Leigh Tesfatsion
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Game Theory: Basic Concepts and Terminology

A **GAME** consists of:

- a collection of decision-makers, called *players*;
- the possible information states of each player at each decision time;
- the collection of feasible moves (decisions, actions, plays,...) that each player can choose to make in each of his possible information states;
- a procedure for determining how the move choices of all the players collectively determine the possible outcomes of the game;
- preferences of the individual players over these possible outcomes, typically measured by a *utility* or *payoff* function.

EMPLOYER

WORKER		**C**	**D**
	C	**(40,40)**	**(10,60)**
	D	**(60,10)**	**(20,20)**

Illustrative Modeling of a Work-Site Interaction as a "Prisoner's Dilemma Game"

D = Defect (Shirk) C = Cooperate (Work Hard),

(P1,P2) = (Worker Payoff, Employer Payoff)

A **PURE STRATEGY** for a player in a particular game is a complete contingency plan, i.e., a plan describing what move that player should take in each of his possible information states.

A **MIXED STRATEGY** for a player i in a particular game is a probability distribution defined over the collection $\mathcal{S}_i$ of player i's feasible pure strategy choices. That is, a mixed strategy assigns a nonnegative probability Prob(s) to each pure strategy s in $\mathcal{S}_i$, with

$$\sum_{s \in \mathcal{S}_i} \mathrm{Prob}(s) \ = \ 1 \ . \tag{1}$$

EXPOSITIONAL NOTE:

For simplicity, the remainder of these brief notes will develop definitions in terms of pure strategies; the unqualified use of "strategy" will always refer to pure strategy. Extension to mixed strategies is conceptually straightforward.

ONE-STAGE SIMULTANEOUS-MOVE N-PLAYER GAME:

- The game is played just once among N players.
- Each of the N players *simultaneously* chooses a strategy (move) based on his current information state, where this information state does *not* include knowledge of the strategy choices of any other player.
- A payoff (reward, return, utility outcome,...) for each player is then determined as a function of the N simultaneously-chosen strategies of the N players.

Note: For ONE-stage games, there is only one decision time. Consequently, a choice of a strategy based on a current information state is the same as the choice of a move based on this current information state.

ITERATED SIMULTANEOUS-MOVE N-PLAYER GAME:

- The game is played among N players over successive iterations $T = 1, 2, \ldots$ TMax.
- In each iteration T, each of the N players *simultaneously* makes a move (action, play, decision,...) conditional on his current information state, where this information state does *not* include the iteration-T move of any other player.
- An iteration-T payoff (reward, return, utility outcome,...) is then determined for each player as a function of the N simultaneously-made moves of the N players in iteration T.
- If T $<$ TMax, the next iteration T+1 then commences.
- The information states of the players at the beginning of iteration T+1 are typically updated to include at least some information regarding the moves, payoffs, and/or outcomes from the previous iteration T.

Note: For ITERATED games there are multiple decision times. Consequently, a choice of a move based on a current information state does not constitute a strategy (complete contingency plan). Rather, a strategy is the choice of a move for the current iteration, given the current information state, together with a designation of what move to choose in each future iteration conditional on every possible future information state.

"PAYOFF MATRIX" FOR A ONE-STAGE SIMULTANEOUS-MOVE 2-PLAYER GAME:

Consider a one-stage simultaneous-move 2-player game in which each player must choose to play one of M feasible strategies $S_1,\ldots,S_M$. The *Payoff Matrix* for this 2-player game then consists of an $M \times M$ table that gives the payoff received by each of the two players under each feasible combination of moves the two players can choose to make.

More precisely, each of the M rows of the table corresponds to a feasible strategy choice by Player 1, and each of the M columns of the table corresponds to a feasible strategy choice by Player 2. The entry in the ith row and jth column of this $M \times M$ table then consists of a pair of values $(P_1(i,j), P_2(i,j))$.

The first value $P_1(i,j)$ denotes the payoff received by Player 1 when Player 1 chooses strategy S_i and Player 2 chooses strategy S_j, and the second value $P_2(i,j)$ denotes the payoff received by Player 2 when Player 1 chooses strategy S_i and Player 2 chooses strategy S_j. **See the 2-player example depicted on the next page**.

This definition is easily generalized to the case in which each player has a different collection of feasible strategies to choose from (different by type and/or number).

EMPLOYER

		C	**D**
WORKER	**C**	**(40,40)**	**(10,60)**
	D	**(60,10)**	**(20,20)**

Illustrative Modeling of a Work-Site Interaction as a "Prisoner's Dilemma Game"

D = Defect (Shirk)   C = Cooperate (Work Hard),

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NASH EQUILIBRIUM FOR AN N-PLAYER GAME:

A specific combination (S_1^*,...,S_N^*) of feasible strategy choices for an N-player game, one strategy choice S_i^* for each player i, is called a *(Pure Strategy) Nash equilibrium* if no player i perceives any feasible way of achieving a higher payoff by switching unilaterally to another strategy S_i'.

DOMINANT STRATEGY FOR AN N-PLAYER GAME:

A feasible strategy for a player in an N-player game is said to be a *dominant strategy* for this player if it is this player's *best response* to *any* feasible choice of strategies for the other players.

For example, suppose S_1^* is a dominant strategy for player 1 in an N-player game. This means that, no matter what feasible combination of strategies $(S_2, \ldots, S_N)$ players 2 through N might choose to play, player 1 attains the highest feasible (expected) payoff if he chooses to play strategy S_1^*.

QUESTIONS:

(1) Does the previously depicted worker-employer game have a Nash equilibrium?

(2) Does either player in this game have a dominant strategy?

(3) What is the key distinction between a dominant strategy and a strategy constituting part of a Nash equilibrium?

PARETO EFFICIENCY:

Intuitive Definition:

A feasible combination of decisions for a collection of agents is said to be *Pareto efficient* if there does *not* exist another feasible combination of decisions under which each agent is at least as well off and some agent is strictly better off.

More Rigorous Definition: N-Player Game Context

For each $i = 1, \ldots, N$, let P_i denote the payoff attained by player i under a feasible strategy combination $S = (S_1,\ldots,S_N)$ for the N players. The strategy combination S is said to be *Pareto efficient* if there does *not* exist another feasible strategy combination S' under which each player i achieves at least as high a payoff as P_i and some player j achieves a strictly higher payoff than P_j. The payoff outcome $(P_1,\ldots,P_N)$ is then said to be a *Pareto efficient payoff outcome*.

QUESTION:

Does the previously depicted worker-employer game have a Pareto efficient strategy combination?

PARETO DOMINATION:

Intuitive Definition: A feasible combination of decisions for a collection of agents is said to be *Pareto dominated* if there *does* exist another feasible combination of decisions under which each agent is at least as well off and some agent is strictly better off.

More Rigorous Definition: N-Player Game Context For each $i = 1, \ldots, N$, let P_i denote the payoff attained by player i under a strategy combination $S = (S_1, \ldots, S_N)$ for the N players. The strategy combination S is said to be *Pareto dominated* if there *does* exist another feasible strategy combination S' under which each player i achieves at least as high a payoff as P_i and some player j achieves a strictly higher payoff than P_j.

QUESTION:

Does the previously depicted worker-employer game have strategy combinations that are Pareto dominated?

COORDINATION FAILURE AS COMMONLY DEFINED IN MACROECONOMICS:

Intuitive Definition: In macroeconomics, a combination of decisions for a collection of agents is commonly said to exhibit *coordination failure* if mutual gains, attainable by a collective switch to a different feasible combination of decisions, are not realized because no individual agent perceives any feasible way to increase their own gain by a unilateral deviation from their current decision.

More Rigorous Definition: N-Player Game Context In macroeconomics, a strategy combination $S = (S_1,\ldots,S_N)$ is commonly said to exhibit *coordination failure* if it is a Pareto-dominated Nash equilibrium.

QUESTIONS:

Does the previously depicted worker-employer game have a move combination that exhibits coordination failure?

Might the *iterative* play of this worker-employer game help alleviate coordination failure problems?

Comments on Coordination Failure Definitions:

In current macroeconomic theory, the focus is on (Nash) equilibria; and, as indicated above, "coordination failure" is commonly used to refer to a feasible Nash equilibrium strategy configuration that is Pareto dominated by some other feasible strategy configuration.

Example: The pure-strategy Nash equilibrium (Defect,Defect) in a one-stage two-player **Prisoner's Dilemma Game**.

In macroeconomics, the focus is on how a group of individuals can become "stuck" in a suboptimal situation because no individual sees any way to improve his/her situation by unilateral actions. Improvements that hurt no one and help at least someone can only come about through strategy changes undertaken simultaneously by a coalition of agents as a result of some kind of newly introduced strategy coordination mechanism (institution).

The study of strategy coordination mechanisms is an important aspect of "mechanism design," a research area pioneered by Leonid Hurwicz, Eric Maskin, & Roger Myerson (winners of the 2007 Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel).

However, there is in fact no single right way to define "coordination failure" for all purposes. In microeconomics, and in game theory more generally, "coordination failure" has been used in a variety of different ways, not all of them consistent with the common macroeconomic usage.

For example:

- Some researchers require the Pareto-dominating strategy configuration to itself be a Nash equilibrium. That is, the focus is on a sequence of two or more Pareto-ranked Nash equilibria, where each successive Nash equilibrium Pareto dominates all earlier ones. **Example:** The Pareto-ranked Nash equilibria along the diagonal of the payoff matrix for a **Stag-Hunt Game**.

- Some researchers say that a game can result in "coordination failure" only if it is possible for players to end up in some kind of *disequilibrium* strategy configuration. **Example:** The possibility of off-diagonal outcomes in the **Battle of the Sexes Game** when each player is uncertain about what the other player will do (and this uncertainty is common knowledge).

- Still other researchers categorize "coordination failure" into distinct categories differentiated by the source of the coordination problem. For example, is the coordination failure due to uncertainty about what other players will do? Or is it due to a lack of alignment in player objectives? Or....?

The following slides present five famous types of one-stage two-player games. For each type of game, the Nash equilibrium and/or Pareto efficiency properties (if any) of the feasible strategy configurations are identified, and the possibility of "coordination failure" is discussed.

Five Types of One-Stage Two-Player Games

Leigh Tesfatsion
Department of Economics
Iowa State University, Ames, IA

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Game Analysis

The game analysis on the following slides is restricted to

- Two-player games
- Choice of pure strategies (as opposed to choice of mixed strategies)
- One-stage games (as opposed to iterated games), so a strategy reduces to a move.

Such games are said to be *symmetric* if

- Both players have the same set of moves;
- If the players swap their moves, they also swap their payoffs.

Five Illustrative One-Stage Two-Player Games

Symmetric Games

- Type 1: Prisoner's Dilemma
- Type 2: Dead Lock
- Type 3: Chicken
- Type 4: Stag Hunt

Non-Symmetric Game

- Battle of the Sexes

Single-Stage Payoff Matrix for a General Symmetric Two-Player Game

Action choices for each player: C or D

Possible payoffs for each player: R, S, T, or P

		Player 2	
		C	D
Player 1	C	(R, R)	(S, T)
	D	(T, S)	(P, P)

Type 1. Prisoner's Dilemma (PD) Game
$T > R > P > S$ ($[T+S]/2 < R$)

	C	D
C	(R, R)	(S, T)
D	(T, S)	(P, P)

Payoffs:

R – Reward for cooperation;

T – Temptation to defect;

S – Sucker's payoff;

P – Punishment for defection

Type 1: PD Game … Continued
T > R > P > S ([T+S]/2 < R)

EXAMPLE:

	C	D
C	(R=3, R=3)	(S=0, T=5)
D	(T=5, S=0)	(P=1, P=1)

PURE-STRATEGY ANALYSIS FOR ONE-STAGE GAME:

(D,D) is the unique pure-strategy Nash equilibrium, and D is a dominant pure-strategy choice for each player. However, (C,C) Pareto-***dominates*** (D,D). The three choice pairs (C,C), (C,D), and (D,C) are all Pareto efficient, but (C,C) is the most ***socially*** efficient choice pair.

Type 2: Dead Lock Game
T > P > R > S

EXAMPLE:

	C	D
C	(R=1, R=1)	(S=0, T=3)
D	(T=3, S=0)	(P=2, P=2)

Type 2: Dead Lock Game … Continued
T > P > R > S

EXAMPLE…Continued:

	C	D
C	(R=1, R=1)	(S=0, T=3)
D	(T=3, S=0)	(P=2, P=2)

PURE-STRATEGY ANALYSIS FOR ONE-STAGE GAME:

(D,D) is a pure-strategy Nash equilibrium and D is a dominant pure-strategy choice for each player, as in the PD game. However, here (D,D) Pareto dominates (C,C), and indeed (D,D) is the most socially efficient outcome. The three choice pairs (D,D), (C,D), (D,C) are all Pareto efficient.

Type 3: Chicken Game

T > R > S > P

EXAMPLE:

	C	D
C	(R=2, R=2)	(S=1, T=3)
D	(T=3, S=1)	(P=0, P=0)

Original Interpretation: Two drivers drive towards each other to see who (if anyone) will "chicken out" by swerving.

"Rebel Without a Cause" (1955 movie) starring James Dean

C = Swerve; D = Drive Straight

Type 3: Chicken Game … Continued

T > R > S > P

EXAMPLE … Continued:

	C	D
C	(R=2, R=2)	(S=1, T=3)
D	(T=3, S=1)	(P=0, P=0)

PURE-STRATEGY ANALYSIS FOR ONE-STAGE GAME:

(C,D) and (D,C) are ***both*** pure-strategy Nash Equilibria, but ***neither*** Pareto dominates the other. ***Neither*** player has a dominant pure-strategy choice. The three choice pairs (C,C), (D,C), and (C,D) are ***all*** Pareto efficient and equally socially efficient.

Type 4: Stag Hunt Game

R > T > P > S

EXAMPLE:

	C	D
C	(R=3, R=3)	(S=0, T=2)
D	(T=2, S=0)	(P=1, P=1)

ORIGINAL STORY (Jean Jacques Rousseau, French Philosopher): Each hunter chooses either C (stay in position to hunt a stag - an adult deer) or D (go after a running rabbit). Hunting stags is quite challenging – to be successful it requires BOTH hunters to choose C and not be tempted by the running rabbit.

ANOTHER STORY: Next to the last day of school, you and your friend decided to do something cool and show up on the last day of school with a crazy haircut. A night of indecision follows ....

Type 4: Stag Hunt Game … Continued
R > T > P > S

EXAMPLE … Continued:

	C	D
C	(R=3, R=3)	(S=0, T=2)
D	(T=2, S=0)	(P=1, P=1)

PURE-STRATEGY ANALYSIS FOR ONE-STAGE GAME:

(C,C) and (D,D) are "Pareto-ranked" pure-strategy Nash equilibria, in the following sense. Both are pure-strategy Nash equilibria, but (C,C) Pareto-dominates (D,D). Neither player has a dominant pure-strategy choice – here each player is better off doing whatever the other is doing. The only Pareto efficient pure-strategy choice is (C,C), which is also socially efficient.

A Non-Symmetric Game: Battle of the Sexes

EXAMPLE:

		Wife	
		Bowl	Ballet
Husband	Bowl	(12, 1)	(0, 0)
	Ballet	(0, 0)	(1,12)

The Basic Story:

A couple has agreed to meet for a date this evening but each has forgotten whether the date is for bowling or attending a ballet (and the fact each has forgotten is common knowledge). The husband prefers bowling and the wife prefers ballet, but both would prefer to be with each other rather than separately attending their preferred type of event. If they cannot communicate, where should each go?

Battle of the Sexes Game … Continued

EXAMPLE…Continued:

		Wife	
		Bowl	Ballet
Husband	Bowl	(12, 1)	(0, 0)
	Ballet	(0, 0)	(1,12)

PURE-STRATEGY ANALYSIS FOR ONE-STAGE GAME:

The husband prefers going bowling to attending a ballet, and the wife prefers attending a ballet to going bowling. However, both players prefer to be with each other rather than going alone to their preferred type of event. Each of the diagonal outcomes (12,1) and (1,12) is a Pareto-efficient Nash equilibrium. However, without knowing what the other will choose, it is possible each player will choose in such a manner that they both end up at an off-diagonal outcome. At such outcomes, *neither* player is at their preferred type of event and *each* player is at this event *alone*.

Extended Game Analysis

- An important (and fun) extension of traditional game analysis is to consider determination of game partners in combination with determination of game strategies.
- Real-world players often have some ability to determine their game partners by two means: (i) *choosing* to direct game offers to preferred game partners; and (ii) *refusing* game offers received from less desirable game partners.

Example: The Trade Network Game (TNG) Laboratory

— Endogenous network formation among strategically interacting buyers, dealers, and sellers able to choose/refuse their trade partners

— Matched trade partners engage in bilateral trades (2-person games)

— Blending of game theory & matching theory

— Demonstration software permits run-time visualization of network formation

— TNG Lab Homepage: http://www2.econ.iastate.edu/tesfatsi/tnghome.htm