

**EVALUATING INDIVIDUAL MARKET POWER IN ELECTRICITY  
MARKETS VIA AGENT-BASED SIMULATION**

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**Abstract.** We use agent-based simulation in a coordination game to analyse the possibility of market power abuse in a competitive electricity market. The context of this was a real application to the England and Wales electricity market as part of a Competition Commission Inquiry into whether two particular generators could profitably influence wholesale prices. The research contributions of this paper are both in the areas of market power and market design policy issues for electricity markets, and in the methodological use of large industry-wide evolutionary simulation models.

*Keywords:* Agent-based, Bertrand game, Coordination game, Electricity, Market power, Simulation.

## 1. Introduction

With the widespread restructuring of electricity industries, moderating the abuse of market power has become a persistent issue for the various regulatory agencies that have been appointed to manage these new competitive markets. For a number of plausible reasons, including preserving security of supply, and (or) enhancing privatisation returns, the restructuring of electricity markets has resulted, in most cases, in too few players to ensure market efficiency. Regulators have often inherited, therefore, the *structural* deficiency of imperfect competition. Under such circumstances, with one or more generating companies evidently possessing market power, monitoring *conduct* and setting a framework for preventing the abuse of market power is a major, on-going, methodological task.

The specific motivation for the study reported here resulted from an initiative by the regulatory agency in Britain (Ofgem) to require all “significant” generators in the market to sign a market abuse licence condition (MALC), which was essentially a “good behaviour clause” in their licence to generate electricity. This had followed a decade of attempts during the 1990s to reduce market power structurally by enforcing divestiture of assets, with relatively little effect (at least by 1999) on wholesale prices, and was being sought (in 2000) ahead of the proposed New Electricity Trading Arrangements (NETA), which was introduced in 2001. Two generators had refused to sign the MALC on the basis that, in their argument, they could not exert a significant influence on market prices. One, British Energy (BE), was quite a large generator with over 10% market share, but being almost all baseload nuclear, claimed it rarely set market prices. The other, AES, had below 10% in market share. Because of this refusal to sign, the case was referred to the Competition Commission for analysis. This specific modelling challenge was therefore to assess, ex-ante, if such players could really

influence market prices profitably by attempting strategic withholding behaviour. Ofgem were clearly anxious that NETA would be introduced with strong restrictions to preclude high prices.

Looking at the literature in electricity markets we notice that several topics have been the focus of debate: a) Market design, e.g., Allaz and Vila (1993), Borenstein and Bushnell (1999), Day and Bunn (2001), and Crampes and Creti (2001); b) Market structure evaluation, e.g., Sweetser (1998), and Borenstein, Bushnell and Knittel (1999); c) Analysis of market equilibrium, e.g., Green and Newbery (1992), Von der Fehr and Harbord (1993); and d) Exploitation of network constraints to gain market power, e.g. Hobbs (1986), Wei and Smeers (1999). In particular, developing a stylised oligopoly model of competition in the England and Wales (E&W) electricity market was achieved by Green and Newbery (1992) using the supply-function equilibrium techniques developed earlier by Klemperer and Meyer (1989). Since then, the supply function equilibrium has become one of the main analytical models used to create competitive price functions for imperfect electricity wholesale markets<sup>1</sup>. The supply-function equilibrium models have been the least stylised of analytical methods, however they do still assume that the supply functions are continuous, whereas in practice generating units are offered in discrete blocks. This limitation leads to a new stream of research that uses discrete supply functions, e.g. Von der Fehr and Harbord (1993), but again analytical constraints require considerable simplifications in the market structure. Moreover, the supply function equilibrium assumes that generators submit a complete function to the market, associating price to quantity. This was the case in the E&W pool, but it is less easy to envision it in a continuously traded bilateral market.

Furthermore, Rothkopf (1999) reminds us that the interactions within an electricity market constitute a repeated game, whereby a process of experimentation and learning changes the behaviour of the firms in the market. A computational technique that could reflect these learning processes and that could model the structure and market-clearing mechanism (with a high level of detail) would therefore appear to be necessary. This technique is agent-based simulation and provides the kind of modelling platform required for company specific market power analysis in practice, such as in the Competition Commission case motivating this study.

The proposed introduction of NETA had already represented a good opportunity to test the usefulness of large-scale agent-based simulation to provide some insights on market design. Bower and Bunn (2000) studies the impact of the introduction of discriminatory pricing on the spot prices under a bilateral market: the agent-based platform enables a detailed description of the market, taking into account discrete supply functions, different marginal costs for each technology, and the interactions between different generators. Bunn and Oliveira (2001) follows that work by developing a simulation platform that represents, with much more detail, the way that market clearing in NETA was designed to function: a) It models the interactions between the Power Exchange (PX) and Balancing Mechanism (BM). b) It considers generators that may own different types of technologies. c) It considers an active demand side, incorporating suppliers. d) It takes into account the learning dynamics underlying these markets as a process by which a player selects the policy to use in the game by interacting with its opponents.

In this work, we adapt and extend the Bunn and Oliveira (2001) NETA simulation platform to analyse if the two companies in the Competition Commission Inquiry had enough market

power to operate against the public interest. In this work, we specifically analyse the capability of these companies to increase market prices<sup>2</sup>. In order to answer the specific question regarding these companies' conduct in the E&W electricity market, we need to take into account some of the important characteristics of electricity markets in general and of NETA in particular.

However, although computational approaches have the capability of allowing the analysis of detailed market models with multiple agents to learn various strategic parameters, this can pose a major problem in understanding and explaining the outcomes of the evolutionary simulations. Therefore, in this paper, we first develop a stylised Bertrand game with constraints, which we use to explore the main strategic decisions faced by the generators in the market, and to motivate the issue of conduct evaluation. The aim of the stylised model is to clarify the exposition of our argument, and to show why, in order to understand more fully the processes governing the market behaviour, we need to use some carefully formulated computational simulations. See Staiger and Wolak (1992), Von der Fehr and Harbord (1993), and Crampes and Creti (2001), for Bertrand games in the electricity industry, and Kreps and Scheinkman (1983) for a game theory analysis. Our stylised Bertrand game adapts Crampes and Creti (2001) framework to a pay-as-bid auction with  $N$ -generators.

The structure of the paper is as follows. In the next section, we describe NETA, as proposed for introduction in 2000. In section 3, we present the formulation of our agent-based simulation model for NETA prices. In section 4 we undertake some analysis of pricing and capacity withholding in a simplified market in order to motivate the need to use computational learning to evaluate conduct in a more realistic market model, which we do in

section 5. Section 6 addresses the important issue of the nature of equilibria that arise from agent-based simulation and section 7 concludes the analysis and case study.

## 2. The New Electricity Trading Arrangements

A good and detailed description of the way that NETA was originally designed to operate can be found in Ofgem (2000, March) and Ofgem (2000, April), and these references were used as the basis for our ex-ante modelling. NETA is based upon unadministered bilateral trading between generators, suppliers, traders and customers, which takes place in the forward markets before gate closure. The Balancing Mechanism (BM) works as a market where the System Operator (SO) buys and sells increments or decrements of electricity, from both generators and suppliers, in order to balance the system as a whole. However, individual generators and suppliers may be out of balance. During the Settlement Process (SP), the SO compares the contract positions (quantities contracted), plus whatever is bought or sold in the BM, with the actual position (quantities generated or consumed) for each one of the suppliers and generators (plant by plant) to calculate the *Imbalances*. The Imbalance may be a *spillage* (if a plant is generating more than it has contracted, or if a supplier is consuming less than it has contracted) or a *top-up* (if a plant is generating less than it has contracted, or if a supplier is consuming more than it has contracted). For both types of imbalances there is a price: if an agent is spilling it will receive as payment for the electricity generated the System Sell Price (SSP); if an agent is topping-up, it will pay the System Buy Price (SBP). The spread between the two prices is intended to provide a penalty for being out of balance: the SSP (SBP) is expected to be considerably lower (higher) than the prices in the forward markets. The day ahead, and within day, balancing schedules are presented in Table 1 that follows Ofgem (2000, April), pp. 160.

**Table 1: Day ahead and within day balancing schedules**

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*The SO day ahead balancing process:*

1. *By 09:00 publishes the day ahead demand forecast.*
2. *By 11:00 receives the Initial Physical Notifications (IPNs).*
3. *Calculates the available national plant margin or shortfall.*
4. *Verifies system security given demand predictions, the submitted IPNs and planned transmission outage.*
5. *By 12:00 issues the total system plant margin data to the market for the day ahead.*
6. *Forecast constraint costs based on the estimated Final Physical Notifications (FPNs) and bid (offer) prices and volumes.*
7. *If necessary calls the most economic Balancing Services contracts to ensure the system security.*
8. *During the following 11:00 hours, receives updates of the Physical Notifications (PNs).*
9. *By 16:00 publishes the revised national plant margin and zonal margin.*

*The SO within day balancing process:*

1. *Publish half hourly averaged demand forecasts for a defined period, until gate closure.*
2. *As participants become aware of changes to their physical position they advise the SO.*
3. *At defined times the zonal and national margins are reassessed and provided to the market.*
4. *Undertake security analysis and reassess the requirements of Balancing Services contracts.*
5. *At gate closure the PNs become FPNs and the SO will have received bids (offers) of the prices and volumes from the participants in the BM.*
6. *During the BM the SO balances the system taking into account: technical constraints; dynamic operating characteristics of generation and demand balancing services.*

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The SO starts the balancing process a day-ahead of time by publishing demand forecasts, both for the system as a whole and for each zone in the network. These forecasts are revised taking into account both the weather and the network conditions, and the *Physical Notifications* (PNs) by generators and suppliers. The SO also forecasts constraint costs based on the

estimated *Final Physical Notifications* (FPNs) and bid (offer) prices and volumes in the BM. If necessary, SO calls the most economic Balancing Services contracts to ensure the system security. Within the daily balancing process, the SO revises its demand forecasts, every half-hour for each defined period, until gate closure. The participants may revise their physical notifications until gate closure. At gate-closure, the PNs will become FPNs and the SO will have received bids (offers) of the prices and volumes from the participants in the BM. During the BM, the SO will balance the system taking into account the technical constraints; dynamic operating characteristics of generation and demand balancing services, and uncertainty in demand.

### 3. The NETA Model

The basic structure of the model follows Ofgem (1999, July), Ofgem and DTI (1999, October), although the details of the BM reflect also the later revisions Ofgem (2000, March) and Ofgem (2000, April). However, to build a detailed model of the balancing process and of its interactions with the power exchange and futures market would present overwhelming complexities and so, for tractability, we adopted some simplifications:

- a) The transmission system was not modelled.
- b) We model only a typical day, taking into account some plant dynamics. The model is aimed at analysing the process of finding the equilibrium solution for the specific daily profile simulated.
- c) We have simplified the continuous nature of trading in NETA, and represented both the forward market and the BM as two, sequential, one-shot markets.
- d) Finally, our model assumes independence between generators and suppliers.

We have built the PX (that in the model represents both the forward bilateral market and the power exchange) using auctions namely the Single Call Market (SCM) (see Cason and Friedman (1997)). In the SCM each player submits a bid or offer price to an auctioneer. The bids and offers have a limit price and the maximum quantity that the trader is willing to buy or sell. The auctioneer calculates the market-clearing price. The clearing-price mechanism used by the auctioneer was slightly modified to reflect the NETA trading principles, with the agents paying the price bid or receiving the price offered, instead of paying (receiving) the clearing price. The same type of auction is also adopted in other models of electricity markets (e.g. Von der Fehr and Harbord (1993) and Nicolaisen, Petrov and Tesfatsion (2001)). An interesting characteristic of this type of auction is that the traders tend to reveal their true values and costs.

The model is organised into trading days, or iterations. A trading day starts with the agents buying (selling) electricity in the PX. In these markets the suppliers try to buy, at a price as low as possible, the amount of electricity needed to fulfil their contract cover objective (the amount of electricity they want to buy in the PX). The generators try to sell at a price as high as possible, given their portfolio of plants, and the amount of electricity they want to save to sell in the BM. After the trading in the PX ends, each agent knows exactly how much it has sold (or bought) and it knows its FPNs, for each one of the hours. Overall, in each day (iteration) 48 auctions take place: in every hour, there is an auction in the PX and another one in the BM.

Then, the trading in the BM begins. Each supplier has an expected demand, and is able to sell or buy in the BM, in order to avoid Imbalances. Each generator knows the amount contracted for each one of its plants and the system position, long or short, and decides if it desires to sell

or buy electricity in the BM. It is noteworthy that, given a certain “system position”, only a certain type of offers is accepted, increments or decrements. The trades in the BM occur between the SO and each one of the generators and suppliers offering (bidding) in the BM. The SO accepts bids or offers from suppliers only for arbitrage reasons, otherwise it takes the offers or bids from generators. After all the trading in the BM is finished, the final positions and imbalances (of each generator and supplier) are calculated. The imbalance prices and costs are then computed. The algorithm is described by the pseudo-code in Table 2.

**Table 2: NETA model pseudo-code**

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*For the number of trading days specified*

- 1. Suppliers predict demand for each hour*
  - 2. Generators define which plants can run*
  - 3. Generators offer in the PX*
  - 4. Suppliers bid in the PX*
  - 5. Trading in the PX and calculation of System Position in each one of the hours*
  - 6. Generators and Suppliers offer (bid) into the BM*
  - 7. Trading in the BM and calculation of imbalance prices*
  - 8. Settlement Process: calculation of imbalances for each one of the suppliers and generators  
(plant by plant)*
  - 9. Learning*
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The conceptualised agents in the model represent generators (i.e. generation companies possibly owning several plants with different generation technologies) and suppliers (i.e. the agents purchasing from wholesale market in order to “supply” end-use customers).

At the end of each trading day, each agent tries to learn from the previous days' accumulated experiences, evaluating the profit got in the PX and in the BM. Then it defines *new policies* to bid or offer in the PX and in the BM, in order to maximise its expected profit. Each agent has some general *strategic objectives* that it tries to achieve, namely profit maximisation and exposure to the BM. An agent learns to improve its performance by using a reinforcement-learning algorithm: see Kaelbling et al. (1996), and Sutton and Barto (1998) for a survey on reinforcement learning.

In our formulation, the following notation is used.  $C_{ij}$  represents the short-run cost of plant  $j$  that is owned by generator  $i$ .  $PO_{ij}$ ,  $PO_{bm_{ij}}$ , ( $PB_{bm_{ij}}$ ) represent, respectively, the price offered in the PX and the price offered (bid) in the BM, by generator  $i$  for plant  $j$ .  $Q_{ij}$ ,  $Q_{x_{ij}}$  and  $QO_{bm_{ij}}$  ( $QB_{bm_{ij}}$ ) represent, respectively, the total electricity sold by plant  $j$ , the quantity sold in the PX and the quantity sold (bought) in the BM.  $PB_i$  ( $PB_{bm_i}$ ) represents the price bid in the PX (BM) by supplier  $i$ , and  $S_i$ ,  $S_{x_i}$ ,  $S_{bm_i}$  ( $SO_{bm_i}$ ),  $Imb_i$  represent, respectively, the market share, the quantity bought in the PX, the quantity bought (sold) in the BM, and the imbalance of supplier  $i$ .  $D$  and  $P_{retail}$ ,  $PB_{bm_i}$  ( $PO_{bm_i}$ ) represent, respectively, the total demand at a certain time, the retailing price and the price bid (offered) by a supplier buying (selling) in the BM.

The objective of each player is to maximise its (average daily) total profit, taking into account the assets it owns, the total demand and the behaviour of other players, and its objective for the BM exposure ( $BMX_i$ ). What is the intuition behind the BM exposure constraint? The significant gap between prices in the BM and in the forward and power exchange markets (the SBP (SSP) is significantly higher (lower) than the clearing price in the bilateral markets) acts as an incentive for suppliers to avoid the BM. A generator is less constrained by the BM system. If there is a shortage of electricity, it may sell in the BM (receiving the price offered,

if accepted by the SO); if there is excessive supply of electricity, the generator may also bid to reduce its generation (being paid the price bid, if accepted). So, why would a generator prefer to sell in the PX? First, baseload plant may prefer to sell its generation before gate closure due to its risk aversion against any interruption in continuous running. Second, since the suppliers have an incentive to trade in the forward and power exchange markets, the mid-merit technologies face a high demand in these markets. In contrast, it seems that the BM is the *natural environment* for expensive and flexible peak-plants looking for price spikes. Let us analyse separately the problem faced by generators and by suppliers.

### 3.1. Generators

A generator faces the problem of finding the best possible policy (how to price its plants), given the behaviour of competitors and of suppliers. Formally, a generator  $i$  tries to maximise daily profit ( $\pi_i$ ), equations (1), by iteratively learning the prices to offer (or bid) in the PX and in the BM (as a simplification, in this representation we assume that a generator is never imbalanced). The iterative process by which a generator tries to approximate the equilibrium represented by equations (1) is described by the pseudo-code in Table 3.

$$\begin{aligned}
 (1) \quad & \text{Max } \pi_i = \sum_j Qx_{ij} \cdot (PO_{ij} - C_{ij}) + QObm_{ij} \cdot (PObm_{ij} - C_{ij}) - QBbm_{ij} \cdot (PBbm_{ij} - C_{ij}) \\
 & \text{st.} \\
 & Q_{ij} = Qx_{ij} + QObm_{ij} - QBbm_{ij} \\
 & \sum_j QObm_{ij} + QBbm_{ij} \leq BMX_i
 \end{aligned}$$

**Table 3: Pseudo-code for generator's learning algorithm**

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*For each plant in the portfolio:*

- 1. Verify that the objectives for the BM exposure. If they were not achieved, penalise the profit obtained.*
  - 2. Given the daily profits in the PX and in the BM revise, for the mark-up used, the:*
    - 2.a) Expected profit in the PX and in the BM.*
    - 2.b) Expected acceptance rate in the PX and in the BM.*
  - 3. Define the new offering policy for the next day.*
  - 4. In the beginning of the next day. Define the quantities and prices offered for each plant, after knowing which plants are available, making sure that all the operational constraints are respected.*
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The expected profits and acceptance rate in Tables 3 and 4 are computed by using reinforcement learning. In this algorithm, the learning is through the *accumulated* history of previous plays. At each one of the iterations, a player updates the value of the expected profits and acceptance rates of each possible mark-up, and decides between following the best policy so far and exploring an alternative one. The full algorithm is presented and analysed in Bunn and Oliveira (2001).

### *3.2. Suppliers*

A supplier tries to find the best possible policy (which price to choose), given the behaviour of the other players. Formally, a supplier  $i$  tries to maximise the total daily profit, equations (2), by learning iteratively the prices to bid (or offer) in the PX and in the BM, and at the same time respecting its objective for the BM. The iterative process by which a supplier tries

to approximate the equilibrium represented by equations (2) is described by the pseudo-code in Table 4.

$$(2) \quad \text{Max } \pi_i = \begin{cases} \text{Sx}_i \cdot (P_{\text{retail}} - PB_i) + \text{Sbm}_i \cdot (P_{\text{retail}} - PBbm_i) + \\ \text{SObm}_i \cdot PObm_i + \text{Imb}_i \cdot (P_{\text{retail}} - SBP) & \text{if } \text{Imb}_i \geq 0 \\ \text{Sx}_i \cdot (P_{\text{retail}} - PB_i) + \text{Sbm}_i \cdot (P_{\text{retail}} - PBbm_i) + \\ \text{SObm}_i \cdot PObm_i - \text{Imb}_i \cdot SSP & \text{if } \text{Imb}_i < 0 \end{cases}$$

*st.*

$$S_i = \text{Sx}_i + \text{Sbm}_i - \text{SObm}_i + \text{Imb}_i.$$

$$\text{Sbm}_i + \text{SObm}_i + |\text{Imb}_i| \leq \text{BMX}_i$$

**Table 4: Pseudo-code for supplier's learning algorithm**

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1. *Verify that the objective for the BM exposure was achieved*
  - If not, penalise the profit obtained in the PX.*
  2. *Given the daily profits in the PX and in the BM revise the:*
    - 2.a) *Expected profit in the PX and in the BM*
    - 2.b) *Expected acceptance rate in the PX and in the BM*
  3. *Define the new bidding policy for the next day.*
  4. *In the beginning of the next day. After calculating predictions for demand, define the quantities and prices bid.*
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4. Conduct evaluation, multi-equilibria and learning

We now look more closely at the issue that motivated this study, namely how to evaluate the conduct of a given generator. It is worthwhile to note that we are not so much interested in analysing if a certain market design, or structure, leads to competitive equilibrium: Instead we

want to analyse whether a certain player, by its conduct, may find it profitable to price and withhold capacity strategically, in order to increase wholesale electricity market prices.

For this analysis, following Crampes and Creti (2001) we use a one-shot Bertrand game with capacity constraints, that we extend to the analysis of a pay-as-bid (each player pays the price bid or receives the price offered)  $N$ -players setting. This model is developed on the following assumptions: a) There is only one market, the power exchange (so that we can disregard the interactions with the BM). b) Each generator only owns one type of technology, characterised by a certain short-run production cost, assumed constant. c) Demand is inelastic,  $D$ . d)  $N$  generators  $a, b, \dots, N$ , with short-run generation costs  $C_a, C_b, \dots, C_N$ , and capacities  $K_a, K_b, \dots, K_N$ . e)  $D$  is such that  $\sum_{i=a}^N K_i > D$  and if  $K_s$  is the capacity of the smallest generator,

$\sum_{i=a}^N K_i < D + K_s$ . f) There is a maximum price,  $\bar{P}$ , that any generator is allowed to charge.

The Bertrand game with capacity constraints is a two-stage model. In the first stage, a generator chooses the amount of capacity available and, in the second stage, it chooses the price offered.

Let us start examining the situation where, in the first stage, all the generators decide to make all their capacity available. In the second stage of the game, each generator needs to decide if it wants to sell all its capacity, undercutting the prices offered by its competitors, or if it offers the highest possible price  $\bar{P}$ , exchanging quantity for price.

Let us assume that a generator  $i$  decides to offer  $\bar{P}$ , then its profit can be represented by equation (3).

$$(3) \quad \pi_i = \left( D - \sum_{j \neq i} K_j \right) \cdot (\bar{P} - C_i)$$

However, if generator  $i$  decides to undercut the price offered by the other generators in order to sell its full capacity, the profit function is represented by equation (4).

$$(4) \quad \pi_i = K_i \cdot (P_i - C_i)$$

At equilibrium, generator  $i$  prefers to charge the maximum price if and only if

$$\left( D - \sum_{j \neq i} K_j \right) \cdot (\bar{P} - C_i) \geq K_i \cdot (P_i - C_i). \text{ Solving this equation in order to get } P_i \text{ we compute the}$$

upper bound ( $\bar{P}_i$ ) on  $P_i$  such that it is better for generator  $i$  not to undercut its competitors, equation (5).

$$(5) \quad \begin{aligned} P_i &\leq \bar{P}_i \\ \bar{P}_i &= \frac{\left( D - \sum_{j \neq i} K_j \right)}{K_i} \cdot (\bar{P} - C_i) + C_i \end{aligned}$$

This means that if, in order to sell its full capacity, a generator  $i$  has to undercut its rivals' offers by offering a price above ( $\bar{P}_i$ ), it should do so, otherwise, if the undercutting price is lower than ( $\bar{P}_i$ ) it should offer the maximum price,  $\bar{P}$ . Without loss of generality, let us assume that for all  $i \neq M$   $\bar{P}_i < \bar{P}_M$ .

Let us analyse the properties of this Bertrand game with capacity constraints<sup>3</sup>. In this one-shot game, how would a generator choose its offer price, given that the capacity was defined in the

first stage? Next, we show that every player can earn a risk-free profit in a possible outcome of the game (see Proposition 1). Further, in Proposition 2 we show that this game has no Nash equilibrium (proofs for both propositions are in the Appendix). Then, by using the security-payoff criterion proposed by Von Neumann and Morgenstern (1972)<sup>4</sup>, we analyse the implications of the risk-free payoff on the behaviour of generators.

**Proposition 1:** *In the one-shot pay-as-bid Bertrand game with capacity constraints, all the generators earn the maximum security-payoff when generator  $M$  offers the maximum price and the other generators undercut the upper bound of generator  $M$ ; i.e.  $P_M = \bar{P}$  and  $P_i = \bar{P}_M - \varepsilon$  for every  $i \neq M$  is the solution at which every agent is earning its maximum security-payoff.*

**Proposition 2:** *In the second-stage (after the capacity available is revealed) of the one-shot pay-as-bid Bertrand game with capacity constraints defined above there is no Nash equilibrium.*

This simple example shows two very interesting characteristics of security-payoff solution for the Bertrand competition with capacity constraints. a) Generators tend to charge prices well above marginal costs, if the market structure allows it. b) Capacity and marginal costs interact to define the pricing strategy of the players. In the example above the highest price is charged by generator  $M$  but this is not necessarily the one with the highest marginal cost.

Moreover, from equation (5) it is trivial to show that if any of the players, or some of them, jointly decide to reduce the available capacity in such a way that  $\sum_i K_i = D$ , then each

generator, at equilibrium, can offer the maximum price, i.e.  $P_a = \dots = P_N = \bar{P}$ . When will a generator withhold capacity? The answer is, whenever this behaviour leads to higher profits. In our example, assuming the security-payoff solution, the players' profits are described by equation (6).

$$(6) \quad \begin{aligned} \pi_i &= K_i \cdot (\bar{P}_M - C_i) \quad \text{for all } i \neq M \\ \pi_M &= \left( D - \sum_{i \neq M} K_i \right) \cdot (\bar{P} - C_i) \end{aligned}$$

Next, Proposition 3 (for which proof is in the Appendix) shows that if a generator has the capability of withholding capacity it may be possible for this generator to increase the security-payoff that can be earned by every generator in the industry. In this case, we analyse the interactions between the decisions in the first and second stages of the Bertrand game with capacity constraints.

**Proposition 3:** *Let  $K_i$ ,  $K_i'$  and  $\bar{P}_M$  stand for, respectively, the full and reduced capacity of generator  $i$ , and the upper bound of generator  $M$ , when generator  $i$  decreases its capacity to  $K_i'$ . Assuming the maximum security-payoff solution of the Bertrand game with capacity constraints, a player  $i \neq M$  can increase its security-payoff by unilateral capacity*

*withholding if and only if  $\bar{P}$  is such that  $\left( D - \sum_{j \neq M} K_j \right) + \frac{C_M - C_i}{\bar{P} - C_M} \cdot K_M < K_i' < K_i$ .*

Hence, Proposition 3 shows that in the conditions defined unilateral capacity withholding may increase the security-payoff that can be earned by a player, leading to higher prices (for the maximum security-payoff solution) above the ones predicted by the Bertrand model with full capacity available (as  $\bar{P}'_M > \bar{P}_M$ ).

As our analysis shows, the profit margin is not a good indicator of abusive behaviour. Assume that a player  $i$  decides to withhold capacity which leads to a generator  $j$  to increase its offer price such that it has the highest profit margin. In this case the market abuser is generator  $i$  and not generator  $j$ . Thus, the task of identifying individual market power is a challenging one, and *the regulator tends to look at capacity availability as the observable variable that indicates abusive behaviour by a certain generator* (as in bilateral contracts prices are private information).

Furthermore, this model also justifies why, in repeated games, we need to take into account learning behaviour to analyse the possible solutions of the game. In a repeated game, a player may try to learn the propensity of the generator  $M$  to undercut its prices. Thus, different solutions can emerge given the different risk propensities of the players. Which solution prevails? How do the players arrive at it? This is where learning theory enters by postulating that, in repeated games, a possible equilibrium solution may emerge from an iterative learning process, by trial and error, where a player tends to revise its pricing behaviour in order to maximise profit. It is noteworthy that Roth and Erev (1995), Feltovich (2000), and Sarin and Vahid (2001) show that learning models predict better than the Nash equilibrium how people behave.

Finally, the aim of this simple Bertrand model is to motivate the analysis of the capacity withholding issue. Even in a situation where capacity is very tight and where each player owns only one type of technology, it may be possible for capacity withholding to be a profitable strategy. The model presented is clearly restrictive as it assumes that there is only one marginal generator, that a generator only owns one type of technology, and that there is a

known maximum price. Even such a model does not have an equilibrium solution. The use of agent-based simulation, as presented next, enables the researcher to develop models of the electricity industry that are more realistic than the ones achieved by analytical game theory.

5. The AES-British Energy Case

In this section we present the experiments used to analyse the capability of AES and (or) BE to influence the wholesale electricity prices. In Table 5<sup>5</sup> we present the market shares of the eight largest generators in the E&W electricity market (in 2000).

**Table 5 : Generation Industry**

Capacity of each Company (% of Total, 59 GW) in 2000						
	Total	Nuclear	Large Coal+CCGT	Small OIL + Pump. Storage	Coal	+OCGT +
PG	16.5		17.8	24.9		
NP	13.9		14.8	22.5		
BE	12.4	58.1	7.1			
Edison	10.6		9.1	30.7		
TXU	9.7		10.5	14.7		
AES	7.8		9.2	6.8		
EDF	4.7	27.8	1.8			
Magnox	3.9		5.1			
Others	20.5	14.1	24.7	0.4		
Total GW	59	7	45	7		

We split the generation capacity owned by each generator into three categories, taking into account the degree of flexibility and running times of each technology: - Nuclear plants,

baseload technology running continuously. - Large Coal and CCGT, the mid-merit technologies. - Small Coal, OCGT, Oil and Pumped storage, the peak plants.

As shown with the simple example in the previous section, a generator’s pricing behaviour is a function not only of the technologies owned (the marginal costs), but also of its market share. Hence, it is important to look at the relative market share of each generator, in each type of technology, in order to understand its *strategic position*. Thus BE had 58% of the Nuclear<sup>6</sup> generation capacity installed in E&W (and 7.1% of the mid-merit capacity), while AES owned both mid-merit (9.2% of mid-merit capacity) and peak plant (7.8% of peak capacity).

The market shares for the retail supply business (in 2000) are presented in Table 6.

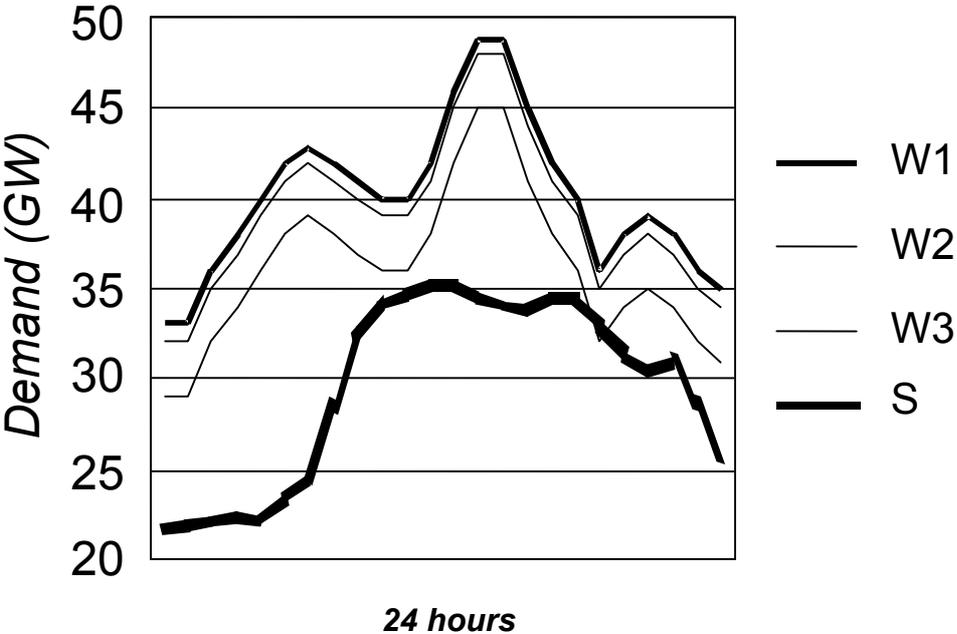
**Table 6: Supply Industry**

Peak Demand 50 GW in 2000			
Market Share of each Company (%)			
London Electricity	12.2	PG	10.2
Eastern	11.2	Scot. & Southern	9.9
NP	10.2	Others	40.7

If we only take into account the market share, the supply business was less concentrated than the generation business. The biggest player was London Electricity with 12% of market share.

The detailed agent-based model was applied to the full system of E&W, as it existed in 2000. Some small gensets were aggregated and overall the model was specified with 80 gensets, owned by 24 generators who sell power to 13 suppliers. Each supplier is an active agent in the

model and is assumed to have a 10% forecasting error on its BM exposures. The total capacity in the system was 59 GW, and three demand profiles for winter (with peaks 49 GW, 48 GW and 45 GW) and one of summer (with three maintenance scenarios 14 GW, 12 GW and 10 GW) were used. The demand profiles are presented in Figure 1.



**Figure 1: Demand profiles.** Lines W1, W2 and W3 stand for winter demand with a peak of 49, 48 and 45 GW respectively. Line S stands for summer demand with a peak of 35 GW.

Active trading between generators and suppliers was simulated as a multistage process through a PX (that works as a proxy for all the forward bilateral markets) and a BM. Settle. All the comparative experiments have simulated 50 iterations (trading days) of the algorithm and the results were averaged over the final 10 days. In most cases, the evolution of prices seemed to settle around a stable position after the initial learning, by about 50 iterations.

We have simulated 6 different *Strategies* for AES and (or) BE in relation to making capacity available, under 6 different *Scenarios*. In strategy ST1, AES and BE offer their full capacity. In strategy ST2, AES saves capacity for the BM (480 MW). In strategy ST3, BE saves capacity for the BM (490 MW). In strategy ST4, AES saves capacity for the BM (480 MW)

and closes a Drax genset (645 MW). In strategy ST5 BE saves capacity for the BM (490 MW) and closes an Eggborough genset (480 MW). Finally, in strategy ST6, AES saves capacity for the BM and closes a genset, and at the same time BE saves capacity for the BM and closes a genset, i.e. strategies (4) and (5) combined. Regarding the possible scenarios, we have three winter day scenarios 1, 2, and 3 with a peak demand of 49 GW, 48 GW and 45 GW, respectively; and three summer demand scenarios, 4, 5, 6, with a maintenance outage of 14 GW, 12 GW and 10 GW, respectively.

Each one of these scenarios was repeated twice with small differences (that is with different random impacts or with different maintenance levels for some plants). We started by comparing the average prices of the six strategies (under the twelve simulations). As expected by the theoretical Bertrand model with constraints, in our simulations the average PX price and SBP are higher than the marginal costs (even with full capacity available).

Hence, we can conclude that the E&W wholesale electricity market had a structure that would give rise to prices above marginal costs, and therefore non-competitive. This implies the existence of market power in the *traditional sense* (capability to rise prices above marginal costs) that results from the overall structure of the industry. In this sense, it is the joint behaviour of all players in an imperfect market, profit maximising without explicit collusion, that results in the higher prices, to the extent that is not possible to say that any individual player is abusing market power. If, however, a player seeks to alter the market structure by deliberately withholding capacity, then the regulatory perspective has clearer grounds for identifying individual market power abuse.

How can we evaluate the capability of AES and BE to manipulate market prices through capacity withholding? As shown in the simple model of the previous section, we need to compare the prices under strategies ST2, ST3, ST4, ST5 and ST6 with the prices under strategy ST1, i.e., we need to evaluate the performance of the market when the players withhold capacity with its performance when full capacity is available.

Unfortunately, this task is not as straightforward as in the Bertrand stylised analysis. In the NETA computational model, we have to take into account the presence of multiple equilibria, and the effects of learning, which make the analysis quite challenging. In order to analyse the results of the simulations we compare the different simulations using t-statistics (for pooled samples). If the t-statistic associated with a certain strategy (ST2, ST3, ST4, ST5 or ST6) is higher than 2.2 (the critical value for 5% significance level with 11 degrees of freedom) then that strategy has a significant impact on prices or profits<sup>7</sup>, otherwise there is no statistical evidence that the particular strategy has an impact on prices and or profits. The t-statistics for the PX price (PXP), the SSP and the SBP are presented in Table 7.

**Table 7 : t-statistics for market prices**

	Strategies				
	ST2	ST3	ST4	ST5	ST6
PXP	1.12	2.21	2.61	2.37	3.55
SSP	0.23	0.83	1.92	0.26	1.7
SBP	-0.15	0.95	0.75	1.98	1.5

The experiments show that AES and BE have enough market power to increase the PXP to levels above the normal prices resulting from the full capacity strategy ST1. Nevertheless,

these players have no capability to influence the SSP or the SBP. The BM prices tend to be more dependent on random impacts and path dependencies than on the manipulation attempts by AES and (or) BE. Therefore, we have statistical evidence, through our simulations, that AES and BE can behave as price makers, but is that synonymous of higher profits? Can they profit from capacity withholding?

In order to analyse the profitability of each strategy (ST2, ST3, ST4, ST5 or ST6) we compare AES' s and BE' s profit contributions under each one of these strategies with their profit contributions under the full capacity strategy ST1. Again, we analyse the profitability issue using t-statistics for the pooled samples. We compare the change in profit for each strategy (when compared with strategy ST1), under each scenario.

**Table 8: t-statistics for AES' s and BE' s profits**

	Strategies				
	ST2	ST3	ST4	ST5	ST6
AES	-0.09	1.77	1.31	1.61	2.29
BE	-1.03	0.32	0.58	1.57	3.04

The t-statistics in Table 8 show that only when both players simultaneously withdraw capacity from the market (and save capacity for the BM) can they profit from price manipulation. These results seem to suggest that profitable price manipulation appears to be harder than expected by the standard market power literature in electricity markets.

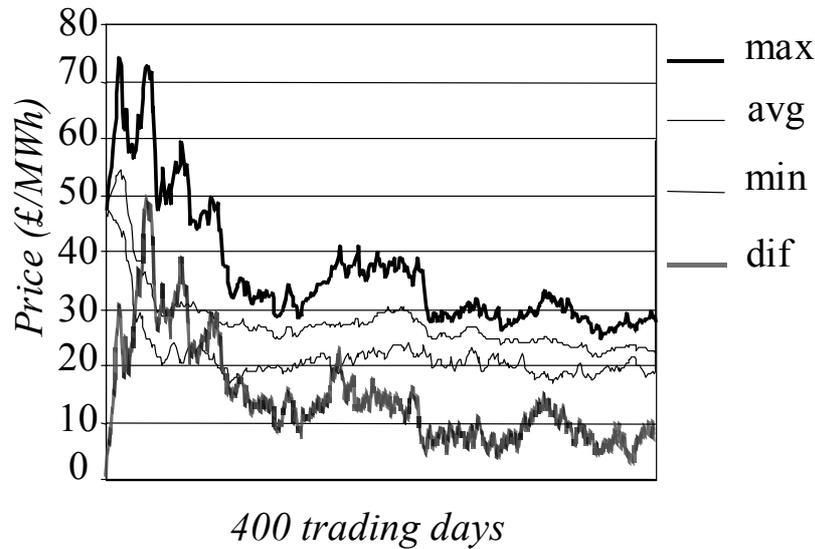
## 6. Equilibrium Analysis

Finally, in order to improve our understanding of the simulation results, let us analyse the trajectory of PXP prices. Given the theoretical intuition of section 4, we would expect to encounter two types of price regimes in the NETA model:

a) Whenever the capacity constraint is so tight that several generators may increase their offers well above marginal costs, we would expect a regime displaying non-convergent behaviour. In this regime, the players need to learn to coordinate their behaviour in such a way that implicitly, by trial and error, they learn the pricing strategy that gives higher profits. However, Staiger and Wolak (1992) shows that collusion is easier in high demand regimes.

b) Whenever there is an upper bound on the maximum price, the uncertainty regarding the maximum price is drastically reduced (see Von der Fehr and Harbord (1993)).

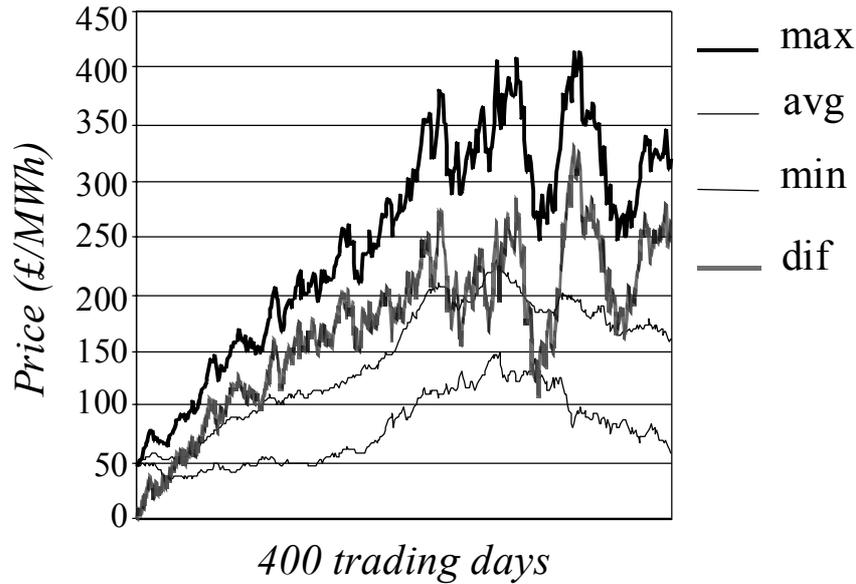
One possible way to analyse the equilibrium properties of the NETA model is to repeat an experiment several times, and check if the prices converge. Similarly, we could run the experiment for a longer period and analyse if prices converge. This was done by undertaking an extensive analysis of the basic scenario of a winter day with peak demand of 49 GW, and total capacity of 59 GW. The experiment was repeated 10 times and, in each one of the runs, 400 trading days were simulated.



**Figure 2: Price behaviour in an hour with high demand (45 GW).** The series *max*, *avg*, and *min* stand for, respectively, the maximum, the average, and the minimum prices in the 10 runs of the simulation at any of the 400 iterations. The series *dif* represents the difference between the max and the min of the 10 runs of the simulation, for each one of the 400 iterations.

In Figures 2 and 3 we analyse the price behaviour in the 10 runs of the model by presenting the average, the maximum, the minimum, and the difference between maximum and minimum prices of the 10 different runs, for each one of the 400 iterations simulated.

The analysis of Figure 2 suggests that average prices converged to equilibrium. For example, at 50 days, we have prices between £18.6/MWh and £34.1MWh, with an average of £24.6/MWh. At 400 trading days the prices range from £17.6 to £22.7/MWh, with an average of £19.8/MWh. This corresponds to an hour of high (45GW), but not peak, demand; and with 59 GW of capacity available, prices achieve an apparently stable, but still significant, mark-up.



**Figure 3: Price behaviour in the hour with peak demand (49 GW).** The series *max*, *avg*, and *min* stand for, respectively, the maximum, the average, and the minimum prices in the 10 runs of the simulation at any of the 400 iterations. The series *dif* represents the difference between the max and the min of the 10 runs of the simulation, for each one of the 400 iterations.

On the other hand, Figure 3 suggests that for the peak demand hours the process presents two distinct periods. In the first one, prices tend to increase steadily. In the second period, the average price stabilises and the difference between maximum and minimum starts to have a more unstable behaviour, with increasing variance, and no convergence. Thus, this seems to be closer to the first type of regime where the maximum price can be influenced by the players' expectations, and where it is harder to coordinate behaviour. This leads to higher prices and unstable behaviour.

## 7. Conclusions

We submit a stylised model of the interactions in an electricity market (modelled as a Bertrand game with capacity constraints) where we show that even when all generators offer their full capacity, and even if the total capacity available is higher than total demand, prices can be much higher than marginal costs (proposition 1). In proposition 2, we show that

unilateral capacity withholding may lead to increased profits and misevaluation of market power abuse, in certain circumstances.

Our research question is the evaluation of the conduct of an individual generator. In the case under study Ofgem, the E&W regulator, was worried about the possibility of AES and BE having the capability to profitably manipulate market prices.

In order to test if AES and BE could manipulate the bilateral markets or the balancing mechanism, and (or) the interactions between them, we simulated several scenarios aimed at producing increased prices, for these generators' possible behaviour. A first contribution of these simulations is to show that the structure of the E&W electricity market was such that prices would significantly tend to rise above marginal costs, even without considering other technical constraints such as ramping rates and transmission. Regarding the specific questions raised by the Competition Commission, our experiments show that:

- BE alone, and BE simultaneously with AES, may significantly increase the Power Exchange prices.
- AES alone cannot significantly influence the wholesale Power Exchange prices.
- The System Sell Price and the System Buy Price cannot be significantly influenced by the strategies implemented by AES and (or) BE.
- Only when acting simultaneously may the firms profit significantly from capacity and price manipulation.

The Competition Commission concluded that it would *not* be against the public interest for the market to operate without modifications of the licence provisions of AES and BE to include a clause seeking to preclude Market Abuse, mainly on the basis of the awkward

principle of this as a regulatory instrument. (Competition law was considered sufficient, by itself, to deal with market abuse, if it did become apparent.)

Overall, the agent-based simulation technique enabled substantial insight to be gained into the behaviour of the new wholesale electricity market in 2000, before its introduction in 2001. This technique enabled the modelling of complex adaptive behaviour in an environment with possible multiple equilibria, with heterogeneous agents and price uncertainty. This shows that models having the functional capability to represent the type of learning occurring in real electricity markets can be used to model the processes underlying the equilibrium selection and behaviour coordination in complex electricity markets.

Furthermore, as we assume a constant demand profile for each scenario of the game, the simulation may be perceived as an iterative approach to compute the solution of the one-shot game. However, since the rewards are learned by interacting with others (and discounted from past experience), these repetitions of the game may change the perceptions an agent has of the others. The learning algorithm aims at capturing this evolutionary process, which may open the door to implicit collusion. This outcome can only be achieved in repeated games. Finally, however, it is plausible that, since this learning process occurs in a stable environment where agents iteratively adapt their behaviour to specific conditions, the potential for strategic coordination can be over-estimated by this type of modelling. In real markets the players have to learn in an environment where demand, fuel costs, and transmission constraints reshape the system continuously, which makes behaviour coordination harder to achieve.

## APPENDIX

**Proof of Proposition 1:** (a) Players  $i \neq M$ .

From equation (5), it follows that each generator may offer a price in the range  $[\bar{P}_i, \bar{P}]$ . Then,

together with condition e)  $\sum_{i=a}^N K_i > D$  and  $\sum_{i=a}^N K_i < D + K_s$  and the assumption that for all

$i \neq M$   $\bar{P}_i < \bar{P}_M$ , it follows that any generator  $i \neq M$  can ensure that its full capacity is sold by

offering in the range  $[\bar{P}_i, \bar{P}_M - \varepsilon]$ . Finally, by equation (4) it follows that by offering  $\bar{P}_M - \varepsilon$

each generator  $i$  earns the maximum security-payoff.

(b) Player  $M$ .

It follows from equation (5) that generator  $M$  may offer a price in the range  $[\bar{P}_M, \bar{P}]$ .

However, if additional conditions are considered, i.e., condition e)  $\sum_{i=a}^N K_i > D$  and

$\sum_{i=a}^N K_i < D + K_s$  and the assumption that for all  $i \neq M$   $\bar{P}_i < \bar{P}_M$ , then as any player  $i$  may offer

a price in the range  $[\bar{P}_i, \bar{P}]$ , it follows that the maximum security-payoff of generator  $M$  is

earned when it offers  $\bar{P}$ . Please note that, given the conditions above, if any generator  $i \neq M$

offers  $\bar{P}$  as well, then  $M$ 's payoff increases as it still sells at the price  $\bar{P}$  and, depending on

the rationing rules, it may increase, but not decrease, its output.  $\square$

**Proof of Proposition 2.** In Proposition 1 it was shown that every generator  $i \neq M$  offers in the

range  $[\bar{P}_i, \bar{P}]$ , and that generator  $M$  offers in the range  $[\bar{P}_M, \bar{P}]$ , where  $\bar{P}_i < \bar{P}_M < \bar{P}$ , for all

$i \neq M$ . Then, since by Proposition 1 it is known that, for every generator  $i \neq M$  the price

$\bar{P}_M - \varepsilon$  gives these players the maximum security-payoff, it follows that they actually offer in

the range  $\left[\overline{P}_M - \varepsilon, \overline{P}\right]$  as the price  $\overline{P}_M - \varepsilon$  dominates the offers in the range  $\left[\overline{P}_i, \overline{P}_M - \varepsilon\right]$ . We now have a matrix game played between each generator  $i \neq M$  and generator  $M$ . Each player  $i$  may decide to offer the price that gives him the security payoff  $\overline{P}_M - \varepsilon$  or to offer in the range  $\left[\overline{P}_M + \varepsilon, \overline{P} - \varepsilon\right]^8$  that, by equations (4) and Proposition 1, leads to a higher payoff than the security-payoff (but also to a higher risk). On the other hand, generator  $M$  may offer  $\overline{P}$ , which by Proposition 1 gives at least the security-payoff, or it may offer in the range  $\left[\overline{P}_M, \overline{P} - \varepsilon\right]$  that, by equation (5), leads to increased payoffs if the player undercuts the price offered by at least one of its opponents (otherwise it leads to lower payoffs). Now, let us analyse the best-response (BR) behaviour of the players:

- a) If every generator  $i \neq M$  offers  $P_i = \overline{P}_M - \varepsilon$  then  $M$ 's BR is  $P_M = \overline{P}$ , by Proposition 1.
- b) If at least one generator  $i \neq M$  offers  $P_i \in \left[\overline{P}_M + \varepsilon, \overline{P} - \varepsilon\right]$  then  $M$ 's BR is  $P_M = P_i - \varepsilon$ : by equation (5). The BR strategy of all other generators  $j$  such that  $j \neq i \neq M$  is to offer  $P_j = \overline{P}_i - \varepsilon$ : by equation (4) and the possible offering range  $\left[\overline{P}_M - \varepsilon, \overline{P}\right]$ .
- c) If generator  $M$  offers  $P_M = \overline{P}$  then the BR of every player  $i \neq M$  is to offer  $P_i = \overline{P} - \varepsilon$ : by equation (4) and the possible offering range  $\left[\overline{P}_M - \varepsilon, \overline{P}\right]$ .
- d) If generator  $M$  offers  $P_M \in \left[\overline{P}_M, \overline{P} - \varepsilon\right]$  then the BR of every player  $i \neq M$  is to offer  $P_i = P_M - \varepsilon$ : by equation (4) and the possible offering range  $\left[\overline{P}_M - \varepsilon, \overline{P}\right]$ .

In this matrix game there is no Nash equilibrium as any of  $M$ 's BR to the strategic choices of every player  $i$  leads at least one of these players to change its behaviour: by a) and c) or by b) and d). Similarly, a player  $i$ 's BR to the strategic behaviour of  $M$  and of all other players  $j \neq i \neq M$  leads to changes in the behaviour of these players: by a) and c) or by b) and d).  $\square$

**Proof of proposition 3.** By equation (4), a player  $i \neq M$  can increase its security-payoff through capacity withholding if and only if  $K'_i \cdot (\overline{P}_M - C_i) > K_i \cdot (\overline{P}_M - C_i)$ . Then we can define

$\overline{P}'_M$  as a function of  $\overline{P}_M$  using equation (5), getting  $\overline{P}'_M = \overline{P}_M + (\overline{P} - C_M) \cdot \frac{K_i - K'_i}{K_M}$ . Replacing

this expression in the profitability condition, we get a condition for profitability depending

only on  $\overline{P}$  and  $\overline{P}_M$ , i.e.  $C_i - \overline{P}_M + (\overline{P} - C_M) \cdot \frac{K'_i}{K_M} > 0$ . Using equation (5), we can get the

profitability condition as a function of  $\overline{P}$  only, i.e.  $K'_i > \left( D - \sum_{j \neq M} K_j \right) + \frac{C_M - C_i}{\overline{P} - C_M} \cdot K_M$ . Finally

$K'_i < K_i$ : by definition of capacity withholding.  $\square$

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## NOTES

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<sup>1</sup> Anderson and Philpott (2002, a, b) analyse the optimal behaviour of a generator offering a supply function under uncertainty.

<sup>2</sup> The Competition Commission defined market power as the ability of a generator, acting independently, to raise prices consistently and profitably above competitive levels.

<sup>3</sup> In the following analysis, we also assume that when two or more generators offer the same price a rationing rule is applied. A commonly used rationing rule is the proportion of the available capacity, i.e., each generator offering the same price serves a proportion of the residual demand equal to the percentage it owns of the total capacity owned by the generators offering that price. Other criteria may also be used such as efficient rationing, i.e., the demand is served by the generators with the lowest marginal cost.

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<sup>4</sup> This solution may be supported by experimental evidence from Van Huyck et al. (1990) and Cooper et al. (1990) that report experiments in which the equilibrium converges to the security-payoff.

<sup>5</sup> A good source of information on the history of the electricity sector in the UK is the Electricity Association: - A brief history of the UK electricity sector (Electricity Association, 2000 a). - A list with a description with all the plants in the UK (Electricity Association, 2000 b). - Company ownership (Electricity Association, 2000 c). - And finally an analysis of the industry's evolution during the 90's (Electricity Association, 1999).

<sup>6</sup> Salama (1997) presents a case study on British Nuclear Fuels (BNFL) analysing the impact of market deregulation on corporate culture and objectives definition.

<sup>7</sup> For the  $t$  test, we pre-tested for normality and could not reject the hypothesis of normality for the sample distribution of the differences, for both prices and profits, using the Kolmogorov-Smirnov test for normality. The overall conclusions for both prices and profits were also corroborated with Wilcoxon nonparametric tests.

<sup>8</sup> The price  $\bar{P}_M$  is dominated by the price  $\bar{P}_M - \varepsilon$  as a generator  $i$  ensures to sell all its capacity without losing any profit, since  $\bar{P}_M \approx \bar{P}_M - \varepsilon$ . Similarly, the price  $\bar{P}$  is dominated by  $\bar{P} - \varepsilon$ , as shown in proposition 1, since by the rationing rule player  $i$  sells less when generator  $M$  also offers  $\bar{P}$ , without compensating the quantity reduction by higher prices as  $\bar{P} \approx \bar{P} - \varepsilon$ .

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