Validation and Calibration in ACE Models: An Investigation on the CATS model.

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Abstract

In this paper we deal with some validation (and a first calibration) experiments on the CATS model proposed in Gallegati et al. (2003a, 2004b).

The CATS model has been intensively used (see, for example, Delli Gatti et al., 2004; Russo, 2004; Gallegati et al., 2003b) to replicate a large number of scaling type stylized facts with a remarkable degree of precision and, for these purposes, the simulation of the model has been performed entering ad hoc parameter values and using the same initial set up for all the agents involved in the experiments.

Nowadays alternative robust and reliable validation techniques for determining whether the simulation model is an acceptable representation of the real system are available (Sargent, 1996; Kleijnen, 1999); furthermore many distributional and goodness-of-fit tests have been developed (see, for example, Prabhakar, 2003; Kleiber and Kotz, 2003) while several graphical tools have been proposed to give the researcher a quick comprehension of actual and simulated data (Embrechts, 1997).

This paper discusses some validation experiments performed with the CATS model. In particular starting from a sample of Italian firms included in the AIDA database, we perform several ex-post validation experiments over the simulation period 1996-2001. In the experiments, the model parameters have been estimated using actual data and the initial set up consists of a sample of agents in 1996. The CATS model is then simulated over the period 1996-2001. Using alternative validation techniques, the simulations' results are ex-post validated respect to the actual data. The results we achieve seem to be quite promising.

JEL classification: C15, C16, D31, E37, L11, L16, O31.

1. Introduction

Mainstream economics adopts the classical mechanics approach of 19th century physics, based upon the reductionist principle, according to which one can understand the aggregate, simply analysing its single elements. The microfoundation of macroeconomics in the (New) Classical tradition is based on the hope that the aggregate behaviour is the magnification of the single agent's behaviour on a larger scale. The application of the reductionist framework implies that the so-called overlapping principle holds true, i.e. the dynamics of a (linear) model can be decomposed into its constituent parts through the representative agent (RA) framework.

The microeconomic foundations of general equilibrium models must be based, according to mainstream economics, on an optimizing RA, fully rational and omniscient. Unfortunately, "there are no assumptions on [...] isolated individuals which will give us the properties of aggregate behavior which we need to obtain uniqueness and stability. Thus we are reduced to making assumptions at the aggregate level which cannot be justified by the usual individualistic assumptions. This problem is usually avoided in the macroeconomic literature by assuming that the economy behaves like an individual. Such an assumption cannot be justified in the context of the standard economic model and the way to solve the problem may involve rethinking the very basis on which this model is founded." (Hildenbrand and Kirman, 1988, p. 239).

The quantum revolution of the last century radically changed the perspective in contemporary physics. According to the holistic approach, the aggregate is different from the sum of its components because of the interaction of particles. In the social sciences, a step in this direction is taken by the agent-based modeling (ABM) strategy.

Agent-based models, which are increasingly applied in economics (Tesfatsion, 2002; Axelrod, 1997), have been developed to study the interaction of many heterogeneous agents. In a sense they are based on new microfoundations, according to a bottom-up approach. They have a holistic methodology as opposed to the reductionist approach of the mainstream economics. One builds a model starting from simple behavioral rules at the single agent level. Through interactions some aggregate statistical regularities emerge and they can not be inferred from the individual level. This emergent behaviour often feeds back to individual agents making their rules change (they may evolve in an adaptive way). According to this approach, macroeconomics is not a set of equations that occurs by summation and averaging of the individual decisions, but it is a SOC (Self-Organized Critical) phenomenon that rises from the micro-level.

As already mentioned, ABM and simulations have been extensively used in many scientific fields, including economics, in the last decade (Axelrod, 1997;

Axtell, 2000). However, only in recent years only, researchers have started being concerned with whether a model and its results may be considered correct. As Sargent (1998) puts it: "This concern is addressed through model verification and validation. Model validation is usually defined to mean substantiation that a computerized model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model". This is not at all a secondary problem, in fact, only a correct model is a good model.

In this paper we deal with some validation experiments of the CATS model proposed in Gallegati et al. (2003a, 2004b).

The CATS model has been intensively used (see, for example, Gallegati et al., 2003b, 2004a, 2005; Delli Gatti, 2004) to replicate a large number of scaling type stylized facts with a remarkable degree of precision and, for these purposes, the simulation of the model has been performed entering ad hoc parameters' values and using the same initial set up for all the agents involved in the experiments. It must be recalled that the above mentioned analyses have been performed following Kaldor's suggestion: "construct a hypothesis that could account for these stylized facts, without necessarily committing himself on the historical accuracy" (Kaldor, 1965, page 178).

In this paper our intentions are a little bit more ambitious: using an initial set up of actual data (a sample of Italian firms in 1996) we aim to verify if the CATS model, simulated over a period for which actual data are available (the interval 1996-2001), is an acceptable representation of the real system. In other words we intend to perform an ex-post validation of the model.

Alternative robust and reliable statistical techniques are currently available for validating simulation models (for a survey see, for example, Sargent 1996 and Kleijnen 1999) and, in our analyses, we use some of the distributional and goodness-of-fit tests discussed in Prabhakar et al. (2003) and Kleiber and Kotz (2003) and the graphical tools (Embrechts, 1997) proposed to give the researcher a quick comprehension of actual and simulated data.

In the validation exercise, over the simulation period 1996-2001, we use a sample of 6422 Italian firms included in the AIDA database. The model parameters have been estimated using actual data and the initial set up consists of the sample data of the year 1996. The CATS model is then simulated over the period 1996-2001 and the simulations' results are ex-post validated with respect to actual data.

We then propose a first simple calibration experiment using a grid method, in order to ameliorate the fitting of data.

Anticipating some conclusions, we may say that the model reproduces, in a short (medium) term horizon, a good percentage (81% in 2001) of the output actual data. The two samples (simulated and observed data) belong to the same distribution with a confidence interval of 95%. Moreover the model also reproduces the firms' growth dynamics at a micro level, while less satisfying is the simulation for the behaviour of the very small and very large firms.

The papers is organized as follows: Section 2 presents the model we have studied and validated; Section 3 describes the database we used and the empirical evidence we aim to investigate; Section 4 shows the proceeding of the validation procedure; Section 5 introduces a first calibration experiment; while Section 6 concludes.

2. The CATS model

Consider a sequential economy[1], with time running $t = 1, 2, ...$, populated by many firms and banks. Two markets are opened in each period: the market for a homogeneous produced good and the market for credit. As in the levered aggregate supply class of models first developed in Greenwald and Stiglitz (1990.1993), our model is fully supply-determined, in the sense that firms can sell all the output they optimally decide to produce. Due to informational imperfections on the equity market, firms can raise funds only on the credit market. The demand for credit is related to investment expenditure and it is fully accomplished at the fixed banks' interest rates: i.e. total credit supply always equals the demand for it.

At any time $t = 1, .6$, the economy consists of N_t firms, each located on an island[2]. Every firm $i \in N_t$ produces the output Y according to a linear production function, in which capital (K_{it}) is the only input[3]:

$$Y_{it} = \phi_{it} K_{it}. \tag{1}$$

For each firm i the productivity ϕ_{it} in $t = 1$ corresponds to its actual productivity (estimated on the AIDA data in 1996) and it evolves to

$$\phi_{it-1} + \varrho_{it} \sqrt{\phi_{it-1}}, \quad where\ \varrho_{it} = \frac{M}{2}, \tag{2}$$

with $M \sim U(0, 2)$, if the firm is small[4], and to

$$\phi_{it} = \phi_{i1}, \tag{3}$$

if large[5].

The demand for goods in each island is affected by an *iid* idiosyncratic real shock. Since arbitrage opportunities across islands are imperfect, the individual selling price in the i-th island is the random outcome of a market process around the average market price P_t of the output, according to the law $P_{it} = u_{it} P_t$, where $E(u_{it}) = \mu$ and $\sigma^2_{u_{it}} < +\infty$. Actual data suggest to split the price generator process into two different processes, depending once again on firms' size. For the sake of simplicity we assume that u_{it} follows two different uniform distributions:

small firms get a high average price and a stronger volatility[6], while big firms face more concentrated prices with a lower mean.

Summarizing, if U_1 is the distribution of u_{it} if i is small and U_2 if i is large, we have that $\mu^{U_1} > \mu^{U_2}$ and $\sigma^2_{U_1} > \sigma^2_{U_2}$.

Since, by assumption, credit is the only external source of finance for firms, the firm can finance its capital expenditure by recurring to net worth (A_{it}) or bank loans (L_{it}), i.e. $K_{it} = A_{it} + L_{it}$. At the exogenous real interest rate $\overline{r}$, at each time t debt commitments for every firm are equal to $\overline{r}L_{it}$. Since, for the sake of simplicity, there are no dividends distributed to equities, financing costs equal debt commitments. Therefore, profit/loss (π_{it}) in real terms is:

$$\pi_{it} = u_{it}Y_{it} - \overline{r}L_{it} \tag{4}$$

In our model a firm goes bankrupt if its net worth become negative, that is to say $A_{it} < 0$. The law of motion of A_{it} is, for hypothesis,

$$A_{it} = A_{it-1} + \pi_{it}. \tag{5}$$

As in Greenwald and Stiglitz (1993), we assume that the probability of bankruptcy ($\Pr^b$) is directly incorporated into the firm's profit/loss function: bankruptcy is costly and increasing with the firm's size:

$$C^b = cY^2_{it} \quad c > 0 \tag{6}$$

Every firm, by maximizing its objective function, determine its optimal capital stock K^*_{it}:

$$\max_{K_{it}} \Gamma_{it} = E(\pi_{it}) - E(C^b). \tag{7}$$

and the demand for credit.

3. The Database and the Empirical Evidence

All our validation experiments, together with the subsequent empirical analysis, are based on firm-level observations from the AIDA database, for the period 1996-2001, AIDA, formerly developed by the Italian Chambers of Commerce, is now a subset of AMADEUS, a comprehensive pan-European database elaborated by Bureau Van Dijk[7].

Thanks to several queries on the database, we have collected a sample of 6422 Italian non-financial firms, all satisfying the following: (i) no missing data in each year; (ii) reliable data for capital, employees and costs. For each firm and year, we have data on equities, long term debts and loans, short term debts, total capital, gearing ratio, solvency ratio, debt ratio, number of employees, cost of employees and revenues.

Recent explorations (Gallegati et al., 2005) in industrial dynamics have detected three empirical regularities, which are so widespread across countries and persistent over time to be characterized as universal laws:

1. The distribution of firms' size is right skewed and can be described by a Zipf or power law probability density function (Gallegati et al., 2003b; Gaffeo et al., 2003; Axtell, 2001; Ramsden, Kiss-Haypal, 2000; Okuyama et al., 1999; Quandt, 1966a-b; Simon, 1955);
2. Firms' growth rates are Laplace distributed, belonging to the Subbotin's Family (Stanley et al, 1996; Bottazzi, Secchi, 2003);
3. There is a power law relation between the variance of the size growth rates and the size itself (Stanley et al., 1996; Gabaix, 2004).

Gallegati et al.(2004b) have analytically shown that 1-3 determine several industrial, financial and business cycle facts (see those papers for a review of the empirical literature.) A model should therefore be able to replicate the empirical evidence 1-3, and our validation exercise is centered according to it.

The following section will present the validation exercise, i.e. if the above presented CATS model successfully deals with the evidence 1-3.

4. Simulation and Results

Our validation exercise is run with a sample of 6422 firms over the period 1996-2001,

The validation procedure we have used is standard and very similar to the methodology presented in Embrechts (1997)[8]. Appendix A contains a quite detailed description of this procedure.

In $t = 1$, every firm is initialized with its actual data from 1996: net worth, loans, productivity and so on. The market interest rate is exogenous and equal for all the firms.[9]

In each period actual data from the AIDA database are compared with the simulated data produced by the model. In particular our analysis can be divided into two different approaches: a pointwise analysis, meant to evaluate the evolution of the single firm, in order to study the predictive power of the model; and a distributional analysis, whose aim is to look for regularities.

Our experiments can be considered a first ex-post validation of the CATS model, that's to say a first step, necessary to develop all the subsequent analysis.

As far as the aggregate output is concerned, the model underestimates it slightly (average aggregate actual output over six years on log scale: 10.3486; average aggregate simulated output over six years on log scale: 10.1132), while the output volatility is almost identical ($\simeq 1.205$ vs. $\simeq 1.207$).

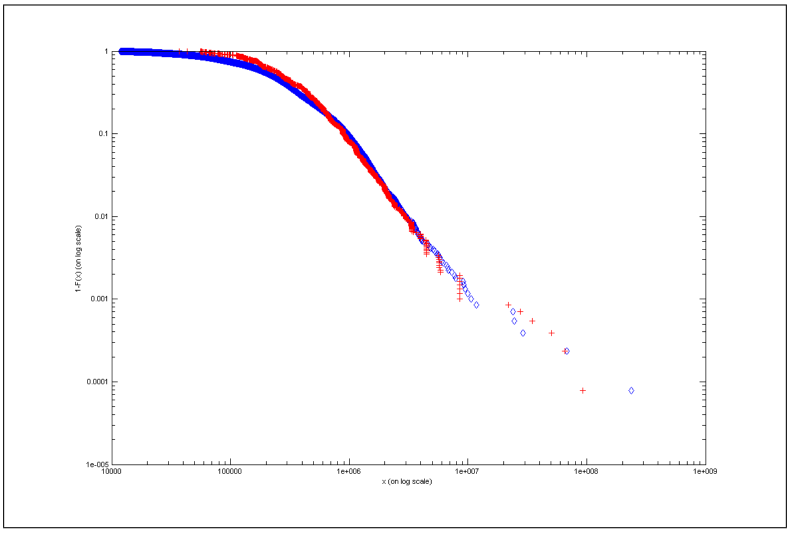

Figure 1: Zipf's Plot of the total capital distributions: observed (plus) and simulated (diamonds).

Let's now consider the total capital dynamics. Accepting a maximum deviation of ±20% between observed and simulated data in 2001 (that's a composite yearly deviation rate of 3.5%), we succeed in reproducing 5201 firms over 6422 (81%). As Figure 1 shows, the tails of the firms' size distribution is not adequately fitted. Similar results can be found in the previous years (in 1997, for example, the percentage of fitted firms is 74%, while in 1999 it's 77%) and analyzing the pooled distributions (78%).[10]

Figure 1 also shows that both observed and simulated capital distributions are particularly skewed, with fat right tails (decreasing linear relationship in the plot). This reproduces a widely accepted result (Zipf, 1932), according to which firms' size is power law distributed[11] (Axtell, 2001; Gaffeo et al., 2003; Gabaix, 2004).

We have performed many graphical and analytical tests to discover if our two samples (observed and simulated data) may be considered belonging to the same distribution.

A first Quantile-Quantile plot (Figure 2) seems to support the idea of a unique distribution for both samples, since there exists a clear linear relationship between observed and simulated data.

Another graphical test, the Box Plot (Figure 3), shows that the two distributions we are considering have many similarities. For example, the median (red

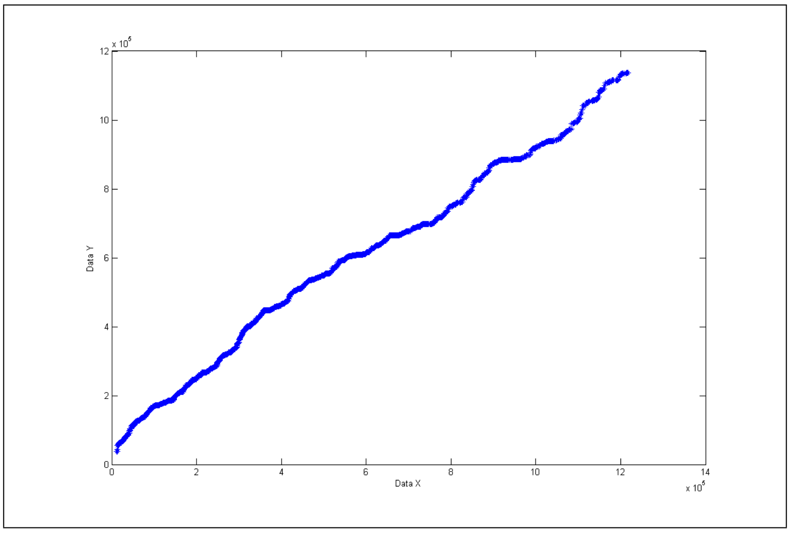

Figure 2: Q-Q Plot of the two Capital distributions

line) is almost the same and it's not centered in the box, indicating two skewed distributions. Moreover, both distributions present a great number of outliers (red plus) in the right tails, underling the possible presence of fat tails.

The same results are supported by the Generalized Kolmogorov-Smirnov Test[12] (Prabhakar et al., 2003) with a confidence interval of 95%. Therefore, it's possible to say that our two samples belong to the same distribution.

In particular, not considering the right Paretian tails (we trimmed them after a threshold study), we found out that our data follow a Generalized Pareto Distribution (GPD)[13], a Pareto II type in particular ($\xi \leq 0$). Figure 4 shows a linear Pareto II Plot[14] of the observed capital distribution (again, after excluding the largest firms).

The presence of a Paretian behaviour in the right tails of the two distributions is also supported by the Mean Excess Function versus Threshold Plot (MEPLOT). In fact, an upward sloping mean excess function, as in Figure 5, indicates a heavy tail in the sample distribution. That is why, thanks to the Hill's method[15], we have decided to estimate the shape parameters of the two sample, in order to see if data have a similar behaviour in the right tails.

Figure 6 contains the Hill's estimates of the shape parameter for the simulated capital, while Figure 7 refers to observed data. In the first case $\alpha = 1.61$, while in the second one $\alpha = 1,68$.So, the two parameters are very similar (Figure 8) and belong to the Paretian field ($0.8 < \alpha < 2$)[16], but we cannot state that the two

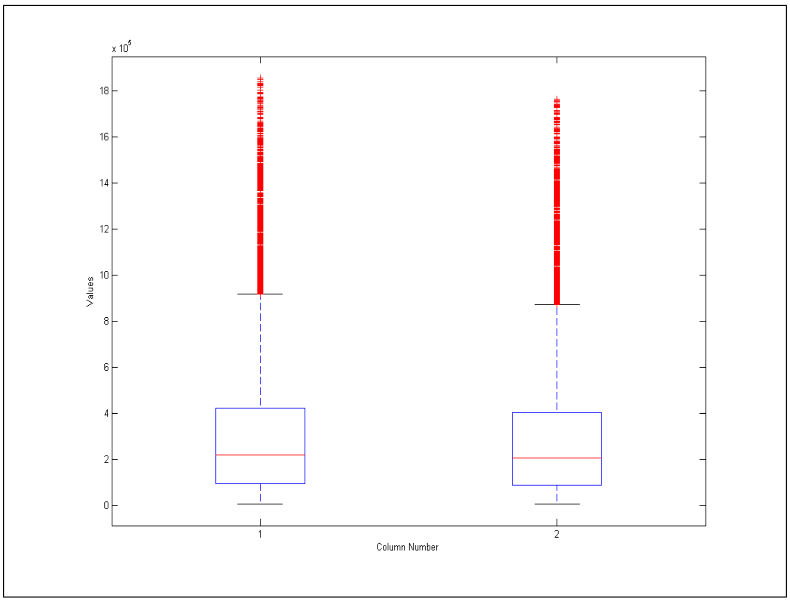

Figure 3: Box Plot of simulated (left) and actual (right) Capital.

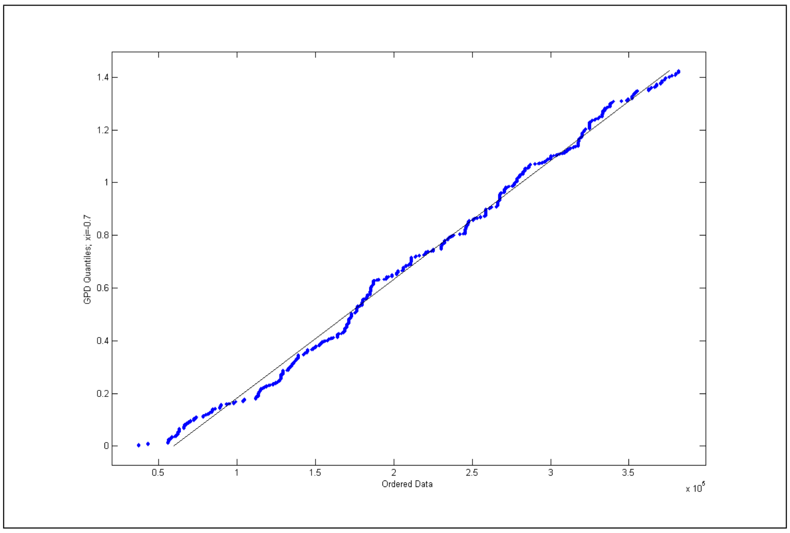

Figure 4: Pareto II Plot of the actual capital (biggest 10% trimmed)

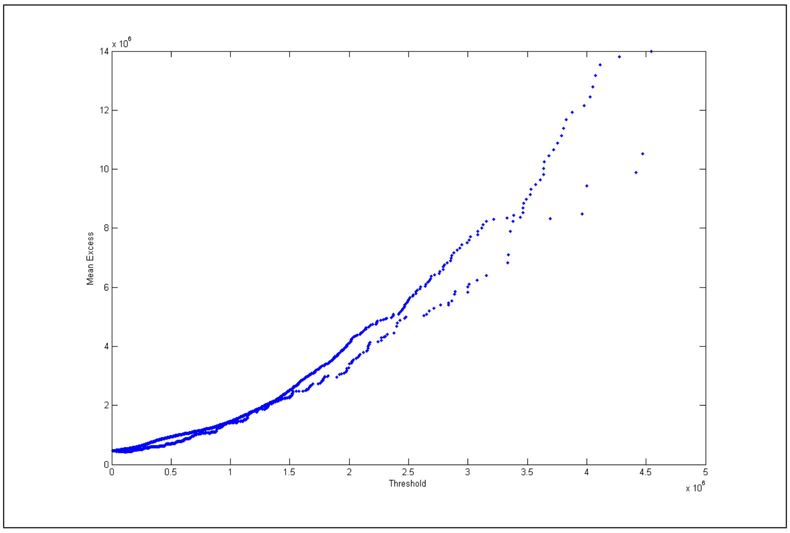

Figure 5: Meplot of observed (above) and simulated (under) Capital.

tails behave in the same way. Simulated capital, in fact, shows a slightly heavier tail (since its alpha is lower[17]), demonstrating that we slightly overestimated the observed values.

As far as net worth is concerned, accepting a maximum deviation of $\pm 20\%$ between actual and simulated data in 2001, we succeed in reproducing 4944 firms over 6422 (77%)[18]. This number is lower than that of total capital, indicating some more problems of fitting.

Other positive results, see Figure 9, are the skewness of the two distributions and the presence of a clear Paretian behaviour in both actual and simulated net worth. Hill's estimates of the shape parameters both show heavy right tails: actual data present $\alpha = 1.52$, while the simulation produces $\alpha = 1.48$.

As far as the possibility of a unique distribution for the two samples, the two-sided generalized Kolmogorov-Smirnov test rejects such a null hypothesis. On the contrary the one-sided right version of the test[19] is not refused, indicating that we get a better fitting of medium and big firms, but we fail in forecasting the smallest one (see in Figure 9).

The results we get about loans are very similar to those of the total capital: we succeed in fitting 5137 firms out of 6422 (80%).

Moreover, as happens for total capital, both graphical and analytical tests support the idea of a unique distribution for both actual and simulated debt data.

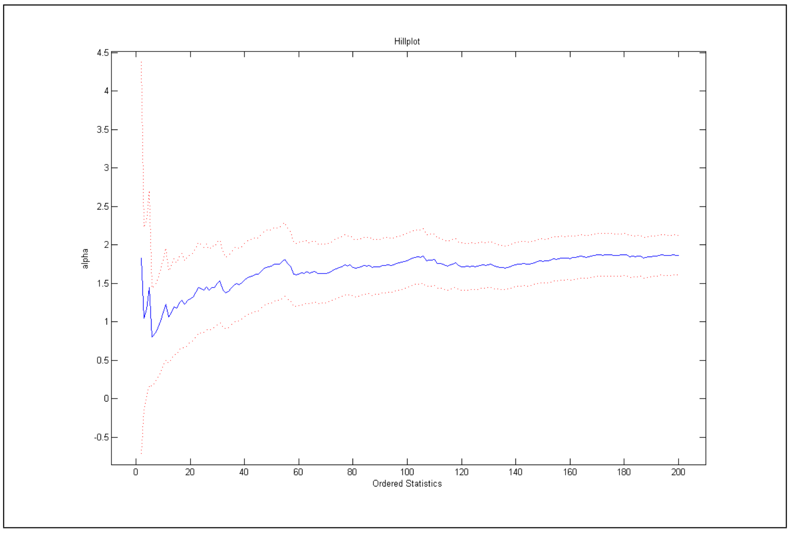

Figure 6: Hill Plot of the simulated Capital

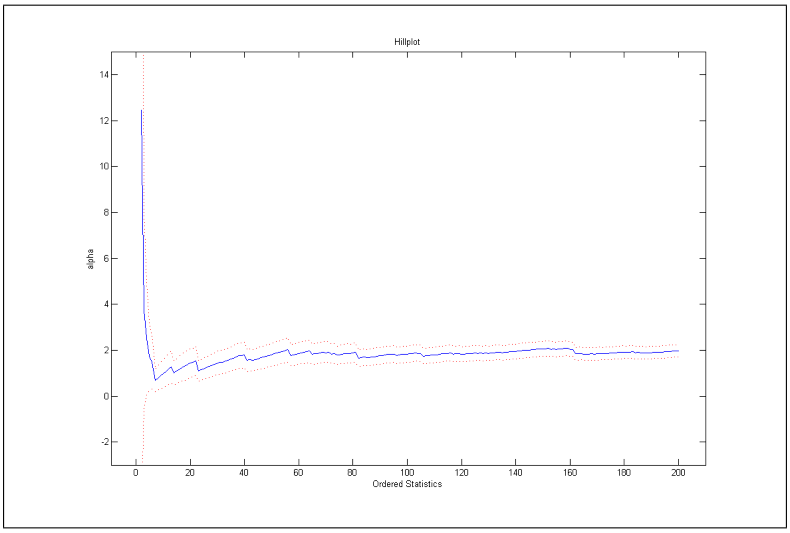

Figure 7: Hill Plot of the actual capital

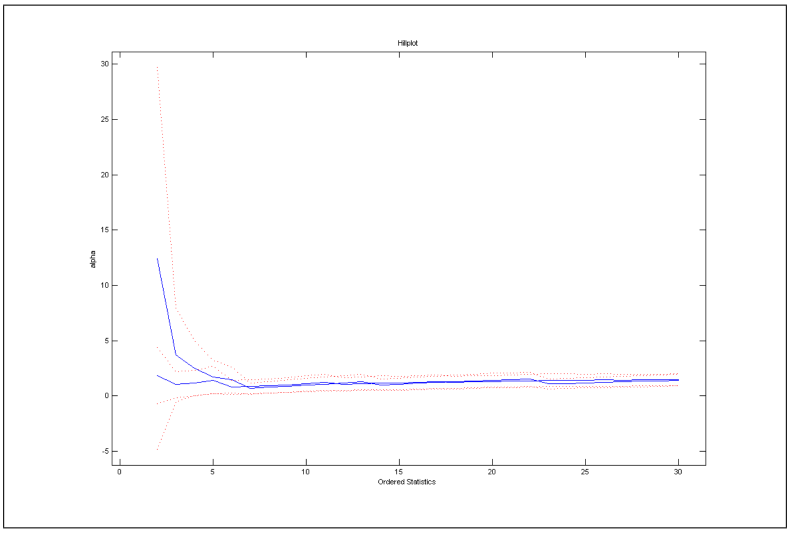

Figure 8: Comparison of the two Hill Plots.

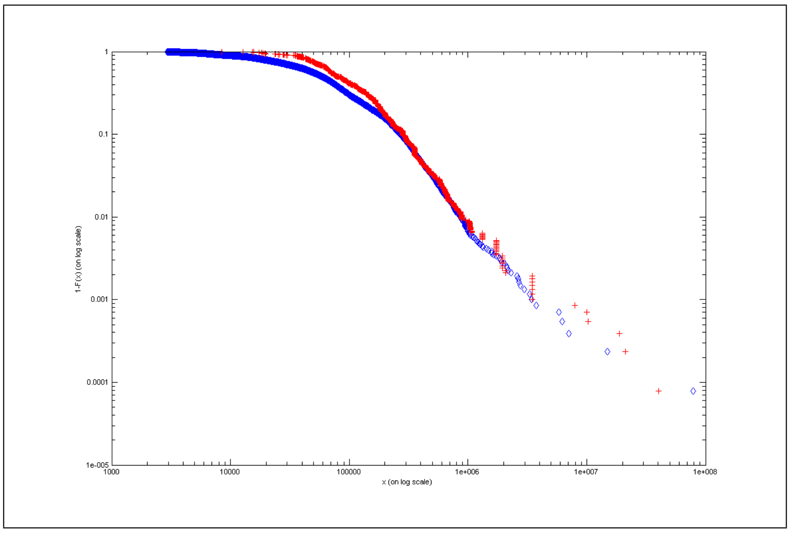

Figure 9: Zipf's Plots of the net worth distributions: observed (plus) and simulated (diamonds) data

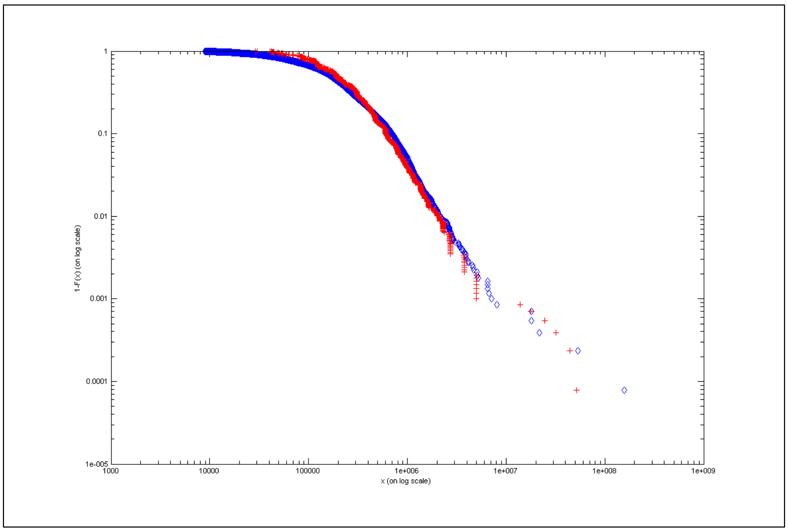

Figure 10: Zipf's plot of loans: observed (plus) and simulated (diamonds).

As in Fujiwara (2003), the distribution of loans is also power law. The Hill's estimates of the shape parameters of the Paretian right tails are $\alpha = 1.71$ for the actual data and $\alpha = 1.58$ for the simulated ones, demonstrating an overestimate of biggest firms.

Finally, analyzing the ratio between net worth and debt we find out that, apart from some exceptions[20], it's almost constant for each firm over time. In other words, if firm i has a ratio of $x\%$ in 1996, it shows a very similar ratio in 2001,

As far as firms' growth rates are concerned, several studies (Axtell, 2001; Bottazzi and Secchi, 2002; Hall, 1987) find a tent-shape behaviour. In particular, the Laplace and Lévy distributions seem to provide the best fitting (Bottazzi and Secchi, 2003; Gabaix, 2004).

We have investigated if the empirical distributions of growth rates (in terms of capital) belong to the well-known Subbotin's family (Subbotin, 1923), which represents a generalization of several particular cases, such as Laplace and Gaussian distributions. The functional form of Subbotin's family is:

$$f(x, a, b) = \frac{1}{2ab^{\frac{1}{b}}\Gamma\left(1 + \frac{1}{b}\right)} e^{-\frac{1}{b}\left|\frac{x-\mu}{a}\right|^{b}}, \tag{8}$$

where μ is the mean, b and a two different shape parameters and Γ is the standard

	observed data	Simulated data
μ	-0.0030 (0.0013)	0.0048 (0.0021)
a	0.0587 (0.0244)	0.0614 (0.0238)
b	1.0184 (0.3495)	1.0626 (0.3664)
$-loglik$	1.1528	1.1549

Table 1: Estimated Subbotin's Parameters (standard errors in brackets)

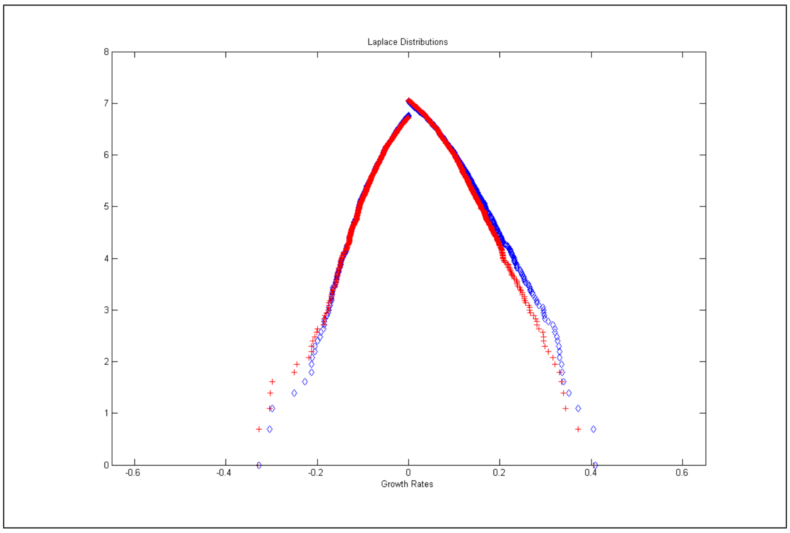

Figure 11: Empirical distributions of actual and simulated growth rates.

Gamma. If $b \to 1$ the Subbotin distribution becomes a Laplace, a Gaussian for $b \to 2$.

Using the maximum likelihood method[21], we have estimated the three Subbotin's parameters on our data. Table 1 contains the results.

At a first glace, observed and simulated growth rates show several similarities:

1. The two means are very close to zero;

2. Since b is very near to 1, both distributions are in the field of attraction of the Laplacian case[22]. Figure 11 supports this evidence since it's tent-shaped;

3. The values of a, the Laplacian shape parameter, are not very different in both cases, even if simulated data show slightly fatter tails (0.061>0.059), see Figure 11.

All in all, the CATS model is able to mimic firms' growth dynamic, once again with some discrepancies as far as the tails are concerned.

In order to analyze the relationship between firms' size and firms' growth rates, we followed the methodology suggested by Gabaix (2004)[23]. Stanley and coauthors and Gabaix find that large firms show a lower volatility of their growth rates; moreover, they show that this volatility (σ_{rates}) linearly decreases with size (S), that is to say

$$\ln \sigma_{rates} = -\alpha \ln S + \beta, \tag{9}$$

with $\alpha \simeq 0.15$.

In order to investigate if this relationship holds true for our actual and simulated data, we divided firms' size in four bins. Then we computed the standard deviation of their growth rates. Finally, we plotted a log-log graph of the average standard deviation of growth rates versus the average size in each bin.

For both observed and simulated data, our results are very similar of those presented in Gabaix (2004). Figure 12 shows how the relationship we have found decreases with size. Our estimates of α are 0.1643 for the actual data and 0.1621 for the simulated ones (not far from Gabaix's 0.15). Once again, the CATS model successfully reproduces the empirical data.

All in all, we may say that the CATS model successfully passes the ex-post validation exercise of this section, with the only exception of very small firms.

5. Calibration: a first experiment

Since the validation results of the CATS model seem to be quite promising, we have decided to try a first calibration experiment using a simple grid method.

Our idea was to perform a sort of sensitivity analysis, in order to discover the most relevant parameters in the model. In particular we have found out that the most sensitive parameters are those concerning with the price generators' processes, that's to say the supports of the uniform distributions.

Using a traditional grid method, we have decided to let the supports change on a grid in order to find those values[24] (supports' inf and sup) which minimize the distances between the distributions of actual and simulated data. In particular, we have used a common quadratic loss function concerning with the shape parameters of the two distributions[25] (Prabhakar et al., 2003).

This first naive calibration show surprisingly good results. As far as total capital in 2001 is concerned, from an initial fitting of 81%, we find out a new value of 88%; while, considering the pooled distributions, from a first 78% we then get a better 83%.

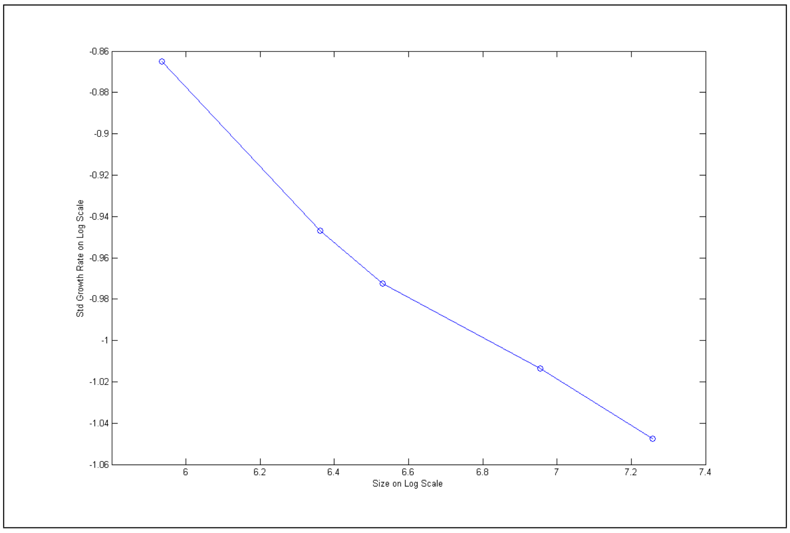

Figure 12: Growth Rates Std vs Size (observed data)

All this makes us believe that more complex and precise calibration methods could get better results. For this reason, in our future studies, we aim to use indirect inference (Gouriéroux and Monfort, 1996) as a calibration tool.

6. Conclusions and what comes next

Even if the results of the ex-post validation experiments discussed in section 4 are preliminary they shows that, in the interval 1996-2001, the simple CATS model, firstly introduced by Gallegati et al. (2003a) and slightly modified for these experiments (see section 2), has good capabilities in replicating empirical evidence, with few exceptions.

More reliable results could be obtained improving the specification of the model, better calibrating some key parameters using simulation based methods discussed, for example, in Gouriéroux and Monfort (1996) and in Klevmarken (1998) and carefully adjusting the dimensions of the sample used in the initial set up.

In future validation experiments, we intend to modify the model specification, endogenising the banking sector (as in the standard CATS model) and the price generator process and including a labor market module. Considering the sample size of the initial set up we are planning to build a new sample. With respect to same key variables (for example, number of employees), we will replicate in our new database firms in order to obtain the same proportions of the firms considered in the INPS (Istituto Nazionale della Previdenza Sociale) sample. Starting with this new set up, we will check whether our new model is able to reproduce, in the simulation interval, the proportions observed in INPS database.

Notes

[1]In a sequential economy (Hahn, 1982) spot markets open at given dates, while future markets do not operate.

[2]In this attempt to calibrate the model, N_t is always equal to 6422 (the total number of firms in our database). In fact, if a firm goes bankrupt, the entry and exit processes guarantee that a new one enters the market.

[3]In this model capital stock never depreciates.

[4]According to the Italian Fiscal Law, to which we referred in writing this paper, a firm is considered: *"small"*, if it has less than 50 employees; *"medium"* if it has between 51 and 250 employees; *"large"* if it has more than 250 employees. In our sample, the percentage of firms is: $\approx 56\%$ small, $\approx 31\%$ medium, $\approx 13\%$ large. In 1996 the smallest firm shows 2 employees, while the largest one 7308.

[5]The evolution of the productivities of small and large firms reproduce an evidence present in our database. Small firms, in fact, show an increasing productivity, while the large ones present an almost constant one.

[6]This assumption about a stronger price volatility has a justification in the greater volatility of small firms' revenues and profits in actual data.

[7]For more information: http://amadeus.bvdep.com

[8]But also in Sargent et al. (2000), Sargent (1998), Kleijen (1998), Gumbel (1958).

[9]It decreases every year starting from 11.5% (1996) and arriving at 10% (2001).

[10]As in Ijiri et al. (1997), the use of pooled distribution is possible since the single distributions show similar slopes.

In this paper, almost all the figures refer to year 2001.

[11]A power law behaviour in firms' size is essentially due to the random iid micro-multiplicative shocks (Solomon, 1995; Gabaix, 2004) and the presence of the (bankruptcy) lower bound we have modelled. As Lutz et al. (1995) show a system with power laws tails distributions have divergent first and second moments, so the law of large numbers does not hold and the system is not ergodic. All this has disruptive consequences for mainstream economics (Davidson, 1986).

[12]The Generalised or *Two Sample* Kolmogorov-Smirnov test is a variation of the classical Kolmogorov-Smirnov test.

Given N data points $Y_1, Y_2, ..., Y_n$ the empirical distribution function (ECDF) is defined as

$$F_N = \frac{n(i)}{N}, \tag{10}$$

where $n(i)$ represents the number of points less than Y_i. As one can see, this step function increases by $\frac{1}{N}$ for each data point.

The Kolmogorov-Smirnov test is based on the maximum distance between the ECDF and the theoretical cumulative distribution one wants to test (F^T):

$$D = \max_{1 \leq i \leq N} \left| F^T(Y_i) - \frac{i}{N} \right|. \tag{11}$$

On the contrary, the two sample K-S test, instead of comparing an empirical distribution function to the theoretical one, compares two different ECDF, that is

$$D = \left| F_1(i) - F_2(i) \right|, \tag{12}$$

where F_i is the empirical distribution for sample i.

The generalised K-S statistic can be defined as:

$H_0 : F_1 = F_2 \rightarrow$ the two samples come from the same distribution

$H_1 : F_1 \neq F_2 \rightarrow$ the two samples come from different distributions

To decide the results of the test, the values of D are compared to the critical values obtained from Kolmogorov and Smirnov's table.

[13]Starting from the well-known Fisher-Tippett Theorem, which deals with the convergence of maxima, the GPD distribution (Pareto, 1986) represents one of the most important limiting cases.

Its functional form is:

$$H(x) = \begin{cases} 1 - (1 + \xi \frac{x}{\beta})^{-\frac{1}{\xi}} & if\ \xi \neq 0 \\ 1 - e^{-\frac{x}{\beta}} & if\ \xi = 0 \end{cases}, \tag{13}$$

where $\beta > 0$ and x is such that $1 + \xi x > 0$ and ξ is the shape parameter (tail index $\alpha = \frac{1}{\xi}$).

There are three different situations:

1. $\xi > 0 \rightarrow$ GPD distribution becomes the classical Pareto distribution and shows fat tails;
2. $\xi = 0 \rightarrow$ GPD distribution converges to the exponential distribution;
3. $\xi < 0 \rightarrow$ GPD distribution it then known as Pareto II.

[14] That's a quantile-quantile plot with Pareto II coefficients.

[15] The well-known Hill's Estimator ξ, together with the Pickard's one, is the most used way to determine the shape parameter $\alpha = \frac{1}{\xi}$ of a distribution belonging to the GEV family.

In particular

$$\xi = \frac{1}{k-1} \sum_{i=1}^{k-1} \ln x_{i,N} - \ln x_{k,N} \quad for\ k \geq 2, \tag{14}$$

where k is the upper order statistics and N the sample size.

[16] Once again the results concerning the pooled distributions are very similar. The reason can be found in the words of Ijiri et al. (1977): "We conclude that when two or more Pareto distributions are pooled together, the resulting distribution is Pareto if and only if all the distributions have similar slopes [...]. This result is important in dealing with the aggregation of empirical firm size distributions."

[17] As clearly showed in Kleiber and Kotz (2003), the Pareto density has a polynomial right tail that varies at infinity with index $(-\alpha - 1)$, implying that the right tail is heavier as α is smaller.

[18] 66% in 1997, 73% in 1999

[19] $H_0 : F_1^+(x) = F_2^+(x)$.

$H_1 : F_1^+(x) > F_2^+(x)$.

[20]While validating our model, we have experienced several experiments on interest rates, finding out an interesting thing.

Those firms showing a decreasing net worth/debt ratio are the same that obviously go bankrupt if the interest rates rise. All this is interesting since the decreasing ratio is almost completely due to a monotonically deteriorating equity ratio (Beaver, 1966; Gallegati et al., 2005). Moreover, surprisingly, all the firms that went bankrupt in our simulations were the same as those that really went bankrupt in 2002, showing a decreasing equity ratio.

Unfortunately, as already said, our data are not complete for 2002, so we prefer not to state all this as a result.

[21]The results are very similar, using the method of moments.

[22]Some authors prefer a truncated Lévy distribution. The *querelle* is open. See Kleiber and Kotz (2003).

[23]But also by Stanley et al. (1996) and Amaral et al. (1997)

[24]The new supports are: $(0.9, 2.7)$ for small firms and $(0.4, 1.85)$ for the big ones.

[25]Since both distributions are in the Paretian case and since their scale is almost equal, if we minimize the distance between their shape parameters, we get two more similar distributions.

A Validation Procedure: some notes

The aim of this appendix is to describe the procedure we have used to validate the CATS model.

All the codes and the programs have been written in Fortran90©, while all the graphics have been developed with Matlab7©.

As far as the simulation of the CATS model is concerned, it can be useful to underline the following aspects:

1. In $t = 1$ (1996), when the simulation starts, every firm is initialized with its actual data from the database. These data are: net worth, loans and productivity. The current version of the model has a recursive structure so that parameters ϕ_{it} have been consistently estimated using, firm by firm, ordinary least squares. Then productivity evolves according to the laws of motions presented in 2 and 3;

2. The parameter M in 2 follows an uniform distribution, whose support $(0, 2)$ has been ad hoc calibrated, thanks to several replications;
3. The interest rate is equal to 11,5% in 1996 ($t = 1$) and decreases every year, arriving at 10% in 2001, This reproduces the average behaviour of the interests paid every year by firms in our database;
4. The two different uniform distributions we have used to model the idiosyncratic shocks on prices show support $(0.8, 2.8)$ for small firms and support $(0.5, 1.7)$ for the large ones. This supports have been inductively calibrated, considering the results of several alternative replications, in order to get the best fitting values;
5. Every year the following data are stored in order to be compared with actual data: net woth, loans, total capital, productivity, growth rates, paid interests, total output, aggregate output.

Our analysis of data can be divided into two different approaches: a pointwise analysis, meant to evaluate the evolution of the single firm, in order to study the predictive power of the model; and a distributional analysis, whose aim is to look for general regularities.

In Embrechts (1997), one can find a quite complete list of all the tests a researcher should perform in analysing data, while Kleijen (1998) deals with the theoretical implications of validation.

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