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A Practical Guide To Inference In Simulation Models

by

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A PRACTICAL GUIDE TO INFERENCE IN SIMULATION MODELS

Thomas Brenner* and Claudia Werker

Abstract

This paper introduces a categorization of simulation models. It provides an explicit overview of the steps that lead to a simulation model. We highlight the advantages and disadvantages of various simulation approaches by examining how they advocate different ways of constructing simulation models. To this end, it discusses a number of relevant methodological issues, such as how realistic simulation models are obtained and which kinds of inference can be used in a simulation approach. Finally, the paper presents a practical guide on how simulation should and can be conducted.

Keywords

Methodology, Simulation Models, Practical Guide

JEL Classification

B41, B52, C63

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1. Introduction

In the last two decades simulation models have become increasingly popular in economics. They have been used for various purposes by employing different modelling strategies and methods. This variety led to a fuzzy picture of this field of research in economics. In particular, a standard model to which everyone can refer to when introducing their own model, such as the standard neoclassical model, is missing. This hampers communication between scholars. Obviously such a disadvantage goes in hand with the advantage that simulation models can cover a much broader range of research questions compared with alternative, traditional methods. Nevertheless, it would be helpful to have a clearer overview about how simulation models are built and how their results are derived.

In order to get a better understanding on how results of simulation models are inferred and how the best match between research questions, research strategy and method can be found, we employ elements from two different scientific fields, i.e. simulation models and methodology. We start with some methodological considerations by first showing how simulation models can be categorized according to their degree of generalization and theoretical considerations (Section 2.1). Moreover, we introduce three general principles of inference (Section 2.2). Based on these methodological considerations we provide an overview of simulation models from the literature (Section 3.1) and derive four principal steps one has to follow when building a simulation model (Seciton 3.2). Based on our insights into the fields of simulation models and methodology, we derive a practical guideline to build simulation models in a methodologically adequate and efficient way (Section 4.). Our goal is to design an approach to building simulations that enables the simulation to address the research question in the most appropriate manner.

2. Methodological Considerations for Simulation Models

2.1 Classification of Simulation Models According to Their Degree of Generalization and of Theory and Data Use

The approaches found in the literature differ strongly in the simulation model set up. Two features are especially important. First, some authors develop a specific model, while others develop a general model. Second, the assumptions, on which models are based, could be obtained with the help of theoretical considerations or empirical data.

We distinguish general and specific models as follows. There are some modellers who assume that the knowledge about the dynamics that are to be modelled is not sufficient to define a specific model. Instead, they try to keep the model as general as necessary or possible. This implies that a model with many unspecified parameters has to be set up. It might even be unclear how the model should be specified, i.e. how dependencies look like and what factors should be included. Hence, either different models have to be considered or the model has to be made more flexible by introducing more parameters. In contrast to a specific model, the unfixed parameters are not varied because their impact is studied, but because the modeller believes that their value is unknown. This approach is used initially, for example, in Bayesian simulations, where inferring knowledge about these parameters from empirical data about the system's dynamics is the main goal.

In contrast, in specific models the modeller tries to detail the model as much as possible. For the purpose of the study, the modeller assumes the possibility to design the model and fix the parameters adequately on the basis of theoretical considerations or empirical data related to the underlying mechanisms. Sometimes certain parameters are varied to conduct counter-factual analyses or to examine the robustness of the results. Similarly, sometimes different model specifications are examined. However, the modeller, in principle, believes that the model can be adequately specified. This is the usual approach in mainstream and heterodox economics.

Of course, most simulation models are neither purely general nor purely specific. These are extreme cases and most simulation models fall somewhere between them. Therefore, we use the specificity-generality dimension of categorizing simulation models as one axis of Figure 1.

With respect to the way, in which the simulation model's assumptions are obtained, we distinguish two approaches: theoretical considerations and empirical estimation. However, there are neither pure theoretical models nor pure empirical models, because every scholar builds assumptions on the basis of empirical knowledge at least in the form of experiences and common knowledge and every empirist builds assumptions on the basis of at least some basic theoretical considerations. Nevertheless, there are models that are called *theoretic* where the modeller does not try to justify the model using empirical data. This includes models based on axioms, which are not proven empirically elsewhere, as well as models that are based on *ad-hoc* assumptions. We call these models "developed according to theoretical considerations".

Similarly, we do not know of any purely empirical model in economics. A purely empirical model would require that the modeller gathers the complete model from empirical studies. Such an approach is taken in natural sciences where the functional forms of dependencies are identified in numerous studies or experiments and put together in models. In economics the possibilities of such an approach are limited or, at least, still very limited. This implies that even if the modeller tries to base the model on empirical studies, it will always contain parts that are based on theoretical considerations. Hence, again there is a continuum of approaches that range from mainly theoretically to mainly empirically-based assumptions in the model set up. We use the theoretical-empirical characteristic as a second dimension to classify the approaches and as the second axis of Figure 1. It denotes the amount of empirical data used in the simulation model set up.

Additional to the two mentioned dimensions we consider how realistically simulation models are represented. We call a simulation model realistic if it is able to exactly reproduce all features and dynamics relevant for the conducted study. If we run simulations, we usually aim at reproducing reality, meaning that we try to set up a model that shows the same features and dynamics as in reality. Usually we are not interested in all features and dynamics that exist in reality but in those of a certain subsystem. In economic studies we can rarely be sure that a developed model exactly represents even the relevant part of reality. However, we might estimate the likelihood that a simulation model exactly matches reality in all relevant aspects. Of course, an exact estimation of this likelihood is not possible. However, we can make a rough estimation. If a certain system is modeled, there are usually many different ways in which this can be done. To clarify, let us assume that there are 20 different ways in which the model could be set up. Let us furthermore assume that there is no empirical knowledge that would indicate which of the models is more likely to be suitable. Nevertheless, let us assume that the modeler tests 5 of these 20 models and finds only one of these to correspond with empirical observations. We could then state that this model is, with a probability of 25%, the correct model because with a probability of 75% the correct model is one of the 15 untested models. In practice, the various model specifications are not usually known. However, it can at least be estimated whether a modeler neglected a lot of plausible alternatives or not. Similarly, it can be judged whether parameters have been arbitrarily fixed or not. Hence, a rough estimate could be obtained on whether the model is realistic.

There are two extreme ways to increase this likelihood: At one end of the spectrum models are developed on the basis of a tremendous amount of empirical knowledge and data (as usually done in natural sciences), hence an empirically based, specific model is developed. At the other end of the spectrum, the model might be kept very general. This means that the parameters of the model are not fixed and different model settings or specifications are analyzed. As more different specifications of the model are included in the analysis, the more likely it is that the real features and dynamics are reproduced. These extreme ways to obtain realistic models correspond to the upper left and lower right edge in Figure 1. Above we stated that these two extreme kinds of models do not exist in economics. However, there is a third way to achieve complete realism. We could use all available empirical data to specify the simulation model. For all aspects that cannot be specified on the basis of empirical data we might generally formulate the model. This means that the model falls onto the line between the upper left and lower right edge in Figure 1 and represents the real system. Hence, if the simulation model is just as specific as the empirical data allows, it is realistic. If more theoretical assumptions are made, the model is located in Figure 1 closer to the lower left edge where its realism decreases. Thus, the third dimension – realism – can be depicted in Figure 1 as the diagonal.

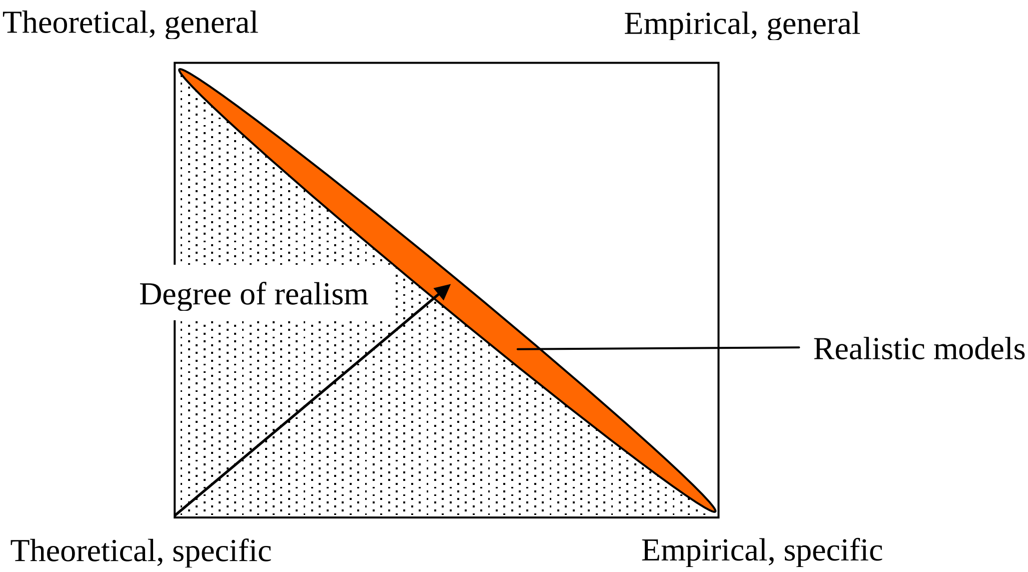

Figure 1: Dimensions according to which simulation models can be classified and their correspondence with reality.

Models that lie above the line of realistic model are not adequate for logical reasons. They are more general than necessary according to the empirical knowledge used. This means that, for example, a parameter that is fitted empirically is, nevertheless, kept unspecified in the simulation approach. A variation of a well-estimated parameter is adequate in a counter-factual analysis but not as a part of setting up a realistic model. Hence, all existing models can be found in the space on and below the line of realistic models.

2.2 Principles of Inference to Build and Analyse a Simulation Model

In preparation for the structured overview on simulation approaches some basic categories and principles of inference are described here. Assumptions and implications are basic elements of inference. Independent from the approach taken, simulations always provide the tool to derive implications from assumptions in an analytical and logical way. Simulations are based on a simulation model that contains all assumptions made by the modeller. Running simulations means that we obtain knowledge about the implication of these assumptions, similar to the analysis of mathematical equations. The different simulation approaches vary in how they embed the simulation in the overall analysis, including such aspects as how many different simulation models are analysed, how assumptions are obtained, how the implications are interpreted, and so on. However, the essence of a simulation approach is the use of simulations to infer implications from assumptions. Premises and definitions are usually part of the assumptions as these elements set the boundaries for modelling. However, sometimes definitions and premises can also be part of the implications, especially if the results of a model indicate that premises and/or definitions have to be revised for further research. Data can be used in both parts of models. In assumptions data provides an empirical basis to start from. Whereas, in implications, data is used to corrobate implications stemming from premises, definitions and logical considerations. Logic is, of course, always at the heart of modelling in all parts and consistently amalgamates all elements of the model.

Three different principles of inference can be distinguished: deduction, induction and abduction. Each principle works in different ways, although with the same result, namely inferring implications from assumptions. *Deduction* is often summarized as inferring “from general to particular” (cf. Lawson, 1997, 24). Let us use, as an example throughout this paper, the impact of different patent laws in different countries on the development of a certain industry in these countries. Deduction would mean in this context that we have or assume a theory about the development of industries dependent on the national patent law, e.g. that stricter national patent laws protect innovators’ property rights better thereby giving more incentives to innovate. From this theory we could deduce that an industry in a country with lax patent laws innovates less compared to the same industry in a country with stricter patent laws. As one can readily see in the example, assumptions within deduction already contain all information available. Generally spoken, deduction sustains information already contained in the assumptions but does not create a new one.

If A = B and B = C, (assumptions)
then A = C. (implication)

In deduction assumptions contain all possible elements of models, like e.g. premises, definitions or causal relationship. Therefore, it is often claimed that in deduction, conclusions stemming from assumptions have to be true. In formal sciences like mathematics this holds, because assumptions are usually provided in the form of axioms, i.e. they are self-evident and need not be proven. However, in social sciences, like economics, such self-evident assumptions do not exist. Implications drawn from premises are in general true but only in the sense that they are logically derived. In social sciences, without self-evident premises available, it is virtually impossible to derive implications that are true in the sense of correctly describing, explaining and prognosticating reality.

Induction is often summarized as inferring "from particular to general" (cf. Lawson, 1997, 24). Its assumptions describe a part of a larger population and then infer conclusions about the characteristics of this larger population. In our example this would mean that we observe the innovative output of a number of industries in a number of different countries with different patent laws. We would then inductively infer general mechanisms, relationships and rules by examining the common characteristics of all observations and could come to the conclusion that industries in countries with stricter patent laws have a larger innovative output. As the inductive principle runs "from particular to general" it is often seen as creating information - however doubtful. The inference in induction says something not contained in the assumptions. If the inference arguments are strong it is probable that the claims made about the conclusions hold. Inductive inference is based on data. However, even if the number of observations in the data set is large it is, in principle, impossible to have all observations available, not the least because future events cannot be observed. This means that the implications derived from data are uncertain. In the future, the same will only happen with an unknown probability. This probability is impossible to gain, because future observations, by definition, cannot be made now.

Abduction - sometimes also called retroduction - classifies “particular events into general patterns” (Lawson, 1997, 24). For our example abduction means that we argue that industries vary in their development and are therefore affected by patent laws in different ways. Abduction means that we start by collecting detailed information about the development of different industries in different countries facing different patent laws. Based on this, we classify the different developments and identify the underlying driving forces. This also enables us to describe, explain and predict developments of other industries in other countries with respect to their patent laws. It is important to notice that abduction requires data based on substantial and detailed observations. Only then it is possible to find meaningful and sensible underlying mechanisms to infer from the assumptions to the implications. So, for example, if we observe that a number of low-tech industries develop in different countries independent of actual patent laws, we might conclude that all low-tech industries are not influenced by any change in the patent laws. Obviously, this is somewhat jumping to conclusions. Abduction requires much more detailed information to infer implications that are likely to hold when confronted with reality. In our example one would wish to know much more about the mechanisms behind the industrial development and the differences between national patent laws. It would especially be important to know what makes an industry’s development independent of patent laws. We could for instance choose to define classes of industries that show similar developments within one class and different developments between classes, e.g. the famous Pavitt taxonomy (Pavitt, 1984). By going back and forth between theorizing and empirical testing we might come to the conclusion that science-based industries profit more from stricter patent laws than scale intensive industries. This would allow transferring the experience to other industries. As more relevant data details are known they can be precisely classified into a general pattern.

Abduction enables us to identify underlying structural elements, which explain observations we make, and to develop a theory of the part of the world we are investigating. This takes us a substantial step further than pure deduction or induction, because abduction helps us to meet theory and data in a creative way. By using the principle of abduction we are able to create new information. According to Peirce (1867/1965, 5, 145f):

"(Induction) never can originate any idea whatever. No more can deduction. All the ideas of science come to it by the way of abduction. Abduction consists in studying the facts and devising a theory to explain them. Its only justification is that if we are ever to understand things at all, it must be in this way."

3. Simulation Models from the Methodological Point of View

3.1 Simulation Models: An Overview from the Methodological Point of View

The simulation method has been widely used in economics. It has served different purposes such as description and explanation of economic processes as well as their prognosis. As we are interested in inference in simulation models it is crucial to understand the relationship between theorizing, on one hand side, and empirical data on the other hand, as well as the degree of generalization used. Economic models are neither free of theory nor free of data. However, the extent to which theory and data is used in simulation models differs considerably (see Section 2.1).

In the following we will present an overview of simulation models, characterising them according to the above defined dimensions of categorisation (see Figure 1). This characterisation of the simulation approaches is complicated by the fact that in some approaches simulation models are set up at the beginning and then modified or specified during the simulation approach. In these cases, the simulation model can be categorized according to Figure 1 at the beginning or at the end of the approach. This topic is also taken up in more detail in Section 3.2.

We distinguish five different types of models: First, we will present models, which are rather specific and use stylised facts to evaluate the simulation results. Second, we will introduce models, which are rather specific and use case study data to specify the simulation model. Third, we will discuss methods based on specific models and the use of comprehensive empirical data. Fourth, we will show a method based on general models and test the simulation model implications using empirical data. Last but not least, we will present an approach in which the aim is neither to develop a specific nor a general model, but in which empirical data is used as much as possible and the model kept as general as necessary. We will show that these types of models usually go hand in hand with the specific use of the principles of inference, i.e. induction, deduction and abduction (see Section 2.2).

In the past, simulation models have often been *specific models, which incorporated data in the form of stylised facts to check the implications.* This is called the *traditional* approach here. The models suggest specific levels of parameters and specific relationships between these parameters. The assumptions of these specific models are based on theoretical considerations, which result from axioms or *ad-hoc* modelling. Hence, these models are located in Figure 1 rather in the lower left edge at the time they are set up (see Figure 2). The aim of these models is usually to either show that they are able to produce certain phenomenon or to study how things vary if certain aspects of the models are changed. Empirical data enters the model via the implications of the simulation model. It is used in the form of stylised facts to check whether the theoretically set up model is realistic. To this end, stylised facts are used in accordance with Kaldor's original idea (Kaldor, 1968, 177f). Stylised facts comprise empirical statements about a wide application area. They mostly rely on common sense and the impression of the scholar using them. The problem with stylised facts is that they “fall from heaven” and often remain unmotivated (for a detailed critique see Schwerin, 2001, 92-98). As it is usually unclear how stylised facts are derived it is not possible to tell whether or not they comprise only the structural elements of economic processes or whether they partly mirror noise in the form of chance elements. Nevertheless, the use of stylised facts enables modellers who want to concentrate on specific theoretical considerations to integrate some empirical reality into their analysis. Often the simulation model is adapted such that it is able to reproduce the stylised facts in a process that is not recorded in the final work. This empirical data is included in the model, so that it moves in Figure 2 to the right. However, it cannot be estimated how often the model has been adapted before the modeller was satisfied with the result. The major principle of inference used in this kind of model is of course deduction, because the focus lies on theoretical considerations. Induction is used by comparing the simulation results with stylised facts.

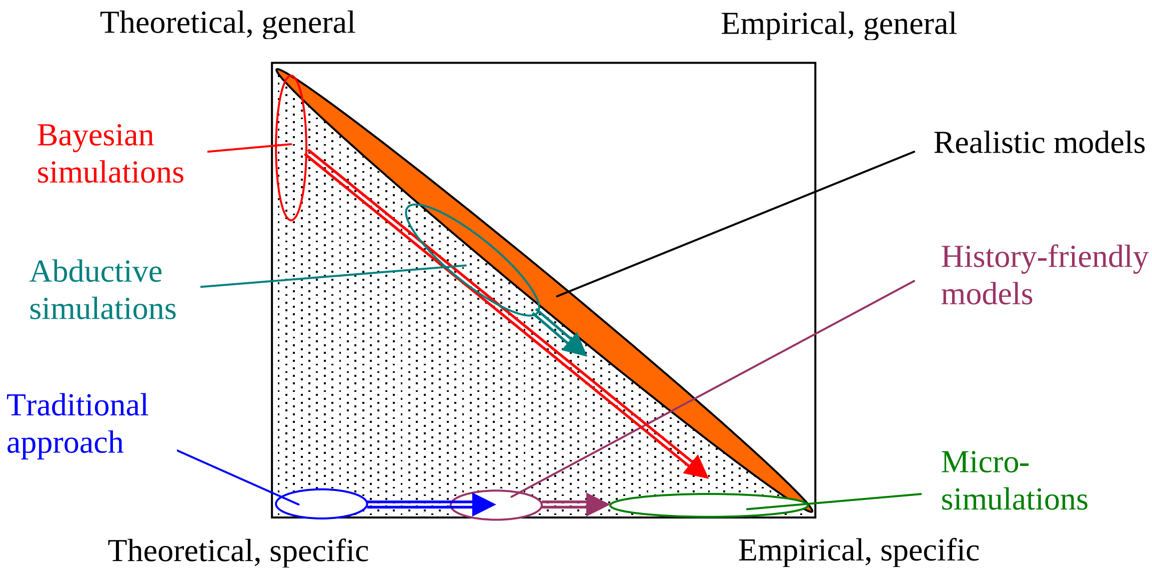

Figure 2: Categorization of five different simulation approaches according to the dimensions theoretical-empirical and specific-general (the ellipses show where these approaches are located initial and the arrows show in which direction they develop if the model is modified during the simulation approach).

Examples of simulation models, which use stylised facts in their modelling, are Harrison (2004), Fagiolo/Dosi (2003), Werker (2003), Winter et al. (2003) and Windrum/Birchenhall (1998). Harrison (2004) presents a simulation model that can reproduce stylized facts in the form of four different kinds of organizational evolution and firm growth. Fagiolo/Dosi (2003) present an endogenous growth model, which is able to reproduce stylized facts in the form of empirically plausible GDP time-series. Windrum/Birchenhall (1998), Winter et al. (2003) as well as Werker (2003) present models, which are able to account for the stylized facts in the evolution of industry life cycles. Harrison (2004), Fagiolo/Dosi (2003), Werker (2003) and Windrum/Birchenhall (1998) model random effects and perform a sensitivity analysis by running the same specifications at least 100 times. They calculate some statistics and give some qualitative impressions of the results. Winter et al. (2003) concentrate more on the analytical results of their model and limit themselves to only a few calculations to test for the sensitivity of their results.

Specific models, which use case study data, are often called history-friendly models. They rely on detailed empirical knowledge about real historical processes and try to find a model that leads to processes with the same characteristics. These models focus mainly on induction, because the case study is the core of the analysis. The case study is used in two ways. On the one hand, it provides some knowledge about the underlying processes that are modelled in the simulation. On the other hand, it provides information about the realistic dynamics of the simulation. Hence, in the model set up some theoretical considerations and some empirical knowledge are involved. The model is therefore located somewhere in the lower middle of Figure 1 when it is first set up. Although this is usually not mentioned in the final publications, different models are tested and rejected by the empirical knowledge on the system dynamics until a model is not rejected. To this, additional empirical knowledge is included and the model moves to the right in Figure 1. How far this process is taken cannot be stated because in the final publications the authors do not usually reflect on this process. Finally, it is argued that the model might describe the mechanisms underlying the known empirical facts. An inference is made from one single case to general mechanisms. This is of course problematic, because - though usually most of the complicated and complex processes involved are depicted - it is not possible to sufficiently distinguish between chance and necessity. This means that scholars using history-friendly models have difficulties in identifying the underlying structural processes, which could be found back in similar historical circumstances. Usually, they provide some sensitivity analyses of their results, which can be and sometimes are explicitly interpreted as counterfactual histories. This gives some idea on the results' stability and whether they crucially depend on random effects. Generally spoken history-friendly models share the advantage as well as the disadvantage with case studies: they give us deep inside knowledge about real economic processes but lack the possibility to generalize their results in a convincing way. Although induction is widely used as a principle of inference, deduction also plays a role in these approaches once a satisfying simulation model has been found. The authors, then, use the developed model to analyse certain model characteristics or conduct counter-factual analyses.

Recent examples of history-friendly models are Eliasson et al. (2004), Kim/Lee (2003), Malerba/Orsenigo (2002), Eliasson and Taymaz (2000) and Malerba et al. (1999). Eliasson et al. (2004) show, with an empirically calibrated micro-macro model for Sweden, how the new economy affects productivity and growth processes. Kim/Lee (2003) construct a history-friendly model for the DRAM industry. Malerba/Orsenigo (2002) and Malerba et al. (1999) concentrate on analyzing a history-friendly model of the computer-industry. Only Eliasson and Taymaz (2000) calculate the sensitivity of their results. All other authors run a number of counterfactual histories in order to account for the sensitivity of their results. Thereby, they produce data that can be statistically analyzed. Consequently, these scholars can use more sophisticated methods and can look into the statistical properties of their models in a systematic way. This gives some insights into how counterfactual histories could have occurred and on how much the results depend on random effects.

The third kind of simulation model are micro-simulations, i.e. *specific models, which are based on comprehensive empirical data*. In this approach the dynamics of the previously studied system is thoroughly examined. Statistical methods are applied to detect the crucial dependencies between variables and the trends in the dynamics of these variables. The findings lead to a simulation model that describes the previously observed dynamics. The crucial theoretical assumption is that, in the future, the same mechanisms and dynamics take place as in the past. We have already stated above, that this is not necessarily given in economics. However, this seems to be the only limitation to the realism of the models in this approach, given that all important processes are considered. In Figure 2 this approach is located in the lower right edge, deviating from the edge only by the fact that the future is theoretically assumed to be the same as the past and by the fact that maybe some important variables or mechanisms are not included in the model. However, the applicability of this approach has some limitations. Two requirements have to be satisfied. First, the dynamics that are studied have to be so simple that they can be completely analysed. Second, sufficient empirical data has to be available to examine the underlying mechanisms. Given that these requirements are satisfied, the obtained simulation model can be used, for example, to make predictions about future developments. Hence, while in this approach the simulation model is obtained in an inductive way, it is used in a deductive way.

Micro-simulation approaches can be frequently found in the literature (surveys and general discussions can be found, e.g., in Merz, 1991, O`Donoghue, 2001, and Creedy/Duncan, 2002). They typically either aim at investigating the effect of certain policies or at predicting future developments. For example, Atkinson et. al. (2002) study the impact of a European Minimum Pension. To this end, they use a simulation model that is based on detailed data about household income in the five largest EU countries and simulate the impact of a European Minimum Pension on these incomes. Based on similar data for the UK and a similar simulation approach, O’Donoghue/Sutherland (1999) study the impact of alternative family tax treatments on the tax paid by these families and work incentives. The different approach of various European countries are compared on the basis of the situation in the UK. Many other works exist in this field that cannot all be presented here.

The fourth kinds of models are Bayesian simulation models, i.e. *general models, which incorporate empirical data by systematically comparing the simulation results to larger sets of empirical observations*. This approach allows for very detailed and systematic analyses of statistical properties and sensitivities towards random effects, because it starts from the assumption that little is known about the models’ parameters and often also about the relationships between the variables of the model. Moreover, the large set of empirical observations, which is typically used, enables the modeller to thoroughly test the different specifications of the simulation model. In Bayesian simulations usually a lot of empirical data is available for the phenomenon but little for the processes that cause this phenomenon. Therefore, a very general simulation model is developed that includes all plausible processes that could cause the phenomenon. Hence, the approach starts with a model that lies in the upper left edge of Figure 2, because the model is general and almost no empirical data is used. However, the Bayesian approach is based on the idea that running all plausible simulation models and comparing their results with empirical data regarding real dynamics can identify the right specification of the simulation model. Hence, the final result is a clearly specified model that is obtained by extensively using empirical data about the dynamics or characteristics of the system. This means that, finally, in the Bayesian approach a model is obtained that is located in the lower right edge of Figure 2. Thus, Bayesian simulations are not that much different from micro-simulations in our categorisation. In both cases a specific simulation model is finally obtained using a large amount of empirical data. The difference is that in the case of micro-simulations, empirical data is directly used to set up the model, while in Bayesian simulations, data about the outcome of the simulation runs is used to induce the correct specification of the simulation model. The intention of Bayesian simulations is to inductively obtain an adequate model specification.

Bayesian inference has become increasingly widespread in econometrics in recent years (for an introduction to Bayesian inference see, e.g., Citro & Hanushek, 1991 and the original work by Zellner, 1971). Examples for Bayesian simulations are, Kaufmann (2000), Tsionas (2000), Jacobson/Karlsson (2004) and Jochmann/Leon-Gonzalez (2004). Tsionas (2000) uses stock market data to check different models about the stochastic movements of prices. To this end, he simulates a number of existing models with different parameters. Since the models are stochastic, for each model and each parameter set, the simulation has to be run numerous times in which he counts how often the results are in line with empirical observations. According to Bayesian inference, this offers some knowledge about the likelihood that each of the parameter specifications and each model is correct. This allows us to judge the adequateness of the various existing models. A similar approach is taken by Kaufmann (2000). Bayesian simulations, however, can also be used to make predictions for future developments (see, e.g., Jacobson/Karlsson, 2004 and Jochmann/Leon-Gonzalez, 2004). Jacobson and Karlsson (2004) use the Bayesian approach to evaluate the relevance of a number of potential indicators in forcasting inflation in Sweden. The resulting knowledge about the adequate indicators can then be used to make predictions. Jochmann and Leon-Gonzalez (2004) predict the demand for health care with a model that classifies the population into classes according to their health status, where they used the Bayesian principle to estimate the parameters and features of the model.

The fifth kind of model are abductive simulation models, i.e. *simulations in which empirical data is used as much as is available to specify the model, while keeping the model as general as necessary*. In this kind of approach, the aim to develop a realistic model is put before all other criteria, such as keeping the model simple or developing a well-specified model. Hence, in contrast to the other four approaches, usually no specific model is obtained at the end of this approach (the approach does not necessarily end in the lower range in Figure 2). This has had quite some impact on the interpretation and analysis of the simulation results.

The approach starts, similar to Bayesian simulations, with the set up of a very general model that should represent all mechanisms and processes that could occur in reality and play a role for the analyzed subject. In contrast to Bayesian simulations, empirical knowledge about the modeled mechanisms and processes is then immediately used to reduce the generality of the model. However, it is crucial in this approach that the generality is reduced only as far as empirical data allows. For example, if empirical evidence is available that shows that certain parameters fall into certain ranges, these ranges are used to restrict the model parameters. As a consequence, the simulations are started with a model based on some empirical knowledge but is still quite general and therefore located in the middle in Figure 2. Because of the model's generality, many different specifications have to be run, as done in the Bayesian approach. Furthermore, empirical data about the real characteristics and dynamics of the modeled system is also used, as in Bayesian simulations. However, it is not used to find the best specification of the model, but only to eliminate those specifications that are unable to produce the real characteristics and dynamics of the system. This means that empirical data about the system's behavior is used to further reduce the model's generality. Nevertheless, the model is usually not completely specified. It is argued that a set of model specifications remains that are all potential candidates for representing reality. Only characteristics that hold for all these specifications can be concluded to be real. Hence, this approach is based on an inference process in which empirical data about specific systems is used to reduce the generality of a simulation model and in which the resulting set of model specifications is used to generate knowledge about the kind of systems that are studied. This is a reductive process.

The approach was first used in Brenner (2004) in order to develop knowledge on the emergence of local clusters. To this end, all processes claimed in the literature to be involved in the emergence of local clusters have been included in a simulation model with varying importance and the common characteristics of the resulting dynamics have been studied. The approach was formalized and applied to the synthetic dye industry by Brenner and Murmann (2003). They used the empirical knowledge available for the synthetic dye industry to model its development from 1856 to 1913. The real development was used to check the realism of different model specifications. Finally, statements about the importance of the university system's responsiveness and the availability of chemists have been inferred. A more elaborate method description is presented in Werker and Brenner 2004.

3.2 Four Steps to Build and Analyse a Simulation Model

Most simulation approaches can be separated into three steps. These steps are usually conducted successively: first, the definition of assumptions and the set up of the simulation model, second, the conduction of the simulations, and third, the analysis of the simulation results. In some cases the simulation results are used to adapt the simulation model. This might be done with or without declaration in the final publication. If it is done according to a clear procedure that is declared in the publication, we treat it as part of the analysis or as a separate four step, which we call the classification of simulation models. The steps are successively presented in detail in the following.

In the *first step of a simulation model* assumptions are defined and the model is set up. Here, we can use the three dimensions explained in Section 2.1: theoretical-empirical and general-specific as well as the likelihood that the model correctly represents reality (see the ellipses in Figure 2). Data might be used for all aspects of the simulation model. We distinguish two very different uses here. A simulation model is always built on the basis of assumptions about the interaction between the model elements. The simulation is then used to transfer these assumptions into dynamics or characteristics of the whole model. Hence, we can distinguish between the underlying interactions and mechanisms built into the simulation model and the resulting dynamics and characteristics of the simulation model. Let us call the former the *underlying level* and the latter the *resulting level*. We might then also distinguish data into *empirical data about the underlying mechanisms* that corresponds with the underlying level and *empirical data about the system's dynamics and characteristics* that corresponds with the resulting level. All data used in simulation approaches is of one of these two types. In the first step, usually only empirical data about the underlying level is used. However, as mentioned above, some approaches, especially traditional simulations and history-friendly models, use some knowledge about the resulting level in the model set-up. This is often done without stating it in the final publications. Different models are developed and tested and the one that fits the empirical knowledge about the resulting level best is taken as final and exclusively presented.

The second step is the *conduction of the simulations*. Once the assumptions are defined and the simulation model is set up, simulations can be run. Simulations are always a deductive act. They provide us with information about the implications of assumptions.

The differences between the approaches taken in the literature in how they conduct simulations are of less interest in this paper. Of course, the simulations are run in different computer languages and on different platforms. Furthermore, in some cases one simulation is run for each setting while, in other approaches, multiple runs are conducted. However, this is mainly caused by the structure of the simulation model. Stochastic models make multiple runs necessary.

Many other differences are caused by the way in which simulation results are analysed. If it is intended to analyse the robustness of the results with respect to some parameters, the simulation has to be run for various values of these parameters. If a general model is used, meaning that the modeller assumes that some parameters cannot be specified, for all possible values of these parameters simulations have to be conducted. Usually some parameter ranges can be defined (a comprehensive discussion can be found in Werker & Brenner 2004). This implies an infinite number of possible values for parameters, so that not all possible values can be simulated. A Monte-Carlo approach is used in such a case.

To sum up, the way in which the simulation runs are conducted depends very much on the set up of the simulation model and the way in which the results are analysed. Therefore, the discussion in this paper concentrates on the first and third step, the set up of the model and its analysis.

In the third step the *simulation results are derived*. There are many different ways in which the results of simulations can be used. The approach taken depends very much on the research question addressed. This is further discussed in Section 2. Here, we technically discuss various analyses and uses possible and used in the literature. We identify three different ways to deal with the results of simulations:

- Characterisation: In many simulation approaches the simulation results are used to study the characteristics of the system that has been modelled. This means that the simulation results are treated similar to empirical data. Sometimes the resulting dynamics or characteristics are simply described. Sometimes they are analysed with the help of statistical tools. Such an analysis also allows to analyse the relationship between parameters, initial conditions, or specifications of the simulation model and the dynamics or outcomes of the simulation runs. This is usually done in order to detect causal relations between the model assumptions and their implications. The aim is to obtain detailed knowledge about the system described by the simulation model. Whether valid knowledge is obtained depends crucially on the realism of the simulation model.

- Comparison: It is also quite common to compare the simulation results to empirical data, in this case empirical data about the system's dynamics and characteristics (*resulting level*). The approaches that use such a comparison conduct the comparison in very different ways. Many simulation approaches in heterodox economics simply use stylised facts to check whether the simulation results are plausible. In contrast, in the Bayesian approach a statistical comparison of the results of various simulation runs with different parameter values or model specifications with empirical data is conducted. The aim is to identify those model settings for which the simulation results are in line with the empirical data. Also many approaches that aim to predict future events use a comparison of simulation results with empirical data to check the adequateness of their simulation model.
- Prediction: Simulations are also frequently used to predict future developments. Both, deterministic and stochastic predictions are possible, depending on the characteristics of the model. In both cases the modeller is usually interested in predicting one or a few variables, so that the analysis concentrates on these variables.

In addition to the usual steps described above, we add a potential *fourth step of classifying the systems of models* here. This fourth step is based on the abductive simulation approach. In this approach the available empirical data on the underlying level is used to specify the simulation model as much as possible. Then, with the help of a Monte-Carlo approach and a comparison of simulation results with empirical data on the resulting level, the model is further specified. The result is a simulation model with some degree of generality. Two ways of a further procedure are possible.

On the one hand, the resulting model can be used for different analyses, such as characterisation, relationship analysis or prediction. On the other hand, if empirical data is available on the resulting level for different real systems, we could separately compare each real system. We could also group these real systems in different classes and conduct to an analyse for each class of systems. Through this we obtain different specifications of the simulation models that refer to different groups of real systems. The characteristics of these specifications can be analysed and compared. We might even do the comparison for each real system separately and use the comparison of the characteristics of the resulting model specifications for the classification of the real system (see Werker & Brenner 2004 for a description of this procedure). In this way, we obtain classes of systems and knowledge about their characteristics and dynamics.

All these proposals imply that we go back and forth between the underlying level and the resulting level with the help of simulations. By this we transfer empirical knowledge from on a level to the other and create new knowledge. This is a process of abduction and we believe that this process is very helpful for understanding economic systems and that simulations are very adequate tools to support this logical process.

To sum up, we classify the above-described frequently used approaches according to the structure that has been developed above. This aims to give an overview on what combinations of the above steps are possible and often used in the literature. It also gives a picture of how much methodological variance there is in the existing literature.

Approach	Step 1	Step 2	Step 3	Step 4
Traditional	Rather theoretical, specific	Usually one specification run	Characterisation	-
Microsimulations	Empirical, specific	One specification run	Prediction, sometimes Comparison	-
Bayesian	Theoretical, general	Many specifications run (sometimes Monte-Carlo)	Comparison	-
History-friendly	Rather	One	Comparison	-

	empirical, specific	specification run, with sensitivity analysis	and Characterisation	
Abductive	As empirical as possible, as general as necessary	Many specification runs (Monte-Carlo)	Comparison and Characterisation	Classification

Table 1: Common simulation approaches and the steps that they use.

4. A Practical Guide to Simulation Models

This section aims to give advice about how to conduct simulations depending on the research question scholars seek to answer. Computational economics still misses some universal standards in how simulation approaches should be conducted. Every researcher proceeds according to her own preferences. Above, we have distinguished a number of different steps that are usually conducted in simulation approaches. A standard would give some advice about which steps have or can be taken and how they should be conducted. One specific standard for all computational economics cannot be established because different research questions require different approaches. However, some advice that differentiates between different research questions can be given. To this end, the simulation steps are discussed in a sequence.

4.1 Set up of simulation model

Above we have shown that the set up of a simulation model can be characterised in a two-dimensional space (see Figure 1). We have also shown that there is a line of realistic models in this space. It is evident that if we want to study real causal relationships, make predictions, or characterise and compare different real systems, we must have a realistic simulation model. Therefore, it is important – also for the reputation of computational economics – that the reader can judge the realism of the model. This means that modellers should state all considerations made and all other models tested to reach the model presented. Furthermore, all assumptions made have to be explained. All assumptions not empirically proven (within the paper or in the literature) move the simulation model away from the realistic frontier. How decisive such assumptions are can only be subjectively estimated. However, the reader should be able to make an individual judgement.

In most cases a completely empirically based model, as used or intended in micro-simulations, is not feasible. There is, however, an alternative option, which is usually neglected in the literature, to deal with insufficient empirical knowledge. The simulation model can be kept as general as necessary, as it is proposed in the abductive approach. This means that in all aspects in which we cannot or will not use empirical data to specify the simulation model we have to keep the model general. In practical terms, we have to allow all parameters that we are not able to fix or restrict on the basis of empirical data, to take all logically possible values. A trade-off between collecting empirical knowledge to restrict the generality of the model and conducting a lot of simulation runs for different parameter values is the consequence. This trade-off is further discussed in Section 4.3.

Sometimes it is claimed that the realism of a simulation model is not an important criterion. There are three arguments in favour of this. First, realistic models are usually very complex, so that it is difficult to understand and analyze the relevant mechanisms in the model. Hence, many authors argue for simple models. However, we should clarify what can be reached with these models. Let us exclude from this discussion situations where simple models adequately represent the studied part of the real world (these situations are rare), and consider situations where reality is very much simplified by the simulation model. In such a case, the simulation model is not realistic. What we obtain by running the simulations is knowledge about the implications of the assumptions put into the model. These assumptions are of a theoretical nature because they do not match reality. Thus, we have an applied mathematical exercise in which we obtain the implications of theoretical assumptions.

This is, of course, of some value, but should not be mixed up with knowledge about reality. It depends on the research question whether such an approach is adequate.

Second, it is sometimes argued that the consequences of hypothetical situations or hypothetical changes of the real situation are to be studied. In this case, the model's realism seems to be less important. However, if the results of the study should not serve purely theoretical purposes, the system modelled should, at least, to some extent, be realistic. This is the case, for example, if questions, such as whether a different market system could change trading efficiency or whether certain previous policies would have changed historical developments, are to be answered. The simulation model should in these cases be realistic, except for the aspects that are explicitly changed to study the impact of this change.

Third, in some cases, for example if the aim is to predict developments, it might be less important to correctly model the underlying processes and mechanisms as long as the resulting dynamics are realistic (this is the old as-if-argument by Freeman). The same holds for parts of a model that are not at the centre of the research question. However, if such an argument is used, it is very important to prove that the model leads to real dynamics and characteristics. A comparison with empirical data about the real system's dynamics and characteristics is, therefore, much more important than with empirical data related to the underlying level. When interpreting the results some care is necessary: Obtaining a good fit of the simulations dynamics and characteristics does not mean that the underlying mechanisms are adequately described. Many different models might be set up that cause the same characteristics and dynamics. What is obtained is a model that does nothing more or nothing less than adequately describes reality on the resulting level.

4.2 Conduction of simulation

The conduction of simulation depends mainly on how Steps 1 and 3 are taken. Hence, we do not differentiate our proposal for the second step because its aim mainly influences the procedure in Steps 1 and 3. Here, only one general aspect is discussed: the use of multiple simulation runs because the real value of parameters is unknown.

It has been discussed above that a realistic model is reached by empirically fitting the structure and parameters of the model and by keeping the model general whenever such a fitting is not possible. In economic modelling, a fitting of the complete structure and all parameters to exact values is not possible in most cases. Let us assume that all aspects of the model's structure and all parameters that cannot be empirically measured are reflected by unfitted parameters. Furthermore, let us assume that for all these parameters ranges have been defined, into which real values fall with a probability of almost one, either by logical arguments or by empirical estimations.

In order to obtain a realistic analysis, we need to run the simulation for each set of parameter values that fall into the defined ranges. If we do this, we can be almost sure that one of the simulation runs represents reality. If the model set up is less general in those aspects that are not empirically proven, the less sure we are that the simulation represents reality. However, if we keep the model sufficiently general, only for one set of parameters, the model will represent reality and we do not know for which parameter set. This has to be kept in mind in interpreting the result. Furthermore, it should be clearly stated in simulation approach if and where the simulation model is specified more than the available empirical data allows.

Furthermore, a trade-off clearly appears here. The collection of empirical data for the set up of a simulation model is usually very cumbersome. The use of empirical data can be substituted by keeping the model very general. However, this has two impacts. First, more simulations have to be run because there are more possible parameter sets. This increases the necessary computer time. Second, only one of these many simulation runs represents reality, although we do not know which simulation run is the most realistic. As a consequence, only those results are reliable that result from all simulation runs for all possible parameter sets. We obtain, in general, less reliable results than with a more specific simulation model.

Finally, we have to address the question of how we treat model parameters that could take an infinite number of different values. While defining ranges for parameters that are not restricted to natural numbers, this might easily happen. A Monte-Carlo approach seems to fit here. In such an approach the parameter values are randomly picked from their range for each simulation run. The more simulations are run in this way, the more likely the realistic parameter set is also used. However, we can never be sure that the realistic parameter set has been randomly picked. But there are methods to calculate the probability that a certain characteristic holds for the realistic model dependent on how many simulations are run and whether this characteristic holds.

To sum up, this section is a plea for the use of empirical data to specify the simulation model. However, we do not claim that every modeller has to do so. What we claim is that if less or no empirical data is used, it must be stated by the modeller including the way in which the model could deviate from being realistic or to keep the model general. A general simulation model is scientifically fine but has some disadvantages as stated above.

4.3 Inference of Simulation Results

The simulation outcomes can be used in a variety of ways depending on the research question. If the aim is to infer knowledge about the reality, such as predictions about the future or knowledge about causal relationships or characteristics of the system, the simulation results can be analysed to obtain this knowledge. This means that in these cases simulations are interpreted as representations of reality. In these cases, the realism of the simulation model is of crucial importance.

A somewhat different situation occurs if the aim is to infer knowledge about mechanisms on the underlying level from the comparison of simulation results with empirical data on the resulting level. This means, in principle, that there are different models about the underlying mechanisms and processes that cause a certain empirically observed system behaviour. The aim is to identify those mechanisms and processes that cause this behaviour. To this end, all possible specifications of the quite general simulation model are simulated and results are compared with the empirical knowledge about the system’s behaviour. If the simulation model is stochastic, a

Bayesian approach can be used. For each studied specification we are able to state whether it is able to produce real behaviour or with what probability it produces real behaviour. If the simulation model is sufficiently general to contain reality with almost certainty (realistic line in Figure 1), we obtain a list of all potential underlying mechanisms or a list of their likelihood to be real. In the case of a simulation model that falls short of being realistic (below the realistic line in Figure 1) we obtain such a list only for the studied underlying mechanisms. There might be others that cause the same system behaviour and one of them could be the real one. Two situations could occur. First, there might be only one specification that is able to reproduce the real system's behaviour. If one can be quite sure that all potential mechanisms and processes on the underlying level are tested, it can be presumed that the suitable model is found. If only part of all potential underlying mechanisms and processes are studied, all that is gained is that some underlying mechanisms and processes can be excluded from the list of potential causes. Second, there might be different specifications that represent the real behaviour of the system. The results can then be used to reduce the generality of the simulation model. The resulting model, in turn, can then be used to study model characteristics – being a representation of reality – as described above. It might also be further used to classify systems of models (see the next section).

4.4 Classification of systems

Above we discussed how empirical knowledge could be transferred from the resulting level to the underlying level. To achieve this, various model specifications have been checked by comparing their results with empirical data on the resulting level. Hence, a simulation approach allows knowledge transfer in both directions between the underlying and resulting level. Let us assume that we start from a very general simulation model with many unspecified parameters. Then, we can use empirical data on the underlying level to estimate parameters or restrict them to certain ranges. As a second step, we can use empirical data on the resulting level to check for which parameter sets the simulation leads to the correct results. This can be used to further restrict the parameters. Finally, the resulting model can be analysed as described above.

Alternatively, we can also use this procedure to classify systems. There are two ways to do this. First, we could already classify the real systems and use for modelling each category of systems (e.g. manufacturing versus service sector) only data that is empirically obtained for this category. We obtain differently specified simulation models for the different categories of systems and can study the resulting differences in their characteristics, relationships between variables and predicted future developments.

Second, we might have a number of different systems (e.g. industries) without knowing how to classify them. Using each time only the empirical data for one of them, we obtain a specification of the simulation model. The resulting simulation models can be examined for similarities and differences and a categorisation can be established on this basis.

In any case a set of categories finally results for which we know or may create via simulations a number of aspects, such as the underlying mechanisms, characteristics, relationships between variables, dynamics and predictions. The different facts are related to each other via the simulation approach. Therefore, knowledge about certain characteristics can be used to categorise a new system, which in turn leads to knowledge about other characteristics. This is called abduction. The simulation approach is perfectly suited to do such abduction, although it is, so far, rarely used in the literature.

5. Conclusions

In this paper we introduced a categorization of simulation models. Moreover, we provided an explicit overview of the steps that lead to a simulation model. This enabled us to highlight the advantages and disadvantages of various simulation approaches in the light of the different ways in which the steps are taken.

There two main conclusions that we draw. First, we argue that computational economists should be much more careful in setting up their model. First of all, we show that empirical data should be used more often where it is available, because this leads to much better founded simulation models. In addition, modelers should be more explicit about the way in which they set up their model, because this makes it easier for others to understand the working of simulation models when reading about them. In particular, if different kinds of models are tested and compared with knowledge about the real dynamics, this should be made explicit. If no empirical evidence is used, the model should ideally be kept general or the lack of evidence should, at least, be stated and also discussed. This would make the restrictions of such an approach visible.

Second, simulations seem to be the perfect tools to transfer (empirical) knowledge from the underlying level to the resulting level and vice versa. This means that in a simulation approach one can go forth and back between the assumptions in the simulation model and the resulting characteristics and dynamics of the system. Empirical data on both sides can be used to improve the knowledge about the system. Even a classification of systems on the basis of such inference is possible. Simulation approaches offer scientific potentials that are far from used in the existing literature, because they can truly use abduction.

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