{K})$ appearing in (24) for each household ψ does not vanish.

Consequently, for TES Management Case 1 with OP = hour 17, the final N(OP)-negotiated retail price (24) for each household $\psi = (u, \phi, j)$ typically deviates from the retail price LMP(b^*,OP) the IDSO commonly sets for all households at the start of N(OP). The specific magnitude and sign of this deviation depend on ψ's specific attributes (u, ϕ, j).

Finally, all households ψ for TES Management Case 1 have the same preference and structural attributes u. However, their connection-line phases ϕ and bus-j locations differ; hence, their power usage can have different effects on distribution network voltages. The IDSO must prevent the violation of the lower limit 0.95p.u. for the phase-a voltage magnitude during OP = hour 17. However, by construction, the negotiation process N(OP) forces the IDSO the satisfy all network constraints in the most efficient manner, i.e., in a manner that results in the smallest possible reduction in household net benefits. Thus, the final N(OP)-negotiated retail price (24) for each household $\psi = (u, \phi, j)$ will typically differ for households that have different connection-line phases ϕ and/or different bus-j locations to account for the different effects of their power usage on the phase-a voltage magnitude.

This explains the *variation* in the TES equilibrium retail prices depicted in Fig. 6 for hour 17.

E. Optimality Verification and Comparison

This subsection poses the following key question: Do the test-case outcomes obtained for the IDSO-managed consensus-based TES design closely approximate the outcomes that would be obtained if the IDSO were able to solve the benchmark complete-information IDSO optimization (17)?

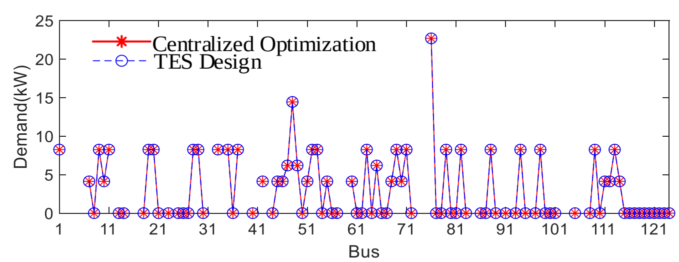

Fig. 7. **TES Management Case 1 (Peak Demand Upper Limit 3200kW):** Comparison of hourly day-D total household TCL outcomes using the consensus-based TES design negotiation process versus the benchmark complete-information IDSO optimization.

Fig. 7 affirmatively answers this question for TES Management Case 1. Hourly day-D total household TCL outcomes are

[13] By [1, App. G, Lemma 1], a dual variable solution for a strictly non-binding inequality constraint must be 0. However, the converse is false.

[14] Since test-case operating hours OP are not partitioned into sub-periods, OP = $\mathcal{K}$ and each household price profile $\pi_\psi(\mathcal{K})$ is a single OP price.

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reported for the IDSO-managed consensus-based TES design versus the benchmark complete-information IDSO optimization (17). The outcomes for the two management approaches are virtually identical.

Finally, Table I reports test-case outcomes for three different customer management methods: IDSO-managed consensus-based TES design; benchmark complete-information IDSO optimization; and a simple IDSO-managed price-reaction method. For the latter method, the IDSO sets the retail price for each hour OP of day D equal to LMP(b^*,OP), the LMP determined in RTM(OP) for the T-D linkage bus b^*.

The constant U = $\sum_{\psi\in\Psi}[u_\psi^{\max}\times NK\times 24]$ appearing in Table I is the *maximum possible* total comfort (utils) that households can achieve during day D, the same for each management method. The reported Net Benefits (utils) are the total net benefits *actually attained* by households during day D under each different management method.

TABLE I
COMPARISON OF DIFFERENT METHODS (PEAK DEMAND LIMIT 3200KW)

	TES Design	Benchmark IDSO Optimization	Price-Reaction
Net Benefits	U - 2.861*10^4	U - 2.861*10^4	U - 2.857*10^4
Privacy issue	No	Yes	No
Scalability issue	No	Yes	No
Network issue	No	No	Yes

As seen in Table I, households attain approximately the same day-D Net Benefits under TES design and benchmark IDSO optimization (17). Both methods require all distribution network constraints to be satisfied. Under TES design, this requirement results in household-specific retail prices (24) for each hour OP of day D that can deviate from LMP(b^*,OP). As seen in Fig. 6, the price deviations for peak hour OP = 17 are positive and relatively large for households $\psi = (u,\phi,j)$ with phase attribute $\phi = a$ and bus location $j \in \{61,\ldots,111\}$.

In contrast, under the simple IDSO-managed price-reaction method, higher day-D Net Benefits are attained. However, as seen in Fig. 3, these higher Net Benefits come at the cost of network reliability constraint violations.

The comparative findings reported in Fig. 7 and Table I for the IDSO-managed consensus-based TES design are promising. They indicate this TES design is capable of achieving outcomes that closely approximate the outcomes for the benchmark complete-information IDSO optimization (17), despite requiring only minimal information about customer attributes and no direct information about local customer constraints.

X. CONCLUSION

The challenging objective of this study has been to provide clear convincing evidence that the proposed IDSO-managed consensus-based TES design is a promising approach to the management of distribution systems electrically connected to transmission systems. In support of this objective, the study largely focuses on the performance of this design for a concrete analytically-formulated ITD system. Within the context of this analytical illustration, convergence and optimality properties of the TES design are first analytically established and then demonstrated by means of numerical test cases.

This study has thus been conducted at DOE Technology Readiness Level 1 (TRL-1). As defined in [29], TRL-1 studies begin the process of translating preliminary research into applied R&D. For example, TRL-1 studies include investigations of basic performance properties for newly conceived rules of operation for electric power systems.

Our intent is to build on the promising findings reported in this study by undertaking performance testing of our proposed TES design within ITD systems modeled with increasing empirical fidelity. This future research will address both conceptual and practical issues.

Regarding conceptual issues, three research directions are planned. First, performance testing of the proposed TES design will be undertaken for ITD systems with meshed distribution networks, distributed generation, and other features critical for achieving lower-emission electric power systems. Second, the TES design will be extended to permit inclusion of aggregators operating as intermediaries between the IDSO and its managed customers to facilitate design scalability. Third, the initial retail prices set by the IDSO at the start of each negotiation process will be carefully tailored to support two goals: reduction of customer exposure to price volatility risk; and preservation of IDSO independence by ensuring IDSO net revenues from distribution system operations are zero on average over time.

Regarding practical issues, we plan to investigate the performance robustness of our proposed TES design in the presence of various practical difficulties. These include: the need to account for power losses; forecast errors for uncontrollable customer loads; highly parameterized models requiring estimation of extensive preference and physical attributes; possible incompatibility of data collection and reporting practices across the distribution network (e.g., substations versus customer smart meters); and communication imperfections, such as delays and packet drops, that could prevent the IDSO-customer negotiation process from reaching consensus.

Attention will also be paid to the possible use of promising new techniques and tools. Examples include data-driven methods to avoid the need for extensive parameter estimation [30], and learning-assisted smart thermostats [31].

APPENDIX: QUICK-REFERENCE NOMENCLATURE TABLE

A. Acronyms, Parameters, and Other Exogenous Terms

$\bar{\boldsymbol{A}}$	Standard incidence matrix (p.u.) for a 3-phase radial network;
$\boldsymbol{B}$	Diagonal matrix with DDA-N(OP) step-sizes along diagonal;
b^*	T-D linkage bus;
$b^p(j)$	Bus immediately preceding bus j along a radial network;
bus 0	Head bus for a radial network;
c_ψ	Conversion factor (utils/$(^oF)^2$) for household ψ;
d	$NK \times NH$;
$\boldsymbol{D}_r$	Block diagonal matrix (p.u.) of line-segment resistances;
$\boldsymbol{D}_x$	Block diagonal matrix (p.u.) of line-segment reactances;

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DDA	Dual Decomposition Algorithm;
DDA-N(OP)	DDA implementation for N(OP);
DSO	Distribution System Operator;
$\boldsymbol{H}_{\psi}(\mathcal{K})$	Household ψ's TCL power-ratio matrix for $\mathcal{K}$;
I_{max}	Max permitted N(OP) rounds;
IDSO	Independent DSO;
ISO	Independent System Operator;
$\ell_j = (i, j)$	Line segment connecting bus i and bus j with $i = b^p(j)$ and $j \in \mathcal{N}$;
LAH(OP)	Look-Ahead Horizon for RTM(OP);
LMP	Locational Marginal Price;
LMP(b^*, t)	RTM LMP (cents/kWh) at b^* for t;
LMP$(\mathcal{K})$	RTM LMP profile for $\mathcal{K}$;
m	Number of explicit constraints for the Benchmark Primal Problem;
$\bar{\boldsymbol{M}}$	Standard incidence matrix (p.u.) for 1-phase radial distribution network;
N	Number of non-head buses for a radial network;
NH	Number of households $\psi \in \Psi$;
NK	Number of sub-periods t forming a partition of OP;
N(OP)	Negotiation process for OP;
N^{ph}_{ψ}	Flag for phase $\phi \in \{a, b, c\}$ of the 1-phase line connecting household ψ to a distribution network bus;
OP	Operating Period;
$\bar{P}$	Peak demand upper limit (p.u.) imposed by IDSO on total household active power usage for each t;
PF$_{\psi}(t)$	Power factor (unit free) in $(0, 1]$ for the HVAC system of household ψ during sub-period t;
p^{max}_{ψ}	Max limit (p.u.) on ψ's TCL active power usage for each $t \in \mathcal{K}$;
$p^{\mathsf{non}}_{\psi}(t), q^{\mathsf{non}}_{\psi}(t)$	Non-TCL active and reactive power usage (p.u.) of ψ during t;
$\mathcal{P}^{\mathsf{non}}_{\psi}(\mathcal{K}), \mathcal{Q}^{\mathsf{non}}_{\psi}(\mathcal{K})$	Non-TCL active and reactive power profiles (p.u.) of ψ for $\mathcal{K}$;
$\boldsymbol{R}_{ij}, \boldsymbol{X}_{ij}$	3-phase resistance & reactance matrices (p.u.) for line segment (i, j);
RTM(OP)	Real-Time Market for OP;
RTO	Regional Transmission Operator;
S_{base}	Base apparent power (kVA) ;
TB^{a}_{ψ}	Bliss (max comfort) inside air temperature (^oF) for household ψ;
TES	Transactive Energy System;
TCL	Thermostatically-Controlled Load;
$\hat{T}^{\mathsf{a}}_{\psi}(0)$	Forecast (^oF) for household ψ's inside air temperature at start-time $s(1)$ for sub-period $1 \in \mathcal{K}$;
$\hat{T}^o(0)$	Forecast (^oF) for outside air temp at start-time $s(1)$ for sub-period $1 \in \mathcal{K}$, same for all households;
$\hat{T}^o(t)$	Forecast (^oF) for outside air temp at end-time $e(t)$ for sub-period $t \in \mathcal{K}$, same for all households;
t	Sub-period of OP;
u^{max}_{ψ}	Household ψ's maximum attainable thermal comfort (utils);
V_{base}	Base voltage (kV);
$\boldsymbol{v_0}(t)$	Vector of 3-phase squared voltage magnitudes (p.u.) at bus 0 for t;
$\boldsymbol{v}^{\mathsf{non}}(t)$	Vector of 3-phase squared voltage magnitudes (p.u.) at all non-head buses for t, assuming zero TCL;
$\boldsymbol{v}_{\mathsf{min}}(t), \boldsymbol{v}_{\mathsf{max}}(t)$	Vectors of min/max limits (p.u.) imposed by IDSO on 3-phase squared voltage magnitudes during t;
α^H_{ψ}	System inertia temp parameter (unit-free) for household ψ;
α^P_{ψ}	Temperature parameter (^oF/kWh) for household ψ;
$\beta_1, \beta_2, \beta_3$	DDA-N(OP) step sizes (unit-free);
$\triangle\tau$	Common duration of each sub-period t, measured in hourly units;
$\eta_{\psi}(t)$	Ratio (unit free) of TCL reactive power to TCL active power for household ψ during sub-period t;
γ_{ψ}	Benefit/cost slider-knob control setting (unit free) in (0,1) for ψ;
μ_{ψ}	Household ψ's marginal utility of money (utils/cent) for $\mathcal{K}$;
ϕ	Circuit phase of a line segment ℓ_j, or of a secondary 1-phase line connecting a household to a bus;
$\psi = (u, \phi, j)$	Household with preference and structural attributes u connected by a secondary phase-ϕ line to bus j.

B. Sets, Sequences, and Profiles

$\mathcal{K} = (1, \ldots, NK)$	Sequence of sub-periods t that partition an operating period OP;
$\mathcal{L}$	Set of all distinct line segments;
$\mathcal{N} = \{1, \ldots, N\}$	Index set for all non-head buses of a radial network;
$\mathcal{N}_j$	Index set for all buses located strictly after bus j for a radial network;
$\boldsymbol{\mathcal{P}}(\mathcal{K})$	Set of household TCL active power profiles for $\mathcal{K}$;
$\boldsymbol{\mathcal{P}}(\boldsymbol{\pi}(\mathcal{K}))$	Set of optimal household TCL active power profiles for $\mathcal{K}$, given $\boldsymbol{\pi}(\mathcal{K})$;
$\mathcal{U}_{\phi,j}$	Set of attributes u such that (u, ϕ, j) denotes a household $\psi \in \Psi$;
$\mathcal{X}_{\psi}(\mathcal{K})$	Set of household ψ constraints for $\mathcal{K}$;
$\Phi = \{a, b, c\}$	Set of line phases ϕ;
$\boldsymbol{\pi}(\mathcal{K})$	Set of household retail price profiles for $\mathcal{K}$;
Ψ	Set of all households ψ.

C. Functions, & Variables

Cost$_{\psi}(\mathcal{P}_{\psi}(\mathcal{K}) \mid \boldsymbol{\pi}_{\psi}(\mathcal{K}))$	Total cost of ψ's TCL active power usage for $\mathcal{K}$, given $\boldsymbol{\pi}_{\psi}(\mathcal{K})$;
$L(\boldsymbol{x}, \boldsymbol{\lambda})$	Lagrangean function for benchmark primal problem;

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$\boldsymbol{P}_{ij}(t), \boldsymbol{Q}_{ij}(t)$	3-phase active and reactive power flows (p.u.) over line segment (i,j) during sub-period t;
$\boldsymbol{P}(t), \boldsymbol{Q}(t)$	3-phase active and reactive power flows (p.u.) over all line segments during sub-period t;
$\boldsymbol{p}_j(t), \boldsymbol{q}_j(t)$	3-phase active and reactive power (p.u.) at bus j for t;
$\boldsymbol{p}(t), \boldsymbol{q}(t)$	3-phase active and reactive power (p.u.) at all non-head buses for t;
$p_\psi(t), q_\psi(t)$	TCL active and reactive power-usage levels (p.u.) of household ψ for t;
$\mathcal{P}_\psi(\mathcal{K}), \mathcal{Q}_\psi(\mathcal{K})$	TCL active and reactive power profiles (p.u.) of ψ for $\mathcal{K}$;
$T^{\mathsf{a}}_{\psi}(p_\psi(t), t)$	Household ψ's inside air temp (^oF) at end-time $e(t)$ for t, given $p_\psi(t)$;
$U_\psi(\mathcal{P}_\psi(\mathcal{K}))$	Total benefit (utils) attained by ψ during $\mathcal{K}$, given ψ's TCL active power profile $\mathcal{P}_\psi(\mathcal{K})$ for $\mathcal{K}$;
$\boldsymbol{v}(t, \boldsymbol{p}_\Psi(t))$	3-phase squared voltage magnitudes (p.u.) at all non-head buses for t;
$\boldsymbol{v}_j(t, \boldsymbol{p}_\Psi(t))$	3-phase squared voltage magnitudes (p.u.) at bus j for t;
$\boldsymbol{\lambda}$	Vector of dual variables (utils/p.u.) for all network reliability constraints for all $t \in \mathcal{K}$;
$\lambda_{\bar{P}}(t)$	Dual variable (utils/p.u.) for max total active power-usage limit for t;
$\boldsymbol{\lambda}_{\bar{P}}(\mathcal{K})$	Vector of dual variables (utils/p.u.) for max total active power-usage limits for all sub-periods $t \in \mathcal{K}$;
$\boldsymbol{\lambda}_{v_{\mathsf{max}}}(t)$	Vector of dual variables (utils/p.u.) for max voltage magnitude limits for sub-period t;
$\boldsymbol{\Lambda}_{v_{\mathsf{max}}}(\mathcal{K})$	Matrix of dual variables (utils/p.u.) for max voltage magnitude limits for all sub-periods $t \in \mathcal{K}$;
$\boldsymbol{\lambda}_{v_{\mathsf{min}}}(t)$	Vector of dual variables (utils/p.u.) for min voltage magnitude limits for sub-period t;
$\boldsymbol{\Lambda}_{v_{\mathsf{min}}}(\mathcal{K})$	Matrix of dual variables (utils/p.u.) for min voltage magnitude limits for all sub-periods $t \in \mathcal{K}$;
$\pi_\psi(t)$	Retail price (cents/kWh) for ψ's TCL active power usage during t;
$\boldsymbol{\pi}_\psi(\mathcal{K})$	Price profile (cents/kWh) of household ψ for $\mathcal{K}$.

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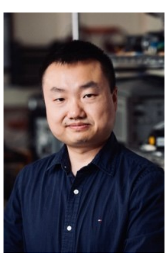

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