Notes on Network Formation

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https://www2.econ.iastate.edu/tesfatsi/

Resource Sites (Links to Network Readings/Software):

[1] **T**rade **N**etwork **G**ame (**TNG**) Lab Home Page

https://www2.econ.iastate.edu/tesfatsi/tnghome.htm

[2] Formation of Economic and Social Networks

https://www2.econ.iastate.edu/tesfatsi/netgroup.htm

Presentation Outline

* Three approaches to the study of network effects

* Two IPD game examples comparing effects of having random vs. preferential partner matching

  - **Preparatory Stuff:** Finite state machine (FSM) representation of IPD player (i.e. strategy) types
  - **Example 1:** IPD game play among ***fixed*** player types
  - **Example 2:** IPD game play among ***evolving*** player types

Three Approaches to the Study of Network Effects

- Agents interact with other agents in a **given interaction network**. Agents do not control with whom they interact, or with what regularity **(e.g. Axelrod Tournament with round-robin PD play).**

- Agents interact with other agents through **given restricted links** but agents have some ability to control the frequency of these interactions **(e.g. Electricity Market).**

- Agents **preferentially decide** with whom they interact and with what regularity **(e.g. Labor Market).**

Network Effects vs. Network **Formation** Effects …

See: Andy Clark, *Being There: Putting Brain, Body, and World Together Again*, MIT Press, 1998

- **Strong Scaffolding**: ***Given*** interaction network; or ***given*** restricted links.
- **Weak Scaffolding**: Agents ***preferentially decide*** with whom they interact, and with what regularity.
- Scaffolding as a substitute for learning and/or thinking?

Network Effects vs. Network **Formation** Effects …

Key Question:

What difference does it make if agents can *preferentially* form their own networks?

Random vs. Preferential Partner Choice: Two Illustrative Examples

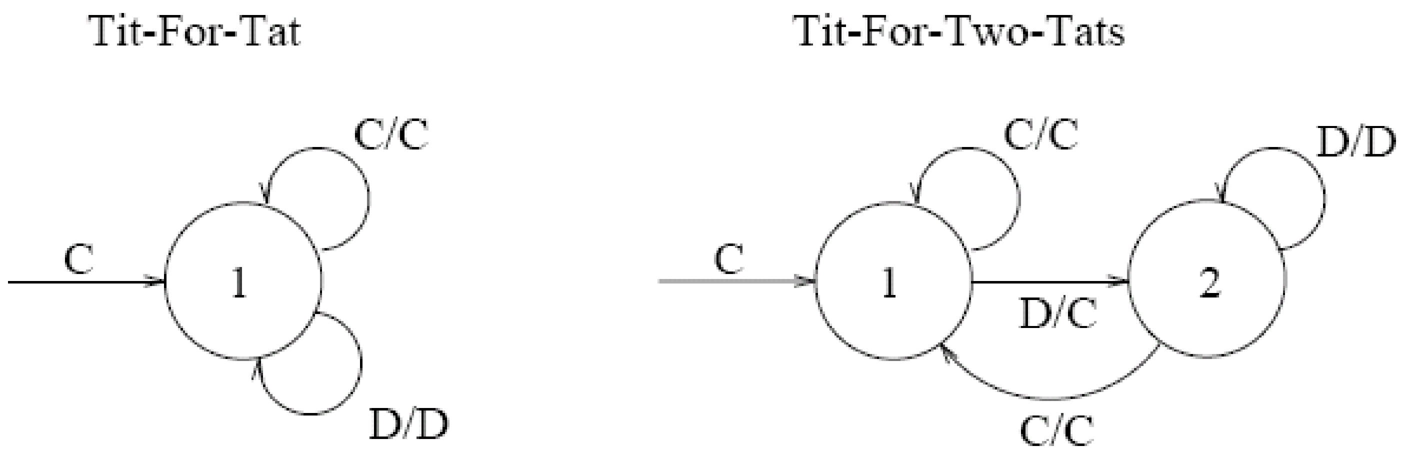
Example 1: IPD game play among fixed player types

Ref.[3]: Leigh Tesfatsion (1997), "How Economists Can Get Alife," pp. 533-564 in W. Brian Arthur, Steven Durlauf, and David Lane (eds), *The Economy as an Evolving System II*, Santa Fe Institute Studies in the Sciences of Complexity, Proceedings Volume XXVII, Addison-Wesley.

Example 2: IPD game play among evolving player types

Ref.[4]: Dan Ashlock, Mark Smucker, E. Anne Stanley, and Leigh Tesfatsion (1996), "Preferential Partner Selection in an Evolutionary Study of Prisoner's Dilemma," *BioSystems*, Vol. 37, Nos. 1-2, pp. 99-125.

Illustrative Finite State Machine Representations for 1-State and 2-State IPD Players



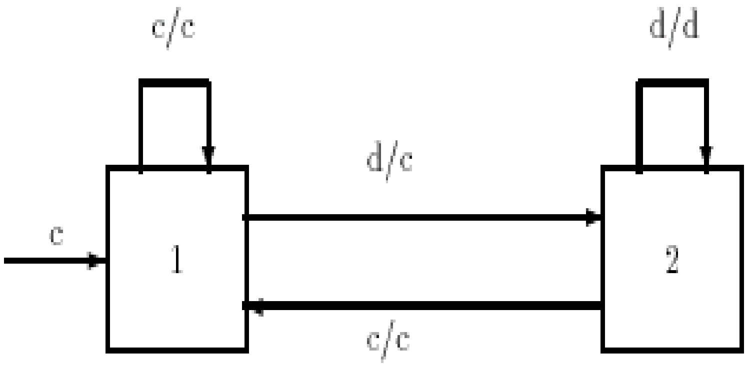
X/Y =: "If my rival's last move was X, I will now play Y."

TFTT (Tit-for-Two-Tats) vs. Rip-Off

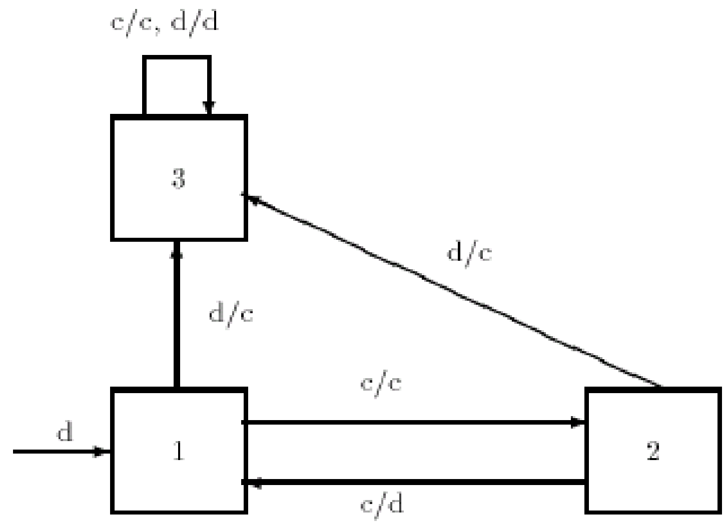
QUESTIONS:

What happens if TFTT is forced to play Rip-Off?

What happens if two Rip-Offs play each other?



(a) Tit-for-Two-Tats



(b) Rip-Off

X/Y =: "If my rival's last move was X, I will now play Y."

Example 1: Fixed-Player IPD Game with Preferential Partner Choice and Refusal

Note: **All Example 1 results are analytically derived**

- Fixed Player Population = 3 TFTTs and 2 Rip-Offs
- Players engage in 150 iterations of an Iterated Prisoner's Dilemma (IPD) Game
- The payoffs for each PD game play are centered about 0, as follows:

  L (Lowest = Sucker Payoff) < **D** (Mutual Defection) < **0**

  < **C** (Mutual Cooperation) < **H** (Highest=Temptation Payoff)

- In addition, PD payoffs satisfy **[L + H]/2 < C**.

Example 1: Payoffs for Each Play of the Prisoner's Dilemma (PD) Game

		Player 2	
		C	D
Player 1	C	(C,C)	(L,H)
	D	(H,L)	(D,D)

L (Lowest) < D (Mutual D) < 0 < C (Mutual C) < H (Highest)

Example 1: Expected Payoff Assessments

- Each player A assigns an ***initial expected payoff U^o*** to each other player B .

- ***Expected payoff assessments U*** are continually updated based on play history (simple averaging) .

- Player A finds player B ***tolerable ($U \geq 0$)*** as long as player A assigns a nonnegative expected payoff U to B .

- Player A stops making play offers to (or accepting play offers from) any player B who becomes ***intolerable ($U < 0$)*** .

Example 1: Preferential Partner Choice and Refusal

- At start of each iteration, each player A makes a play offer to a ***tolerable*** player B he judges to offer the currently highest expected payoff U.

- Player A "flips a coin" to settle ties and goes inactive if he judges every other player to be ***intolerable (U < 0).***

- If player A has a play offer ***refused*** by a tolerable player B:
  - He experiences ***refusal disutility*** – specifically, a ***decrease in his utility given by - R ≤ 0*** -- due to shame and wasted effort.
  - If possible, he redirects his offer to a different tolerable player B' that he assesses to have the next highest expected payoff U.
  - If he assesses all remaining players to be intolerable, he goes inactive.

Example 1: Preferential Partner Choice and Refusal …

- Each player A updates his expected payoff U(B) for another player B whenever he receives a payoff from an interaction with B -- either refusal disutility (- R) or a game payoff {L, D, C, H} -- starting from an initial "stance towards strangers" expected utility assessment $U^o(B)$ for B.

  Example: If player A has played B twice in the past and received payoffs p1 and p2, his current expected payoff U(B) for player B is

$$U(B) = [\, U^o(B) + p1 + p2 \,]/3$$

- If U(B) falls below 0, player B is deemed ***intolerable***

➡ player A will refuse any future play offers received from player B, and player A will not direct any future play offers to player B.

Example 1: Key Issues

- Fixed population consisting of two agent types:
  3 TFTTs & 2 Rip-Offs
- With random partner choice, Rip-Offs will chew TFTTs to pieces
- How does the introduction of preferential partner choice affect the ***long-run fitness (accumulated points)*** of TFTTs vs. RipOffs?
- How does the **initial expected payoff level U^o** affect long-run fitness outcomes?

Example 1: Visualization of Findings

Network Visualization:

- **Boxes** = Players
- **Box size** = Long-run fitness level
- **Lines** = Persistent interactions

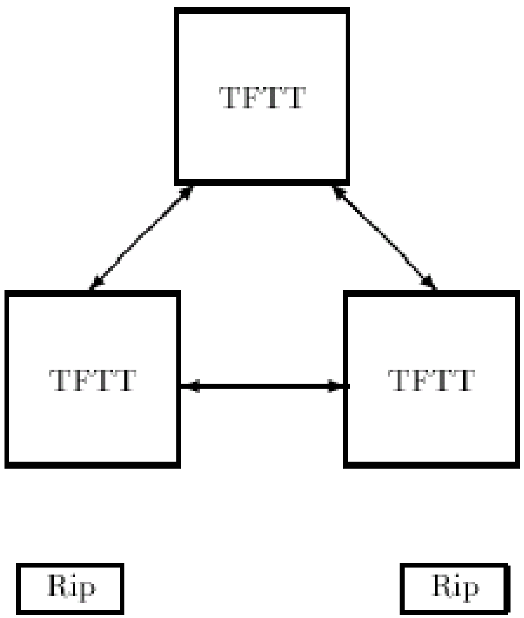
Treatment Factor ("Stance Towards Strangers"):

Initial exogenously-set expected payoff U^o

Analyzed Range of Values for Initial Expected Payoff U^o:

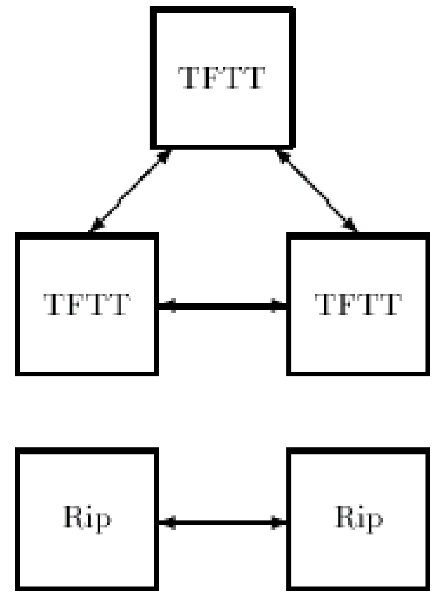
Low to high (relative to PD-game payoffs)

Outcomes Visualized for TFTT vs. Rip-Off with Relatively Low U° Values



(a) Case (CP.1)

$0 < U^{o} < -D$

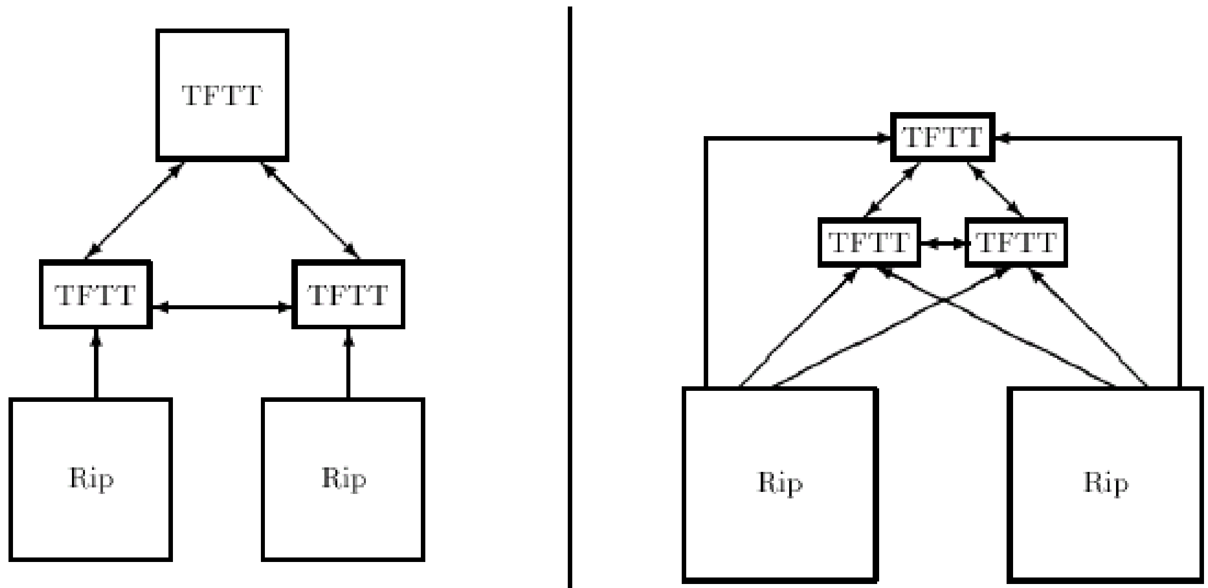


(b) Case (CP.2) or (CP.3)

$(-D \leq U^{o} < -L)$ or $(C < -L \leq U^{o})$

Note: **A larger box indicates a relatively higher long-run fitness.**

Outcomes Visualized for TFTT vs. Rip-Off
with Relatively High U^o Values



(c) **Case (CP.4): $-L \leq C$ with $-L \leq U^o < (H+C)/2$**

(d) **Case (CP.4): $-L \leq C$ with $(H+C)/2 \leq U^o$**

Figure 2: Long-Run Trade Networks Under Assumption (CP) for the Illustrative 5-Tradebot TNG. A relatively larger box indicates a definitely higher fitness score for a sufficiently long trade cycle loop. In case (d), the Rip-TFTT interactions are stochastic if $(H+C)/2 = U^0$ and deterministic if $(H+C)/2 < U^0$.

Example 2: Evolutionary IPD Game with Preferential Partner Choice and Refusal

Note: **All Example 2 results are computationally derived**

Key Issue Studied:

What happens in an IPD game if players preferentially choose and refuse their partners, as in Example 1, ***and*** the players also evolve their "types" (game strategies) over time based on their realized payoff outcomes?

Example 2: Maintained Parameter Specifications

PD Payoff Settings: Sucker (C played, opponent plays D) = 0,

Mutual Defection (D played, opponent plays D) = 1,

Mutual Cooperation (C played, opponent plays C) = 3

Temptation (D played, opponent plays C) = 5

Initial Expected Payoff: U^o ("Stance Towards Strangers") = 3

Intolerable Partner: Expected payoff U for this partner is less than $\tau = 1.6$

Refusal Disutility: $-R = -1.0 < 0$

Example 2: Benchmark Case

Evolutionary IPD Game with Random Partner Matching

- **Initial Strategies:** Each player in an initial population of 30 IPD game players starts with an exogenously specified IPD game strategy.

- **Random Matching:**

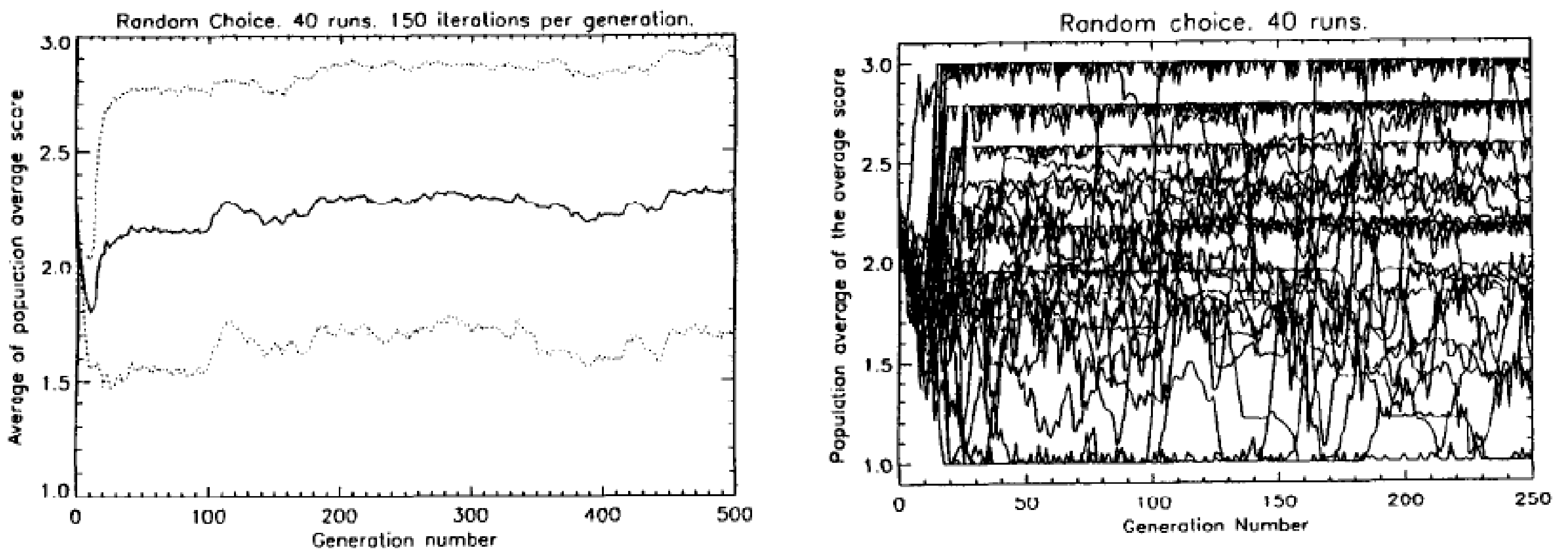
  Each player is ***randomly*** matched in each iteration with another player to play a PD game

→ *no* choice/refusal of partners is permitted, *no* refusal disutility is experienced, and *no* intolerability assessments are made.

Example 2: Benchmark Case … Continued

- **Initial Generation:**
  - **30 IPD game players** with **initially set IPD strategies** engage in 150 iterations of PD game play using random partner matching.
  - A **Genetic Algorithm (GA)** is then used to evolve a ***new*** IPD strategy for each player, given the fitness outcomes resulting from use of the current 30 IPD strategies.
- **Next 499 Generations:**
  - The 30 IPD game players with **evolved** IPD strategies engage in 150 iterations of PD game play using random partner matching.
  - A GA is then used to evolve a ***new*** IPD strategy for each player, given the fitness outcomes resulting from use of the current 30 IPD strategies.
- **Forty runs (500 generations each) are conducted in total**
  - The **P**seudo-random **N**umber **G**enerator (**PNG**) for each run is initialized with a different seed value.

Example 2: Benchmark Case … Sample Outcomes



(a) (b)

Fig. 3. Random choice evolved for 500 generations. Each player chooses exactly one partner at random on each of the 150 iterations comprising an IPD tournament. (a) The overall average fitness achieved by successive generations across 40 runs. The dashed lines (error bounds) show this overall average fitness plus or minus one standard deviation. (b) Each line shows the average fitness achieved by successive generations during one of the 40 runs. Note the wide spread and the horizontal bands. The bands tend to occur because populations become genetically homogeneous and mutants tend to do poorly.

Example 2: Evolutionary IPD Game with Preferential Partner Choice & Refusal

- IPD game play proceeds as in Example 2 (Benchmark Case) with **one important difference:**

- Each IPD game player in each generation uses **preferential partner choice and refusal**, as follows:

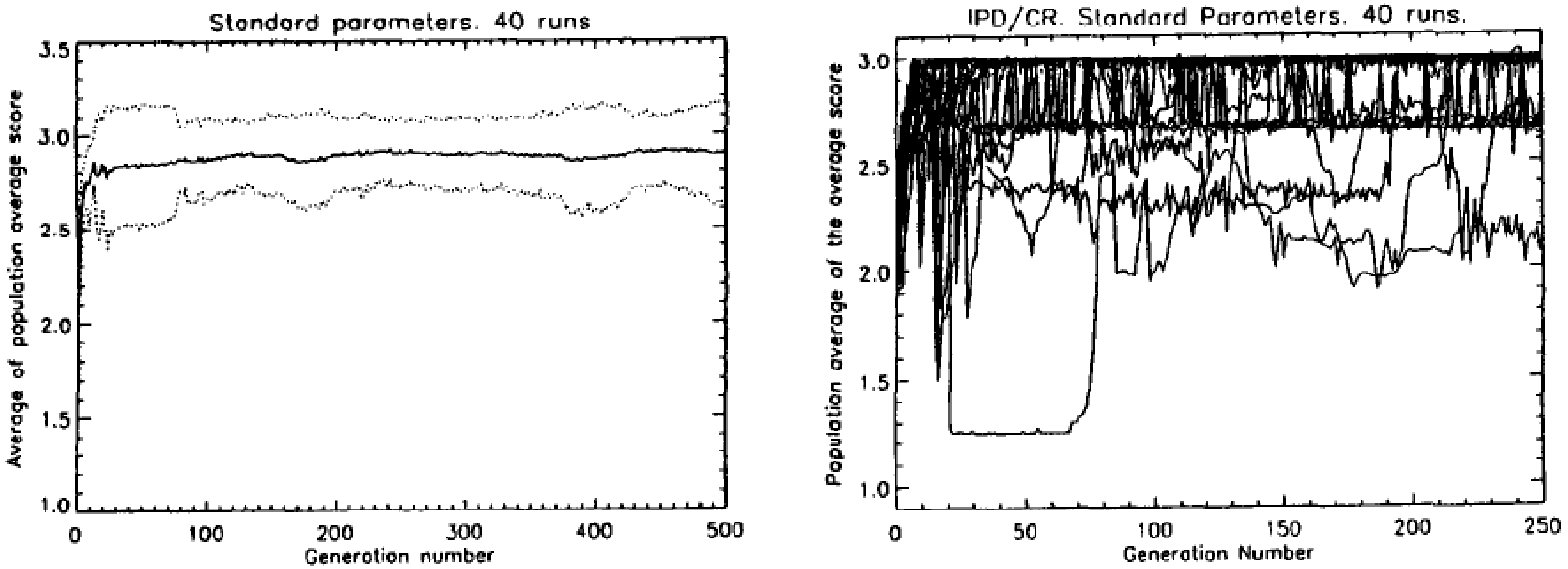
  – In each iteration of the IPD game, each player A makes PD game-play offers to preferentially favored other players while also accepting/refusing PD game-play offers received from other players.

  – These choice/refusal decisions are based on continually updated expected payoff assessments U(B) for each other player B, determined as weighted averages of payoff outcomes experienced in past PD game-plays with B.

  – Each player refuses to play with intolerable players ($U < 1.6$).

  – Each player A's expected utility assessment U(B) for a player B decreases by $R = 1.0$ if B refuses to accept a PD game-play offer received from A.

Example 2: Outcomes for Evolutionary IPD Game with Preferential Partner Choice and Refusal



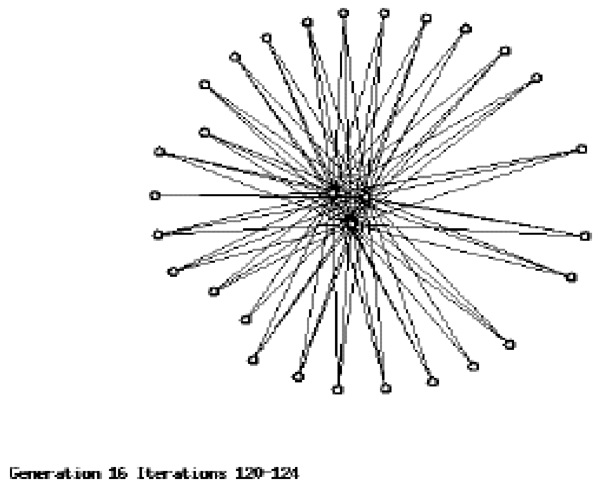
(a) (b)

Fig. 4. IPD/CR evolved for 500 generations with all parameters at their standard scenario levels. As in Fig. 3, each player chooses at most one partner in each iteration. (a) Overall average fitness across 40 runs and error bounds. (b) Average fitness achieved by successive generations for 40 individual runs. Note how few fitness levels are achieved in comparison to Fig. 3. The jumps in average fitness from the fitness region near 2.69 to a level above the mutual cooperation fitness region at 3.0 are observed frequently, and indicate the Raquel-and-the-Bobs phenomenon discussed in the text.

Example 2: Emergence of "Raquel-and-the-Bobs" Pattern for the Evolutionary IPD Game with Preferential Partner Choice and Refusal

Actual Slice-in-Time Picture:

Inner grouping of 3 "Raquels" playing ≈ C:C (i.e., start with Cooperation (C), then continually play C) with outer grouping of 27 latched "Bobs" playing ≈ D:C (i.e., start with Defection D, then continually play C)



Homogenous population of 30 Bobs → Rise of mutant Raquels until fitness of Bobs > fitness of Raquels → Decimation of Raquels → Back to homogeneous population of 30 Bobs → cycle repeats

Example 2: Summary of Findings for Evolutionary IPD Game with Preferential Partner Choice and Refusal

Main Conclusions:

- Introduction of choice and refusal of partners (in place of random matching) ***accelerates the emergence of mutual cooperation*** in the evolutionary IPD game.

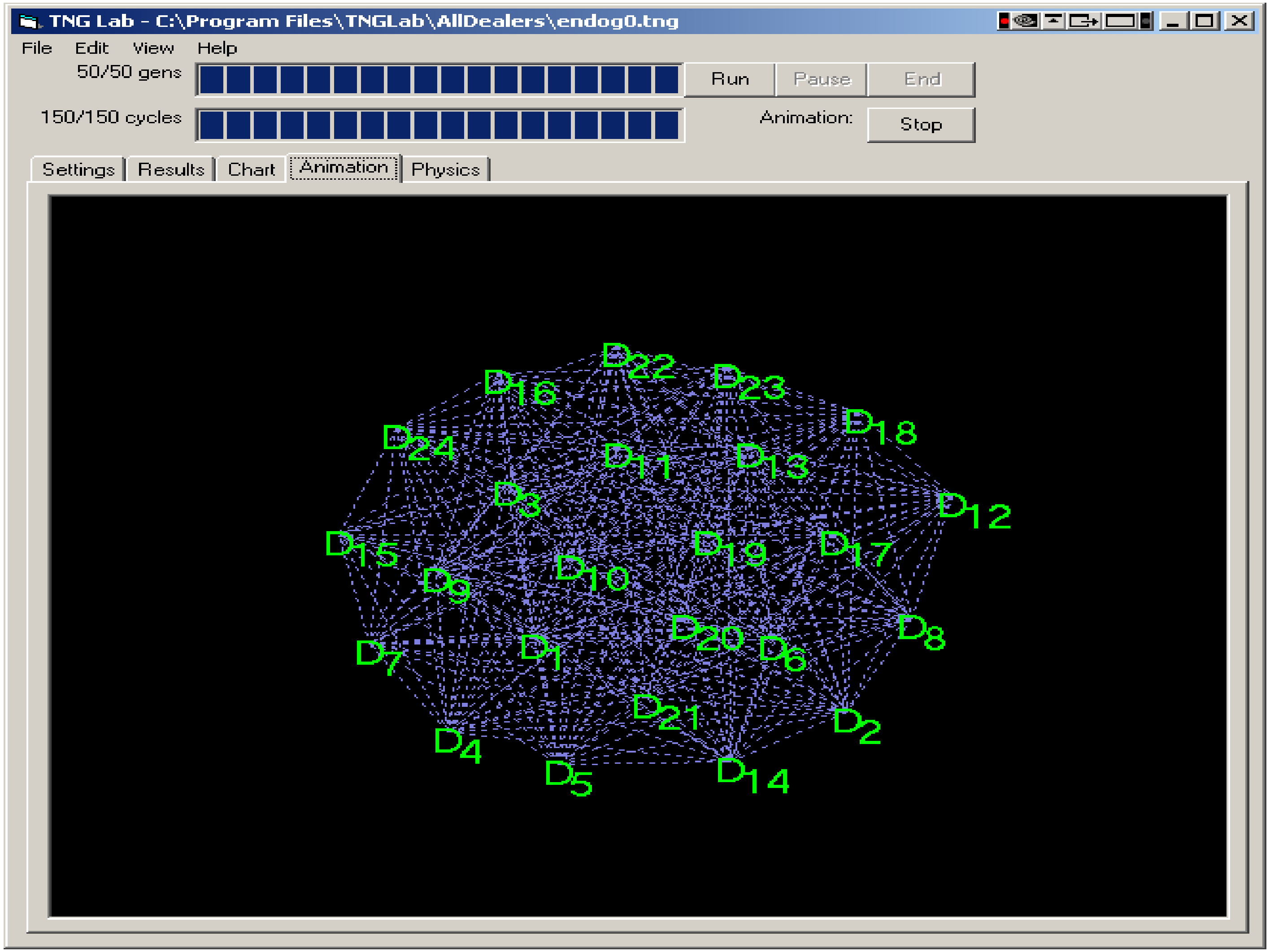
- However, in general, mutual cooperation can be supported by a ***wide variety of underlying trade networks*** (latched, star, recurrent, disconnected, etc.)

Note: Below for illustration are various types of trade networks – evolved using the **T**rade **N**etwork **G**ame (**TNG**) Lab – that support persistent mutual cooperation among "latched" and/or "stochastically interacting" subsets of trade partners.

Stochastic All-Dealer Trade Network

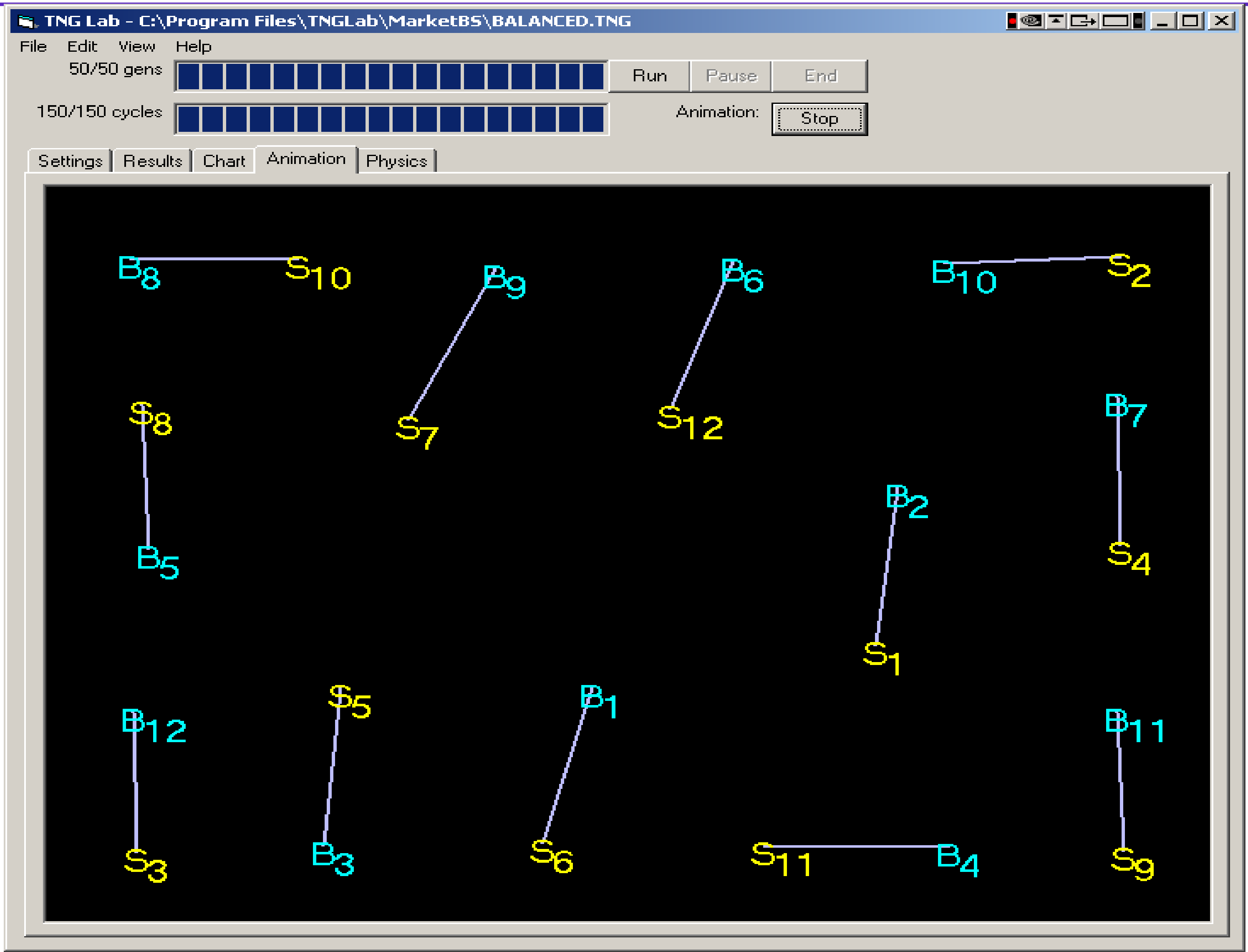
Evolved by the TNG Lab

https://www2.econ.iastate.edu/tesfatsi/tnghome.htm



Bi-Lateral (Buyer-Seller) Trade Network

Evolved by the TNG Lab



Tri-Lateral (Buyer-Dealer-Seller) Trade Network

Evolved by the TNG Lab