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Monitoring Daily Hedge Fund Performance

When Only Monthly Data is Available

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Abstract

This paper introduces a new approach to monitoring the daily risk of investing in hedge funds. Specifically, we use low-frequency (monthly) models to forecast high-frequency (daily) hedge fund returns. This approach addresses the common problem that confronts investors who wish to monitor their hedge funds on a daily basis—namely, that disclosure of returns by funds occurs only at a monthly frequency, usually with a time lag. We use monthly returns on investable assets or factors to fit monthly hedge fund returns, then forecast daily returns of hedge funds during the following month using the publicly observed daily returns on the explanatory assets. We show that our replication approach can be used to forecast daily returns of long-short hedge funds. In addition, for diversified portfolios such as hedge fund indexes and funds of hedge funds, our approach forecasts daily returns very accurately. We illustrate how our simple replication approach can be used to 1) hedge daily hedge fund risk and 2) estimate and control value-at-risk.

Introduction

Investors have long understood the value of frequent disclosure of information from portfolio managers. For example, arguments for increased disclosure by consumer groups representing individual investors helped to bring about new regulations requiring that U.S. mutual funds disclose complete portfolio lists at the end of each fiscal quarter, rather than twice per year, starting in May 2004. However, portfolio managers usually resist high-frequency disclosure, due to the potential for harmful effects, which include front-running of fund trades by outsiders as well as free-riding by investors on the costly expenditures made to research securities—by replicating fund holdings instead of investing in the mutual fund itself (Wermers 2001).

Hedge fund management companies have argued successfully that the costs of frequent portfolio or return disclosure far outweigh the benefits. As a result, unlike mutual funds, which provide daily per-share net asset values (NAVs) widely to the media within two hours of the New York Stock Exchange close, hedge funds typically report NAVs monthly, sometimes with a significant delay.[1] Further, hedge fund database vendors (e.g., TASS, HFR) provide only monthly performance, also with a delay. In addition, hedge funds routinely implement barriers to inflows and outflows, such as subscription or redemption periods, which serve to further slow any response of investors to such infrequent information disclosure (Getmansky et al. 2009). With such barriers in an infrequent disclosure environment, investors are limited in their ability to respond to dramatic market shifts, such as the recent market turmoil of 2008–2009.

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Ironically, hedge fund investors may face an even greater need to understand their risk exposures than mutual fund investors. Hedge funds, while often less volatile in general market conditions, can experience greater volatility during market crises due to their exposures to common strategies (and the risks from correlated flows that such common strategies create—see, for example, Khandani and Lo 2011). Also, each hedge fund investor is exposed to negative externalities imposed by the actions of all other investors, such as the costs imposed by investors who are first to exit a fund during a liquidity crisis (see Chen et al. 2009, for evidence of runs in illiquid mutual funds). Thus, timely information can be of significant value to hedge fund investors who wish to understand daily hedge fund gains/losses, to plan redemptions or contributions, or even to consider potential hedging strategies. For example, while many investors expected poor performance from their hedge fund investments during October 2008, they had little ability to evaluate the magnitude of losses before receiving a performance report after the end of the month, making prompt risk management impossible.

Accordingly, this paper develops a new approach for hedge fund investors to infer timely high-frequency (i.e., daily) performance from low-frequency (i.e., monthly) returns data. This new method relies on relatively simple computation methods that use only observed monthly hedge fund returns. One may view our approach as a high-frequency (daily) replication strategy for hedge fund returns that uses only investable assets for which daily returns information is widely available with little time lag. As such, our approach is related to a long line of literature that attempts to replicate monthly hedge fund returns with primitive securities, factor exposures, or distribution-based replication procedures (see Gupta et al. 2008, for a survey of these approaches). For example, Hasanhodzic and Lo (2007) perform in-sample and out-of-sample replication of monthly returns of individual hedge funds, and compare return distributions of original funds and their "clones."

However, our goal is different from prior replication techniques. Rather than creating a portfolio that replicates the long-term returns of hedge funds, we wish to create a simple approach to allow investors to model the risk of their hedge fund investments on a daily basis, using commonly observed investable assets with liquid, daily market prices. Such daily monitoring is especially important to investors during periods of high market volatility, such as during the financial crisis of fall 2008. Specifically, we project the daily performance of hedge funds by creating synthetic replication portfolios based on a monthly factor model that uses common investable indexes. As suggested by Amenc et al. (2010), we implement several models to improve the quality of our replication. Instead of the traditional moving-window return-based style analysis (RBSA), we implement an improved model that uses dynamic style analysis (DSA), which captures dynamic index exposures without a moving window. We also run a factor selection model to select best-fit factors through a pool of more than 100 market indexes and factors.

Once a high-quality in-sample replication is achieved, we then use daily returns of replicating assets to create a daily proxy of the hedge fund. Naturally, our methodology builds on and contributes to a large and growing literature on the exposure of hedge funds to market indexes. A number of studies, including Fung and Hsieh (1997, 1999, 2001); Brown et al. (1999); Ackermann et al. (1999); Liang (1999); Agarwal and Naik (2000, 2004); and Markov et al. (2006) have examined the relation between hedge fund returns and broad market returns. The contribution of our study is that we develop an efficient approach, using DSA, for forecasting daily hedge fund returns using monthly data, while past research has focused on forecasting monthly returns using observed past monthly returns.

To backtest the effectiveness of the proposed methodology, we calculate the tracking errors between actual (ex post) daily hedge fund returns and the corresponding (ex ante) daily replication portfolios (using common investable indexes) created using models that use lagged monthly returns. We find that our approach closely tracks actual daily hedge fund index returns with low tracking error; this small tracking error demonstrates that our replication portfolio can successfully mimic the daily variation of actual hedge fund index returns using only monthly data as inputs. For instance, the median daily tracking error for all ten HFRX indexes is as small as 18 basis points (bps) during 2005–2006. And, the correlation between our daily projection returns and actual index returns are 76 percent.

We also apply our technique to a sample of forty-seven individual mutual funds with hedge-fund-like strategies. We chose these funds because we can track their actual performance with daily returns; thus we can backtest the success of our replication approach that fits monthly models to replicate daily returns. Our replication technique generates median daily tracking errors of 39 bps for long-short equity funds, with a median correlation of 60 percent. Even more dramatic is that we use generic style factors to replicate the daily returns of an equal-weighted portfolio of the twenty-two long-short funds and obtain daily tracking errors of 9 bps, with 96-percent correlation. This result indicates that funds-of-funds managers can closely track their daily performance with only general knowledge of the styles that might be employed by their individual hedge fund managers.

Our methodology has several potential applications. First, these replication hedge fund portfolios could serve as an early warning system, thereby enabling investors to make prompt investment and risk management decisions at any time, instead of waiting until the end of the month.[2] As an example, we illustrate with an application where we hedge unwanted risks of a hedge fund. Second, this methodology could be used to improve the Value-at-Risk (VaR) measure of hedge funds. Jorion (2008) argues that estimating VaR with the realized monthly return distribution fails to capture the dynamic risk of hedge funds. And Goetzmann et al. (2000) show that monthly models fit to funds that use daily dynamic strategies can be problematic. We demonstrate that the daily ex ante VaR measures (created using the projection technique) successfully capture the dynamics of hedge fund risks that are not reflected in monthly risk measures. Finally, our projection methodology is generic enough to apply to any investment portfolio that reports only monthly or lower-frequency performance data. In addition to hedge funds and hedge funds of funds, this group also includes institutional and separately managed accounts, which are left for future research.

Data

HFRX Indexes

HFRX indexes are a series of benchmarks designed to represent the performance of a larger universe of hedge fund strategies. The indexes are designed to be investable, offer full transparency, daily pricing, and consistent underlying fund selection. In this study, we use ten HFRX indexes, which offer the most frequently reported performance on a day $t + 1$ basis, with a common period back to June 2004. These ten indexes cover all major hedge fund strategies.

Hedged Mutual Funds

Another way to proxy actual daily hedge fund returns is to use hedged mutual funds (HMFs). Hedged mutual funds are mutual funds that use hedge-fund-like trading strategies, but, unlike hedge funds, they are regulated by the Securities and Exchange Commission (SEC) in exactly the same way as traditional mutual funds. According to Agarwal et al. (2009), hedged mutual funds outperform traditional mutual funds.

We collect a sample of hedged mutual funds following Agarwal et al. (2009). In brief, we begin by including all HMFs under the long-short equity, equity market neutral, and short bias categories in the Morningstar and Lipper mutual fund databases, while filtering out index funds and all but one of the multiple share classes of a single fund. For HMFs under other strategies, such as event-driven and absolute-return strategies, we searched both Morningstar and Lipper databases and the Internet using the key words list in the appendix of Agarwal et al. (2009). In total, we collected forty-seven HMFs with common period return from June 2008 to May 2011.

Methodology and Related Literature

Methodology

Our proposed daily projection methodology involves replicating the monthly returns of an individual hedge fund with market indexes and/or factors that have daily pricing available to the public with little delay. The best mimicking replication portfolio from the in-sample period is then passively held during the following month to estimate daily returns of the hedge fund.

In short, our daily projection can be summarized as a two-step process:

Step 1: Calibration of a best factor portfolio that minimizes the tracking error with respect to monthly hedge fund returns,

$$\begin{cases} R_t^{HF} = \sum_{k=1}^{K} \hat{\beta}_k F_{kt} + \hat{\varepsilon}_t \\ s.t. \sum_{k=1}^{K} \hat{\beta}_k = 1 \end{cases} \tag{1}$$

where R_t^{HF} is the return of a hedge fund during month t, $\hat{\beta}_k$ is the estimated exposure of the hedge fund to factor k, F_{kt} is the return during month t for factor k, and $\hat{\varepsilon}_t$ is the estimated specific risk of the hedge fund. If all K factors represent investable market indexes, there is a constraint that the summation of all asset loadings (betas) equals one (the budget constraint).

Step 2: Projection of the daily returns of the hedge fund using daily returns on the underlying factors identified in step 1:

$$R_t^d = \sum_{k=1}^{K} \hat{\beta}_k^{m-1} F_{kt}^d, \ t = T_1^m, \ldots, T_T^m \tag{2}$$

where R_t^d is the daily projection return of the hedge fund at date t, $\hat{\beta}_k^{m-1}$ is the estimated loading on factor k at the end of the previous month, $m-1$, F_{kt}^d is the day t return of factor k, and T_1^m and T_T^m are the first and last trading days of month m, respectively. The calibration in step 1 is repeated every month, after new monthly hedge fund returns become available. The daily hedge fund returns are projected each day during the following month with updated data by following equation 2.

Related Literature

Our daily projection methodology is closely related to literature on hedge fund replication. Two major approaches to hedge fund replication exist: factor-based replication and distribution-based replication. Proposed by Amin and Kat (2003) and more recently extended by Kat and Palaro (2005), the latter approach attempts to replicate the distribution of hedge fund returns. By dynamically trading futures, Kat and Palaro (2005) show that it is possible to generate returns that are distributed very similarly to the actual returns generated by hedge funds. Since our focus is on matching the unconditional distributional properties of hedge fund returns, as opposed to their time-series properties, the distribution-based approach does not fit our purpose for daily hedge fund return projection.

Our methodology is a direct extension of the factor-based replication. In a review of the extensive literature, we found that people report mixed results in attempts to replicate monthly hedge fund returns.[3] The seven-factor model, proposed by Fung and Hsieh (2004), can explain up to 80 percent of the return variation of funds-of-funds and various hedge fund indexes. Jaeger and Wagner (2005) implement a multilinear asset class factor model and find that some hedge fund strategies (such as long-short equity, short selling, and event driven) can be replicated much more successfully than others. Amenc et al. (2008) reproduce the results reported in Hasanhodzic and Lo (2007) on fourteen different hedge fund strategy indexes. They question the ability of simple factor models to be used in the context of replicating out-of-sample hedge fund returns.

Challenges and Improvements for Factor-Based Replication[4]

Factor selection. The main challenge for factor-based replication is to find a properly selected set of factors such that the replication can capture the hedge fund returns of different strategies under different market conditions. Fung and Hsieh (2001, 2002a,b) go beyond generic asset-class indexes and develop equity factors for equity hedge funds, bond factors for fixed-income hedge funds, and trend-following factors for commodity and managed futures funds. Agarwal and Naik (2004) create buy-and-hold and option-based risk factors for eight hedge fund strategies.

In this paper, we address this challenge by using an extensive set of more than 100 factors similar to those in Dor et al. (2006). These factors reflect the returns to various asset classes, sectors, geographical regions, and currencies. We also include the strategy-specific factors covered by the literature above (a complete description of the factors is given in appendix A). To decide which factors should be included in the replication, we employ a technique that selects the best set of factors by running all possible combinations of them through the model.[5] To account for the change of hedge fund factor exposure through time, the factor selection procedure is run in every month to generate the best fit in equation 1 above.

Dynamic style analysis. The most common technique to account for a hedge fund's dynamic factor exposures through time is a rolling-window methodology. It assumes that exposures estimated over a rolling fixed window (usually the previous twenty-four months) hold for the next month. The average exposure over the past two years is the proxy for next month's exposure. However, with a simulated portfolio, Bacmann et al. (2008) illustrates that the explanatory power of replication decreases sharply if the dynamics of the factor exposures are too high.

Improved dynamic models have been introduced recently in an attempt to address the limitations of rolling-window methodology. For example, Swinkels and van der Sluis (2006) use a Kalman filter to model time-varying exposures of mutual funds explicitly. Bodson et al. (2010) apply a similar technique on hedge fund indexes. Their out-of-sample test indicates a significant improvement over the traditional rolling-window approach.

In this paper, we use a dynamic filtering technique called dynamic style analysis (DSA), developed by Markov et al. (2004a,b, 2006). The technique is based on the groundwork of Kalaba and Tesfatsion (1989), in which they develop a time-varying linear regression model using flexible least squares methodology. Detailed description of DSA methodology is available in appendix B.

Empirical Results

Monitoring Daily Hedge Fund Performance

To test how closely our projection system can monitor returns of individual hedge funds, we applied our methodology to a sample of hedged mutual funds that employ strategies similar to those of hedge funds. These mutual funds have daily returns available through Lipper/Reuters, allowing us to measure the success of our daily replication strategy. Following the selection process detailed above, we identified forty-seven hedged mutual funds with five different hedge fund strategies. Table 1 reports the tracking errors and correlations between our projection returns and the actual daily returns of these forty-seven funds.

Results in Different Strategies

Overall, our daily projection returns achieve low tracking errors against their actual daily fund returns. Among all five strategies, absolute return and long-short equity strategies have the highest median tracking error at 0.39 percent/day. Event driven and equity market neutral have the lowest median tracking error at 0.20 and 0.30 percent/day, respectively. However, the correlations between actual and projected returns among these two strategies, on average, are also very low, indicating a relatively weak fit of the projection model. Judging by both tracking error and correlation, short bias and long-short equity have the best daily projection quality, with both low tracking error and high correlation.

Weekly Aggregation of Results

We also performed a weekly aggregation of tracking results to demonstrate that a significant portion of the daily tracking error (TE) could have come from other sources unrelated to the model estimation and prediction quality, such as pricing fluctuations, accounting errors, fees, etc. For an independent and identically distributed (IID) process, one would expect weekly volatility to be $\sqrt{5} = 2.24$ times the daily volatility. Therefore, if a significant portion of the tracking error is indeed related to noise in daily fund NAV calculations, it is likely that the noise will cancel out if the daily data are aggregated weekly, and the resulting weekly TE will be smaller than 2.24 times the daily TE.

In table 1, the weekly columns represent tracking errors and correlations between weekly return series. We didn't redo any monthly exposure computations; we simply compounded each daily return series used in the daily portion of the table to obtain weekly returns. We observe that weekly aggregation indeed improved the projection. The median ratios of weekly TE to the daily TE for long-short equity and short bias strategies are 1.56 and 1.73. These figures indicate that up to one-half of the daily observed projection discrepancies could cancel out in the long run, indicating the share of daily pricing error in the actual series. Therefore, investors should expect much smaller cumulative under/over performance (weekly, monthly, annually) than the daily TE numbers suggest due to the reduction in actual fund pricing errors over longer periods.

Attributing Projection Quality

To understand what portion of the projection error could be attributed to using monthly data instead of daily data for estimation of the tracking portfolio, we create an ideal daily benchmark. Such a benchmark represents a tracking portfolio with index exposures estimated using daily returns rather than monthly, assuming that both fund and index returns are available. By definition, such a portfolio will track a hedge fund more closely than one created using monthly data. To

Note: Table 1 shows the daily and weekly tracking errors and correlations between projection returns and actual returns of hedged mutual funds from Lipper Mutual Fund database. Monthly factor loadings were estimated with expanding-window dynamic style analysis (DSA). Factor selection is used to select a handful of factors within more than 100 market indexes/factors listed in appendix A. Daily projection returns for each month were calculated using previous monthly estimated factor loadings times the current month's daily market factor returns. The daily tracking error is calculated as the standard deviation of actual returns relative to projection returns. Correlation is also between actual returns and projection returns. For benchmark tracking error, a forty-day trailing window daily analysis is used to calculate daily estimated factor loadings. Daily projected returns are aggregated to weekly frequency to calculate weekly statistics. EW Port. represents an analysis of the equally weighted portfolio consisting of all listed funds within each strategy.

TABLE 1: DAILY PROJECTION FOR HEDGED MUTUAL FUNDS

Absolute Return	June 2010–May 2011						
	Daily			Weekly			
Fund	TE,%	Corr.	Benchmark TE,%	TE,%	Corr.	Benchmark TE,%	Annual Turnover
Nakoma Absolute Return	0.39	0.20	0.21	0.79	0.14	0.49	126.00
Quaker Akros Absolute Strategies A	0.22	0.16	0.18	0.45	0.16	0.27	373.76
Oppenhmr Abs Return;A	0.42	0.38	0.25	0.82	0.49	0.43	409.00
Nakoma Abs Return	0.39	0.20	0.21	0.79	0.14	0.49	126.00
Absolute Opptys;Inst	0.27	0.24	0.15	0.55	0.28	0.33	630.00
mean	0.34	0.24	0.20	0.68	0.24	0.40	332.95
median	0.39	0.20	0.21	0.79	0.16	0.43	373.76
EW Port.	0.19	0.51	0.09	0.37	0.64	0.19	
Event Driven	**June 2010–May 2011**						
AQR Diversified Arbitrage	0.13	0.26	0.10	0.25	0.32	0.18	361.00
Arbitrage R	0.25	–0.15	0.11	0.48	–0.10	0.24	371.00
Gabelli ABC	0.10	0.82	0.07	0.19	0.82	0.12	363.00
Gabelli Entpr Mergers & Acquisitions	0.20	0.88	0.12	0.35	0.88	0.21	228.00
GAMCO Mathers AAA	0.11	0.32	0.07	0.20	0.56	0.16	475.00
Merger	0.24	–0.19	0.08	0.36	0.38	0.18	192.21
Quaker Event Arbitrage A	0.27	0.24	0.22	0.55	0.32	0.46	138.58
mean	0.19	0.31	0.11	0.34	0.45	0.22	304.11
median	0.20	0.26	0.10	0.35	0.38	0.18	361.00
EW Port.	0.10	0.66	0.07	0.19	0.66	0.11	
Equity Market Neutral	**June 2010–May 2011**						
Calamos Market Neutral Income A	0.08	0.96	0.08	0.14	0.98	0.15	87.80
American Century Equity Mkt Neutral A	0.42	–0.13	0.23	0.90	0.02	0.60	140.00
Highbridge Statistical Mkt Neutral A	0.19	0.07	0.15	0.38	0.11	0.31	1,017.00
JPMorgan Multi-Cap Market Neutral A	0.26	0.08	0.17	0.51	0.25	0.35	146.00
JPMorgan Research Market Neutral Instl	0.23	0.02	0.15	0.53	–0.03	0.30	182.00
Vanguard Market Neutral I	0.41	0.02	0.18	0.86	0.14	0.42	153.00
Virtus Market Neutral A	0.30	0.20	0.22	0.66	0.31	0.42	155.00
James Market Neutral	0.54	0.11	0.30	1.19	0.14	0.78	44.00
TFS Market Neutral	0.34	0.55	0.15	0.67	0.59	0.32	703.00
mean	0.31	0.21	0.18	0.65	0.28	0.41	291.98
median	0.30	0.08	0.17	0.66	0.14	0.35	153.00
EW Port.	0.16	0.20	0.08	0.32	0.31	0.19	
Long-Short Equity	**June 2010–May 2011**						
Glenmede:Long/Short	0.27	0.84	0.21	0.54	0.84	0.42	636.00
Old Mutual Analytic Z	0.44	0.73	0.35	0.68	0.85	0.55	168.45
Absolute Strategies I	0.12	0.44	0.08	0.26	0.53	0.15	192.00
Bull Path Long Short I	0.48	0.49	0.27	0.96	0.50	0.44	287.00
Robeco Long/Short Eq I	0.55	0.88	0.27	0.99	0.90	0.55	81.00
Dunham Monthly Distribution A	0.25	0.60	0.14	0.43	0.70	0.29	370.00
Diamond Hill Long-Short A	0.47	0.72	0.21	0.73	0.85	0.39	43.00
Gateway A	0.25	0.76	0.13	0.44	0.84	0.17	7.00
Hussman Strategic Growth	0.62	0.39	0.24	1.23	0.38	0.42	111.00
ICON Long/Short I	0.25	0.96	0.18	0.60	0.94	0.49	136.50
Driehaus Active Income	0.11	0.67	0.08	0.19	0.82	0.11	51.00
Aberdeen Equity Long-Short C	0.39	0.56	0.17	0.52	0.83	0.33	152.09
Direxion Evolution All-Cap Equity Inv	0.85	0.58	0.39	1.78	0.57	0.86	1,677.00
Quaker Long-Short Tactical Allc A	0.98	0.02	0.56	1.80	0.13	1.12	2,474.00
Rydex|SGI Multi-Hedge Strategies A	0.30	0.49	0.18	0.56	0.58	0.41	993.00
Rydex|SGI Alpha Opportunity A	0.61	0.91	0.22	1.02	0.95	0.49	954.00
Wasatch Long/Short	0.43	0.82	0.22	0.85	0.84	0.41	60.00
Caldwell & Orkin Market Opportunity	0.48	0.20	0.25	1.03	0.17	0.68	662.00
Schwab Hedged Equity	0.32	0.90	0.16	0.58	0.92	0.35	115.00
Direxion Spectrum Select Alternative Svc	0.29	0.36	0.19	0.37	0.75	0.28	192.00
Federated Market Opportunity A	0.39	0.60	0.20	0.61	0.76	0.45	180.00
SSgA Directional Core Equity Fund	0.30	0.91	0.18	0.52	0.94	0.41	197.00
mean	0.42	0.63	0.22	0.76	0.71	0.44	442.68
median	0.39	0.64	0.21	0.61	0.83	0.42	186.00
EW Port.	0.09	0.96	0.06	0.13	0.98	0.11	
Short Bias	**June 2010–May 2011**						
Comstock Capital Value A	0.34	0.96	0.22	0.58	0.98	0.31	1,239.00
Federated Prudent Bear A	0.34	0.93	0.19	0.60	0.95	0.32	379.00
PIMCO StocksPLUS TR Short Strat Instl	0.24	0.98	0.20	0.37	0.99	0.23	483.00
Grizzly Short	0.41	0.96	0.28	0.63	0.98	0.48	N/A
mean	0.33	0.96	0.22	0.55	0.98	0.34	700.33
median	0.34	0.96	0.21	0.59	0.98	0.32	483.00
EW Port.	0.22	0.98	0.13	0.32	0.99	0.19	

construct such a benchmark, first, a forty-day (approximately two-month) trailing window daily RBSA analysis is used to calculate daily estimated factor loadings. Then, the following-day return is forecasted using the fitted model and the following-day factor return.

We present the tracking error between the out-of-sample replication portfolio (the synthetic portfolio using daily estimated asset loadings times daily out-of-sample index returns) and the actual fund's daily returns in table 1 as the "Benchmark TE." We observe that the tracking errors of such a daily benchmark are, on average, half the size of our tracking errors, which are based on monthly out-of-sample projections. Surprisingly, we lose only about one-half of the precision by using monthly data in estimation versus daily for individual hedge funds.

> "Clearly, such a high level of trading activity presents a challenge to using monthly estimations for projections of daily returns, unless funds are diversified and maintain stable overall factor or sector exposures."

The Impact of Portfolio Turnover

To further understand the impact of portfolio turnover on the quality of return projections, we provided annual turnover figures for each fund, where available. These represent trailing twelve-month turnover figures, as of the latest available date before year-end.[6] The data were obtained using various fund filings and data vendors, such as Morningstar. Investors rarely have access to hedge fund turnover figures, and these data provide some useful insights.

Based on the long-short equity strategy in table 1, two outliers in both the tracking error (TE) and correlation stand out: Direxion Evolution and Quaker Long-Short. These funds have the highest daily TE and one of the lowest daily correlations to the projection portfolio. A closer analysis of these funds reveals strategies with an unusually high level of annual turnover: 1,677 percent for Direxion and 2,474 percent for Quaker.

Clearly, such a high level of trading activity presents a challenge to using monthly estimations for projections of daily returns, unless funds are diversified and maintain stable overall factor or sector exposures. For example, Glenmede Long/Short has a high annual turnover at 636 percent, yet its daily projection also achieves a below-average TE at 0.27 and a high correlation at 0.84.

Portfolio Aggregation

Within each strategy, we created an equal-weighted (EW) monthly rebalanced return series of all the funds and performed all the steps in analyzing it as we did for every fund in the table. The results are presented in the row denoted "EW Port." One would expect the tracking error (if it's entirely diversifiable nonsystematic) to decrease by the factor of $\sqrt{N}$, where N is the number of funds, due to diversification. In reality, we observe a very similar decline in TE of the analyzed portfolio. For example, the TE of an EW portfolio in long-short equity is 9 bps, decreased by a factor of 4.3 from its median TE of all twenty-two funds. Despite the fact that many of the funds in the portfolio have very significant annual turnover of 1,000–2,000 percent and that we weren't able to successfully project several funds, the long-short equity portfolio incorporating all such funds has a TE less than 10 bps. This should be encouraging for investors in diversified portfolios, such as funds of funds.

Monitoring Daily Performance of Hedge Fund Indexes

We applied the same analysis we used to monitor daily hedge fund performance to all HFRX indexes; the results are reported in table 2. Similar to the prior section, we selected factors by running our factor selection model using more than 100 factors listed in appendix A. The monthly factor loadings were estimated by an expanding-window dynamic style analysis.

Results during Different Time Periods

The long history of HFRX indexes allows us to examine our daily projection quality under different market conditions. For the six years from 2005 through 2010, we divided the available history from 2005 through 2010 into three two-year time intervals to represent before, during, and after the past financial crisis.

As shown in table 2, the median daily TE before and after the crisis is 18 bps and 24 bps, respectively. The median TE during 2007–2008—at the height of the market crisis—is 54 bps, three times as high as the TE two years prior. This is due to the significant increase in market volatility. (The daily volatility of the S&P 500 Index also tripled from 0.64 to 1.96 during the same time period.) However, the correlation, another measure of projection quality, largely stays the same between these two time periods.

Results for Different Strategies

Regarding daily projection quality across different hedge fund strategies, our results are consistent with Jaeger and Wagner (2005) in monthly frequency. The hedge fund composite indexes, such as equal-weight strategy and global hedge strategy, have the lowest TE and the highest correlation. Among individual strategies, equity hedge and market directional outperform others in tracking under different market conditions. It is worthwhile to note that the numbers for the convertible arbitrage strategy are apparent outliers, as

TABLE 2: DAILY PROJECTIONS FOR ALL HFRX HEDGE FUND INDEXES

HFRX indexes(# of funds)	Daily			Weekly		
2005–2006	**TE,%**	**Corr.**	**Benchmark TE,%**	**TE,%**	**Corr.**	**Benchmark TE,%**
Absolute Return (14)	0.10	0.39	0.08	0.29	0.57	0.19
Convertible Arbitage (3)	0.21	0.07	0.18	0.54	0.16	0.32
Equity Hedge (17)	0.14	0.92	0.09	0.26	0.87	0.22
Equity Market Neutral (4)	0.21	0.28	0.15	0.32	0.36	0.21
Event Driven (13)	0.17	0.83	0.11	0.36	0.85	0.27
EW Strategy Index (42)	0.12	0.86	0.06	0.33	0.75	0.12
Global Hedge Fund (42)	0.13	0.87	0.07	0.30	0.91	0.19
Macro Index (8)	0.37	0.68	0.18	0.82	0.65	0.53
Market Directional (14)	0.24	0.84	0.14	0.54	0.88	0.42
Relative Value (10)	0.19	0.29	0.11	0.37	0.51	0.23
Mean	0.19	0.60	0.12	0.41	0.65	0.27
Median	0.18	0.76	0.11	0.35	0.70	0.23
2007–2008						
Absolute Return (14)	0.43	0.40	0.13	0.92	0.28	0.32
Convertible Arbitage (3)	0.74	0.02	0.34	6.63	0.44	1.03
Equity Hedge (17)	0.57	0.80	0.21	0.93	0.83	0.51
Equity Market Neutral (4)	0.42	0.09	0.37	0.98	0.43	0.90
Event Driven (13)	0.50	0.74	0.26	0.94	0.82	0.44
EW Strategy Index (42)	0.41	0.74	0.16	1.09	0.65	0.42
Global Hedge Fund (42)	0.59	0.67	0.16	1.03	0.74	0.40
Macro Index (8)	0.79	0.56	0.43	1.53	0.71	0.89
Market Directional (14)	0.44	0.80	0.23	0.87	0.88	0.57
Relative Value (10)	0.83	0.40	0.37	1.47	0.68	0.82
Mean	0.57	0.52	0.27	1.64	0.65	0.63
Median	0.54	0.62	0.25	1.01	0.70	0.54
2009–2010						
Absolute Return (14)	0.19	0.40	0.08	0.34	0.24	0.17
Convertible Arbitage (3)	0.17	0.45	0.15	0.46	0.71	0.46
Equity Hedge (17)	0.43	0.87	0.20	0.38	0.87	0.31
Equity Market Neutral (4)	0.41	0.18	0.24	0.68	0.52	0.49
Event Driven (13)	0.19	0.45	0.16	0.36	0.71	0.31
EW Strategy Index (42)	0.18	0.75	0.11	0.31	0.75	0.22
Global Hedge Fund (42)	0.22	0.75	0.11	0.39	0.78	0.26
Macro Index (8)	0.49	0.17	0.24	0.97	0.19	0.57
Market Directional (14)	0.26	0.52	0.16	0.46	0.77	0.41
Relative Value (10)	0.52	0.46	0.33	1.14	0.71	0.78
Mean	0.31	0.50	0.18	0.55	0.63	0.40
Median	0.24	0.46	0.16	0.43	0.71	0.36

Table 2 shows the daily and weekly tracking errors between projection returns and actual returns of all HFRX hedge fund indexes with more than six years of data available. Numbers in parenthesis after the indexes indicate the number of funds composed of the indexes, composite means using all hedge funds for the index construction, but with a different weighting scheme. For details, refer to HFRX index methodology. Monthly asset loadings were estimated with expanding-window dynamic style analysis (DSA) by running factor selection for each index. All other calculations are the same as in table 1.

the strategy itself suffered a breakdown in 2008 because of the market turmoil and some rule changes by the U.S. Securities and Exchange Commission (Walker and Baum 2008). The huge distortion in performance makes consistent projection of this strategy almost impossible.

Applications

Hedging Market Exposures

In addition to passively tracking hedge fund performance, investors can proactively hedge unwanted risk (if they expect their investment is deteriorating due to specific market exposures). As an example, we estimate that HFRX Equity Hedge Index had about 20-percent international equity exposure, represented by EAFE, using dynamic style analysis (DSA) with expanding window ending at August 31, 2008. If investors foresee further decline in this sector, they could actively hedge this sector by shorting 20-percent MSCI Europe, Australasia, and Far East (EAFE) by taking, for example, a position in an exchange-traded fund (ETF) such as EFZ (NYSE-listed ProShares Short MSCI EAFE ETF). Figure 1 presents two projection returns for HFRI EH: one with EAFE exposure hedged (green) and one without (red). The portfolio with EAFE hedged using EFZ significantly outperformed the actual HFRX EH index. Moreover, the hedged portfolio also has a much smaller volatility; the annualized standard deviation is 5.48 percent for the EAFE hedged portfolio, compared to 18.24 percent for HFRX EH index during September 2008.

Unlike in our hypothetical hedging example above, in some cases, using indexes involved in the analysis as hedging instruments could be either impossible or impractical. For instance, small-cap equity indexes could be very expensive to short. Nevertheless, a hypothetical analysis using generic indexes provides an important benchmark for further steps, when such indexes are replaced with actual hedging instruments. In this way, we are able to attribute the loss of hedging precision to the selection of more efficient (albeit less precise) hedging instruments.

Estimating Daily VaR

Hedge funds usually have a short history, provide only low-frequency return data (typically monthly), and employ dynamic investment strategies. This presents challenges for Value-at-Risk as an effective risk measure. Jorion (2008) argues that, for hedge funds, using monthly returns to compute ex post risk measures such as standard deviation and/or VaR is insufficient to capture dynamic portfolio risks. For example, Goetzmann et al. (2000) show that monthly models fit to funds using daily dynamic strategies can be problematic. Further, Lo (2001) calculates the required sample size for accurate VaR estimation and concludes that, to achieve a 95-percent confidence level, one needs more than 475 data points. Most hedge funds don't meet this data requirement.

In this section, we apply our replication methodology to generate daily data to calculate daily VaRs. This provides enough return data points for us either to fit in a parametric distribution function or use empirical quantiles for VaR estimation.[7] As illustrated below, daily VaR also reveals a much more dynamic picture for hedge fund risk through time, as opposed to monthy VaR.

We demonstrate the method, again using HFRX EH as an example. To estimate the daily VaR value for day T_m in month m, we construct a daily hypothetical portfolio by multiplying the previous month-end estimated factor

FIGURE 1: HFRX EH ACTUAL RETURNS VS. EAFE HEDGED RETURNS

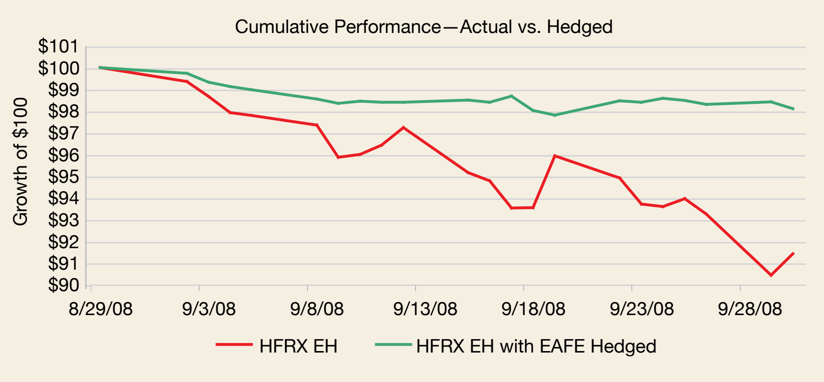
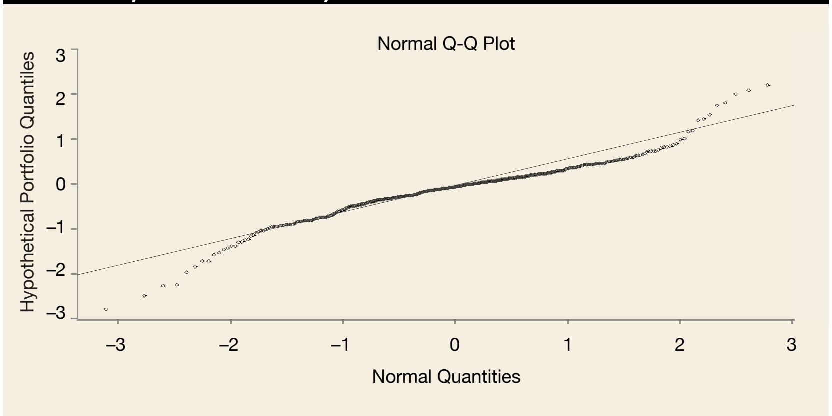


FIGURE 2: Q-Q PLOT OF HYPOTHETICAL PORTFOLIO RETURNS, AUGUST 16, 2006–AUGUST 15, 2008



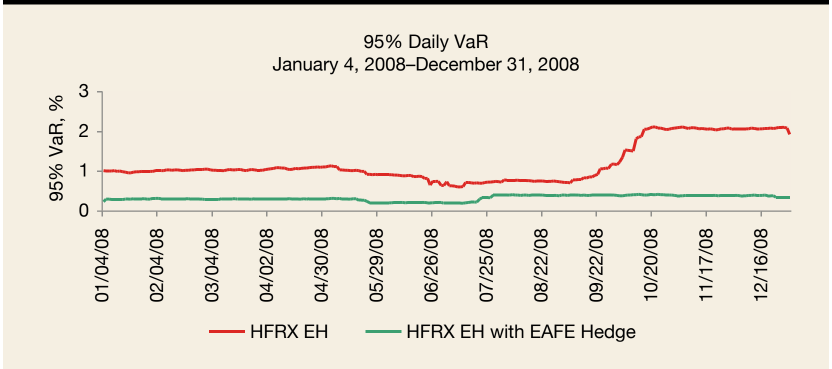
loadings by the time series of historical daily factor returns, that is,

$$R_t^d = \sum_{i=1}^{N} \hat{\beta}_{i,\,m-1} F_{i,t}^d ,\ t = T_m - W + 1, \ldots, T_m \quad (3)$$

where F_t^d represent daily historical factor returns; $\hat{\beta}_{i,\,m-1}$ are the factor loadings estimated using style analysis in equation 1 and applying it to available monthly return data through month $(m - 1)$. As a result, R_t^d as defined in equation 3 does not represent an actual portfolio but rather constructs the daily history of a hypothetical portfolio having constant factor loadings. The VaR estimate as of the end of day T_m is then calculated using W daily hypothetical returns $\{R_t^d\}$ within the VaR estimation window ending on T_m.

One advantage of this hypothetical portfolio is that it could account for fat tails to the extent that they are present in the daily historical data. To illustrate, we construct the hypothetical portfolio to compute VaR on September 15, 2008, for the HFRX EH index. We use the same estimated factor loadings in August as those in figure 1, and the hypothetical portfolio returns are constructed using a two-year (503-day) VaR estimation window from September 18, 2006, to September 15, 2008. Figure 2 depicts the Q-Q plot of these returns against a standard normal distribution. Accordingly, the plot substantially deviates from linearity at both ends and indicates the existence of fat tails for hypothetical portfolio return distribution. The Jarque-Bera test also rejects the normality hypothesis with a p-value close to zero. As a result, the 95-percent daily VaR for September 15, 2008, is 0.91 percent using an empirical quantile method and 0.7 percent by assuming a normal distribution. Therefore, empirical and parametric methods produce very different VaR estimates.

FIGURE 3: DAILY VALUE-AT-RISK



To test whether our daily VaR provides a reasonable risk measure of the HFRX EH index, we performed an out-of-sample test by computing daily VaR values for each day in 2008 and compared them with the next-day actual index returns. More specifically, for each month $m = 1,...,12$ in year 2008, we used twenty-four monthly returns preceding month m to estimate the effective asset loadings $\{\hat{\beta}_{i,\,m-1}\}$ for the month m daily VaR calculation. We then computed the daily VaR according to equation 2, using computed factor loadings and daily factor returns for each day in our VaR estimation window W. According to the VaR definition, the percentage of exceptions when the observed actual daily loss is greater than the estimated VaR value should be close to the theoretical probability value for which VaR was computed (e.g., 5 percent of exceptions for 95-percent VaR). The test statistic that is commonly used to measure the quality of VaR backtesting is Kupiec's (1995) Proportion of Failures (POF), as defined in equation 4:

$$POF = 2\,ln\left(\left(\frac{1-\hat{\alpha}}{1-\alpha}\right)^{T-x}\left(\frac{\hat{\alpha}}{\alpha}\right)^{x}\right) \qquad (4)$$

where T is VaR estimation window, x is the number of times actual loss exceeds VaR value (exception), α is the significance level for VaR (0.05 for 95-percent VaR in our test), and $\hat{\alpha}$ is the observed frequency of exceptions, which equals $\frac{x}{T}$. The null hypothesis is H0: $\hat{\alpha} = \alpha$. Accordingly, POF is asymptotically distributed as chi-squared with one degree of freedom. With 253 daily 95-percent VaR values estimated for year 2008 using empirical quantiles, we observed fourteen daily index losses exceeding their daily VaR estimates, so that the observed frequency $\hat{\alpha} = 0.055$. The corresponding POF statistic is equal to 0.15 and the null hypothesis of the 95-percent VaR estimate being accurate is accepted with p-value = 0.69.

As stated in Jorion (2008), daily VaRs would also reveal some volatility that cannot be captured in monthly VaRs. In figure 3, we plot daily VaR values of the HFRX EH index during 2008. The daily risk profile is nothing but volatile, and it shows the VaR value spiking significantly after October 2008. Such daily level risk information is important but hard to come across with monthly VaR estimation. If only monthly data were used to compute VaR, an increase in VaR may only be noticed in several months. However, an investor monitoring risk using our daily VaR would discover this much more quickly.

Similar to the first hedging application in this section, investors can hedge the unwanted risk exposure if they foresee the further deterioration to this specific exposure. In figure 3, we also plot the daily 95-percent VaR of the HFRX EH index, but with EAFE position hedged**.** As depicted, this simple hedging strategy places daily VaRs within reasonable limits.

Conclusion

This paper presents a methodology for using low-frequency (monthly) hedge fund returns to model high-frequency (daily) out-of-sample returns. We show that our technique successfully tracks the actual out-of-sample daily returns of a diversified portfolio of hedge funds, such as the HFRX EH index, as well as the out-of-sample daily returns of several individual long-short mutual funds. We compare several regression estimation methods and find that dynamic filtering techniques provide an improvement over static regressions.

Our simple and easy-to-implement methodology has important and valuable applications. It allows hedge fund investors and analysts to monitor daily hedge fund proxy returns and to make proactive investment decisions (e.g., allocating inflows and outflows) intra-month, rather than after they receive month-end performance results from the fund. For hedge fund investors faced with redemption restrictions (e.g., lockups and gate clauses), the proposed methodology provides a means to implement risk controls and to effectively hedge unwanted risks. Portfolio managers and investors can apply this approach to improve their existing risk management measures, such as value-at-risk.

Obviously, this is a clear approximation of daily performance and the model does not attempt nor claim to understand the trades, leverage, or positions that a hedge fund could take on a daily basis (which can greatly alter the risk exposure of the fund). However, given the lack of actual daily hedge fund returns or holdings, our proposed approach attempts to provide investors with some insight into how their hedge funds might be performing each day. JIC

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Appendix A

TABLE A.1: LIST OF FACTORS FOR REPLICATION

Equities	Category	Examples
U.S.	Broad Market	Russell 3000, Wilshire 5000
	Style	Russell Top Value, Russell Small Growth
	Sector	S&P 500 Industrial, S&P 500 Consumer Staple
Developed ex. U.S.	Broad Market	MSCI EAFE + Canada
	Region + Style	MSCI Japan Value, MSCI Europe Growth
Emerging	Broad Market	MSCI Emerging Market
	Region + Style	MSCI China Small Cap
Fixed Income	**Category**	**Examples**
U.S.	Broad Market	Barclays Capital U.S. Agg. Bond
	Treasury	BoA ML U.S. Treasury Bills, Treasury 10 Years
	Credit	BoA ML U.S. Corporates 1–3 Years, 7–10 Years
	High Yield	BoA ML U.S. High Yield Master II, CCC and Lower Rated
	MBS/ABS	BoA ML Mortgage Masters, BoA ML ABS Floating Rate
Developed ex. U.S.	Broad Market	BoA ML Global Market ex. U.S.
	Treasury	BoA ML Japan Govt, BoA ML EMU Direct Govt
	Credit	BoA ML Euro Corporate
	High Yield	BoA ML Euro High Yield
Emerging	Sovereign	BoA ML Global Em Mkt Sovereigns Asia
	Credit	BoA ML Global Em Mkt Credit Latin Am
Others	**Category**	**Examples**
	Currencies	JPY, EUR
	Commodities	S&P GS Precious Metal, Gold
	Volatility	CBOE VIX
	Options	CBOE PutWrite, BuyWrite
	Momentum	AQR Int'l Momentum, Small Cap Momentum
	Trend Following	Fung & Hsieh Trend-Following Factors

Appendix B

Dynamic Style Analysis (DSA) Methodology Overview

In this appendix we describe the basic elements of DSA.[8] In Sharpe (1992) RBSA the return on a portfolio $r_t^{(p)}$, $t = 1, 2, 3,...,T$ is approximated by the return on a linear combination of indexes $\boldsymbol{r}_t = \left(r_t^{(1)},..., r_t^{(N)}\right)$ with the factor loadings $\boldsymbol{\beta} = \left(\boldsymbol{\beta}^{(1)},..., \boldsymbol{\beta}^{(N)}\right)$ and the intercept α such that

$$r_t^{(p)} \cong \alpha + \sum_{i=1}^{N} \boldsymbol{\beta}^{(i)} r_t^{(i)} = \alpha + \boldsymbol{\beta}' r_t$$

for $t = 1, 2, 3,...,T$. The parameters α and $\boldsymbol{\beta}$ are determined by solving the following least-squares problem[9]:

$$\begin{cases} (\alpha, \boldsymbol{\beta}) = \arg\min \sum_{t=1}^{T} \left(r_t^{(p)} - \alpha - \boldsymbol{\beta}' r_t\right)^2 \\ \text{s.t. } \boldsymbol{\beta}' 1 = 1 \end{cases}$$

We note that in Sharpe's RBSA factor loadings $\boldsymbol{\beta}$ are assumed constant within the estimation window T.

In contrast, in the DSA method factor loadings evolve slowly over time, satisfying the relationship:

$$\boldsymbol{\beta}_{t+1} \cong \boldsymbol{V}_t \boldsymbol{\beta}_t$$

where the transition matrixes, $\boldsymbol{V}_t$, determine the dynamics (Hidden Markov Model) of the factor loadings. In DSA, the parameters α and $\boldsymbol{\beta}_1, ..., \boldsymbol{\beta}_T$ are determined through time-varying regressions referred to as Flexible Least Squares (Kalaba and Tesfatsion 1989).[10]

$$\begin{cases} (\alpha, \boldsymbol{\beta}_1,..., \boldsymbol{\beta}_T) = \arg\min_{\alpha, \boldsymbol{\beta}_1,..., \boldsymbol{\beta}_T} \sum_{t=1}^{T} \left(r_t^{(p)} - \alpha - \boldsymbol{\beta}' \mathrm{r}_t\right)^2 \\ + \lambda \sum_{t=1}^{T-1} (\boldsymbol{\beta}_{t+1} - \boldsymbol{V}_t \boldsymbol{\beta}_t)' \boldsymbol{U}_t (\boldsymbol{\beta}_{t+1} - \boldsymbol{V}_t \boldsymbol{\beta}_t) \\ \text{s.t. } \boldsymbol{\beta}_t' 1 = 1 \end{cases}$$

We observe that the objective function consists of two terms, each term penalizing a different component of the model specification error. The first is the sum of the squared residuals, measuring the goodness of fit of the regression. The second term provides penalty for non-smoothness of the dynamic factor loadings. The matrixes $\boldsymbol{U}_t$ are weighting matrixes. The positive parameter λ measures the relative importance between the goodness of fit and the smoothness of the regression coefficients.

We use leave-one-out cross-validation to find an optimal λ. For this purpose, the optimal dynamic model is constructed for the data after one observation has been removed from the sample, and the prediction error is calculated on the removed observation. We repeat this procedure for each observation in the sample, and the sum of squared errors is computed. A linear complexity algorithm to perform such cross-validation is presented in (Markov et al. 2006). Assuming we omit the s-th observation, we denote the optimal DSA solution as follows:

$$\left(\alpha, (s, \lambda), \boldsymbol{\beta}_1 (s, \lambda), \boldsymbol{\beta}_2 (s, \lambda), ..., \boldsymbol{\beta}_T (s, \lambda)\right)$$

The residual error of the regression is calculated for each omitted observation. The so-called cross-validation estimate of the noise variance is found as the average over all the local squared prediction errors:

$$D(\lambda) = \frac{1}{T} \sum_{s=1}^{T} \left(r_s^{(p)} - \alpha(s, \lambda) - \boldsymbol{\beta}_s' (s, \lambda)\, \mathrm{r}_s\right)^2$$

We define the Predicted R^2 by

$$PR^2(\lambda) = 1 - \frac{D(\lambda)}{D(r^{(p)})} = 1 - \frac{\frac{1}{T}\sum_{s=1}^{T}\left(r_s^{(p)} - \alpha(s,\lambda) - \boldsymbol{\beta}'_s(s,\lambda)\, \mathrm{r}_s\right)^2}{\frac{1}{T}\sum_{s=1}^{T}\left(r_s^{(p)}\right)^2}$$

and choose the parameter λ such that the Predicted R^2 is maximized.

Endnotes

1 Our discussions with funds-of-funds managers indicate that individual hedge funds offer an estimate of the end-of-month NAV by the fifth business day of the following month, and that the official NAV is usually issued before the next month ends (usually during the second and third week of that month). This includes a capital balance statement from an administrator.

2 Usually investors must notify hedge funds for redemption forty-five to sixty-five days before quarter end. However, this process could start earlier with such a warning system. Also, investors can apply other methods to hedge their hedge fund exposures if they foresee a potential loss in the short term.

3 See Gupta et al. (2008) and Amenc et al. (2008) for summaries of various approaches and results.

4 See also Bacmann et al. (2008) for a summary of challenges when using factor-based replication.

5 Instead of selecting one factor at a time as in stepwise regression, our factor selection runs through all possible combinations of a subset of factors and selects the combination that achieves the highest explanatory power. The technique is consistent with Bai and Ng (2002, 2006). For a comprehensive discussion of factor selection in hedge fund replication, see also Darolles and Mero (2009) and Weisang (2011).

6 In some cases, the turnover data may not represent the exact year-end but rather the third-quarter report.

7 A large sample size is required to obtain meaningful quantiles. For instance, a 95-percent daily VaR estimated over a window of 100 days only produces five observations in the tail on average. According to Jorion (2007) Chapter 10, in practice, most banks use periods between 250 and 750 days for daily VaR calculations.

8 See Markov et al. (2004a,b) for a detailed description of the methodology.

9 RBSA non-negativity constraints on $\hat{a}$ are typically relaxed or removed when analyzing hedge funds and the budget constraint is not utilized when regressors represent noninvestable factors.

10 Here we assume static intercept for simplicity. The intercept also can be assumed time-varying within the same model.

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