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Modeling Economic Systems as Locally-Constructive Sequential Games

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Abstract

Real-world economies are open-ended dynamic systems consisting of heterogeneous interacting participants. Human participants are decision-makers who strategically take into account the past actions and potential future actions of other participants. All participants are forced to be locally constructive, meaning their actions at any given time must be based on their local states; and participant actions at any given time affect future local states. Taken together, these essential properties imply real-world economies are locally-constructive sequential games. This paper discusses a modeling approach, Agent-based Computational Economics (ACE), that permits researchers to study economic systems from this point of view. ACE modeling principles and objectives are first concisely presented and explained. The remainder of the paper then highlights challenging issues and edgier explorations that ACE researchers are currently pursuing.

Keywords: Economic systems, local constructivity, sequential game, agent-based computational economics

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1. Introduction

Real-world economies exhibit five essential properties. First, they consist of *heterogeneous interacting participants* characterized by distinct local states (data, attributes, methods) at each given time. Second, they are *open-ended dynamic systems* whose dynamics are driven by the successive interactions of

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their participants. Third, human participants are *strategic decision-makers* whose decision processes take into account past actions and potential future actions of other participants. Fourth, all participants are *locally constructive*, i.e., constrained to act on the basis of their own local states at each given time. Fifth, the actions taken by participants at any given time affect future local states and hence induce *system reflexivity*.[1]

These five essential properties imply that real-world economies are *locally-constructive sequential games*.[2] Two key questions can thus be posed. Do modeling tools exist that permit real-world economies to be represented and implemented as locally-constructive sequential games? If so, can these modeling tools usefully advance the state of economic knowledge?

This study answers both of these questions in the affirmative, focusing specifically on *Agent-based Computational Economics (ACE)* for concrete illustration.[3] ACE is the computational study of economic processes, including whole economies, as open-ended dynamic systems of interacting agents (Tesfatsion, 2017a). The driving concern in the development of ACE has been to provide a flexible modeling approach that enables a researcher to specify and implement a model for a problem at hand with a degree of empirical verisimilitude appropriate for this problem. In particular, modelers should not be forced to rely on *a priori* model specifications whose only justification is analytical tractability.

ACE is a specialization to economics of the more broadly conceived approach referred to as *Agent-Based Modeling (ABM)* (Axelrod and Tesfatsion,

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[1]See Davis (2007, 2016) for a more precise definition and discussion of reflexivity for economic systems.

[2]*Sequential games*, also referred to as *extensive-form games*, are dynamic games in which multiple players undertake sequential decision making. Since decision-making players are informed, at a minimum, about their own decision histories, their states evolve over time. For a basic introduction to game theory in general, and sequential games in particular, see Eatwell et al. (1989).

[3]This study is an extended version of a keynote address given at the Duke Forest Conference (Durham, NC, Nov. 11-13, 2016), titled "Economic Systems as Constructively Rational Games: Oh, the Places We Could Go!." Some of the materials in this study are adapted from Axelrod and Tesfatsion (2006), Borrill and Tesfatsion (2011), LeBaron and Tesfatsion (2008), and Tesfatsion (2017a). Annotated pointers to ACE tutorials, publications, demos, software, research groups, and research area sites are posted at the ACE website (Tesfatsion, 2017a). For broad ACE/ABM overviews, see Arthur (2015), Chen (2016), Epstein (2006), Kirman (2011), and Tesfatsion (2006).

2006). Although the precise meaning of ABM continues to be debated in the ABM literature, seven specific modeling principles have been developed for ACE that carefully distinguish it from other types of modeling and that highlight its particular relevance for the study of economic systems.

The seven modeling principles underlying ACE model design are presented and explained in Section 2. Taken together, they express the fundamental goal of many agent-based modelers: namely, to be able to study real-world systems as historical processes unfolding through time.

Section 3 discusses four key objectives currently being pursued by ACE researchers: empirical understanding; normative design; qualitative insight and theory generation; and methodological advancement. The achievement of each of these objectives is critical for the overall success of ACE as a modeling approach.

The next three sections highlight challenging opportunities for ACE modelers. Section 4 considers distinct empirical validation aspects that researchers tend to weight differently, depending upon their purpose: input validation; process validation; in-sample fitting; and out-of-sample forecasting. Although differential weighting by purpose is commonly done, it is argued that ACE modeling permits researchers to strive for a more comprehensive approach to empirical validation that simultaneously considers all four aspects.

Section 5 discusses the increasingly important role that ACE models are playing as computational laboratories for the development and testing of policy initiatives in advance of implementation. A taxonomy of *Policy Readiness Levels (PRLs)* is proposed for policy initiatives ranging from conceptual policy formulation (PRL 1) to real-world policy implementation (PRL 9). ACE modeling is helping to bridge the difficult gap between conceptual policy research (PRLs 1-3), typically undertaken at universities, and large-scale policy models incorporating numerous real-world features (PRL 7) that are favored by industry, government, and regulatory agencies as a prelude to field studies (PRL 8) and real-world policy implementations (PRL 9).

An additional potential benefit of the PRL taxonomy is addressed in Section 6: namely, it could facilitate the development of presentation protocols for economic policy models that appropriately take into account model purpose and level of model development. This is particularly important for newer modeling methodologies, such as ACE, which do not yet have established presentation practices familiar to large numbers of economists.

Section 7 considers ways in which ACE permits edgier explorations of critical real-world systems. These include: (i) the study of labor markets as evolutionary sequential games with endogenous hiring, firing, and quits (Section 7.1); (ii) the study of macroeconomies with anticipatory learning by locally-constructive consumers and firms attempting to achieve intertemporal objectives (Section 7.2); (iii) the study of risk-management by strategically interacting rural and urban decision-makers residing within a watershed affected by climate and hydrological processes (Section 7.3); (iv) the study of new market design features for U.S. electric power systems (Section 7.4); and (v) the use of ACE modeling principles as design principles guiding the development of decentralized "transactive energy" architectures for U.S. transmission and distribution systems (Section 7.5).

In Section 8 it is shown how ACE can be viewed as a limit point of a broad spectrum of experiment-based modeling approaches ranging from 100% human subject to 100% computer agent. By design, any decision-making agent in an ACE model can be replaced by a real person. This opens up huge mix-and-match opportunities to study individual and group behaviors in realistically rendered contexts. Concluding remarks are provided in Section 9.

2. ACE Modeling Principles

The following seven modeling principles collectively characterize the ACE modeling approach:

(MP1) *Agent Definition:* An *agent* is a software entity within a computationally constructed world capable of acting over time on the basis of its own *state*, i.e., its own internal data, attributes, and methods.

(MP2) *Agent Scope:* Agents can represent individuals, social groupings, institutions, biological entities, and/or physical entities.

(MP3) *Agent Local Constructivity:* The action of an agent at any given time is determined as a function of the agent's own state at that time.

(MP4) *Agent Autonomy:* Coordination of agent interactions cannot be externally imposed by means of free-floating restrictions, i.e., restrictions not embodied within agent states.

(MP5) *System Constructivity:* The state of the modeled system at any given time is determined by the ensemble of agent states at that time.

(MP6) *System Historicity:* Given initial agent states, all subsequent events in the modeled system are determined solely by agent interactions.

(MP7) *Modeler as Culture-Dish Experimenter:* The role of the modeler is limited to the setting of initial agent states and to the non-perturbational observation, analysis, and reporting of model outcomes.

Considered as a collective whole, modeling principles (MP1)–(MP7) embody the idea that an ACE model is a computational laboratory permitting users to explore how changes in initial conditions affect outcomes in a modeled dynamic system over time. This exploration process is analogous to biological experimentation with cultures in petri dishes. A user sets initial conditions for a modeled dynamic system in accordance with some purpose at hand. The user then steps back, and the modeled dynamic system thereafter runs forward through time as a virtual world whose dynamics are driven by the interactions of its constituent agents.

The explicit statement of these modeling principles permits ACE to be distinguished more clearly and carefully from other modeling approaches, such as standard game theory and general equilibrium modeling within economics, and standard usages of state-space modeling by economists, engineers, and physicists. It also permits more precise comparisons between ACE and important historical antecedents, such as system dynamics (Rahmandad and Sterman, 2008) and microsimulation (Richiardi, 2013).

Modeling principle (MP1) provides a concise definition of an ACE agent as a software entity capable of taking actions based on its own local state. Here, "state" refers to three possibly-empty categories characterizing an agent at any given time: data (recorded physical sensations, empirical observations, statistical summaries,...); attributes (physical conditions, financial conditions, beliefs, preferences...); and methods (data acquisition, physical laws, data interpretation, logical deduction, optimization routines, learning algorithms, decision rules, ...). There is no presumption here that the data acquired by an agent are accurate or complete, or that the methods used by an agent to process data are without error.

An agent's state represents the *potential* of this agent to express various types of behaviors through its actions. The agent's *actual* expressed behaviors within its virtual world, while conditioned on the agent's successive states, are also constrained and channeled by interactions with other agents.

An important corollary of (MP1) is that agents in ACE models are *encapsulated* software entities, i.e., software entities that package together data, attributes, and methods. This encapsulation permits an agent's internal aspects to be partially or completely hidden from other agents.

A person familiar with *object-oriented programming (OOP)* might wonder why "agent" is used in (MP1) instead of "object," or "object template" (class), since both agents and objects refer to encapsulated software entities. "Agent" is used in ACE, and in ABM more generally, to stress the intended application to problem domains that include entities capable of various degrees of self-governance, self-directed social interactions, and deliberate masking of intentions. In contrast, OOP has traditionally interpreted objects as passive tools developed by a user to aid the accomplishment of a user-specified task.

The "state" conceptualization in (MP1) differs in two important ways from state depictions in standard state-space modeling:

(i) *Diversity of State Content:* The expression of an agent's state in terms of data, attribute, and method categories is broader than the standard depiction of states as vectors of real-valued variables;

(ii) *Variability of State Dimension:* An agent's state is not restricted to lie within a fixed finite-dimensional domain.

Regarding (ii), an agent's state can evolve over time in open-ended ways. For example, an agent can continue to augment its data $\mathbb{D}(t)$ over successive times t without need to rely on fixed-dimensional sufficient statistics, and its attributes $\alpha(t)$ can also vary over time. Moreover, the agent's methods $\mathbb{M}(t)$ might include a domain $\mathbb{R}(t)$ of possible decision rules plus a genetic algorithm g that involves mutation and recombination operations. When g operates on $\mathbb{R}(t)$, given $\mathbb{D}(t)$ and $\alpha(t)$, the result $g(\mathbb{R}(t); \mathbb{D}(t), \alpha(t))$ could be a modified decision-rule domain $\mathbb{R}(t + \Delta t)$ that has different elements than $\mathbb{R}(t)$ and possibly also a different dimension than $\mathbb{R}(t)$.

Modeling principle (MP2) expresses the intended broad scope of the agent definition provided in (MP1). In particular, in contrast to many agent definitions proposed in the general ABM literature, (MP2) makes clear that ACE agents are not restricted to human representations. Such a restriction would require modelers to make unnecessary distinctions between human actions and the actions of all other kinds of entities. Instead, (MP1) is in accordance with the standard dictionary meaning of agent as any entity able to take actions that affect subsequent events. ACE researchers can thus represent agents at different ontological levels in accordance with their purposes (Gräbner, 2015).

Another way of viewing (MP2) is that it calls for a broad *agent taxonomy*, i.e., a broad classification of agents into ordered groups or categories. As illustrated in Fig. 1, agents in ACE models can span all the way from passive physical entities with no cognitive function to active social entities with sophisticated decision-making capabilities. Moreover, agents can have other agents as constituent members, thus permitting the modeling and study of hierarchical systems (Simon, 1962).

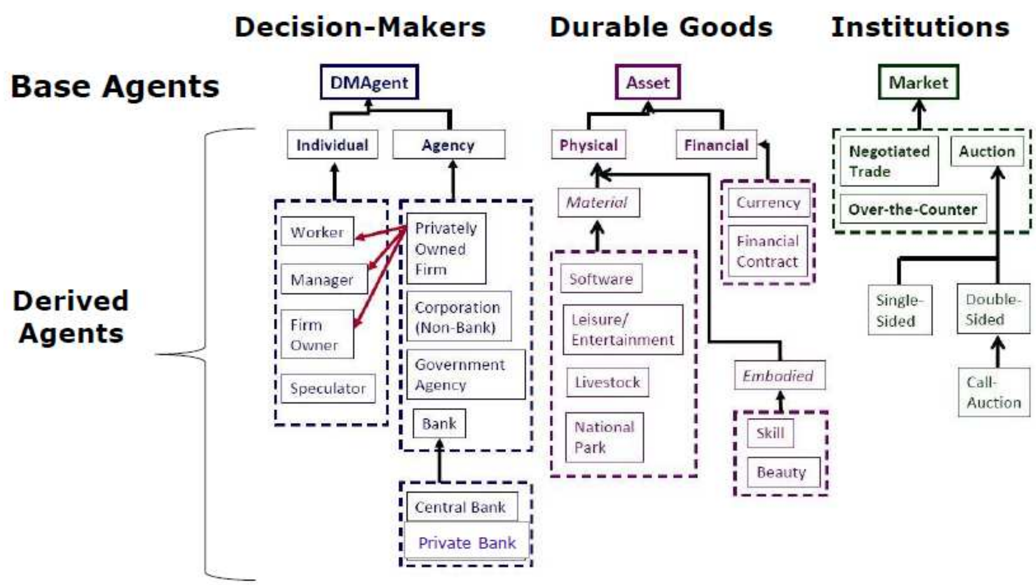

Figure 1: Partial agent taxonomy for an ACE macroeconomic model. Up-pointing arrows denote "is a" relationships and down-pointing arrows denote "has a" relationships.

The process of economic theorizing and model building would be on a firmer footing if it could routinely be based on an empirically grounded taxonomy. What types of human needs and desires are relevant for understanding particular types of economic phenomena? What types of goods and services meet or could meet these human needs and desires? What types of facilities exist or could exist to produce these goods and services, and who participates in these production activities? What kinds of institutions exist or could exist to distribute these goods and services, and who participates in these distribution activities? And what types of entities (if any) oversee the design and/or operation of these institutions, and for what purposes?

ACE modeling per se provides no answers to these questions; it is a methodological approach, not a theory. However, ACE modeling provides a systematic way to incorporate whatever agent taxonomy a researcher believes is useful for the exploratory study of a particular economic phenomenon. The researcher is freed from the constrictive binds of analytical tractability and from the need to rely on narrow fragmented taxonomies arising from artificial disciplinary boundaries.

The remaining five modeling principles (MP3)–(MP7) imply that ACE models are *state-space models in initial value form* (Tesfatsion, 2016). Specifically, an ACE model specifies how an ensemble of agent states varies over time starting from a given ensemble of agent states at an initial time.

However, these five principles further require this state process to embody essential real-world properties, such as local constructivity and system historicity. Ideally, an ACE model should satisfy the following *constructive Turing-test requirement*: Any ACE agent should be replaceable by a real-world counterpart, linked into the model via a suitable interface. Thus, an ACE representation of a human decision-maker should be replaceable by a real person. An ACE representation of a stock market should be replaceable by an actual stock market. An ACE modeling of an air-conditioning (A/C) system should be replaceable by an actual A/C system. And so forth.

Modern economic theory also relies heavily on state-space models. However, these models typically incorporate modeler-imposed rationality, optimality, and equilibrium conditions that could not (or would not) be met by locally constructive agents interacting within an historical process. For example, rational expectations assumptions require ex ante agent expectations to be consistent with ex post model outcomes. Consequently, as detailed in (Tesfatsion, 2017b, Section 5), the derivation of rational expectations solutions is a global fixed-point problem requiring the simultaneous consideration of all time periods without regard for local constructivity and historical process constraints.

The five modeling principles (MP3)-(MP7) also require an ACE model to be *fully* agent based. That is, all entities capable of acting within an ACE computationally-constructed world must be modeled as some form of agent. This requirement has two key advantages. First, it enhances conceptual transparency; all factors affecting world events must be clearly identified as an agent or agent component. Second, it facilitates plug-and-play model scalability. The number of previously-typed agents can easily be increased, since this does not require changes to the interfaces between agent types.

Also, *high-level architectures (HLAs)*[4] can be designed for ACE models that facilitate enlargement of their scope through inclusion of new agent types.

Despite being fully agent based, ACE models can still exhibit *emergent phenomena*, here defined to mean system attributes or events that arise from agent states but that cannot be directly and simply inferred from the forms of these states.[5] Dramatic forms of emergent phenomena include wars and financial market crashes. Less dramatic forms include the spatial distribution of cities, segregation patterns, and the persistence of wealth and income inequality across successive generations of households. The difficulty in inferring such phenomena from agent states is that their proximate causes can be a complex swirl of agent interactions arising from agent states that also cause further changes in these states.

For ACE researchers, as for economists in general, the modeling of decision methods for decision-making agents is a primary concern. Here it is important to correct a major misconception still being expressed by some commentators uninformed about the powerful capabilities of modern software: namely, the misconception that ACE decision-making agents cannot be as rational (or irrational) as real people.

To the contrary, the constraints on agent decision making implied by modeling principles (MP1)–(MP7) are constraints inherent in every real-world economic system. As seen in the ACE learning research linked at (Tesfatsion, 2017c), the decision methods used by ACE agents can range from simple behavioral rules to sophisticated anticipatory learning algorithms for the approximate achievement of intertemporal objectives. A more extended discussion of this point is provided in Section 7.2.

A second common misconception is the incorrect belief that (MP1)–(MP7) rule out any consideration of stochasticity. To the contrary, stochastic aspects can easily be represented within ACE models. Agent data can include recorded realizations for random events, agent attributes can include beliefs based on probabilistic assessments, and agent methods can include *pseudo-random number generators (PRNGs)*. A PRNG is an algorithm, initialized by a seed value, that is able to generate a number sequence whose

[4]An HLA is a general purpose framework that manages the interactions among a "federation" (collection) of "federates" (simulation entities) (IEEE, 2010). The goal is to promote the interoperability and reuse of simulation systems.

[5]See Harper and Lewis (2012) for a survey exploring the conceptualization and use of "emergence" within contemporary economics.

properties mimic the properties of a random number sequence.

Regarding the latter, PRNGs can be included among the methods of decision-making agents, permitting them to "randomize" their behaviors. For example, decision-making agents can use PRNGs to choose among equally preferred actions or action delays, to construct mixed strategies in game situations to avoid exploitable predictability, and to induce perturbations in action routines in order to explore new action possibilities.

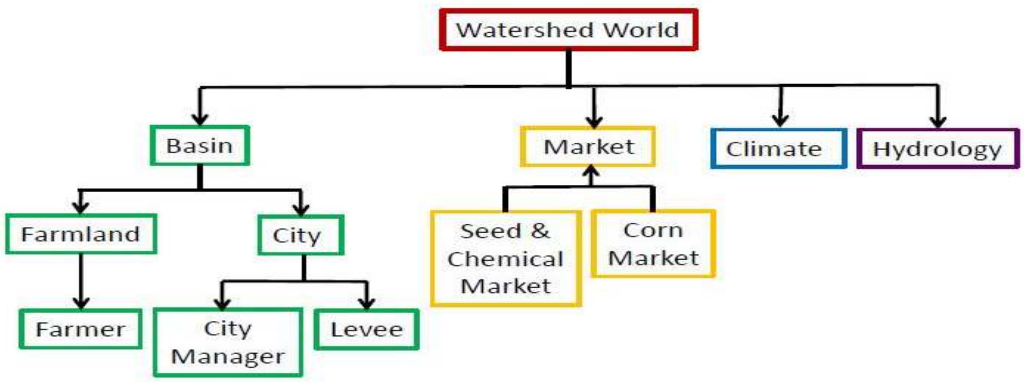

Figure 2: Agent taxonomy for an ACE watershed model. Up-pointing arrows denote "is a" relationships and down-pointing arrows denote "has a" relationships. Source: Tesfatsion et al. (2017).

Also, PRNGs can be included among the methods of other types of agents, such as physical or biological agents, in order to model stochastic processes external to decision-making agents. For example, Fig. 2 depicts the agent taxonomy for an ACE watershed model (Tesfatsion et al., 2017) with upstream farmland and a downstream city subject to flooding. The Climate agent uses a PRNG to generate a weather pattern (hourly rainfall amounts) for each simulated year, which in turn affects Hydrology outcomes (river water flow), Basin outcomes (water run-off), Farmland outcomes (bushels per acre), Farmer outcomes (land allocation decisions), and City Manager outcomes (budget allocation decisions).

An important constraint affecting the ACE modeling of stochasticity is that the modeling principles (MP1)–(MP7) require an ACE model to be dynamically complete. Thus, ACE modelers must identify the *sources* of any stochastic shocks affecting events within their modeled worlds, not simply their impact points, because all such shocks must come from agents actually residing within these worlds. This requirement encourages ACE modelers to think carefully about the intended empirical referents for any included stochastic shock terms. It also facilitates successive model development. For example, a Climate agent represented as a highly simplified stochastic process in a current modeling effort can easily be modified to have a more empirically compelling representation in a subsequent modeling effort.

Another key issue is whether the modeling principles (MP1)–(MP7) imply ACE models are necessarily pre-statable. As stressed by (Longo et al., 2012; Koppl et al., 2015), the real world "bubbles forth" with an ever-evolving state space, driven in part by random (acausal) events. This renders infeasible the pre-statement of accurate equations of motion for real-world state processes.

ACE modeling addresses this issue in two ways. First, there is no requirement in ACE modeled worlds that the agents residing within these worlds be able to accurately depict laws of motion for their states in equation form, or in any other form. Second, data can be streamed into ACE models in a manner that prevents even the modeler from being able to accurately pre-state future model outcomes.

More precisely, suppose an ACE model has no run-time interaction with any external system during times $t \in [t^o, T]$ for some finite horizon T. Then, in principle, the modeler at time t^o could pre-state all model outcomes over the time interval $[t^o, T]$, conditional on a given specification of agent states at time t^o, in the same manner that he could in principle pre-state all possible plays of a chess game with a given closure rule.

Nevertheless, modeling principles (MP1)–(MP7) do not imply ACE agents have complete state information. Thus, ACE agents can experience events that they have no way of knowing in advance.

For example, suppose an ACE model consists of a Climate agent interacting over times $t \in [t^o, T]$ with a variety of other agents, as depicted in Fig. 2. At the initial time t^o the modeler might know the weather pattern the Climate agent will generate over $[t^o, T]$, or be able to pre-state this weather pattern based on the modeler's time-t^o knowledge (or control) of the Climate agent's data, attributes, and methods. However, if other agents have no access to the Climate agent's internal aspects, they will experience weather over $[t^o, T]$ as a stochastic process.

Alternatively, an ACE model can have run-time interactions with an external system. For example, as discussed by LeBaron and Tesfatsion (2008, Section III) and Borrill and Tesfatsion (2011, Section 2.1), an ACE model can be *data driven*; that is, it can include conduit agents permitting external data to be streamed into the model during run-time that are unknown (or unknowable) by the modeler at the initial time t^o. In this case the modeler at time t^o will not be able to pre-state future model outcomes, even in principle.

A particularly intriguing case to consider is when the data streamed into an ACE modeled world include sequences of outcomes extracted from real-world processes. For example, real-world weather data could be streamed into a Climate agent that this agent then uses to generate a weather pattern for its virtual world. These weather data could include thermal or atmospheric noise data accessible to decision-making agents, enabling them to use "truly random" numbers in place of PRNGs for their decision-making processes.

Finally, another important issue for ACE modelers is the representation of time. ACE agents experience time as a sequence of event realizations. In order for multiple agents to be able to coordinate their future actions on the basis of time, they must have a common agreed-upon way to measure the passage of time in terms of event realizations. For example, these event realizations could be signals separately received from Clock agents with synchronized digital displays, or the commonly observed cyclical motions of a Sun agent. An open question is the extent to which this event-based modeling of time reflects the real-world nature of time.[6]

3. ACE Objectives and Scope

Current ACE research divides roughly into four strands differentiated by objective. One primary objective is *empirical understanding*: What explains the appearance and persistence of empirical regularities? ACE researchers seek possible causal mechanisms grounded in the successive interactions of agents operating within computationally-rendered virtual worlds. A virtual world capable of generating an empirical regularity of interest provides a candidate explanation for this regularity. If distinct virtual worlds are found to have equivalent generative capability, further work must be done to adjudicate among these candidate explanations based on the empirical plausibility of their inputs and modeled processes (Epstein, 2006, pp. 8-10).

A second primary objective of ACE researchers is *normative design*: How can ACE models be used as computational laboratories to facilitate the design of structures, institutions, and regulations resulting in desirable system

[6]As discussed by Borrill and Tesfatsion (2011, Sec. 4.2), philosophers of science and physicists are still debating the real-world nature of time. For an important recent entry into the debate, see Unger and Smolin (2014).

performance over time? The ACE approach to normative design is akin to filling a bucket with water to determine if it leaks. A researcher constructs a virtual world that captures salient aspects of a system operating under a proposed design. The researcher identifies a range of initial agent state specifications of interest, including seed values for agent PRNG methods. For each such specification the researcher permits the virtual world to develop over time driven solely by agent interactions. Recorded outcomes are then used to evaluate design performance.

One key issue for ACE normative design is the extent to which resulting outcomes are efficient, fair, and orderly, despite possible attempts by strategic decision-making agents to game the design for personal advantage. A second key issue is a cautionary concern for adverse unintended consequences. *Optimal* design might not always be a realistic goal, especially for large complex systems; but ACE models can facilitate *robust* design for increased system resiliency, a goal that is both feasible and highly desirable.

A third primary objective of ACE researchers is *qualitative insight and theory generation*: How can ACE models be used to study the *potential* behaviors of dynamic systems over time? Ideally, what is needed is a dynamic system's *phase portrait*, i.e., a representation of its potential state trajectories starting from all feasible initial states. Phase portraits reveal not only the possible existence of equilibria but also the basins of attraction for any such equilibria. Phase portraits thus help to clarify which regions of a system's state space are credibly reachable, hence of empirical interest, and which are not. An ACE modeling of a dynamic system can be used to conduct batched runs starting from multiple feasible initial states, thus providing a rough approximation of the system's phase portrait.

A fourth primary objective of ACE researchers is *method/tool advancement*: How best to provide ACE researchers with the methods and tools they need to undertake theoretical studies of dynamic systems through systematic sensitivity studies, and to examine the compatibility of sensitivity-generated theories with real-world data? ACE researchers are exploring a variety of ways to address this objective ranging from careful consideration of modeling principles to the practical development of programming, visualization, and empirical validation tools.

4. Enabling Comprehensive Empirical Validation

Modelers focused on the scientific understanding of real-world systems want their models to have empirical validity ("consistency with real world data"). Below are four distinct aspects of empirical validation which, ideally, a model intended for scientific understanding should simultaneously achieve:

EV1. Input Validation: Are the exogenous inputs for the model empirically meaningful and appropriate for the purpose at hand? Examples of exogenous model inputs include functional forms, random shock realizations, data-based parameter estimates, and/or parameter values imported from other studies.

EV2. Process Validation: How well do the physical, biological, institutional, and social processes represented within the model reflect real-world aspects important for the purpose at hand? Are all process specifications consistent with essential scaffolding constraints, such as physical laws, stock-flow relationships, and accounting identities?

EV3. Descriptive Output Validation: How well are model-generated outputs able to capture the salient features of the sample data used for model identification? *(in-sample fitting)*

EV4. Predictive Output Validation: How well are model-generated outputs able to forecast distributions, or distribution moments, for sample data withheld from model identification or for data acquired at a later time? *(out-of-sample forecasting)*

In practice, economists relying solely on standard analytical modeling tools do not place equal weight on these four aspects of empirical validation. Particularly for larger-scale economic systems, such as macroeconomies, analytical tractability issues and a desire to adhere to preconceived rationality, optimality, and equilibrium ideals have forced severe compromises.

In contrast, an ACE model is an open-ended dynamic system. Starting from an initial state, outcomes are determined forward through time, one state leading to the next, in a constructive manner. This process does not depend on the determination, or even the existence, of equilibrium states. ACE thus provides researchers with epistemological flexibility, permitting modeled agents to be tailored for particular purposes.

In particular, ACE researchers can match modeled biological, physical, institutional, and social agents to their empirical counterparts in the real world. This ability to match modeled agents to empirical counterparts, important for scientific understanding, is also critical for normative design purposes. Robustness of proposed designs against strategic manipulation can only be assured in advance of implementation if the modeled decision-making agents used to test the performance of these designs have the same degree of freedom to engage in strategic behaviors as their empirical counterparts.

ACE modeling thus permits researchers to strive for the simultaneous achievement of all four empirical validation aspects EV1 through EV4. This pursuit of comprehensive empirical validation will of course be tempered in practice by data limitations. Even in an era of Big Data advances, data availability and quality remain important concerns (D'Orazio, 2017).

Computational limitations such as round-off error, truncation error, and error propagation are also a concern. Advances in computer technology and numerical approximation procedures are rapidly relaxing these limitations. In the interim, however, as elegantly expressed by Judd (2006, p. 887), numerical error must be traded off against specification error:

> "The key fact is that economists face a trade-off between the numerical errors in computational work and the specification errors of analytically tractable models. Computationally intensive approaches offer opportunities to examine realistic models, a valuable option even with the numerical errors. As Tukey (1962) puts it, 'Far better an approximate answer to the right question ... than an exact answer to the wrong question ...'."

Empirical validation of ACE models in the sense of EV1 through EV4 is a highly active area of research. Extensive annotated pointers to this research can be found at (Tesfatsion, 2017d).

5. Avoiding Premature Jumps to Policy Implementation

Ideally, changes in a society's current institutional and regulatory policies should be guided by research that is strongly supported by empirical evidence. Reaching a point where a proposed new policy is ready for real-world implementation will typically require a series of modeling efforts at different scales and with different degrees of empirical verisimilitude. Moving too soon to policy implementation on the basis of over-simplified models entails major risk of adverse unintended consequences.

Consider, for example, the *Policy Readiness Levels (PRLs)*[7] proposed in Table 1 for research directed towards the design of institutional and/or regulatory policies. Due to relatively limited data and computational capabilities, policy researchers at universities tend to work at PRLs 1-3. In contrast, policy researchers within industry, government, and regulatory agencies tend to work at PRLs 7-9.

The interim PRLs 4-6 thus constitute a "valley of death" that hinders the careful step-by-step development and testing of policy proposals from conceptual formulation all the way to real-world implementation. Fortunately, ACE modeling is well suited for bridging this valley because it facilitates the construction of computational platforms[8] permitting policy model development and testing at PRLs 4-6. Examples are discussed in Section 7.

All nine levels in the PRL taxonomy are essential for ensuring conceptual policy ideas are brought to real-world fruition. Explicit recognition and acceptance of this tiered model valuation could encourage policy researchers to become more supportive of each other's varied contributions.

Another important point is that the PRL taxonomy does not necessarily have to represent a one-way road map from initial concept to completed application. Rather, PRLs 1-9 could constitute a single concept-to-application iteration in an ongoing *Iterative Participatory Modeling (IPM)* process.

An IPM process is an open-ended collaborative learning process in which researchers join with stakeholders in a repeated looping through a multi-stage process involving conceptual formulation, model development, and real-world testing. The objective is to help stakeholders manage complex problems over time through a continuous learning process rather than to attempt the delivery of a definitive problem solution. Extensive annotated pointers to IPM studies can be found at (Tesfatsion, 2017d).

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[7]These PRLs mimic, in rough form, the *Technology Readiness Levels (TRLs)* devised by the U.S. Department of Energy (DOE, 2011, p. 22) to rank the readiness of proposed new technologies for commercial application.

[8]In the current study the term *computational platform* is used to refer to a software framework together with a library of software classes that permit the plug-and-play development and study of a family of computational models.

Table 1: Policy Readiness Level (PRL) Classifications for Policy Models

Development Level	**PRL**	**Description**
Conceptual policy idea	PRL 1	Conceptual formulation of a policy with desired attributes
Analytic formulation	PRL 2	Analytic characterization of a policy with desired attributes
Modeling with low empirical fidelity	PRL 3	Analysis of policy performance using a highly simplified model
Small-scale modeling with moderate empirical fidelity	PRL 4	Policy performance tests using a small-scale model embodying several salient real-world aspects
Small-scale modeling with high empirical fidelity	PRL 5	Policy performance tests using a small-scale model embodying many salient real-world aspects
Prototype small-scale modeling	PRL 6	Policy performance tests using a small-scale model reflecting expected field conditions apart from scale
Prototype large-scale modeling	PRL 7	Policy performance tests using a large-scale model reflecting expected field conditions
Field study	PRL 8	Performance tests of policy in expected final form under expected field conditions
Real-world implementation	PRL 9	Implementation of policy in final form under a full range of operating conditions

6. Towards Standardized Protocols for Policy Model Presentations

The classification of policy models in accordance with policy readiness levels (PRLs), as proposed in Section 5, could also help resolve another key issue facing ACE policy researchers. Specifically, how can ACE models and model findings undertaken for policy purposes be presented to stakeholders, regulators, and other interested parties in a careful, clear, and compelling manner (Wallace et al., 2015, Sections 3-4,6)?

Most ACE models are not simply the computational implementation of a model previously developed in equation form. Rather, ACE modeling often proceeds from agent taxonomy and flow diagrams, to pseudo code, and finally to software programs that can be compiled and run. In this case the software programs *are* the models. On the other hand, it follows from the modeling principles presented in Section 2 that ACE models are initial-value state space models. Consequently, in principle, the software program for any ACE model can equivalently be represented in abstract form as a system of discrete-time or discrete-event difference equations, starting from user-specified initial conditions.[9] These analytical representations become increasingly complex as the number of agents increases. An example of such a representation is given in Tesfatsion et al. (2017, Section 6.3).

The practical challenge facing ACE policy researchers then becomes how best to present approximations of their models to interested parties who are unable or unwilling to understand these models in coded or analytical form. Most ACE policy researchers resort to verbal descriptions, simple graphical depictions for model components and interactions, Unified Modeling Language (UML) diagrams,[10] and/or pseudo-code expressing the logical flow of agent interactions over time. Anyone wishing to replicate reported results is

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[9] As discussed in Section 2, the ability to represent an ACE model in abstract analytical form at an initial time t^o does not necessarily imply that the future *outcomes* of this model are pre-statable at t^o, even in principle.

[10] The *Unified Modeling Language (UML)* is a general-purpose modeling language intended to provide a standard way to design and visualize conceptual and software models. UML diagrams enable partial graphical representations of a model's structural (static) and behavioral (dynamic) aspects. UML has become increasingly complex in successive version releases and is not specifically tailored for dynamic systems driven by agent interactions. Perhaps for these reasons, UML as a general modeling tool has not been widely adopted by ACE/ABM researchers to date; see Collins et al. (2015) for further discussion of these points.

pointed to the original source code.

The lack of presentation protocols for ACE policy models (and for ACE models more generally) has been severely criticized by economists who directly specify their models in analytical or statistical terms using commonly accepted approaches. At the very least, it complicates efforts to communicate model features and findings with clarity, thus hindering the accumulation of knowledge across successive modeling efforts.

Fortunately, the development of presentation protocols for agent-based models is now an active area of research (Tesfatsion, 2016, 2017e). For example, the ODD (*O*verview, *D*esign concepts, and *D*etails) protocol developed by Grimm et al. (2006, 2010) has been widely adopted by ecologists who use agent-based models.

To date, however, proposed protocols such as ODD have attempted to provide "one size fits all" requirements for the presentation of models, regardless of purpose and development level. The insistence on a single set of presentation requirements can make it difficult to apply these protocols to particular modeling efforts.

For example, a serious effort is undertaken by Tesfatsion et al. (2017) to present an ACE watershed model in strict accordance with the ODD protocol. As explained in Grimm et al. (2010, Table 1), the "overview" part of the ODD protocol requires researchers to present a relatively detailed model summary that covers purpose, entities, state variables, scales, processes, and process scheduling. The "design concepts" part of the ODD protocol requires researchers to discuss the basic principles underlying their model designs and the extent to which these designs embody ten specific design principles (e.g., emergence, adaptation, ...). The "details" part of the ODD protocol requires researchers to provide complete technically accurate descriptions of their models, including initialization, input data, and submodels (processes).

While the effort by Tesfatsion et al. (2017) to abide by the ODD protocol did significantly improve the clarity of their model presentation, the following serious problems were encountered:

- *Too many reporting requirements for journal outlets:* The three ODD protocol requirements could not be met within the normal page limits for *Environmental Modelling & Software*. Fortunately the editor permitted publication even though the length of the published paper exceeded normal page limits.

- *Breakage of encapsulation:* The ODD protocol defines agent states solely in terms of agent attributes and requires separate reporting of agent attributes from agent data and agent methods. In contrast, an important property of ACE models is agent encapsulation, i.e., the modeling of an agent as a bundle of data, attributes, and methods that can be partially or completely hidden from other agents. Although the authors fulfilled this ODD protocol requirement, they also presented figures and text descriptions that expressed and explained agents as encapsulated entities.

- *Presentation requirements not suitable for general readers:* The "Details" part of the ODD protocol requires submodels (processes) to be presented in their entirety, using actual code names for parameters, variables, and functional forms. The authors deviated from this requirement to preserve general readability. The processes for the ACE watershed model are carefully expressed in analytical form using standard mathematical notation for functions and variables. Readers desiring to replicate and extend results reported for test-case[11] simulations of the ACE watershed model are directed to a website repository providing complete source code and test-case initialization data.

Given these difficulties with the "one size fits all" ODD protocol, a better way to proceed would seem to be the development of multiple standardized presentation protocols tailored to the purpose and development level of a modeling effort. For example, for policy models, use could be made of the PRL taxonomy presented in Table1.

Specifically, a protocol for PRL 1-3 models could require a complete model presentation within the confines of a typical journal article. In contrast, a protocol for PRL 4-7 models could consist of two sets of presentation requirements: one set for a summary model presentation to be reported within the confines of a typical journal article; and a second set for a complete model presentation (source code plus documentation) to be reported at a supplementary website repository. Another important need for PRL 4-7 models is the development of presentation protocols for test-case simulation studies, a need not specifically addressed by the ODD protocol.

[11] A *test case* for an ACE model is a particular numerical specification for initial agent states that permits the model to be simulated on a computer.

7. Edgier Explorations

In the early years of agent-based computational modeling, researchers created and explored agent-based models populated with agents whose features far exceeded traditional economic specifications. Agents could communicate with each other using adaptively scripted messages, endogenously form their interaction networks based on choice and refusal of interaction partners, move around in spatially configured landscapes, use vision to locate resources, and even reproduce in a manner mimicking genetic procreation. See, for example, Epstein and Axtell (1996), Belew and Mitchell (1996), Arthur et al. (1997), and the Schelling-Sakoda checkerboard-model research reviewed by Hegselmann (2017).

In the process of tailoring studies for publication in economic journals, these features have been trimmed or eliminated to the point that computational agents simply appear to be boundedly rational variants of traditional economic agents. Readers could rightly wonder: Where is the revolution?

This section highlights a number of ways in which ACE models incorporating non-traditional specifications for computational agents could both advance traditional economic research goals and expand research horizons.

7.1. ACE Modeling of Labor Markets as Evolutionary Sequential Games

The rightly celebrated work by Pissarides (2000) exploits data on job and worker flows at a micro level in order to provide a better understanding of critical labor market issues, such as endogenous job creation and destruction. This work relies on the use of an exogenously specified aggregate job-matching function to approximate the process by which workers match with employers in actual labor markets.

A number of ACE researchers have extended this work by permitting workers and employers to match through an endogenous process. For example, in (Tesfatsion, 2001) a group of workers and employers participate in a sequential labor market game with adaptive job search and incomplete contracts, implemented in C++ via the open-source Trade Network Game Laboratory (Tesfatsion, 2017f). The Pissarides aggregate job-matching function is replaced with an endogenous preferential worker-employer matching process based on the Gale-Shapley deferred acceptance algorithm (Roth, 2008).

More precisely, as depicted in Fig. 3, at the beginning of each work period the workers communicate work offers to their most preferred employers

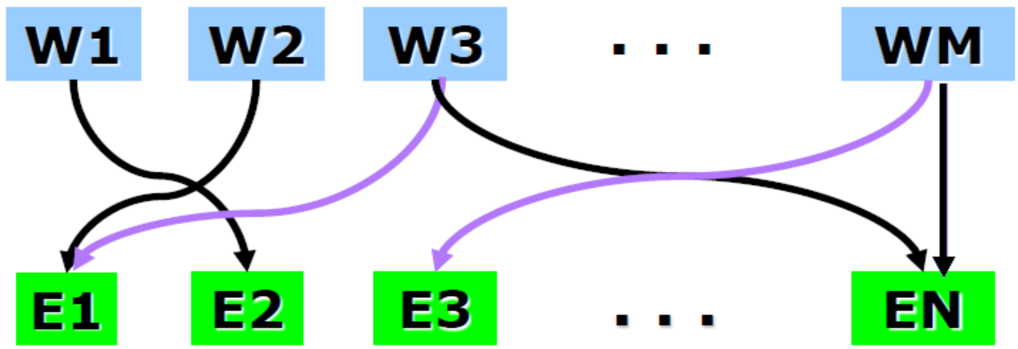

Figure 3: An ACE sequential labor market game. Hiring, firing, and quits are endogenously determined by the successive strategic decisions of workers (W) and employers (E) based on past work-site interactions. Dark (black) arrows indicate accepted work offers, and lighter (purple) arrows indicate refused work offers. Source: Tesfatsion (2001).

deemed acceptable on the basis of past work-site interactions (or prior judgments). Employers immediately refuse work offers from applicants deemed unacceptable on the basis of past work-site interactions (or prior judgments) and place all remaining work offers in their applicant pools. Refused workers either redirect their work offers to acceptable next-most-preferred employers (if any such employers exist) or become unemployed. Once all employer applicant pools have stabilized, each employer accepts work offers from among the most preferred workers in his applicant pool (up to his hiring capacity) and refuses the rest. This determines worker-employer matches, vacancies, and unemployment for the current work period.

Matched workers and employers then participate in work-site games. Workers and employers use the outcomes of their work-site game interactions to update: (i) their preference orderings over potential work-site partners, hence the manner in which they make, choose, and/or refuse work offers in the next work period; and (ii) their game strategies for work-site interactions in the next work period.

In subsequent mentions of this work in the economics literature, the following non-traditional aspects have typically not been noted. Workers and employers engage in direct adaptive communication to implement an endogenous matching process. Workers and employers use genetic algorithms to evolve "new ideas" regarding appropriate game strategies for use in work-site interactions. Matching theory is blended with game theory in order to permit the *endogenous* choice and refusal of *game* partners.

Various other ACE modelers have exploited agent capabilities to open up new labor market research directions as well. See Tesfatsion (2017g) for annotated pointers to this research.

7.2. ACE Macroeconomic Modeling with Anticipatory Learning

A growing number of researchers have become interested in the study of macroeconomic systems for which agents are forward-looking optimizers with incomplete knowledge about the structure of the economy. As surveyed in Evans and Honkapohja (2013), the standard context assumed in this literature is that a representative consumer with learning capabilities resides in a dynamic world consisting of itself, a representative firm, and a government policy-maker. The representative consumer has incomplete information about the structure of its world, and it behaves as an econometrician in its attempts to learn about its world from observed data.

Specifically, the representative consumer makes consumption and labor decisions in each successive time period conditional on intertemporal budget constraints. These budget constraints depend on current state variables (e.g., financial and physical asset values), on current and forecasted future values for system variables (e.g., goods prices, wages, and interest rates), and on current and forecasted future values for government policy variables (e.g., tax rates). The consumer's system variable forecasts are obtained from a reduced-form econometric model. The consumer estimates and updates the parameters of this econometric model over time, often by means of a least-squares or Bayesian learning method. The consumer's government policy variable forecasts are generated by means of the latest announced government policy rule, assumed to be credible common knowledge.

Functional forms and calibrated maintained parameter values are specified in the initial time period to guarantee the existence of a steady-state solution, assumed to be common knowledge. A temporary equilibrium solution for the macroeconomic model is then approximately determined in differenced form (i.e., differenced from steady-state values) in each successive time period. A key concern is to analyze how different learning specifications affect the properties of these temporary equilibria, e.g., will they converge over time to the steady-state solution (in either a global or local stability sense), or will they persistently deviate from this solution.

Clearly this literature takes an important step towards more realistic macroeconomic modeling by recognizing the constrained information and computational capabilities of decision-making agents. Nevertheless, locally-constructive decision-making is still not ensured; external coordination, equilibrium, and optimality conditions are imposed on agents both intertemporally and cross-sectionally in order to obtain model solutions. Examples of such conditions include: single representative consumer assumptions; the guaranteed existence of steady-state solutions; non-constructive common knowledge assumptions; the assumed coordination of agents on a single temporary equilibrium solution in each time period; the assumed satisfaction of non-constructive transversality conditions for consumer optimization; the assumed absence of interest-rate arbitrage opportunities; the assumed absence of ponzi-game opportunities such as persistent debt roll-over; and the assumed absence of excess supplies and demands in markets.

This failure to ensure local constructivity has important consequences. Real-world decision makers are forced to be locally constructive, which places limits on their expressed behaviors. Idealized behavioral benchmarks are useful for the specification of performance metrics, providing upper limits for such metrics; but an insistence on idealized behavioral specifications for all modeled economic decision-makers hinders understanding of real-world economic systems. For scientific modeling purposes, the relevant set of decision-making processes lies within the set of locally-constructive decision-making processes; and "rationality" should properly be defined relative to this set.

ACE macroeconomic modeling permits the systematic study of locally-constructive decision processes in macroeconomic contexts. Extensive annotated pointers to this work can be accessed at (Tesfatsion, 2017h).

To date, however, most ACE macroeconomic researchers have postulated decision rules for decision-making agents that are not explicitly derived as solutions for optimization problems, although they are sometimes motivated as heuristic approximations for such solutions. This has led some macroeconomists to dismiss ACE macroeconomic modeling based on the incorrect belief that ACE decision-making agents must necessarily be reactive stimulus-response agents with myopic objectives. To the contrary, however, as concretely demonstrated by Sinitskaya and Tesfatsion (2015), the locally-constructive decision processes used by ACE decision-making agents can range from simple fixed rules to intertemporal optimization based on sophisticated anticipatory learning algorithms.

Specifically, Sinitskaya and Tesfatsion (2015) take a standard macroeconomic model in which consumers and firms have intertemporal utility-maximizing and profit-maximizing objectives and introduce four important

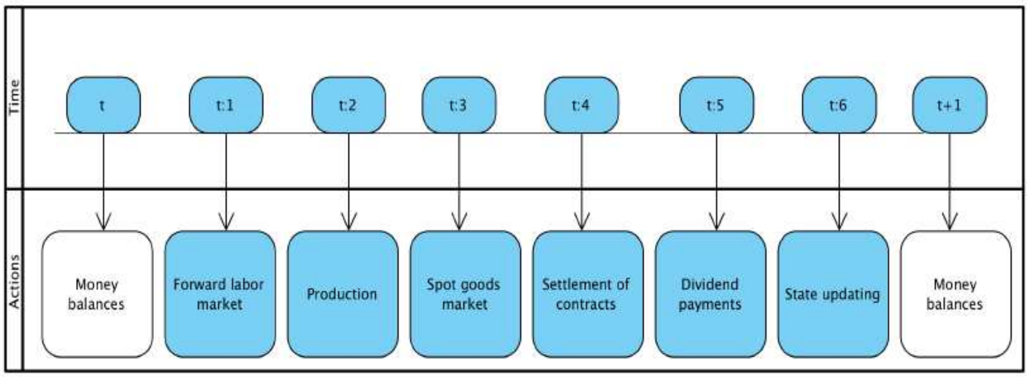

Figure 4: Locally-constructive trading sequence in each period t for consumers and firms in an ACE macroeconomic model who have learning capabilities and seek to maximize intertemporal utility and profits. Source: Sinitskaya and Tesfatsion (2015).

changes. First, all externally imposed market clearing and rational expectations assumptions are removed. Second, consumers and firms are modeled as locally-constructive agents with learning methods. Third, as depicted in Fig. 4, consumers and firms attempt to satisfy their objectives through participation in an open-ended sequence of locally-constructive market activities for which every purchase must be backed by actual purchasing power (money balances). Fourth, firms that become insolvent must exit the economy.

Four learning methods are tested for consumers and firms in the resulting *Dynamic Macroeconomic (DM) Game*: (i) A simple reactive reinforcement learning algorithm due to Roth and Erev (1995) that asks "if this happens, what should I do?"; (ii) Q-learning (Watkins, 1989), a well-known anticipatory learning algorithm that asks "if I do this, what will happen?"; (iii) an anticipatory rolling-horizon learning method; and (iv) an anticipatory stochastic dynamic programming learning method involving the adaptive updating of value functions approximated by basis functions.[12]

Welfare outcomes for consumers and firms are generated under 44 different learning-method combinations. The best combination turns out to be when all consumers and firms engage in rolling-horizon learning; the welfare outcomes for this combination are shown to constitute a Pareto-optimal Nash

[12]Source code (C++) and initialization data for this DM Game sensitivity study can be accessed at (Sinitskaya, 2015).

equilibrium relative to the set of 44 tested learning combinations.

7.3. ACE Modeling of Coupled Natural and Human Systems

ACE modeling enables researchers to study economic processes as critical components of *coupled natural and human (CNH)* systems. This permits consideration of a broader range of potential causal factors for economic outcomes of interest. Extensive annotated pointers to ACE CNH research can be accessed at (Tesfatsion, 2017i).

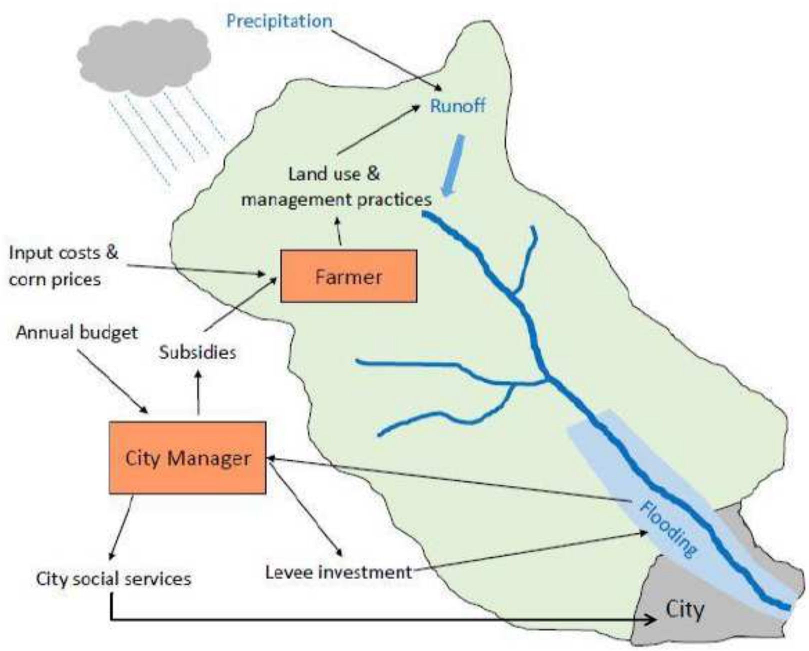

Figure 5: An ACE watershed test system, implemented by means of the WACCShed Platform, that focuses on local governance issues for the Squaw Creek watershed in central Iowa. Source: Tesfatsion et al. (2017).

For example, Tesfatsion et al. (2017) develop an open-source ACE computational platform in Java for the study of watersheds under evolving climate conditions, referred to as the *W*ater *A*nd *C*limate *C*hange Water*shed (WACCShed)* Platform. This platform permits a careful modeling of the natural and institutional environment that shapes and channels the actions of human watershed participants. In turn, as advocated by An (2012), the platform permits a watershed environment to be affected by the actions of its human participants.

An ACE watershed test system implemented by means of the WACCShed Platform is presented in detail in order to demonstrate, in concrete terms, the capabilities and use of the WACCShed Platform. As depicted in Fig. 2 and Fig. 5, this test system captures, in highly simplified form, the structural attributes of the Squaw Creek watershed in central Iowa.

The test system restricts attention to two types of decision makers, a representative farmer and a city manager, in order to identify with care the manner in which their strategic interactions and risk-management practices result in an intrinsic dynamic coupling of natural and human systems over time. Illustrative findings are reported showing the sensitivity of farmer and city social welfare outcomes to changes in three key treatment factors: farmer land-allocation decision method; farmer targeted savings level; and levee quality effectiveness for the mitigation of city flood damage.

Source code and initialization data for the ACE watershed test system can be accessed at the WACCShed homepage (Jie et al., 2016).

7.4. ACE Modeling of Critical Infrastructure Systems

As stressed in Section 5, ACE computational platforms are increasingly being used to study proposed policies in advance of implementation. This section illustrates this use for a complex critical infrastructure system: namely, U.S. centrally-managed wholesale electric power markets.

The AMES (*A*gent-based *M*odeling of *E*lectricity *S*ystems) Test Bed is an open-source ACE computational platform permitting the open-ended dynamic study of wholesale electric power markets operating over transmission grids subject to grid congestion (Tesfatsion, 2017j). AMES incorporates, in simplified form, the core features of the two-settlement system design proposed by the U.S. Federal Energy Regulatory Commission for U.S. wholesale electric power markets. To date, this design has been adopted in seven U.S. energy regions (CAISO, ERCOT, ISO-NE, MISO, NYISO, PJM, SPP) encompassing over 60% of U.S. generation capacity.

The latest released version of AMES (V4.0) is a modular extensible platform developed in Java and Python. As depicted in Fig. 6, the key features of AMES (V4.0) are as follows:

1. **Market Participants:** These include: *Load Serving Entities (LSEs)* that demand wholesale electric power in order to service the *loads (power demands)* of their retail customers; and *generators* that produce and supply electric power from both dispatchable and non-dispatchable

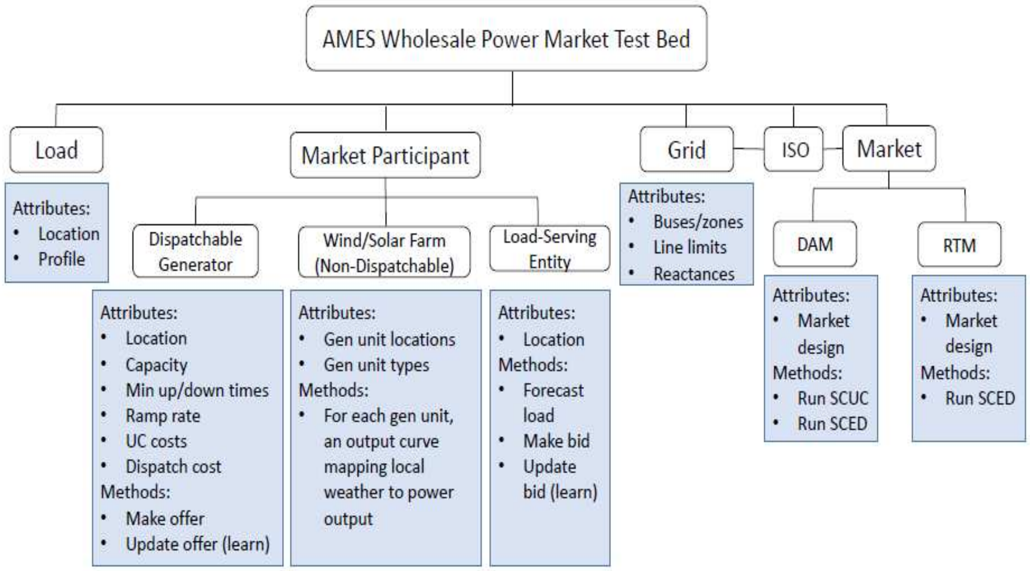

Figure 6: An ACE computational platform permitting study of U.S. ISO-managed wholesale electric power markets. Source: Tesfatsion (2017j).

resources. Each participant is modeled as a private business entity whose goal is to secure the highest possible net earnings from its market activities over time. At the beginning of each simulation run, the user-specified methods of the LSEs and dispatchable generators include demand bid and supply offer functions, and they can also include learning algorithms permitting the endogenous updating of these functions over time.

2. **Central Management:** Grid and market operations are centrally managed by a non-profit *Independent System Operator (ISO)* whose goal is to maintain the reliable, efficient, and fair operation of the wholesale electric power system over time.
3. **Two-Settlement System:** On each successive day the ISO conducts a bid/offer-based *Day-Ahead Market (DAM)* to determine hourly resource commitments and dispatch levels for next-day operations as well as a *Real-Time Market (RTM)* to correct for any imbalances between day-ahead dispatch schedules and real-time power needs. Each market is separately settled by *Locational Marginal Pricing (LMP),* i.e., the pricing of power by the timing and location of its withdrawal from, or injection into, the transmission grid.

4. **AC Transmission Grid.** The LSEs and generators are located at user-specified locations across a user-specified *alternating current (AC)* transmission grid. Grid congestion is managed via LMP.

The AMES Test Bed has been used to study a number of policy issues, including: stochastic versus deterministic market-clearing methods for day-ahead markets; effects of increasing wind power penetration and price-responsive demands on the efficiency, reliability, and fairness of system operations; and the effects of locational marginal pricing on the amounts of net surplus extracted from market operations by LSEs, generators, and the ISO. Software downloads, manuals, tutorials, and publication links can be accessed at the AMES homepage (Tesfatsion, 2017j).

AMES is a key component of an integrated transmission and distribution system platform currently under development. As discussed in Section 7.5, the purpose of this platform is to facilitate the study of new types of "transactive energy system" designs for electric power systems that are based more fully on economic transactions.

7.5. ACE Modeling Principles as Real-World Design Principles

As stressed by Borrill and Tesfatsion (2011, Section 2.1), real-world systems whose architectures are designed in accordance with agent-based modeling principles can, in turn, be simulated by agent-based models that embody their basic architecture and constituent agent types. This duality provides unprecedented opportunities for deep empirical validation.

In recent years, agent-based modeling principles are increasingly being used to design "bottom up" *Transactive Energy System (TES)* architectures for electric power systems (GAC, 2015; Tesfatsion, 2017k). TES architectures are decentralized market-based mechanisms that permit electric power systems to operate more fully in accordance with core economic principles.

More precisely, a TES architecture is a set of economic and control mechanisms that permits the balancing of power supplies and demands across an entire electric power system, consistent with system reliability. The intent is to enhance value for all participants subject to physical and security constraints. A key characteristic of TES architectures is a stress on decentralization. Information technology is viewed as the nervous system that will permit management of an electric power system to be shared by a society of distributed resource owners, customers, and other stakeholders. The ultimate TES objective is to achieve "a loosely coupled set of controls with just enough information exchange to allow for stability and global optimization through local action" (GAC, 2015, p. 10).

For example, TES researchers such as Kok (2013) are investigating the possibility of introducing various forms of aggregators able to harness ancillary services from collections of *distributed energy resources (DERs)* owned by businesses and households connected to distribution grids. Examples of DERs include distributed generation (e.g., rooftop photovoltaic panels), battery storage systems, and household appliances with energy storage capabilities and flexible power needs that permit collections of these appliances to function as *prosumers*, i.e., as entities that can produce or consume power depending on local conditions.

Harnessing of services from DERs for real-time operations requires voluntary participation by DER owners, hence it requires the creation of value streams that can be used to compensate DER owners appropriately for provided DER services. In addition, it requires technological developments such as local device management by intelligent software agents to ensure DER response to real-time electronic signals in a timely and accurate manner.

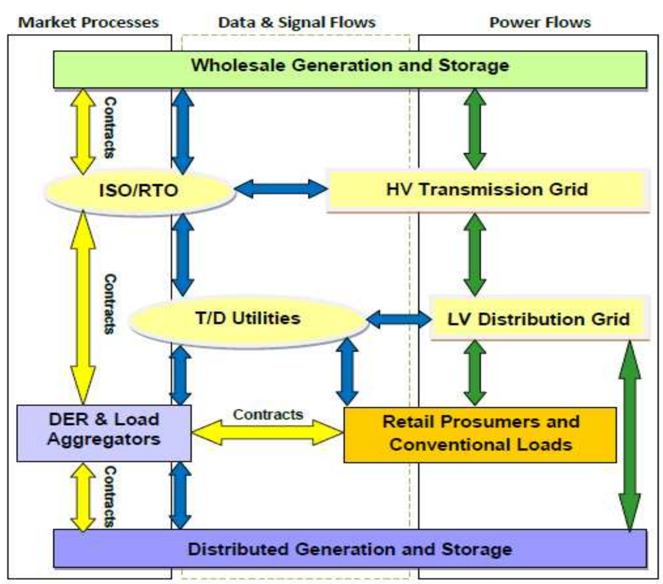

Figure 7: An ACE computational platform permitting the study of transactive energy system designs for U.S. electric power systems. Source: Tesfatsion (2017k)

Figure 7 depicts an open-source ACE computational platform currently under development by research teams from Iowa State University and Pacific

Northwest National Laboratory. The purpose of this platform is to facilitate the performance study of integrated transmission and distribution (T/D) systems operating under TES designs.

Key aspects of this platform are as follows. An ISO-managed wholesale electric power market operating over a high-voltage (HV) transmission grid is tightly linked through DER/load aggregators to a distribution system operating over a lower-voltage (LV) distribution grid. The participants in the distribution system include DERs (distributed generation, storage, prosumers), locally managed by intelligent price-responsive software agents, as well as conventional (non-price-responsive) loads. An important characteristic of such a system is that data, signals, and power can flow up as well as down between the wholesale/transmission and retail/distribution sectors.

8. A Spectrum of Experiment-Based Models

As depicted in Fig. 8, it is now feasible to develop experiment-based models that permit subjects to range all the way from 100% real people to 100% computer agents. The key question is whether it is worthwhile.

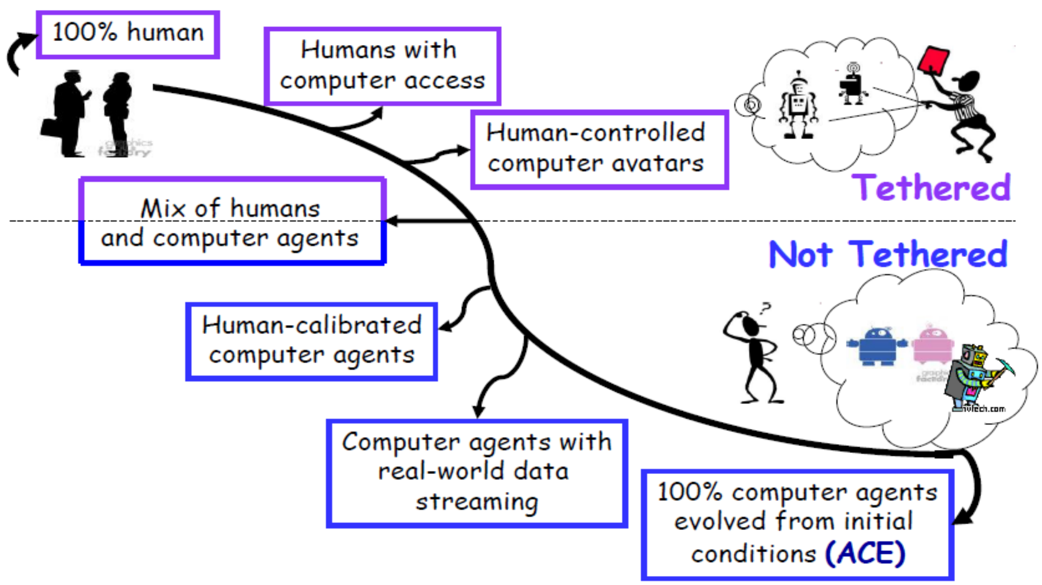

Figure 8: Spectrum of experiment-based models with subjects ranging from 100% human to 100% computer agents.

The arguments in favor are many. Experiments based entirely on human subjects must typically be kept short, with relatively simple experimental settings, to avoid excessive subject compensation costs and subject misunderstanding of instructions. In contrast, experiments based solely on computer agents can generate simulated outcomes for complex problem environments over long time horizons at relatively low expense. Nevertheless, if human behaviors are misspecified, these simulated outcomes can be seriously misleading.

Mixed experiment-based models that combine human subjects and computer agents permit experimenters to postulate more realistic experimental settings for their human subjects by letting computer agents represent critical but complicated real-world aspects. They also permit the systematic study of human behaviors within controlled group settings, from small to large, by letting computer agents represent "others" in these groups. In addition, they provide a way for computer agents to be trained *in situ* to embody human decision-making behaviors before they are used to represent human decision-making behaviors in longer-run studies. Annotated links to economic research using mixed experiment-based models can be found at (Tesfatsion, 2017l).

An interesting related topic is *serious gaming*, i.e., the development of game environments that are designed for teaching, training, and research purposes and not simply for entertainment. While by no means a new topic, serious gaming has attracted increased attention since 2002 due to both economic and technological factors (Djaouti et al., 2011). Current serious game releases typically take the form of computerized games that involve mixed play among human and computer-implemented participants.

9. Concluding Remarks

This study starts with the claim that real-world economic systems are locally-constructive sequential games. It then strives to demonstrate, through general analysis and concrete examples, that ACE modeling provides a practical useful way for economists to represent and study real-world economic systems as locally-constructive sequential games.

The ultimate goal of this study is the routine inclusion of ACE modeling within the toolkits of economists as a useful *complement* to current modeling approaches. Expansions of toolkits with new modeling tools providing additional research capabilities should be embraced, not fought, as Pareto-improving evolutions of economic methodology.

Acknowledgements

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