Locational Marginal Pricing: When and Why Not?

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***Abstract*—This study establishes that Locational Marginal Pricing (LMP) is conceptually problematic for grid-supported centrally-managed wholesale power markets transitioning to decarbonized grid operations with increasingly diverse participants, hence with increasingly uncertain and volatile net loads. LMP assigns a common per-unit price LMP(b,T) ($/MWh) to each "next" unit (MWh) of grid-delivered energy, conditional on delivery location b and delivery period T. However, the valuation of this "next" unit by a market participant or system operator will typically depend strongly on the specific dynamic attributes of the path of power injections and/or withdrawals (MW) used to implement the delivery of this "next" unit at b during T. One option is to muddle through, forcing market participants and system operators to express benefit and cost valuations for "next" units of grid-delivered energy (MWh) in per-unit form ($/MWh) without regard for the true benefits and costs of flexible dynamic power delivery. Another option, illustrated in this study, is to explore alternative conceptually-coherent product definitions, settlement rules, and bid/offer contract formulations that permit electric power grids to function efficiently as flexibility-support insurance mechanisms enabling just-in-time power deliveries to meet just-in-time customer power demands and grid reliability requirements.**

***Index Terms*—Locational marginal pricing, grid-supported RTO/ISO-managed wholesale power markets, benefit/cost many-to-one measurement issues, problematic short-to-long emphasis, supply-offer rigidity, conceptually-consistent alternative market design, grids as flexibility-support insurance mechanisms.**

I. INTRODUCTION

LOCATIONAL Marginal Prices (LMPs) are dual-variable solutions for nodal power-balance constraints in Security-Constrained Economic Dispatch (SCED) optimizations carried out for grid-supported centrally-managed wholesale power markets, conditional on a designated ("committed") collection of generation units. Given suitable regularity conditions, the LMP determined at a grid-node b for an operating period T is the rate of change of the optimized SCED objective function for T with respect to a change in the constraint-constant for the power balance constraint at b [1].

Conceptual and computational-fragility cautions regarding marginal-price settlements for electric power systems have been reported for decades.

For example, in a seminal 1949 study [2], Boiteaux considers 'long-run" marginal pricing of energy (Wh) within a highly stylized market framework for the joint risk-free selection of a plant-size and a plant-conditional optimal energy production level to match a known constant or recurrent "normal load" (Wh). However, Boiteux qualifies his optimality conclusions carefully, highlighting the strength of his assumptions: no attention is paid to *physical grid* constraints, to *volumetric grid risk*, and to possible *load-forecasting difficulties* due to load volatility and uncertainty arising from weather conditions, active customer choice, or other factors.

In a well-known 1996 study, Wu *et al.* [3] identify various anomalous LMP properties within the context of a grid-supported centrally-managed wholesale power market. For example, if grid congestion (i.e., an active line-capacity constraint) occurs for some transmission line ℓ_i with a positive power flow, the LMP value at the start-node for ℓ_i must separate from the LMP value at the end-node for ℓ_i; but additional LMP separation can also arise for transmission lines ℓ_j that are *not* congested. This confounds the use of LMP separation as a direct indicator of grid-congested lines.

In a 2011 study [4], Li and Tesfatsion investigate the welfare ramifications of a key distinction between classic economic marginal-price settlements and LMP settlements in U.S. RTO/ISO-managed day-ahead markets. In the classic framework, market efficiency is measured on a 0 to 1 scale as the *actual* total net surplus extracted by buyers and suppliers divided by the *maximum* total net surplus that buyers and suppliers *could* feasibly extract; the implicit "market auctioneer" that determines competitive (marginal benefit = marginal cost) market-clearing prices never takes a cut of this total net surplus. In contrast, in the presence of grid congestion, the "congestion-rent" portion of total net surplus accruing to the RTO/ISO due to LMP separation can be surprisingly large with difficult-to-measure buyer/supplier welfare implications.

In research undertaken during 2008–2023, Sioshansi *et al.* [5], Eldridge *et al.* [6], and Feng [7] demonstrate that calculated LMP solutions can vary erratically in response to seemingly small changes in computational procedures. For example, changes in the stopping-rules implemented in the Security-Constrained Unit Commitment (SCUC) optimizations used to determine approximate convergence to optimal generation-unit commitments can induce changes in the resulting "optimal" unit commitments that in turn induce large changes in subsequent LMP solutions conditioned on these unit commitments. These variations in LMP solutions can result in substantial systematic (non-random) wealth transfers across market participants.

In a 2018 EPRI webinar, Ela [8] presents in dismaying detail the complex idiosyncratic sequence of if-then steps followed

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in each U.S. RTO/ISO-managed wholesale power market to transform initial LMP determinations into ultimate financial settlements for market participants. These steps include: dispatch deviation penalties, make-whole payments for recovery of start-up and no-load/min-gen costs, and uplift/make-whole payments for day-ahead profit assurance, price-volatility, alternative fuel usage, and lost opportunity costs not compensated through prices.

Finally, the 2021 study [9] discusses how LMP settlements appear to be a key factor contributing to three worrisome trends for U.S. RTO/ISO-managed wholesale power markets: (i) proliferation of "participation models" (market eligibility rules) functioning as artificial market entry barriers; (ii) proliferation of conceptually problematic "flexibility products" intended to facilitate the balancing of increasingly volatile and uncertain net load;[1] and (iii) proliferation of out-of-market make-whole payments to suppliers in response to increasing supplier revenue insufficiency.

Despite these expressed concerns, LMP settlement has been adopted and retained as the core energy pricing mechanism in all seven grid-supported U.S. RTO/ISO-managed wholesale power markets. A commonly expressed view is that, in analogy to competitive (marginal benefit = marginal cost) pricing for commodity spot markets, LMP settlement is guaranteed to achieve market efficiency for these markets apart from practical implementation issues. Thus, attention should remain focused on the resolution of these implementation issues, not on fundamental settlement-rule changes.

Nevertheless, in April 2022 the U.S. Federal Energy Regulatory Commission (FERC) initiated a formal call [10] for a fundamental reconsideration of current U.S. RTO/ISO-managed wholesale power market operations. This formal call explicitly requests that dependence on LMP settlements be included in this reconsideration.

As detailed in Report [11], prepared in response to [10], the LMP two-settlement system reflects the misleading static viewpoint that RTOs/ISOs are fiduciary managers for weakly cross-correlated collections of competitive short-run energy markets, conditional on location and time. Cross-correlation arises from the need to schedule feasible transfers of generation across delivery locations to ensure the continual balancing of scheduled energy demands and supplies at each location.

The dynamic reality is far more daunting. RTOs/ISOs are fiduciary "conductors" tasked with orchestrating the availability and subsequent possible dispatch of "power-paths" produced by increasingly diverse power resources to service just-in-time power demands of increasingly diverse customers while meeting just-in-time power requirements for grid reliability. As illustrated in Fig. 1, a *power-path* is a sequence $\mathbf{p}_m(\mathrm{T}) = (p_m(t) \mid t \in \mathrm{T})$ of injections and/or withdrawals of power $p_m(t)$ (MW) by a market participant m at a *single* grid location $b(m)$ during a designated time-interval T, where $b(m)$ denotes m's electrical point-of-connection to the grid.

[1] *Net load* for a transmission grid consists of power withdrawals and inadvertent power losses *net* of non-dispatched power injections by Intermittent Power Resources (IPRs) – such as wind and PV solar power facilities – whose power injections are not fully controllable by centrally-managed dispatch.

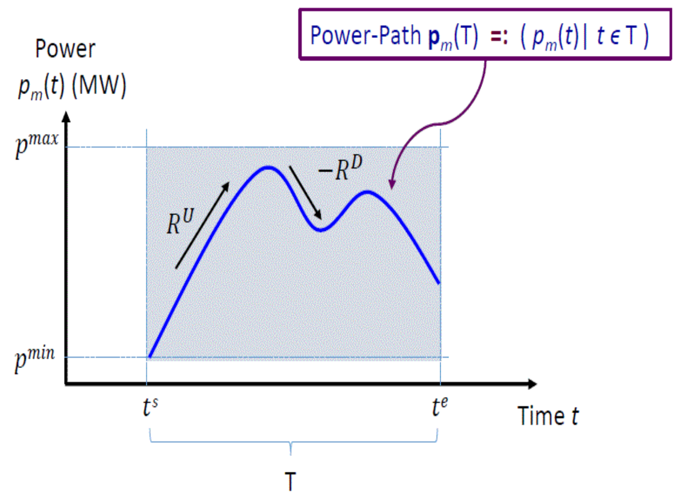

Fig. 1. ***Depiction of a power-path*** for a grid-supported centrally-managed wholesale power market. A power-path $\mathbf{p}_m(\mathrm{T}) = (p_m(t) \mid t \in \mathrm{T})$ is a sequence of injections and/or withdrawals of power $p_m(t)$ (MW) by a market participant m at a single grid-location $b(m)$ during a designated time-interval T.

Report [11] identifies and illustrates seven conceptually-problematic aspects of the LMP two-settlement system for RTO/ISO-managed markets, summarized as follows:

Product-Definition and Pricing Issues:

(1) Conceptually-problematic focus on grid-delivered amounts of *energy* (MWh) as the basic transacted product.

(2) Conceptually-problematic use of per-unit LMP (\$/MWh) settlements for *grid-delivered* energy.

(3) Conceptually-problematic introduction of ancillary "flexibility" products – such as Ramp (MW/min) and Capacity (MW) – intended to support net-load balancing for scheduled grid-delivered energy (MWh) despite being strongly correlated with grid-delivered energy.

Settlement-Timing Issue:

(4) Successive provisional forward-market determination of LMP settlements in advance of final ex-post LMP settlements for actual real-time dispatched performance results in time-inconsistent settlements, hence in unnecessarily complex and confusing settlement rules.

Supply-Offer Formulation Issues:

(5) Suppliers are *forced* to express variable supply costs as functions of grid-delivered energy amounts (MWh).

(6) Suppliers are *not required* to distinguish their avoidable costs from their non-avoidable ("sunk") costs.

(7) Suppliers are *unable* to specify their supply offers in a manner that ensures their *revenue sufficiency*, i.e., in a manner that ensures their *market*-attained earnings are sufficient to cover their *market*-incurred avoidable costs.

Issues **(1)**–**(2)** – essential underpinnings for issues **(3)**–**(7)** – raise serious concerns about the continued reliance on LMP settlements in U.S. RTO/ISO-managed wholesale power markets. An accurate assessment of these concerns requires careful multi-disciplinary understanding of fundamental measurement and economic concepts relevant for the physical operation of real-world grid-supported electric power markets.

The goal of the current study is to provide this multi-disciplinary understanding in a clear yet rigorous manner, with no reliance on "proofs by assertion" or "proofs by authority."

Section II provides a concise description of the basic LMP two-settlement system at the core of current U.S. RTO/ISO-managed wholesale power market operations. Basic measurement concepts are defined and illustrated in Section III, and basic economic concepts are defined and illustrated in Section IV. These concepts are used in Section V to highlight and explain the extremely strong assumptions underlying the definition and derived optimality properties of competitive markets based on marginal per-unit pricing.

Adverse implications of these strong assumptions for the reliance on LMP settlements in current U.S. RTO/ISO-managed wholesale power markets are carefully developed and illustrated in Section VI. An alternative conceptually-consistent "swing-contract" design [9] for grid-supported centrally-managed wholesale power markets is briefly reviewed in Section VII as proof-of-concept that other promising paths could be taken. Section VIII provides concluding remarks.

II. Legacy Core Design of U.S. RTO/ISO-Managed Wholesale Power Markets

The development of the legacy LMP two-settlement system supporting current U.S. RTO/ISO-managed wholesale power market operations can be traced in a series of reports released by the U.S. Federal Energy Regulatory Commission (FERC), culminating in a 2003 White Paper [12].

In [12], FERC envisions grid-delivered energy (power accumulations) at designated grid delivery locations during designated operating periods to be the basic transacted product. These grid-delivered energy quantities are to be determined by means of a daily bid/offer-based LMP-settled RTO/ISO-managed Day-Ahead Market (DAM) operating in tandem with a daily bid/offer-based LMP-settled RTO/ISO-managed Real-Time Market (RTM); see Fig. 2.

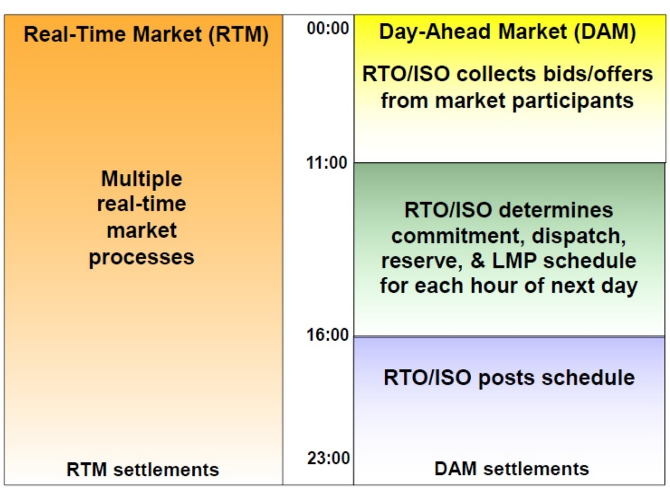

Fig. 2. Illustrative depiction of daily DAM/RTM operations under an LMP two-settlement system.

The overall goal of the LMP two-settlement system is to permit energy transactions at designated grid delivery locations during designated operating periods to be efficiently determined by the demand bids and supply offers of energy buyers and suppliers. With this overall goal in mind, the LMP two-settlement system is designed to reflect the determination of market-clearing prices and quantities in *competitive commodity spot markets* to an extent consistent with maintaining the reliability of a supporting physical transmission grid susceptible to transmission-line congestion.

The purpose of the RTO/ISO-managed DAM held on each day D is to commit RTO/ISO-dispatchable generation units for day D+1 that permit the RTO/ISO to ensure efficient continual net-load balancing during day D+1.

Load-Serving Entities (LSEs), acting on behalf of managed customers, submit demands bids into the day-D DAM for the purchase of energy at grid delivery locations for each hour H of day D+1. Each such demand bid can take the form of a *fixed (non-dispatched must-service) energy demand*; and it can also include or take the form of a *price-sensitive energy demand schedule*.

Generation units submit supply offers into the day-D DAM for energy sales at grid delivery locations for each hour H of day D+1. A supply offer can take the form of a *fixed (non-dispatched must-service) energy supply*. It can also include or take the form of a *dispatchable price-sensitive energy supply schedule* if the generation unit has installed real-time telemetry permitting the RTO/ISO to incrementally adjust the unit's energy supply by real-time dispatch signals.

The RTO/ISO conducts a bid/offer-based SCUC/SCED optimization conditional on current state conditions, submitted bids and offers, and forecasts for non-dispatched IPR injections/withdrawals of power at each grid delivery location during each hour H of day D+1. The optimization is also subject to system constraints that include a net-load power balance requirement at each grid delivery location for each hour H of day D+1.

The optimization determines a binary (yes/no) commitment solution for each dispatchable generation unit for each hour H of day D+1 indicating whether or not this generation unit is required to be available for possible RTO/ISO-dispatch during hour H of day D+1. It also determines anticipated dispatch schedules for price-sensitive energy bids and/or offers at each grid delivery location for each hour H of day D+1.

Settlements for cleared bids and offers are determined by *locational marginal pricing* [13]; that is, by the pricing of grid-delivered energy (MWh) conditional on grid delivery location and delivery period, subject to system constraints. The *Locational Marginal Price* LMP(b, H, D+1) ($/MWh) determined in a day-D DAM SCED optimization for scheduled energy deliveries at a grid delivery location b during some hour H of day D+1, conditional on previous SCUC-determined generation-unit commitments, is the dual variable solution for the power-balance constraint at b for hour H; see [1].

An RTM is a daily collection of sub-markets for near-term future time-periods τ with relatively short durations (e.g., 5 minutes). These RTM sub-markets are cleared by RTO/ISO-managed SCED optimizations conditional on previously-determined unit commitments plus RTO/ISO forecasts for fixed (non-dispatched must-service) energy bids and offers for τ. RTM SCED optimizations are similar in form to DAM SCED optimizations except that RTMs impose stricter restrictions on the submission of *price-sensitive* demand bids; see, for example, [14, Sec. 4.3].

The purpose of these RTM sub-markets (plus supplemental unit-commitment processes) is to permit the successive updating of previously determined optimal SCUC/SCED solutions to take into account updated RTO/ISO forecasts as well as unanticipated changes in other relevant factors. Any adjustments needed in the scheduled energy deliveries determined in the day-D DAM for some hour H of day D+1, as indicated by the solutions for RTM sub-markets conducted after the close of the day-D DAM but prior to hour H, are settled using the LMPs determined in these RTM sub-markets.

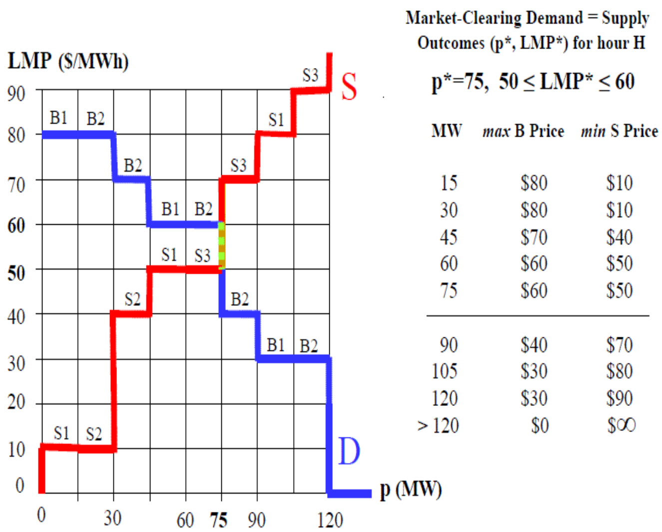

Fig. 3. Illustrative depiction of the optimal market-clearing (demand = supply) solution for the *maintained* power-withdrawal levels of cleared buyers and the *maintained* power-injection levels of cleared suppliers for a given hour H during operating day D+1, as determined by a bid/offer-based RTO/ISO-managed DAM SCED optimization conducted on day D.

Figure 3 illustrates the determination of an optimal market clearing demand-equals-supply (D=S) solution for a given hour H on day D+1 by means of a bid/offer-based RTO/ISO-managed DAM SCED optimization conducted during day D. For ease of depiction, absence of grid congestion and line losses is assumed to hold at this optimal solution.

The depicted solution for hour H of day D+1 consists of a *set* of points with a common optimal power level $p^* = 75$ (MW) and a range of optimal price levels LMP* ($/MWh). This optimal price indeterminacy arises because the demand bids and supply offers submitted to this DAM take a required step-function form that results in flat vertical and horizontal segments for the aggregate demand and supply schedules D and S.

The depicted aggregate demand and supply schedules D and S are constructed[2] from the LSE demand bids and generation-unit supply offers submitted to the day-D DAM by two LSE buyers (B1,B2) and three generation-unit suppliers (S1,S2,S3).

Note the optimal market-clearing outcomes depicted in Fig. 3 are indeed outcomes for an *energy* market, despite the appearance of power levels (MW) along the quantity axis. These power levels represent possible choices for a *maintained* power level p (MW) *during* operating hour H (1h). Hence, choice of a power level p is equivalent to choice of a grid-delivered energy-block $p \cdot 1h$ (MWh).

Finally, FERC [12, p. 11] explicitly delegated the management of ancillary support for grid reliability to the individual states participating in each RTO/ISO. Consequently, procurement and settlement processes for ancillary support products differ widely across the seven U.S. RTOs/ISOs.

III. Essential Measurement Concepts

A. Units of Measurement

DEFINITION D1: *Standard Unit of Measurement*. Specified positive amount u of a phenomenon that is commonly used (by law or by convention) to measure the magnitude of general amounts of this phenomenon in a comparable manner.

Seven Standard International (SI) Base Units of Measurement for Physical Phenomena: *Length* measured by meter (m); *Mass* measured by kilogram (kg); *Time* measured by second (s); *Electric Current* measured by Ampere (A); *Thermodynamic Temperature* measured by degree Kelvin (K); *Amount of Substance* measured by mole (mol); *Luminous Intensity* measured by candela (cd).

Examples of Units Defined as Functions of SI Base Units: *Pound* (lb) =: Unit for Weight: 1lb =: 0.45359237kg; *Metric Ton* (mt) =: Unit for Weight: 1mt =: 1000kg; *Watt* (W) =: Unit for Power: 1W =: $[1\text{kg}][1\text{m}]^2[1s]^{-3}$; 1kW =: 1000W; 1MW =: 1000kW; *Volt* (V) =: Unit of Electric Potential: 1V =: $[1\text{W}][1\text{A}]^{-1}$; *Hour* (h) =: Unit for Time: 1h =: 60s; *Watt-hour* (Wh) =: Unit for Energy: 1Wh =: [1W][1h]; 1kWh =: 1000Wh; 1MWh =: 1000kWh; *Hertz* (Hz) =: Unit for Frequency: 1Hz =: $[1 \text{ cycle}] \cdot [1\text{s}]^{-1}$.

Other Commonly-Used Units of Physical Measurement: *Degree Fahrenheit* (^{o}F), a normalized temperature unit such that water freezes at 32^{o}F and boils at 212^{o}F; *British Thermal Unit* (Btu), a unit of heat defined to be the quantity of heat required to raise by 1^{o}F the temperature of 1lb of liquid water currently at the temperature ($\approx$ 39^{o}F) at which water has its greatest density; *Person-Hour*, a unit of human labor defined to be one hour of work by one person.

[2]The aggregate demand schedule D in Fig. 3 gives, from left to right, the highest *purchase reservation value* ($/MWh) – i.e., the highest *maximum willingness to pay* ($/MWh) – for each successive unit (MW) increase in the maintained power level p for H, where this highest purchase reservation value is calculated across all buyers (here B1 and B2). Conversely, the aggregate supply schedule S in Fig. 3 gives, from left to right, the lowest *sale reservation value* – i.e., the lowest *minimum acceptable payment* – for each successive unit (MW) increase in the maintained power level p for H, where this lowest sale reservation value is calculated across all suppliers (here S1, S2, and S3). The optimal (market-clearing) solution points are then given by the intersection points of D and S with all horizontal and vertical segments included. For an extended discussion of these fundamental economic concepts, see [9, Ch. 12].

B. Asset Definitions: Unit Measurement Distinctions

DEFINITION D2: ***Asset***. Anything in physical or financial form that can function as a store of value. In principle, an asset can be constructively characterized as a vector of multiple value-relevant possibly-correlated attributes.

Asset Example: *Apple* =: (location; time; taste; weight; shape; color; crispness; freshness; ...)

Human Asset Examples: *Health*; *Hand-Grip Strength*; *Intelligence*.

Social Asset Examples: *Beauty*; *Labor-Capability*; *Verbal English Language Fluency*.

Physical Asset Examples: *Electric Power Quality*; *Mineral Hardness*; *Fruit*; *Battery*; *House*; *Grid-Delivered Energy*.

Financial Asset Examples: *Personal Loan*; *Fire Insurance Contract*; *Home Mortgage Contract*; *U.S. Treasury Bill*; *Common Stock Share*.

DEFINITION D3: ***u-Asset***. An asset A that has a standard unit of measurement u.[3]

Human u-Asset Examples: *Hand-Grip Strength* measured by standardized test score; *Intelligence* measured by Intelligence Quotient (IQ).

Social u-Asset Examples: *Labor-Capability* measured by person-hour; *Verbal English Language Fluency* measured by standardized test score.

Physical u-Asset Examples: *Mineral Hardness* measured by Mohs scale; *Fruit* measured by pounds (lb); *Battery* measured by number of batteries; *House* measured by number of houses; *Grid-Delivered Energy* measured by Watt-hour (Wh);

Financial u-Asset Examples: *Home Fire Insurance Contract*, measured by coverage amount ($); *30-Year Fixed Home Mortgage Contract at 7.12% Interest*, measured by mortgage principal ($); *1-year U.S. Treasury Bill*, measured by redemption value ($); *Share of Duke Energy Common Stock (NYSE:DUK)*, measured by current market value ($/share).

Examples of Assets that are *Not* u-Assets: *Health*; *Beauty*; *Electric Power Quality*.

DEFINITION D4: ***Commodity***. A physically exchangeable u-asset Q such that, conditional on location and time, each Q-trader (supplier and/or buyer) considers all Q-units u available for trade to be *perfect substitutes* for each other; that is, to have the same economic value.[4]

[3]A u-asset is a new asset categorization introduced in Report [11] to help highlight the distinct measurement characteristics of grid-delivered energy. The set of all u-assets (Definition **D3**) is strictly nested between the set of all assets (Definition **D2**) and the set of all commodities (Definition **D4**).

[4]The "same economic value" assigned to all units u of a commodity Q available for trade at a given location and time *can differ* across different Q-traders. Nevertheless, Q-trading is facilitated as follows: Commodity Q can be sold by a Q-supplier i to a Q-buyer j in *bulk (multi-unit)* amount q' (measured in u) at a *common per-unit price* π' (measured in $/$u$) as long as: (i) π' is greater or equal to the common economic value assigned by supplier i to each possible "last" Q-unit he supplies at q'; and (ii) π' is less than or equal to the common economic value assigned by buyer j to each possible "last" Q-unit he procures at q'.

Commodities Defined by Legally-Enforceable Standards:

Agricultural Examples: *HoneyCrisp Apples* measured by pound (lb); *No. 1 Hard Red Winter Wheat* measured by metric ton (mt); *Champagne* measured by millilitres (ml);

Industrial Examples: *DURACELL AA 1.5v Batteries*, measured by number of batteries; *Henry Hub Natural Gas (Louisiana)*, measured by metric million Btu (mmBtu).

Financial Examples: *1-year U.S. Treasury Bills issued on 1 January 2022 with $100 Redemption Value*, measured by number of bills; *Duke Energy Common Stock (NYSE:DUK)*, measured by number of outstanding shares.

Examples of u-Assets that are *Not* Commodities: *Hand-Grip Strength*; *Intelligence*; *Labor-Capability*; *Verbal English Language Fluency*; *Mineral Hardness*; *Fruit*; *House*; *Grid-Delivered Energy*; *Fire Insurance Contract*; *Home Mortgage Contract*.

C. Unit/Per-Unit Calculations Can Mask Conceptual Error

Let R denote the set of real numbers. The standard algebraic operators that act on elements of R include: addition ($+$); subtraction ($-$); multiplication ($\times$); division ($\div$); and equality ($=$). The set R together with its standard algebraic operators is hereafter referred to as the *Real Number System*.

The International System of Units (SI) is commonly referred to as the *Metric System*. The Metric System consists of the seven real-valued SI Base Units listed in Section III-A together with real-valued units derived from these seven real-valued SI Base Units by means of standard algebraic operators.

The seven SI Base Units $\{m, kg, s, A, K, mol, cd\}$ are each defined in terms of a latest internationally agreed-upon value for a physical constant pertaining to some physical aspect of the Real World, where these physical constants are assumed to be mutually independent of each other. For example, the SI Base Unit for length is a meter (m), defined in terms of the latest internationally agreed-upon value of the speed of light in vacuum space. The SI Base Unit for mass is a kilogram (kg), defined in terms of the latest internationally agreed-upon value for the Planck constant, $\hbar$. The SI Base Unit for electric current is an Ampere (A), defined in terms of the latest internationally agreed-upon value for the electrical charge carried by an electron.

Unit and per-unit calculations must be undertaken with great care. Consider the status (True, False, Undecidable, Ambiguous, Undefined,...) assigned to each of the following five statements:

Statement S1: *10 = 10*

Status: True statement within the Real Number System. Ambiguous statement (10 of what?) within both the Metric System and the Real World.

Statement S2: *10 pounds of apples = 10 pounds of apples*

Status: Undefined statement within the Real Number System (what is a pound? what is an apple?) and the Metric System (what is an apple?). Ambiguous statement about the Real World: no two separate apples are physically identical, and physical differences can affect production cost, eating preferences, and consumption benefits; thus, what type of "equality" is "=" meant to signify?

Statement S3: *2MWh = 2MWh*

Status: Undefined statement within the Real Number System (what is a MWh?). True statement within the Metric System. Ambiguous statement about the Real World (what type of "equality" is "=" meant to signify?).

Regarding Real World ambiguity, consider the following possibilities. The energy (2MWh) on each side of the operator "=" could be identically stored energy located at a grid location b at a particular point in time; thus, the operator "=" could represent physical equivalence. Alternatively, the energy (2MWh) on each side of the operator "=" could represent energy that has been grid-delivered at b during the course of some operating day D, i.e., the accumulation of a flow of power (MW) injected at b during D. For example, these power injections might have occurred: (i) *throughout* all 24 hours of day D at a constant level 1MW/12; or (ii) *only during the first 12 hours of day D* at a constant level 1MW/6; or (iii) *every other half hour during day D* at a constant level 1MW/6. The operator "=" could thus signify customer indifference regarding the exact manner in which energy (2MWh) has been delivered at their grid-node location *b during* operating day D as an accumulated flow of power.

Important Remark: In Schweppe et al. [13, App. F.1] and [15, fn, p. 1153], the proposed *Frequency Adaptive Power Energy Rescheduler (FAPER)* is carefully restricted to *energy loads* ("energy-type usage devices") characterized by: (1) a need for a certain amount of energy over a period of time T in order to fulfill their functions (or purposes); and (2) indifference as to the exact times within T during which the energy is furnished. *Power loads* are characterized as the loads of devices requiring power at specific times during a period of time T in order to full their functions (or purposes). Surprisingly, however, the critical importance of the distinction between energy loads and power loads for the hourly nodal "spot pricing" approach proposed in the main chapters of [13] is not addressed.

Statement S4: *1 DURACELL AA 1.5v Battery = 1 DURACELL AA 1.5v Battery*

Status: Undefined statement within both the Real Number System and the Metric System (what is a "DURACELL AA 1.5v Battery"?). Ambiguous statement within the Real World; even for a single brand and type of battery, no two distinct manufactured batteries are ever *exactly* the same in terms of their physical attributes. Thus, what type of "equality" is "=" meant to signify?

Statement S5: *Let Q denote a commodity (Definition* ***D4****) with Q-amounts q measured by a standard unit of measurement u; and let the algebraic operator "=" signify "is a perfect substitute for". Then, conditional on a location and time, 10u = 10u for each Q trader.*

Status: Undefined statement within the Real Number System (what is u?) and the Metric System (what is a commodity?). True statement (by definition) for the Real World.

D. Implications of Essential Measurement Concepts

The results reported in Sections III-A through III-C have important implications, formalized below as three lemmas.

LEMMA III.1: *The use of the operator "=" to equate amounts of two u-assets measured in terms of the same standard unit of measurement u can result in conceptual error.*

Outline of Proof for Lemma III.1: By design, the standard unit of measurement u for a u-asset A typically measures the "amount" of A based on *only one attribute* of A, such as: weight measured in pounds (lb): energy measured in megawatt-hours (MWh); and economic value measured in U.S. dollars ($). No attempt is made to ensure that u characterizes the attributes of A in a physically or economically complete manner. Thus, the "same" u-amounts for two physically-distinct or economically-distinct u-assets with the same standard unit of measurement u can have *substantially different* physical effects when used within a physical application, and *substantially different* economic effects when used as consumption goods or as inputs to a production process. //

LEMMA III.2: *The standard use of per-unit (p.u.) calculations in economics and power engineering can mask conceptual error.*

Outline of Proof for Lemma III.2: Conditional on a location and time, suppose: (i) assets A' and A'' are u-assets that share a common standard unit of measurement u; (ii) a' is an amount of A' measured in u; (iii) a'' is an amount of A'' measured in u; (iv) $a' = a''$ measured in u; but (iv) the u-units for assets A' and A'' are *not* equivalently exchangeable for a purpose at hand.

For example, the u-assets on each side of "=" could be equal apple amounts (measured in pounds) for two distinct apple varieties offered for sale at a given location and time, as in **S2**; or, as in **S3**, they could be equal energy amounts (measured in MWh) that have each been grid-delivered at a grid location b during an operating-period T as the accumulation of power-injection sequences with distinctly different physical attributes (e.g., different ramp-rate *profiles* during T, different capacity *profiles* during T, different *delivery timing* within T, ...).

Dividing the u-amounts on each side of "=" by a common "base u-value" (for example, "1 pound of apples" for the apple example, or "1MWh of energy" for the grid-delivered energy example), one is left with "per-unit" equations such as "10 = 10" for the apple example and "2 = 2" for the grid-delivered energy example. These appear to be correct equations because they are true statements for the Real Number System.

Any differences in the full collection of attributes characterizing the two underlying u-assets A' and A'' that conceptually invalidate the unqualified use of an equality operator "=" in the original versions of these equations – that is, the use of "=" without the qualification "measured in u" – are now lost from sight. //

LEMMA III.3: *A conceptually-meaningful real-line "quantity axis" cannot be constructed for an asset A conditional on location and time* ***unless*** *asset A is a u-asset whose u-units are equivalently exchangeable for the purpose at hand, conditional on this location and time.*

Outline of Proof for Lemma III.3: Suppose an asset A is *not* a u-asset. Then there is no way to measure "amounts" of A along a real-line "quantity axis" by measuring these amounts in terms of a real-valued unit-of-measurement u.

Suppose, next, that a u-asset A is to be used as an input for a physical and/or economic process Z to take place at a location b at a start-time t. However, suppose the u-units of A are *not* equivalently exchangeable for process Z. Finally, suppose a process manager is tasked with the construction of a function mapping different amounts of input A (measured in u) into corresponding physical and/or economic outcomes for process Z, taking as given a particular configuration of all other process inputs.

As a first task-step, the manager sets about the construction of a "quantity axis" for A by identifying each real number $r \geq 0$ along the real-line with an amount of A of size r (measured in u units). Unfortunately for the manager, the precise selection of u-units comprising each given amount r of A can affect the resulting physical and/or economic outcomes of process Z because, by assumption, the u-units of A are not equivalently exchangeable for process Z.

Thus, the physical and/or economic outcomes for process Z cannot be expressed as a conceptually well-defined function of the "amount" of input A represented as a non-negative r-value along the real line. //

The fundamental issues highlighted in **Lemmas III.1–III.3** regarding the representation of real-world quantity amounts as points along Cartesian coordinate axes suggest the desirability of considering alternative constructive mathematical approaches permitting "holistic" representations of real-world phenomena and their interactions. See, for example, the discussion of this point in Tesfatsion [16, Sec. 3].

Crucial ramifications of **Lemmas III.1–III.3** for LMP will be highlighted in Section V after definitions for several essential economic concepts are first reviewed in Section IV.

IV. Essential Economic Concepts

A. Basic Market Definitions

DEFINITION BM1: ***Spot Market.*** Market for an asset A such that transacted amounts of A, payments for these transacted amounts of A, and deliveries of these transacted amounts of A all occur at the same location and time ("on the spot").

DEFINITION BM2: ***Forward Market.*** Market for an asset A such that transacted amounts of A and payment obligations for these transacted amounts of A are determined in advance of the delivery of these transacted amounts of A.

DEFINITION BM3: ***Non-Avoidable Fixed Cost ("Sunk Cost").*** Conditional on location and time, the *non-avoidable fixed cost* (\$) of a supplier i in the process of selecting a non-negative supply-level a (measured in u) for a u-asset A is the fixed cost SC_i^o (\$) that supplier i has incurred to date that cannot be modified by any current or future decision that supplier i makes, including selection of a.

DEFINITION BM4: ***Avoidable Fixed Cost.*** Conditional on location and time, the *avoidable fixed cost* (\$) of a supplier i in the process of selecting a non-negative supply-level a (measured in u) for a u-asset A is the fixed cost AFC_i^o (\$) that supplier i incurs *if and only if* supplier i selects a *positive* supply-level a.

DEFINITION BM5: ***Variable Cost.*** Conditional on location and time, the *variable cost* (\$) of a supplier i in the process of selecting a non-negative supply-level a (measured in u) for a u-asset A is the a-dependent cost $VC_i(a)$ (\$) that supplier i would have to incur for each selection of a, where $VC_i(0) = 0$.

DEFINITION BM6: ***Total Avoidable Cost.*** Conditional on a location and time, the *total avoidable cost* (\$) of a supplier i in the process of selecting a non-negative supply-level a (measured in u) for a u-asset A equals 0 (\$) if supplier i selects $a = 0$ and equals the summation $[AFC_i^o + VC_i(a)]$ of supplier i's avoidable fixed cost (\$) and variable cost (\$) if supplier i selects $a > 0$.

DEFINITION BM7: ***Revenue Sufficiency.*** A supplier i participating in a market M for a u-asset A is *revenue sufficient* for M if the total earnings (\$) that supplier i attains from participation in M suffice to cover the total avoidable cost (\$) that supplier i incurs from participation in M.

DEFINITION BM8: ***Purchase Reservation Value.*** Conditional on location and time, a buyer j's *purchase reservation value* (\$) for an item z available for purchase from a supplier i is the *maximum payment* (\$) that buyer j is *willing to make* to supplier i for item z.

DEFINITION BM9: ***Sale Reservation Value.*** Conditional on location and time, a supplier i's *sale reservation value* (\$) for an item z that supplier i is offering for sale to a buyer j is the *minimum payment* (\$) that supplier i is *willing to accept* from buyer j for item z.

B. Commodity Market Definitions

DEFINITION CM1: ***Commodity Spot Market.*** Spot market for a commodity.

DEFINITION CM2: ***Futures Market.*** A forward market for a commodity.

DEFINITION CM3: ***Ordinary Demand Schedule.*** Conditional on location and time, a buyer j's *ordinary demand schedule* for a commodity Q with standard unit of measurement u is a function that maps each non-negative Q-unit price π (measured in \$/u) into the maximum Q-amount $q = D_j^o(\pi)$ (measured in u) that buyer j is willing to procure at price π.

DEFINITION CM4: ***Benefit Function.*** Conditional on location and time, a buyer j's *benefit function* for a commodity Q with a standard unit of measurement u is a function that maps each non-negative Q-amount q (measured in u) into the benefit $B_j(q)$ (measured in \$) that buyer j would attain from procurement of q.

DEFINITION CM5: ***Marginal Benefit Function.*** Conditional on location and time, a buyer j's *marginal benefit function* for a commodity Q with a standard unit of measurement u is a function that maps each non-negative Q-amount q (measured in u) into the incremental benefit $MB_j(q)$ (measured in \$/u) that buyer j would attain from procurement of a *next* Q-unit, given that buyer j has already procured q.

DEFINITION CM6: ***Inverse Demand Schedule.*** Conditional on location and time, a buyer j's *inverse demand schedule* for a commodity Q with a standard unit of measurement u is a function that maps each non-negative Q-amount q (measured in u) into the maximum Q-unit price $D_j(q) = \pi$ (measured in \$/u) that buyer j is willing to pay to procure a *next* Q-unit, given that buyer j has already procured q.[5]

DEFINITION CM7: ***Ordinary Supply Schedule.*** Conditional on location and time, a supplier i's *ordinary supply schedule* for a commodity Q with a standard unit of measurement u is a function that maps each non-negative Q-unit price π (measured in \$/u) into the maximum Q-amount $q = S^o_i(\pi)$ (measured in u) that supplier i is willing to supply at price π.

DEFINITION CM8: ***Total Avoidable Cost Function.*** Conditional on location and time, a supplier i's *total avoidable cost function* for a commodity Q with a standard unit of measurement u is a function that maps each of supplier i's feasible non-negative Q-supply levels q (measured in u) into the total avoidable cost $C_i(q)$ (measured in \$) that supplier i would have to incur to supply q.

DEFINITION CM9: ***Marginal Cost Function.*** Conditional on location and time, a supplier i's *marginal cost function* for a commodity Q with a standard unit of measurement u is a function that maps each non-negative Q-amount q (measured in u) into the incremental cost $MC_i(q)$ (measured in \$/u) that supplier i would have to incur to supply a *next* Q-unit, given that supplier i is currently supplying q.

DEFINITION CM10: ***Inverse Supply Schedule.*** Conditional on location and time, a supplier i's *inverse supply schedule* for a commodity Q with a standard unit of measurement u is a function that maps each of supplier i's non-negative Q-supply levels q (measured in u) into the minimum non-negative Q-unit price $S_i(q) = \pi$ (measured in \$/u) that supplier i is willing to be paid for a *next* Q-unit, given supplier i has supplied q.[6]

V. Marginal Pricing Requires Commodities

DEFINITION CM11: Let Q denote a commodity with standard unit of measurement u, and let CSM denote a commodity spot market for Q. Then CSM is a ***Competitive Commodity Spot Market (CCSM)*** for Q if:[7]

(CCSM1) The participants in CSM consist of a fixed set of Q-buyers j and a fixed set of Q-suppliers i.

(CCSM2) Each buyer j and supplier i is a price-taker.[8]

(CCSM3) Each buyer j has a non-increasing *ordinary demand schedule* $D^o_j(\pi)$ that maps each non-negative Q-unit price π (\$/u) into a non-negative Q-demand $q_j = D^o_j(\pi)$ (u).

(CCSM4) Each supplier i has a non-decreasing *ordinary supply schedule* $S^o_i(\pi)$ that maps each non-negative Q-unit price π (\$/u) into a non-negative Q-supply $q_i = S^o_i(\pi)$ (u).

(CCSM5) The equilibrium concept for CSM is *competitive equilibrium*, defined as follows. Let $q = D^o(\pi) =: \sum_j D^o_j(\pi)$ denote the *(ordinary) aggregate demand schedule* for Q, and let $q = S^o(\pi) =: \sum_i S^o_i(\pi)$ denote the *(ordinary) aggregate supply schedule* for Q. Then a price-quantity pair $e^* = (\pi^*, q^*)$ with $q^* > 0$ is a ***competitive equilibrium*** for CSM if e^* is an intersection point of the aggregate demand and supply schedules $q = D^o(\pi)$ and $q = S^o(\pi)$ plotted in the (π, q) plane; that is, if e^* satisfies the following condition (1):

[5] The following regularity conditions are sufficient to ensure an *inverse* demand schedule $D_j(q) = \pi$ for a buyer j, as defined in **CM6**, can be inverted to obtain a well-defined *ordinary* demand schedule $q = D^o_j(\pi)$ for buyer j as defined in **CM3**, and vice versa, where $D_j(q)$ coincides with buyer j's marginal benefit function $MB_j(q)$; see [9, Sec. 9.3.4] for extended discussion. Suppose buyer j has a *benefit function* $B_j(q)$, defined as in **CM4**, that is non-decreasing, differentiable, and *concave* over $q \geq 0$. Buyer j's *marginal benefit*, evaluated at any $q' \geq 0$, is then the non-negative derivative of buyer j's benefit function $B_j(q)$ with respect to q, evaluated at $q = q'$. This mapping $D_j(q')$ of q' into a non-negative *incremental* benefit evaluation $\partial B_j(q')/\partial q =: MB_j(q')$ $=: \pi'$ (\$/u) is buyer j's *inverse demand schedule for Q*. Finally, if buyer j's marginal benefit function $MB_j(q)$ is a *strictly* decreasing function of q for $q \geq 0$, a common "diminishing marginal returns" assumption for commodity spot markets, it can be inverted over $q \geq 0$ to give a *strictly* decreasing *ordinary* demand schedule $q = D^o_j(\pi)$ for buyer j. In this case, by construction, the price π' that satisfies $q' = D^o_j(\pi')$ is the marginal benefit of buyer j evaluated at the Q-demand level q'. Economists studying competitive commodity spot markets typically work with ordinary demand schedules mapping prices into quantities because, as will be seen below in definition **CM11**, all buyer participants in such markets are assumed to be price-takers. However, in U.S. RTO/ISO-managed wholesale power markets, demand schedules ("demand bids") are typically expressed in inverse form, as mappings from quantities into prices.

[6] The following regularity conditions are sufficient to ensure an *inverse* supply schedule $S_i(q) = \pi$ for a supplier i, as defined in **CM10**, can be inverted to obtain a well-defined *ordinary* supply schedule $q = S^o_i(\pi)$ for supplier i as defined in **CM7**, and vice versa, where $S_i(q)$ coincides with supplier i's marginal cost function $MC_j(q)$; see [9, Sec. 8.2] for extended discussion. Suppose supplier i has a *total avoidable cost function* $C_i(q)$, defined as in **CM8**, that is non-decreasing, differentiable, and *convex* over $q \geq 0$. Evaluated at any Q-supply level $q' \geq 0$, supplier i's marginal cost $MC_i(q')$ (measured in \$/u) defined as in **CM9** is then the derivative of supplier i's total avoidable cost function $C_i(q)$ with respect to q, evaluated at $q' \geq 0$. This mapping of $q' \geq 0$ into a non-negative *marginal* cost evaluation $\partial C_i(q')/\partial q =: MC_i(q') =: \pi'$ (\$/u) is supplier i's *inverse supply schedule for Q*. Finally, if supplier i's marginal cost function $MC_i(q)$ is a *strictly* increasing function of q for $q \geq 0$, a common "increasing marginal cost" assumption for commodity spot markets, it can be inverted over the range $q \geq 0$ to give an *ordinary* supply schedule for supplier i; that is, to give a strictly-increasing function $S^o_i(\pi)$ mapping each non-negative Q-unit price π' (measured in \$/u) into a non-negative Q-supply $q' = S^o_i(\pi')$ (measured in u). In this case, by construction, the Q-unit price π' that maps into q' is supplier i's marginal cost $MC_i(q')$ (measured in \$/u), evaluated at Q-supply level q'. Economists studying competitive commodity spot markets typically work with ordinary supply schedules mapping prices into quantities because, as will be seen below in definition **CM11**, all supplier participants in such markets are assumed to be price-takers. However, in U.S. RTO/ISO-managed wholesale power markets, supply schedules ("supply offers") are typically expressed in inverse form, as mappings from quantities into prices.

[7] See [9, Ch. 12] for a detailed illustrated definition for the standard economic concept of a CCSM, as well as for key related concepts such as net surplus extraction and market efficiency.

[8] A participant in a spot market for a commodity Q with a standard unit of measurement u is a *price-taker* if the participant behaves as if his own market transactions have no effect on the market-determined Q-unit price π (measured in \$/u).

Competitive (D=S) Market Clearing Condition *at $e^* = (\pi^*, q^*)$ with $q^* > 0$:*

$$q^* \ = \ D^o(\pi^*) \ = \ S^o(\pi^*) \quad . \tag{1}$$

The next lemma, an immediate implication of definitions **CM3–CM10** in Section IV-B, establishes an important alternative "marginal pricing" form for the Competitive (D=S) Market Clearing Condition (1).

Lemma V.1 *Suppose regularity conditions*[9] *hold such that:*

- **(a)** *Each buyer j has an inverse demand schedule $D_j(q_j) = \pi$ that can be inverted to give a well-defined ordinary demand schedule $q_j = D^o_j(\pi)$ for buyer j, and vice versa, where $D_j(q_j)$ coincides with buyer j's marginal benefit function, i.e., $D_j(q_j) = MB_j(q_j)$.*
- **(b)** *Each supplier i has an inverse supply schedule $S_i(q_i) = \pi$ that can be inverted to give a well-defined ordinary supply schedule $q_i = S^o_i(\pi)$ for supplier i, and vice versa, where $S_i(q_i)$ coincides with supplier i's marginal cost function, i.e., $S_i(q_i) = MC_i(q_i)$.*

Then the Competitive (D=S) Market Clearing Condition (1) is equivalent to the following marginal-pricing condition:

Competitive (MB=MC) Spot-Pricing Rule *at $e^* = (\pi^*, q^*)$ with $q^* > 0$: For each buyer j and supplier i such that $q^*_j > 0$ and $q^*_i > 0$,*

$$\pi^* \ = \ \mathrm{MB}_j(q^*_j) \ = \ \mathrm{MC}_i(q^*_i) \tag{2}$$

Outline of Proof for Lemma V.1: To see the claimed equivalence, given properties **(a)** and **(b)**, apply appropriate *inverse* demand and supply function operations to the terms in condition (1), and apply appropriate *ordinary* demand and supply function operations to the terms in condition (2). //

A CCSM for which the Competitive (D=S) Market Clearing Condition (1) is equivalent to the Competitive (MB=MC) Spot-Pricing Rule (2) will be called a ***Marginal-Pricing CCSM***, or ***MP-CCSM*** for short. MP-CCSMs have a variety of attractive efficiency and optimality properties. Several of these properties are stated below as lemmas for later reference.

Lemma V.2: *All fixed cost for each supplier i participating in an MP-CCSM is sunk cost, i.e., non-avoidable fixed cost.*

Proof for Lemma V.2: By definition, an MP-CCSM is a commodity spot market that takes place at a *given* location and time for a *given* set of participants whose demand and supply schedules are *automatically* submitted to the MP-CCSM and *instantly* cleared (or not cleared) to determine competitive equilibrium outcomes. Thus, no supplier participating in an MP-CCSM is a decision-maker able to avoid (or not avoid) some cost depending on a decision the supplier makes at this given location and time. //

Lemma V.3: *Revenue sufficiency* holds for an MP-CCSM. That is, at any competitive equilibrium $e^* = (\pi^*, q^*)$ for an MP-CCSM, the total revenue earned by each supplier i is sufficient to cover supplier i's total avoidable cost.

Outline of Proof for Lemma V.3: The equivalent defining conditions (1) and (2) for an MP-CCSM competitive equilibrium $e^* = (\pi^*, q^*)$ imply that the total (possibly-zero) revenue earned by each supplier i (whether supplier i is cleared or not) is sufficient to cover the total sum (or integral) of the possibly-zero marginal costs that supplier i incurs at e^*.[10] By Lemma V.2, the *total variable cost* of supplier i at e^* coincides with the *total avoidable cost* of supplier i at e^*. It follows from definition **BM7** that supplier i is revenue sufficient. //

Major Caution: The commodity requirement in **Definitions CM3–CM11** and **Lemmas V.1-V.3** is essential. If this commodity requirement fails to hold, it follows from **Lemma III.3** that there would be no way to construct conceptually-coherent real-line "quantity axes" for the functions appearing in these definitions and lemmas.

Important implications of this caution specifically for marginal pricing are summarized in two additional lemmas:

Lemma V.4: *Any buyer j that receives an additional unit of a commodity Q at a given location and time is indifferent with regard to which precise additional Q-unit he receives because, by assumption, the incremental benefit (economic value) that buyer j gains from this receipt is the same for all Q-units.* ***This indifference is a necessary condition for buyer j to have a conceptually well-defined marginal benefit function for Q at the given location and time.***

Lemma V.5: *Any supplier i that supplies an additional unit of a commodity Q at a given location and time is indifferent with regard to which precise additional Q-unit he supplies because, by assumption, the incremental cost (lost economic value) that supplier i incurs from this supply is the same for all Q-units.* ***This indifference is a necessary condition for supplier i to have a conceptually well-defined marginal cost function for Q at the given location and time.***

To understand the import of **Lemmas V.4–V.5** in more concrete terms, consider the following situation. At a given location and time, an experimental economist plans to use a sealed bag containing a mixture of HoneyCrisp Apples and Dole Mandarin Oranges to construct an ordinary demand schedule for fruit for a human subject called "buyer j." The standard unit of measurement u for fruit is taken to be a piece of fruit; hence, fruit-quantities q are measured by the number of included fruit pieces u, and fruit-unit prices π are measured by U.S. dollars per fruit-piece ($/$u$).

In accordance with condition **CCSM3** in definition **CM11**, the experimenter hands buyer j an ordered list of successively higher fruit-unit prices π and asks buyer j to report the maximum fruit-quantity $q = D^o(\pi)$ that he would be willing to buy at each listed fruit-unit price π. At the end of the experiment, one of the listed fruit-unit prices, π^*, will be randomly announced, the bag of fruit will be unsealed, and buyer j will be required to pay $\pi^* \times q^*$ ($) for a fruit-quantity $q^* = D^o(\pi^*)$ that the experimenter draws randomly from the unsealed bag.

[9] Regularity conditions ensuring that properties **(a)** and **(b)** in **Lemma V.1** both hold are provided in Footnotes 5 and 6.

[10] See Footnote 2 in Section II and Tesfatsion [9, Ch. 12].

Unfortunately for the experimenter, suppose buyer j does *not* consider a HoneyCrisp Apple to be a perfect substitute for a Dole Mandarin Orange; that is, suppose the specific apple-versus-orange attribute of a fruit-piece *matters* to buyer j? *In this case, the economic value that buyer j attains from any procured fruit-quantity q will depend on the specific apple-orange composition of q*.

Consequently, the maximum fruit-quantity q that buyer j is willing to purchase at each listed fruit-unit price π will depend on how buyer j resolves his uncertainty regarding two related aspects of the experiment. First, what is the apple-orange composition of fruit-pieces in the *sealed* bag? Second, given this composition, what will be the likely apple-orange composition of the fruit-quantity q^* that is randomly drawn from the *unsealed* bag if buyer j reports that $q^* = D^o_j(\pi^*)$ is the maximum fruit-quantity he is willing to purchase at the announced price π^*?

The bottom line is that an ordinary *demand* schedule **CM3** is *not* well-defined for fruit for a fruit-buyer j at a given location and time *unless* buyer j considers all pieces of fruit available for purchase at this location and time to be perfect substitutes. Analogous arguments can be used to demonstrate that an ordinary *supply* schedule **CM7** is *not* well-defined for fruit for a fruit-supplier i at a given location and time *unless* supplier i considers all pieces of fruit available for supply at this location and time to be perfect substitutes.

What about *inverse* demand and supply schedules for fruit, defined as in **CM6** and **CM10**? Here the cautionary need for **Lemmas V.4–V.5** is even simpler to understand.

Suppose apples and oranges are *not* perfect substitutes for a fruit-*buyer* j. How can buyer j express his *maximum acceptable purchase price* for a *next* piece of fruit, given he has already procured a fruit-quantity q, without knowing: (i) which specific fruit piece, apple or orange, is to be his next-procured fruit piece; and (ii) what is the specific apple-orange composition of his already-procured fruit-quantity q?

Suppose apples and oranges are *not* perfect substitutes for a fruit-*supplier* i. How can supplier i express his *minimum acceptable sale price* for a *next* piece of fruit, given he has already supplied a fruit-quantity q, without knowing: (i) which specific fruit piece, apple or orange, is to be his next-supplied fruit piece; and (ii) what is the specific apple-orange composition of his already-supplied fruit-quantity q?

VI. Implications for Locational Marginal Pricing

Grid-delivered energy is a u-asset with standard unit of measurement u =: MWh. However, grid-delivered energy conditional on delivery location and time *does not function as a commodity* within the context of U.S. RTO/ISO-managed wholesale power markets. That is, participants in such markets typically do *not* view 1MWh of grid-delivered energy to be a perfect substitute for any other 1MWh of grid-delivered energy, conditional on delivery location and time.

To the contrary, power producers and power customers typically care about the *dynamic* attributes of the *power-paths* they inject/withdraw at their grid locations during successive operating periods; see Fig. 1. For example, power producers dispatched to inject power at a grid location b during an operating period T might reasonably care about the amount of equipment wear-and-tear cost they will incur due to ramping needed to match dispatch set-points. Power customers electrically connected at a grid location b might reasonably care about the degree of flexibility they will have to determine the attributes of their withdrawn power-paths at b during T to meet their diverse local just-in-time power requirements.

Moreover, RTOs/ISOs are tasked with maintaining reliable grid operations, which requires continual nodal balancing of grid net load. This fiduciary obligation requires RTOs/ISOs to ensure the *advance availability* of *diverse dispatchable power-path production capabilities* at grid-nodes b for *possible* RTO/ISO dispatch *during* future operating periods T, not simply the static ex-post demand-equals-supply balancing of grid-delivered energy (MWh) at grid-nodes b for each T.

Thus, if a producer, customer, or RTO/ISO were asked to assign in advance a cost or benefit valuation to a specific quantity of grid-delivered energy E* (MWh) to be delivered at a designated grid location b during a designated future operating period T, typically they would not be able to do so without knowing the power-path to be used for this delivery.

For example, what valuation would each of these entities assign to the delivery of each of the four following power-path options $\mathbf{p}_b(\mathrm{T}) = \big(p_b(t) \mid t \in \mathrm{T}\big)$ at a designated grid location b during a designated future 24-hour operating-period T, where each power-path option accumulates to the *same* quantity E$^*(b,$T) = 12MWh of *grid-delivered* energy:

Power-Path (a): Power (MW) is injected (or withdrawn) at b at level $p = 24$ during the *first half-hour* of operating-period T and at level $p = 0$ during the remainder of T.

Power-Path (b): Power (MW) is injected (or withdrawn) at b at level $p = 0.5$ throughout operating-period T.

Power-Path (c): Power (MW) is injected (or withdrawn) at b at level $p = 1$ during *every other* hour of operating-period T, with $p = 0$ during the remaining hours of T.

Power-Path (d): Power (MW) is injected (or withdrawn) at b during operating-period T in a flexible manner that is entirely up to the entity, apart from the requirement that the resulting total injected (or withdrawn) energy must be 12MWh.

Presumably, any producer, customer, or RTO/ISO would assign to the flexible power-path option **(d)** a value that is at least as high as the value assigned to any of the rigid power-path options **(a)–(c)**, because option **(d)** strictly encompasses all three of these rigid options. Indeed, option **(d)** would presumably be assigned at least as high a value as *any* other power-path option for possible injection (or withdrawal) at b during a future operating-period T subject only to E$^*(b,$T) = 12MWh since option **(d)** encompasses all such options.

The clear implication of these examples is that grid-delivered energy E(b,T) (MWh) conditional on delivery location b and operating-period T does *not* function as a *commodity* within U.S. RTO/ISO-managed wholesale power markets. Hence, as seen in Sections II–V, the following functions essential for LMP settlement of E(b,T) in these markets are *not conceptually well-defined*: demand schedule (ordinary or inverse); supply schedule (ordinary or inverse); marginal cost function; and marginal benefit function.

Attempts to justify LMP settlement for grid-delivered energy by pointing to the efficiency and optimality properties of competitive (MB=MC) spot-pricing (2) for commodity spot markets are thus conceptually problematic.

Many *non-commodity* u-assets – i.e., u-assets with non-homogeneous units u, conditional on location and time – are successfully transacted in real-world markets. Indeed, product innovation typically proceeds through gradually introduced variations in the attributes of existing product units in a search for at least temporary market advantage. Examples of widely-transacted types of non-commodity u-assets include houses, mortgages, property and life insurance, and human-provided services such as dental care, haircuts, and wage-labor measured in person-hours.

U.S. RTO/ISO-managed markets are necessarily *forward* markets due to the speed of real-time grid operations. The key to the conceptually-consistent design of these markets is to consider with care how real-world *forward* markets transact *non-commodity* products.

As carefully discussed in [9], reviewed in [11, Sec. 6], and briefly illustrated in the following section, the answer is *appropriate contract design*.

VII. A Conceptually-Consistent Alternative

A. Introduction

Given the necessarily forward nature of grid-supported U.S. RTO/ISO-managed wholesale power markets, the basic products transacted in these markets are forms of *reserve (physically-covered insurance)* to reduce volumetric grid risk[11] for *future* operating periods. This reserve provides valuable risk-reduction services whether or not it is subsequently used by the RTO/ISO to provide actual physical support for real-time balancing of net load.

Insurance in other contexts is routinely offered by means of legally-enforceable two-part-pricing contracts. These contracts ensure coverage of *avoidable fixed costs* for the insuring party through contractually-specified insurance premiums (in lump-sum or amortized form) and coverage of *variable costs* (if any) for the insuring party through contractually-specified terms such as co-payments, deductibles, and negligence penalties.

It would thus seem reasonable for reserve offers in U.S. RTO/ISO-managed wholesale power markets to take the form of *two-part pricing insurance contracts* permitting reserve suppliers to be *separately* compensated for two distinct types of cost: (i) *avoidable fixed cost* incurred to maintain reserve availability, hence reduction of volumetric grid risk, for *future* operating periods; and (ii) *variable cost* (if any) incurred for RTO/ISO-dispatched delivery of reserve, hence for *actual* physical down/up power-provision during real-time operations to meet just-in-time customer power demands and grid reliability requirements.

This is the approach taken by the *Linked Swing-Contract Market Design*, developed and tested in [9].

[11] *Volumetric grid risk* for a grid-supported centrally-managed wholesale power market is systemic risk of grid collapse due to physical imbalance between net load and centrally-dispatched power injection at one or more grid locations. A *systemic risk* is a *system-wide* risk, i.e., a correlated risk arising for system operations as a whole.

B. The Linked Swing-Contract Market Design

The Linked Swing-Contract Market Design includes three innovative features to facilitate the conceptually-consistent, efficient, and reliable operation of grid-supported RTO/ISO-managed wholesale power markets with increasingly decarbonized grid operations and with more active participation by demand-side power resources.

The first innovative feature is the conceptualization of a *power-path* as the basic transacted product for an RTO/ISO-managed wholesale power market, and for grid-supported electric power systems more generally; see Fig. 4. As illustrated earlier in Fig. 1, a power-path $\mathbf{p}_m(\mathrm{T}) = \left(p_m(t) \mid t \in \mathrm{T} \right)$ is a sequence of injections and/or withdrawals of power $p_m(t)$ (MW) by a market participant m at a *single* grid-location $b(m)$ during a designated time-interval T. This product conceptualization permits market design features to be envisioned and developed from the distributed vantage points of market participants as well as the centralized vantage point of the RTO/ISO, thus facilitating incentive alignment.

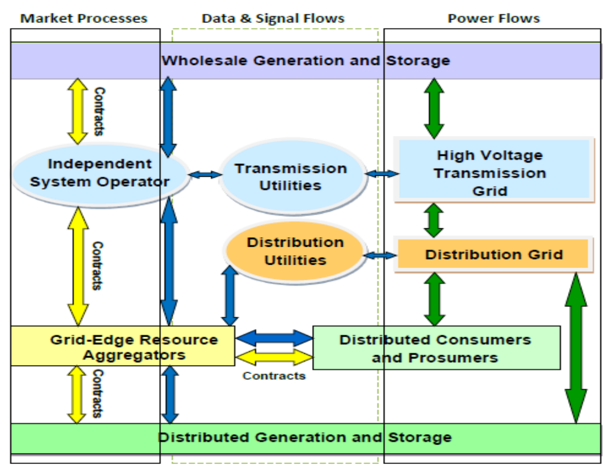

Fig. 4. *Grid-supported T&D electric power systems as support mechanisms for flexible nodal-based power-path transactions.*

The second innovative feature is a fundamental change in the envisioned resource-management role for the RTO/ISO from a *short-to-long* focus to a *long-to-short* focus.

In current U.S. RTO/ISO-managed wholesale power markets, the *primary* role of the RTO/ISO is the scheduling of balanced demands and supplies for grid-delivered energy in *short-run* (day-ahead and intra-day) forward markets, supported by ancillary service procurement, supplementary unit-commitment procurement, and – with the exception of ERCOT – supplemental capacity (MW) procurement. In contrast, in the Linked Swing-Contract Market Design the *primary* role of the RTO/ISO is the continually orchestrated procurement and deployment of reserve (dispatchable power-path production capabilities) in forward markets M(T) for future operating periods T to protect against volumetric grid risk. The RTO/ISO-specified look-ahead horizon LAH(T) between the close of M(T) and the start of T can range in duration from years to seconds, as can the duration of T itself.

The third innovative feature is the two-part pricing swing-contract form of the reserve offers that Dispatchable Power

Resources (DPRs) submit into a linked collection of RTO/ISO-managed forward reserve markets M(T) for future operating periods T. This reserve-offer form facilitates flexible reserve availability for the RTO/ISO as well as time-consistent revenue-sufficient settlements for the DPRs.

More precisely, a reserve offer $\mathrm{SC}_m(\mathrm{T})$ submitted by a DPR m into a market M(T) for a future operating period T is a two-part pricing swing-contract consisting of four m-specified components: an **offer price (insurance premium)** $\alpha_m(\mathrm{T})$ to be paid to m in lump-sum or amortized form if $\mathrm{SC}_m(\mathrm{T})$ is cleared, permitting m to ensure it receives appropriate compensation for the *avoidable fixed cost* that m must incur to guarantee reserve availability for T; an **exercise set** $\mathbb{T}^{\mathrm{ex}}_m(\mathrm{T})$ of possible $\mathrm{SC}_m(\mathrm{T})$ contract exercise times for the RTO/ISO; a **production possibility set** $\mathbb{PP}_m(\mathrm{T})$ in "digital twin" form that physically characterizes the dispatchable power-paths $\mathbf{p}$ offered by m for possible RTO/ISO dispatch during T; and a **performance payment method** $\phi_m(\cdot,\mathrm{T})$ mapping each power-path $\mathbf{p}$ in $\mathbb{PP}_m(\mathrm{T})$ into a dollar amount $\phi_m(\mathbf{p},\mathrm{T})$ denoting the variable-cost compensation that m would require for the RTO/ISO-dispatched delivery of $\mathbf{p}$ during T.

The holistic form of the performance payment method $\phi_m(\cdot,\mathrm{T})$ helps to ensure conceptually-coherent settlements in grid-supported RTO/ISO-managed wholesale power markets. It permits the variable-cost assessment $\phi_m(\mathbf{p},\mathrm{T})$ for the period-T delivery of each power-path $\mathbf{p} \in \mathbb{PP}_m(\mathrm{T})$ to depend on the *correlated* physical attributes of $\mathbf{p}$. These correlated attributes include $\mathbf{p}$'s period-T capacity (MW) *profile*, ramp-rate (MW/min) *profile*, and power-factor (MVA/MVAr) *profile*, as well as $\mathbf{p}$'s period-T grid-delivered energy (MWh).

As explained and illustrated in [9, Ch. 16], transitioning to this alternative design would require changes in product definitions, settlement rules, and bid/offer contract formulations, but *not* in real-time operations. Thus, these required changes could be introduced gradually, without disruption of real-time operations.

The conceptual and illustrative findings presented in [9] for the Linked Swing-Contract Market Design have recently been buttressed by favorable 118-bus test-case findings reported for this design in a study by Li and Wang [17]. The key contributions of the latter study relative to previous work on this design are as follows:

- Conceptual development and analytic modeling of a Linked Swing-Contract (SC) Market consisting of a linked RTO/ISO-managed SC Day-Ahead Market (DAM), SC Hour-Ahead Market (HAM), and SC Real-Time-Market (RTM);
- Conceptual development, analytic modeling, and systematic performance testing of an SC offer for privately owned and offered battery storage;
- Use of 118-bus test cases to undertake systematic performance testing for Linked SC DAM-HAM-RTM operations, given high renewable power penetration firmed by privately owned and offered battery storage;
- Use of 118-bus test cases to undertake systematic comparative performance testing for Linked SC DAM-HAM-RTM operations and current U.S. DAM-HAM-RTM operations under high renewable power penetration.

C. Other Approaches

The Linked Swing-Contract Market Design [9] reviewed in Section VII-B provides an alternative way to think about product definition, settlement rules, and bid/offer contract formulations for U.S. RTO/ISO-managed wholesale power markets. What about other proposed approaches?

For example, what about the use of *Extended Locational Marginal Pricing (ELMP)* in RTO/ISO-managed short-run (day-ahead and intra-day) markets to reduce the need for the RTO/ISO to make *Out-of-Market (OOM)* make-whole payments to cleared suppliers to ensure their revenue sufficiency?

As carefully explained by Schiro et al. [18], ELMP is a "convex hull pricing" method for an RTO/ISO-managed short-run market. The goal of ELMP is to minimize the OOM "side-payments" that cleared suppliers of grid-delivered energy and operating reserve (unencumbered generation capacity) would need to be paid to ensure that their received revenues provide "incentive compatible" coverage of their short-run avoidable fixed costs (e.g., start-up, no-load, and energy opportunity costs) as well as any variable costs they incur for grid-delivered energy in accordance with dispatch instructions.

Thus, ELMP is an incomplete resolution for settlement issues in grid-supported RTO/ISO-managed wholesale power markets because it only ensures cost coverage for a narrowly-construed set of *short-run* costs incurred by suppliers of energy and operating reserve in *short-run* markets. As detailed in [9, Sec. 15.3], a key cause of supplier revenue insufficiency in current U.S. RTO/ISO-managed wholesale power markets is insufficient coverage of avoidable fixed costs incurred by *reserve* suppliers over *longer* look-ahead horizons. These longer-horizon avoidable fixed costs, listed by category in [11, App. A.4], include capital investment costs, transaction costs (e.g., employee-search expenses, licensing fees, billings for contract preparation), and longer-run opportunity costs arising from large-scale asset commitments.

On the other hand, recent work (e.g., [19]) on the *multi-interval* formulation, pricing, dispatch, and settlement of supply offers for RTO/ISO-managed wholesale power markets appears to have intriguing connections with the following key features of the Linked Swing-Contract Market Design: namely, the characterization of *reserve* as nodal-based *multi-interval* power-paths available for possible RTO/ISO dispatch during future operating periods; the use of swing ("flexibility") contracts for reserve offers; and the inclusion of performance-payment methods in these reserve offers that permit the holistic *multi-interval* valuation of power-paths based on their correlated dynamic physical attributes.

VIII. Concluding Remarks

This study establishes that LMP settlements for grid-delivered energy in U.S. RTO/ISO-managed wholesale power markets are conceptually problematic due to a fundamental many-to-one measurement problem for benefit/cost valuations.

More precisely, for these LMP settlements to be conceptually consistent, each market participant m electrically connected at a grid location $b(m)$ during an operating period T would need to consider all possible "next" units of grid-delivered energy E (MWh) at $b(m)$ during T to be perfect substitutes for each other. This *unit homogeneity property* is necessary for the existence of conceptually well-defined buyer marginal-benefit functions MB(E:*b*,T) ($/MWh) and supplier marginal-cost functions MC(E:*b*,T) ($/MWh) for grid-delivered energy E (MWh), conditional on delivery location *b* and delivery period T, hence for the existence of conceptually well-defined competitive (MB=MC) per-unit prices ($/MWh) for E (MWh), conditional on *b* and T.

In reality, however, the marginal benefit/cost valuation that a market participant or RTO/ISO assigns to each possible "next" unit of grid-delivered energy E (MWh), conditional on delivery location *b* and delivery period T, will typically depend strongly on the *dynamic* properties of the power-path used to implement the delivery of this "next" unit at *b* during T – for example, the power-path's ramp-rate *profile* at *b* during T. Consequently, any attempt to determine a *single* per-unit price LMP(*b*,T) ($/MWh) for grid-delivered energy E (MWh) at *b* during T could entail serious mis-measurement of the true benefit/cost valuations of market participants and system operators.

Interestingly, Kirkham et al. [20] argue that important physical operational issues for electric power networks can be traced back to a fundamental many-to-one measurement problem for key power concepts defined in IEEE Standard 1459 (2019): namely, active power, reactive power, apparent power, and power factor. Specifically, these definitions – expressed in static root-mean-square (RMS) time-averaged form for an electric power network during a conventionally designated time-period T – can correspond to multiple underlying dynamic realities with important distinct effects on the actual physical operation of the electric power network during T.

Implications of these two related many-to-one measurement problems for the design and operation of grid-supported RTO/ISO-managed wholesale power markets will be explored in future studies.

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