Productivity Growth: Theory and Measurement

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INTRODUCTION

Productivity growth has received greater attention from economists and policy makers in Asia in the 1990s. This is partly due to the work of Alwyn Young (1992, 1995) and Paul Krugman (1994), who argued that economic growth in Asia is driven by the accumulation of the inputs in the production process rather than by increases in productivity. In other words they, in particular Krugman, believe that the Asian economic miracle is largely attributable to an increase in the quantity and not the quality of the factors of production. Further analysis and evidence showed that as countries become more developed and move closer to the limits of factor accumulation, they rely more and more on increasing productivity to sustain the economic growth process. In fact, some studies (such as Rao and Owyong 1997) indicate that this process of productivity growth is already occurring in the more developed economies in the region. Nonetheless, policy makers and economists alike have begun to recognize more fully the importance of technology and productivity in economic growth. In order to facilitate this process of understanding the movements in productivity and designing the right policies to enhance it, it is critical to be able to first get a handle on what exactly productivity is and how to measure it. This will be the focus of this article.

SOME BASIC DEFINITIONS OF PRODUCTIVITY

At its most basic level, productivity is based on the economics of the firm. It is measured as the ratio of output to input. Historically, productivity is often expressed as the ratio of output to the most limited or critical input, with all the other inputs held constant. Agricultural productivity is usually measured in bushels of wheat or corn per acre. In industries that require skilled labor, which is often in relative shortage, output per worker is considered as the most appropriate measure of productivity. However, such single-factor-based measures of productivity suffer from obvious limitations. First, in most industries or sectors there may be several factors of production that are of almost equal importance, in which case it might be difficult to choose among them. Second, the relative importance of inputs may change over time. For instance, the relative importance of labor may be low in the initial stages of development when unemployment is high, but may become critical as the country becomes more developed because of declining birth rates and an aging labor force.

Total factor productivity is the the combined productivity of all inputs, and hence avoids the problems faced by measures based on just one factor. One does not have to choose any factor on which to base productivity growth, since all factors are included. Furthermore, as will be made clearer later, the impact of each input on total factor productivity is allowed to vary, hence taking into account the possibility that the relative importance of factors may change over time. As a result of these advantages, total factor productivity is the most commonly known and widely used method of productivity measurement.

TOTAL FACTOR PRODUCTIVITY: SOME MEASUREMENT CONCEPTS

Simply defined, total factor productivity is the weighted average productivity of all inputs, where the weights to these inputs are their shares in the total cost of production. Suppose for the moment that output is measured in some physical unit, say tons. Then TFP is measured as the ratio of output Y to aggregated input X:

$$\mathrm{TFP} = \frac{Y}{X} \tag{1}$$

Since there are multiple inputs, X has to be computed by aggregation. Using the definition of Divisia indexes, the growth rate of the aggregated input is equal to the weighted sum of the individual inputs' growth rates:

$$\frac{dX}{X} = \sum_{i=1}^{I} v_i \frac{dx_i}{x_i} \tag{2}$$

where x_i is quantity of input i and v_i is the weight assigned to input i.

$$v_i = \frac{\text{Unit cost of input } i \times \text{Units of input } i \text{ employed}}{\text{Total expenditures for all inputs}} \tag{3}$$

Consider that instead of having just a single type of output, there are multiple outputs. Using Divisia indexes again, it therefore follows that

$$\frac{dY}{Y} = \sum_{i=1}^{J} w_j \frac{dy_j}{y_j} \tag{4}$$

where y_j is the quantity of the jth output produced, with the weight w_j being the share of total revenue contributed by the jth output. Combining (2) and (4) leads to the following expression for TFP growth:

$$\hat{\text{TFP}} = \sum_j w_j \hat{y}_j - \sum_i v_i \hat{x}_i \tag{5}$$

where the hats represent growth rates and the weights are functions of the relevant prices and quantities:

$$w_j = \frac{q_j y_j}{\Sigma_j q_j y_j} \quad \text{and} \quad v_i = \frac{p_i x_i}{\Sigma_i p_i x_i} \tag{6}$$

where q_j and p_i are the prices of the jth output and ith input, respectively. The firm is assumed to maximize profits subject to the constraint of the production technology, which is given by

$$(y_1, \ldots, y_J) = F(x_1, \ldots, x_I) \tag{7}$$

where profits are given as follows.

$$\pi = \sum q_j y_j - \sum p_i x_i \tag{8}$$

If the production technology follows constant returns to scale, then

$$\sum_j q_j y_j = \sum_i p_i x_i \tag{9}$$

Totally differentiating the last equation with respect to time and dividing both sides by the corresponding total value yields

$$\sum_j w_j[\hat{q}_j + \hat{y}_j] = \sum_i v_i[\hat{p}_i + \hat{x}_i] \tag{10}$$

Equation (5) implies that the rate of growth of TFP is equal to the aggregate growth rate of output minus the aggregate growth rate of inputs. In addition when (10) is used, it may be shown that

$$\hat{\text{TFP}} = \sum_i v_i.\hat{p}_i - \sum_j w_j.\hat{q}_j \tag{11}$$

which is to say that the rate of TFP growth is equal to the average rate of growth of input prices less the average rate of growth of output prices.

Relationship among TFP Growth, Output Growth, and Labor Productivity Growth

For small changes in a variable the rate of change from one time period t to $t+1$ is closely approximated by the corresponding difference in logarithms. Thus, for any variable Z,

$$\hat{Z} = \frac{Z_{t+1} - Z_t}{Z_t} \approx \ln\ Z_{t+1} - \ln\ Z_t \tag{12}$$

Given this useful result, we can reformulate our measure of TFP in (5) by replacing all growth rates by the corresponding log differences. It follows that the growth rate of TFP is

$$\hat{\text{TFP}}_t = \ln\ \text{TFP}_t - \ln\ \text{TFP}_{t-1} = \sum_i \bar{v}_{i,t}\,(\ln(Y_t/x_{i,t}) - \ln(Y_{t-1}/x_{i,t-1})) \tag{13}$$

where average expenditure share $\bar{v}_{i,t} = 0.5(v_{i,t} + v_{i,t-1})$. In this form it becomes clear that the growth of total factor productivity is the weighted sum of the growth rates of all single factor productivities. Put another way, output growth is equal to the sum of the TFP growth rate and the growth rate of the average input.

$$\hat{Y}_t = \sum_i \bar{v}_{i,t}\hat{x}_{i,t} + \hat{\text{TFP}}_t \tag{14}$$

Finally, the growth rate of the productivity of any input can be expressed in terms of the rates of growth of the ratios of all other inputs to that input, and the growth of TFP. For the case of labor productivity, this implies that

$$
\begin{aligned}
\text{Growth rate of labor productivity} &= (\ln Y_t - \ln x_{l,t}) - (\ln Y_{t-1} - \ln x_{l,t-1}) \\
&= (\ln Y_t - \ln Y_{t-1}) - (\ln x_{l,t} - \ln x_{l,t-1}) \\
&= \sum_{i \neq l} \bar{v}_{i,t}\hat{x}_{i,t} - (1 - v_l)\hat{x}_{l,t} + \hat{\text{TFP}}_t \\
&= \sum_{i \neq l} \bar{v}_{i,t}(\hat{x}_{i,t} - \hat{x}_{l,t}) + \hat{\text{TFP}}_t \qquad (15)
\end{aligned}
$$

where the second last equality follows from (14) and the last equality uses $1 - \bar{v}_{l,t} = \Sigma_{i \neq l}\bar{v}_{i,t}$.

Presence of a Nonvariable Factor

We now relax the assumptions of instantaneous adjustment of all inputs. It is assumed that there is one quasi-fixed factor, whose adjustment is hindered by adjustment costs or institutional factors. This quasi-fixed factor is usually taken to be capital. Hence the firm's problem is to minimize the variable cost of producing output $(y_1, \ldots, y_J)$ subject to the fixed prices of the variable inputs as well as the fixed capital input $\bar{k}$. Mathematically this is expressed as

$$
\begin{gathered}
\min\ VC = f(x_1, \ldots, x_I, \bar{k}; y_1, \ldots, y_J) \\
\text{subject to } \sum_j q_j y_i \geq \sum_i p_i x_i \qquad \text{and} \qquad \partial(VC)/\partial y_j \leq q_j
\end{gathered}
$$

where the first constraint implies that total variable cost must be met or exceeded by total revenues, with the second requiring that price must exceed marginal cost. The shadow price to the quasi-fixed input is

$$
p_k = \frac{\Sigma_j\, q_j y_j - \Sigma_i\, p_i x_i}{\bar{k}} \geq 0 \qquad (16)
$$

Hence this implies that in the short run when the capital input is often fixed, TFP is measured by

$$
\begin{gathered}
\hat{\text{TFP}} = \sum_j \mathrm{w}_j \hat{\mathrm{y}}_j - \sum_i \mathrm{v}_i \hat{\mathrm{x}}_i - \mathrm{v}_k \bar{\mathrm{k}} \\
\text{where } v_k = p_k \bar{k} \Big/ \left(\sum_i p_i x_i + p_k \bar{k} \right) \qquad (17)
\end{gathered}
$$

Extension to the Economy Level

At the level of the economy, we can think of an aggregate production function, which for the moment is assumed to follow the Cobb-Douglas form:

$$\log\, Y_t = \alpha + \beta \log\, K_t + (1 - \beta)\, \log\, L_t + \log\, u_t \tag{18}$$

where the Cobb-Douglas assumption is taken to be a first approximation to a potentially much more complex relationship. Differentiating the above expression with respect to time yields

$$\hat{Y}_t = \beta \hat{K}_t + \gamma \hat{L}_t + \mathrm{T\hat{F}P_t} \tag{19}$$

The parameters β and γ represent the share of total input cost in the Cobb-Douglas formulation, which is in accordance with the weights that were used earlier in the article. Measures of total factor productivity may then be obtained by deducting the input growth rates from output growth. This approach to decompose the total growth of output in the economy into its different potential factors is called growth accounting. These factors are to explain output movements; what is left unexplained (often called the residual) is considered as total factor productivity.

To see more clearly the implicit restrictions of the simple Cobb-Douglas production function, consider a production function of the general form $Y_t = f(x_{1,}t, \ldots, x_{I,t}, t)$, where the time trend variable t may be thought of as a proxy for total factor productivity growth. We can then write

$$\frac{dY}{dt} = \sum_{i=1}^{I} \frac{\partial f}{\partial x_i} \frac{dx_i}{dt} + \frac{\partial f}{\partial t} \tag{20}$$

Dividing both sides by Y, and recognizing that $\partial f / \partial x_i$ is by definition the marginal product of factor i (MP_i), we get

$$\begin{aligned} \mathrm{T\hat{F}P} &= \frac{\partial Y/\partial t}{Y} = \frac{dY/dt}{Y} - \sum_i \frac{MP_i . x_i}{Y} \cdot \frac{dx_i/dt}{x_i} \\ &= \hat{Y} - \sum_i \frac{MP_i . x_i}{Y}\, \hat{x}_i \end{aligned} \tag{21}$$

Assuming perfect competition, the values of the marginal products equal factor prices, i.e., $P.MP_i = p_i$, where P is the output price and p_i is the price of input i. The last expression can therefore be written as

$$\mathrm{T\hat{F}P} = \hat{Y} - \sum_i \frac{p_i x_i}{PY}\, \hat{x}_i \tag{22}$$

where the weights of the input growth rates are the ratios of input payments to total revenues. These weights will be equal to the share of total input costs if we assume constant returns to scale, in which case $P.Y = \Sigma_i p_i x_i$. Given this and by restricting the number of inputs to capital and labor, the last equation (22) reduces to the Cobb-Douglas form in equation (19).

Measurement of Capital and Other Inputs

The data for generating measures of productivity growth should include as many outputs and inputs of the firms as possible in order to reflect all production and costs. Output is usually measured as an aggregate of all types of production activities. The categories of inputs generally identified are capital, labor, energy, nonenergy intermediate materials, and sometimes purchased services. Inputs such as land and inventories are often included in the measure of capital. The two potentially most problematic issues that arise in data construction involve the measurement of capital and aggregation. Aggregation is a problem because capital is clearly not homogeneous. As regards its measurement and the construction of a capital series, it is also problematic since it requires rethinking the idea of current input use. As a durable input the services from the available stock of capital, and the rental or user prices of these services, are relevant values for the construction of productivity growth measures, and neither of these is readily observable. Developing capital measures also requires consideration of what types of inputs should be included as components of the capital stock, which is sometimes unclear.

Measurement of Output

Output measurement for a single-output firm is fairly straightforward, since for a single output there is only one type of unit involved, say the number of pairs of shoes or tons of steel. In this case, therefore, an average price per pair or ton can generally be specified in dollars as total sales divided by the quantity of the output, and thus quantity and price indexes can directly be computed.

Even for this simple case there are problems involved. For example, it is not immediately clear how changes in quality can be handled. In a few cases (tons of steel might be an example) this is not a critical issue since the product is quite homogenous. However in most other cases, such as the number of computers produced, the quality of a particular unit might change over time or across companies (as in different brands).

Another problem that may complicate the measurement of output is the existence of inventories. Data are generally reported in terms of sales, whereas actual production is the relevant output for the measurement of productivity. Inventories constitute the difference between these two figures. For the measurement of output, therefore, sales data should ideally be adjusted by net inventory change. In other words, the correct output series to use is obtained from adding sales to inventory change.

For a firm that produces multiple outputs, there are further difficulties: how to add together goods that are measured in different units is a standard index-number problem? It is not an easy problem to deal with. While determining the total value of production is relatively straightforward, dividing this value into its aggregate quantity and price components is not. This aggregation issues will be dealt with in greater detail below.

Measurement of Labor

Labor input is relatively easy to measure compared to other inputs, since labor statistics are generally presented in terms of wage bill paid and the number of workers or person-hours. By dividing the wage bill by the number of workers or person-hours, we obtain an estimate of the average wage rate. The number of person-hours is generally a better measure of true labor input than number of workers, since the latter does not reflect changes in the hours worked per worker.

Measurement of Capital

The most problematic input to measure is probably capital. First, the categories are often not clearly defined. Although buildings and structures, machinery and equipment, etc. are often accounted for, other categories that are potentially important are ignored. One such example is research and development, which might be considered a long-term investment, and therefore a component of the capital stock. The main difficulty of measuring capital, however, is how to deal with an input that provides a stream of services over time, and is often not considered as part of the explicit costs of the firm.

A relevant measure of the available capital stock is computed as what is left of the capital investment in past time periods for the firm. This is generally written for each capital asset x_k as

$$x_{k,t} = \sum_{\tau=0}^{T} x_{k,t,t-\tau} = \sum_{\tau=0}^{T} s_{k,t,\tau} z_{k,t-\tau} \tag{23}$$

where T is the life of the durable good, $x_{k,t,t-\tau}$ is the stock of x_k in time period t still remaining from investment in period $t - \tau$, $s_{k,t,\tau}$ is defined as the physical survival rate for age τ investment in time period t for asset k, and $z_{k,t-\tau}$ is gross investment in asset k at time $t - \tau$. This summation must be done for each asset individually, and then the assets must be aggregated based on their user costs, as will be discussed below.

Determining the level of $x_{k,t}$ for each asset therefore requires finding a benchmark level of the stock in period 0, deflating the value of investment by relevant deflators (to convert to constant dollars) and cumulating the investment from that point on based on some assumption about survival rates.

Finding a benchmark is sometimes difficult, often requiring some judgment together with past data and numbers from other studies. As for the deflators to be used in the second step, they may be obtained from the output price series for the supplying industries, such as office equipment. The most difficult is step three, which requires us to characterize $s_{k,t,\tau}$, the physical survival rate. There are a number of possible assumptions for this: (a) one-hoss shay (the machine runs at full tilt until it dies), (b) constant exponential decay (i.e., the decay per time period is a constant percentage, say δ%, which implies that $s_\tau = (1 - \delta)^\tau$), (c) straight line or linear depreciation (e.g., 5% of the initial capital stock in its time period), (d) decelerated depreciation (any method where the age-price profile declines slower than concave). The most common method is a form of exponential decay called the perpetual inventory method, based on geometric deterioration. This assumption implies that capital services never actually reach zero so every unit of investment is perpetually a part of the stock of capital. The perpetual inventory method essentially requires that

$$K_t = (1 - \delta_t)K_{t-1} + I_{t-1} \tag{24}$$

where K_t is the capital stock at the beginning of time t and I_{t-1} is the investment in period $t - 1$. Often a constant exponential rate is assumed for δ_t, which makes it fall under the category of constant exponential decay.

Next a price for the capital good needs to be obtained. Since the underlying theory specifies the service flow from capital as the relevant input to measure, it is necessary to construct corresponding data series measuring the service flow price. This concept leads to notion of the user cost of capital, which not only includes the investment price, but also adjusts it by the interest rate, the depreciation rates, and government taxes and incentives. Mathematically this is represented by the following equation:

$$c_t = TX_t[r_t J_{t-1} + \delta\ J_t - \Delta\ J_t] + b_t \tag{25}$$

where b_t represents the effective property tax rate, J_t is the asset price at time t, $\Delta\ J_t = J_t - J_{t-1}$ denotes the capital gains, r_t is the rate of interest, δ is the depreciation rate, and TX_t is the effective rate of taxation on capital income given by

$$TX_t = (1 - T_t\Theta_t - \kappa_t)/(1 - T_t) \tag{26}$$

where T_t is the effective corporate income tax rate, Θ_t is the present value of depreciation deductions for tax purposes on a dollar's investment over the lifetime of the good, and κ_t is the effective rate of the investment tax credit.

Econometric Issues

Many different function forms have been used for the econometric estimation of productivity growth. The choice among different functional forms is generally based on the type of analysis to be carried out. Some functions simplify computation of elasticity formulas and specification of constraints such as constant returns to scale, some facilitate consideration of dynamic interactions, some allow curvature conditions to be directly imposed, and some enhance the ability to identify the difference between short-run and long-run behavior. Most modern studies of production technology, however, do rely on some type of flexible functional form, which allows generality in terms of interactions among arguments of the function, such as substitution among inputs.

One example of a flexible functional form which has been used extensively for the analysis of production is the translog function. The translog production function, assuming instantaneous adjustment of all inputs is of the form:

$$\begin{aligned}\ln\ Y_t = {} & \alpha_0 + \alpha_K \ln\ K_t + \alpha_L \ln\ L_t + \alpha_t t + 0.5\ B_{KK}(\ln\ K_t)^2 \\ & + B_{KL}(\ln\ K_t)(\ln\ L_t) + B_{Kt}(\ln\ K_t).t + 0.5 B_{LL}(\ln\ L_t)^2 \\ & + B_{Lt}(\ln\ L_t).t + 0.5 B_{tt}\ t^2\end{aligned} \tag{27}$$

where the assumption of constant returns to scale implies that

$$\alpha_K + \alpha_L = 1, \quad B_{KK} + B_{KL} = B_{LL} + B_{KL} = B_{Kt} + B_{Lt} = 0 \tag{28}$$

It is clear from observation that the translog function is a generalization of the Cobb-Douglas functional form. The Cobb-Douglas form is restrictive in terms of the implicit substitution assumptions: elasticities of substitution between all inputs are one and shares of the inputs are constant. Extending the Cobb-Douglas to the translog function enables these constraints to be relaxed because cross-effects between inputs are recognized and therefore more complex substitution patterns can then be captured.

CONCLUSION

With the increasing recognition that productivity growth is the key to sustained economic expansion, measuring productivity is becoming important to economists and policy makers alike. The accurate measurement of productivity growth plays an important role in providing the information economists need to put forth better policy recommendations and for policy makers to make the right decisions. In this article we have considered some of the ways to capture this elusive concept of productivity. Although much further research remains to be done in this area, it is hoped that this article will prove helpful by clarifying some of the concepts on productivity and by documenting some of the measurement methods employed by economists.

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