Understanding Interest Rates

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Notes on Mishkin Chapter 4: Part A

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Mishkin Chapter 4: Part A -- Selected Key In-Class Discussion Questions and Issues

- Five basic types of **debt (or credit market) instruments**. Who pays what, to whom, and when?
- Why is **present value (PV)** considered to be one of the most important concepts in finance?
- Why is **yield to maturity (YTM)** considered to be the most important measure of an interest rate?
- PV and YTM -- **what's the connection?**
- **Illustrations**

Five Basic Types of Debt Instruments

1. Simple Loan Contracts
2. Fixed-Payment Loan Contracts
3. Coupon Bond
4. (Zero-Coupon) Discount Bond
5. Consol (or Perpetuity)

Type 1: (One-Year) Simple Loan Contract

- Borrower issues to lender a contract stating a loan value (principal) LV ($) and interest payment I ($).

- Today the borrower receives LV from lender.

- One year from now the lender receives back from the borrower an amount LV+I.

- ***Example:* One-Year Deposit Account**

  Deposit LV = $100; Interest payment I = $10

  Borrower's end-of-year payment = $100 + $10 .

Type 2: Fixed Payment Loan Contract

- Today a borrower issues to a lender a contract with a stated loan value LV ($), an annual fixed payment FP ($/Yr), and a maturity of N years
- Today the borrower receives LV from the lender.
- For the next N successive years, the lender receives from borrower the fixed payment FP.
- FP includes principal and interest payments

***Example:* 30-year fixed-rate home mortgage**

Type 3: Coupon Bond

- Today a seller offers for sale in a bond market a bond with stated annual *coupon payment* C ($/yr), *face* (or *par*) value F ($), and a remaining *maturity* of N years.

- Today the bond seller receives from a buyer a price P ($/bond) ***as determined in the bond market.***

- For next N successive years, the bond holder receives the fixed annual payment C ***from original bond issuer.***

- At maturity, the bond holder also receives the face value F ***from the original bond issuer.***

Examples: **30-year corporate bond, U.S. Treasury notes (1-10yrs) and bonds (≥ 10yrs)**

Type 4: Discount Bond

- Today a seller offers for sale in a bond market a bond with a stated face value F ($) and remaining maturity of N years.

- Today the bond seller receives from a buyer a price P ($/bond) ***as determined in the bond market.***

- At the end of N years the bond holder receives the face value F ***from the original bond issuer.***

- ***Example:*** **Treasury Bills** Maturity < 1yr., typically offered in 1mo., 3mo., & 6 mo. maturities. The U.S. Treasury stopped offering 1yr (52-week) bills in 2001.

Type 5. Consol (or Perpetuity)

- Today a seller offers for sale in a bond market a bond with a stated annual coupon payment C ($/Yr) and no maturity date (i.e., bond exists “in perpetuity”).

- Today the bond seller receives from a buyer a price P ($/bond) ***as determined in the bond market.***

- In each future year the bond holder receives the coupon payment C ***from the original bond issuer.***

- ***Example:*** **Consols were originally issued by UK in 1751, and remain a small part of UK’s debt portfolio**.

Interest Rates and the Yield to Maturity

- **Interest Rate:** Measure of cost of borrowing money
- The most important interest rate that economists calculate is the "Yield to Maturity" (YTM):

YTM for an asset A = The interest rate i that equates the "**current value**" of A with the "**present value**" of all future payments received by the owner of A

- What does "**Current Value (CV)**" mean?
- What does "**Present Value (PV)**" mean?

Calculating Present Value (PV)

- **PV** is the value today of future received money
- Suppose the annual interest rate is i.
- The present value of \$100 to be received N years in the future is

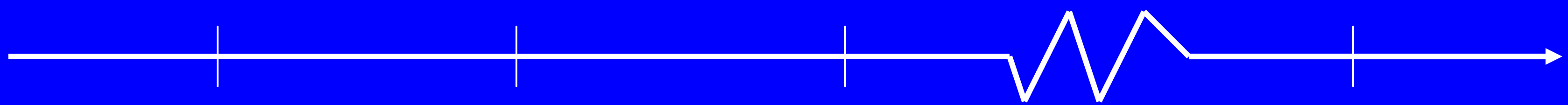
$$PV = \$100/(1+i)^N$$

Why?

- If $PV = \$100/(1+i)^N$ is deposited today, and left to accumulate interest for N years, the amount at end of N years is $(1+i)^N \cdot [\$100/(1+i)^N] = \$100$

Timing of Payments

Cannot directly compare payments received at different points in time:



	\$100	\$100	\$100	\$100
Year	0	1	2	N
PV	100	$100/(1+i)$	$100/(1+i)^2$	$100/(1+i)^N$

Numerical Examples

- If i = 10%, and $1 is received one year from now,

  $PV = \$1/(1+.10)^1$

  $PV \approx \$0.91$

- If i = 10%, and $1 is received two years from now,

  $PV = \$1/(1+.10)^2$

  $PV \approx \$1/1.21$

  $PV \approx \$0.83$

- If i = 10%, $4 is received at end of year 1, and $5 is received at end of year 2, the PV of ($4,$5) is

$$\$4/(1+.10) + \$5/(1+.10)^2 \approx \$3.64+\$4.15 \approx \$7.79$$

Numerical Examples…Continued

- Suppose the annual interest rate is i = 10%.
- You will receive \$3 at the end of one year, \$5 at the end of 3 years, and \$110 at the end of eight years.
- Your payment stream is

  (\$3, 0, \$5, 0, 0, 0, 0, \$110)

- The PV of your payment stream is

$$\$3/(1+.10) + \$5/(1+.10)^{3} + \$110/(1+.10)^{8}$$

Yield to Maturity Again

- The "Yield to Maturity (YTM)" on a debt instrument A is defined as follows:

  YTM on A = The interest rate i that equates the "current value" of A with the present value (PV) of all future payments received by the owner of A

- ***Current Value (CV) of A*** = Amount someone is actually willing to pay today to own A.

- CV is determined either by loan contract terms or through a market process.

YTM for (One Year) Simple Loans: Example

- LV = Loan value (Principal) = $1000
- Maturity N = 1 Year
- Interest Payment I = $10
- Current Value (CV) for loan contract = LV
- Equate CV with PV of total payment stream:

  $$CV = \$1000 = \left[\$1000/(1+i) + \$10/(1+i) \right] = PV$$

- The value of i that solves this formula is the YTM for the simple loan:

  $$i^* = \$10/\$1000 = 0.01 \quad (1\ \%)$$

YTM for 1-Year Simple Loans: General Formula

- Loan value = LV
- Maturity = 1 Year
- Interest Payment = I
- Current Value (CV) = LV
- Equate CV with PV of total payment stream:

$$LV = [LV + I]/(1+i)$$

- The value of i that solves this formula is the YTM:

$$i^* = I/LV$$

YTM for a Fixed Payment Loan: Example

- Loan value (LV) = $1000
- Annual fixed payment FP=$126 for 25 years
- Current Value (CV) = LV
- Equate CV with PV of total payment stream:

$$1000 = \$126/(1+i) + \$126/(1+i)^2 + \ldots + \$126/(1+i)^{25}$$

- The value of i that solves this formula is the ***YTM*** for the fixed payment loan:

$$i^* \cong 0.12 \ (12\%)$$

YTM for a Fixed Payment Loan: General Formula

$$CV = FP/(1+i) + FP/(1+i)^2 + \ldots + FP/(1+i)^N$$

- CV = Loan Value (LV)
- FP = Annual fixed payment
- N = Number of years to maturity
- The value i* that satisfies this formula is the ***YTM*** for the fixed payment loan

YTM for a Coupon Bond: Example

- A coupon bond has an annual coupon payment C=$100, a face value F=$1000, and it matures in 10 years
- The current price of the bond is P = $1200
- Current Value (CV) = $1200
- The YTM is the value of i that solves CV = PV:

$$\$1200 = \$100/(1+i) + \$100/(1+i)^2 + \ldots + \$100/(1+i)^{10} + \$1000/(1+i)^{10}$$

- The ***YTM*** is i* = 0.07135 (7.135%)
  www.moneychimp.com/calculator/bond_yield_calculator.htm

YTM for a Coupon Bond: General Formula

$$P = C/(1+i^*) + C/(1+i^*)^2 + \ldots + C/(1+i^*)^N + F/(1+i^*)^N$$

- P = Bond market price = Current Value (CV)
- C = Annual coupon payment
- F = Face value
- N = Maturity
- Solve formula for i* = ***YTM***

✷ Note there is an **INVERSE** relationship between the bond market price P and the YTM i* "all else equal" (that is, for any *given* face value F, coupon payment C, and maturity N)

Inverse Relationship Between Price P and YTM for a Coupon Bond

Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000) **NOTE: Coupon Rate = C/F**

Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

Four Interesting Facts in Table 1:

1. **The bond price P and the YTM are negatively related.**
2. **When P equals the face value F=$1000, the C/F (10%) equals the YTM.**
3. **P/F > 1 implies C/F (10%) > YTM .**
4. **P/F < 1 implies C/F (10%) < YTM.**

A simple way to remember relationship among P, F, YTM, and Coupon Rate C/F:

- Consider the coupon bond formula for YTM i* for N=1:

$$P = C/(1+i^*) + F/(1+i^*) = (F+C)/(1+i^*)$$

- Divide each side by the face value F

$$P/F = (1 + C/F)/(1+i^*)$$

- It follows that

$$P/F > 1 \quad \text{if and only if} \quad C/F > i^*$$

$$P/F = 1 \quad \text{if and only if} \quad C/F = i^*$$

$$P/F < 1 \quad \text{if and only if} \quad C/F < i^*$$

YTM for a One-Year Discount Bond

- Face value F
- Maturity N=1

Note: No explicit "interest payment"

- Current Value CV = P (bond market price)
- YTM is the value i* that solves the formula

  $P = F/(1+i^*)$, or equivalently, $i^* = (F - P)/P$

- ***Example:*** If P=$900 and F=$1000, then

  $i^* = (\$1000 - \$900)/\$900 \approx 0.11$ (11%)

YTM for a Consol (or Perpetuity)

- Consol has fixed coupon payment C **forever**
- As explained in Mishkin (footnote 3, page 77, 2nd Bus School Edition), for any given i,

  PV of (C,C,C,….) = C/i

- Current Value (CV) = P (market price)
- The YTM is the value i* that solves

  P = C/i*

- Therefore i* = C/P

The Power of the YTM Concept

- Suppose you observe a person today buying a coupon bond ($C=\$100$, $F=\$1000$, $N=10$) at a current market price $P=\$1200$.

- You then calculate that the YTM is $i^* = 0.07135$

- How might i^* be used to estimate what CV the ***same*** person would be willing to pay today for a discount bond with face value $F=\$3000$ and maturity $N=2$?

- Can estimate $CV = \$3000/[1+i^*]^2 \cong \$2,613.70$