

The Cobb-Douglas Production Function: An Antipodean Defence?

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ABSTRACT

In this paper a re-examination of the original time-series data sets used by Professor Douglas and associated researchers to establish the existence of an aggregate production function is undertaken. Particular attention is paid to the issue of whether the data provide deductive support for the 'Laws of Production' as claimed by Douglas (1948). Various statistical methods are used to analyse the data to see if the claims of Douglas are justified. Only the New South Wales data and to a lesser extent the New Zealand data yield results that support the assertions of Douglas - hence the Antipodean defence.

1. INTRODUCTION

It is more than 50 years since the 1947 American Economic Association presidential address, 'Are There Laws of Production?' by Professor Paul Douglas was published in the *American Economic Review*. It summarised approximately 30 econometric studies that inductively investigated marginal productivity theory. It was this body of work that yielded the Cobb-Douglas production function that is used extensively in theoretical and applied research. Douglas undertook this research because he had reached the conclusion that economic theorists had become lazy in how they illustrated marginal productivity curves. Douglas (1948) argued that economic theorists showed little interest in determining the actual position and slope of marginal productivity curves when drawing them. He observed that they were happy to draw them downward sloping left to right and that this was a symptom of, 'intellectual slovenliness, unfortunately set in' (p.5).

The central finding of his research was that the estimated exponents of the Cobb-Douglas production function were very similar to the expected factor shares. This allowed Douglas to argue that the findings provided strong support for marginal productivity theory - *The Laws of Production*. Douglas (1948) makes this point as follows:

The fact that on the basis of fairly wide studies there is an appreciable degree of uniformity, and that the sum of the exponents approximates unity, fairly clearly suggests that there are laws of production which can be approximated by inductive studies and that we are at least approaching them. (pp. 20-1)

The (apparent) robustness and strength of the results also allowed Douglas to argue that the likelihood of obtaining these findings by chance was as implausible as Bertrand Russell's claim, 'that all the books in the British Museum were written by monkeys pounding typewriters at random'. (p.40, fn.30).

Douglas' research has been challenged on several fronts; the Cambridge capital theory controversies (Harcourt, 1972), the restrictive assumptions to ensure aggregation (Fisher, 1969), and the humbug production function view espoused by (Shaikh, 1974). A defence of the findings of Douglas provided by the instrumental approach, which although acknowledging the legitimacy of the above criticisms, argues that they are of no practical relevance because of the (apparent) good statistical fit of the data achieved in the original body of research. However, it is an interesting question to ask whether or not the original work and claims made by Douglas can be maintained when the original data sets are re-examined using current statistical techniques. In this paper we focus specifically on the five time-series studies: Cobb and Douglas (1928), Douglas (1934), Handsaker and Douglas (1937) and Williams (1945). The reason for focusing on the time-series data sets is that they are the most frequently cited in relation to the results derived by Douglas.²

The re-examination of the data undertaken in this paper is as follows. Firstly, we begin by examining collinearity diagnostics and time-series properties of the data. It is shown that all data are subject to collinearity and that the time-series properties raise questions as to the statistical robustness of the estimates presented by Douglas.³ Secondly, we employ Ordinary Least Squares (OLS) and Generalised Maximum Entropy (GME) (Golan, Judge and Miller, 1996) to estimate the production functions. The reason for using GME is that it yields meaningful estimates when data are subject to collinearity because it does not use traditional inversion procedures. We find that some of the Australian and New Zealand data provides some support regarding the claims of Douglas - hence, the Antipodean defence in the title of this paper.

The findings may well be criticised in that it is easy in hindsight to see the methodological mistakes made by earlier researchers. Indeed, the work of various esteemed contemporary econometricians of Douglas, such as Moore and Schultz, is also open to criticism with the same benefit of hindsight (Stigler, 1962 and Epstein, 1987). But, like Moore and Schultz, the contribution that Douglas made should not necessarily be judged solely in terms of the statistical and theoretical robustness of his work.

However, although we can respect the efforts of Douglas, this should not prevent us from examining and evaluating the contribution made. It is also noted that there is no reason *a priori* to assume that the results of research conducted before the introduction of more sophisticated econometric techniques will be meaningless. For example, Cook (2000) demonstrates that, in relation to consumption function research undertaken in the 1940s, the original estimation specification is the most appropriate.

The research undertaken by Douglas was also not beyond challenge by his contemporaries.⁴ For example, Slichter (1928), the discussant for the original presentation by Cobb and Douglas at the 1927 AEA meeting, questioned whether over a number of years it is credible to assume that the relative factor shares for capital and labour would remain fixed. Subsequently Williams (1945), Douglas (1948) and McCombie (1998) found that the estimates for capital and labour factor shares are extremely sensitive to the inclusion or otherwise of certain data points. Slichter also noted that the measurement of capacity utilisation is likely to vary over the dura-

tion of a business cycle and that this will not be adequately captured by the measure of stock of capital.

The structure of this paper is as follows. In Section 2 we provide an overview of the Cobb-Douglas production function and various extensions to the original estimation formulation considered appropriate now. In the next section we begin by examining the various data sets to be used. We then consider collinearity diagnostics (i.e., condition indexes and variance-decomposition proportions), and the time-series properties of the data. In Section 4 we present OLS results with and without the inclusion of a time trend. We then briefly explain GME estimation in Section 5 and present GME estimates. Finally, in Section 6 conclusions are offered.

2. THE COBB-DOUGLAS PRODUCTION FUNCTION

The Cobb-Douglas production function used and estimated by Cobb and Douglas (1928), and in each of the subsequent time-series papers, takes the following form:

$$Q = A L^{b_1} K^{b_2} \quad (1)$$

where Q , L and K are output, labour and capital respectively, and A , b_1 and b_2 are constants.⁵ They assumed constant returns to scale (CRS) with $b_1 + b_2 = 1$. By imposing CRS, it was only necessary to estimate b_1 , effectively avoiding any potential problem of collinearity in estimation. The imposition of the CRS restriction without testing is econometrically unsatisfactory and the restriction was subsequently relaxed by Douglas (1934), without any real impact on the estimated values of b_1 and b_2 .

A more important problem with the original specification of the functional relationship is the omission of technical change. The need to take account of technical change in estimation was noted by Handsaker and Douglas (1937) and Williams (1945). Although Williams noted a method to proxy technical change, no effort was made in either of these studies to address this issue. Unless it is feasible to assume that over the entire data period there existed constant technology (i.e., A is constant) then there is a need to re-estimate the data with an additional explanatory variable. A standard procedure for introducing the possibility of technical change is to include a time trend (T). This captures observed changes in the technology although it is assumed exogenous to the estimated specification. Importantly the introduction of T de-trends the data without which it is likely that the regression estimates only capture historical growth rates in the data.

$$Q = A(t) L^{b_1} K^{b_2} \quad (2)$$

where $A(t) = Ae^{dt}$. A and d are constants. d is a measure of the proportionate change in output per time period when input levels are held constant (i.e. the proportionate change in Q that happens as a result of technical progress). This specification incorporates neutral technical change - there is no impact on the marginal rate of substitution between capital and labour. This formulation implies that technical change is exogenous and disembodied.

Equation (3) is usually estimated as follows:

$$\ln Q = a + dT + b_1 \ln L + b_2 \ln K + e \quad (3)$$

where e is an error term. The log-linear specification means that the estimates of b_1 and b_2 are elasticities and to assess CRS simply requires a hypothesis test on the sum of b_1 and b_2 .

3. PRELIMINARY DATA ANALYSIS

In this section we begin by describing the time-series data used by Douglas. We then examine collinearity diagnostics to assess the impact on the parameter estimates when employing OLS. Finally, we consider the time-series properties of the data and the implications for estimation.

3.1 Data

The data used in this paper are taken from Cobb and Douglas (1928) for the USA, Douglas (1934) for Massachusetts (MASS) and New South Wales (NSW), Handsaker and Douglas (1937) for Victoria (VIC), and Williams (1945) for New Zealand (NZ). The salient features of the data are described in Appendix A. An examination of Appendix A reveals significant differences in the data used by Douglas. For example, some labour series include salaried employees, supervisory officials and working proprietors (e.g., MASS, VIC, NSW) but these are excluded from the USA data. There are also differences in the various measures of capital and output used.

However, it needs to be understood that the data used in this paper employ particular versions of the data used by Douglas. He invested much time and effort in refining the data, especially the measure of capital in an effort to ‘improve’ the estimates derived. This point is neatly borne out by the results presented by Douglas (1964) who presents for the USA four sets of parameter estimates based on different measurement specifications for the variables used.

To give the reader a feel for the data used in this paper time-series plots are presented in Figure 1.

A particularly striking feature is that for the USA, MASS and to a lesser extent VIC there is a rapid growth in the capital series that is not accompanied by the same rate of increase in output. The MASS capital series also exhibits a decline towards the end of the series. It is difficult to explain this feature and it raises serious questions as to the interpretation of the capital estimates. In comparison, both NSW and NZ show all data series moving together. Also the NZ data exhibit a downturn in all series when the Great Depression occurred. The importance of these features of the data will become apparent.

3.2. Collinearity Diagnostics

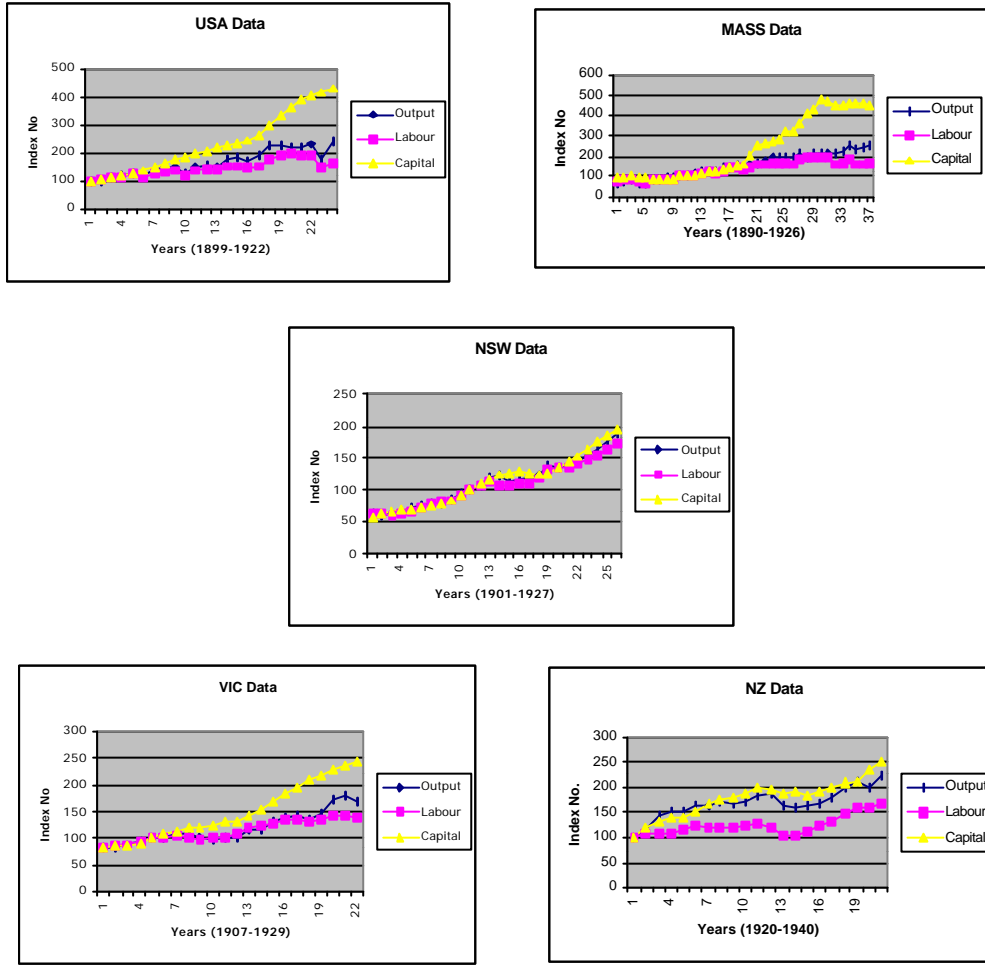
3.2.1 Theory

Belsley (1991) argues that many of the frequently employed methods for detecting collinearity are limited in their usefulness e.g., correlation coefficients. Here we employ the systematic approach developed by Belsley to examine collinearity uses the eigensystem. Firstly, eigenvalues are used to form condition indexes that provide information as to the strength of collinearity. Secondly, eigenvalues and eigenvectors are used to form variance-decomposition proportions that assist in identifying which regressors are collinear. It is the information provided simultaneously by condition indexes and variance-decomposition proportions that we use here to examine the number of collinear terms (columns) in the $n \times p$ data matrix $\mathbf{X}^o [\mathbf{X}_1, \dots, \mathbf{X}_p]$.

Condition Indexes

If there are near linear dependencies among columns of \mathbf{X} this will result in a small eigenvalue (λ) of the cross-product matrix $\mathbf{X}^T \mathbf{X}$. However, as Belsley (1991) notes, we need a meaningful way to determine what is meant by small. By ordering the eigenvalues in descending order we

Figure 1



can assess the smallness of the minimum eigenvalue relative to the maximum eigenvalue. It is this information that is used to define condition indexes.

The k th condition index of \mathbf{X} is defined as:

$$h_k = \frac{I_{\max}}{I_k} \quad (5)$$

for $k=1 \dots p$. Therefore, it follows that an eigenvalue (λ_k) that is small relative to the maximal eigenvalue (λ_{\max}) will yield a high condition index. By way of experimental observation Belsley (1991) states that weak dependencies are associated with condition indexes of between 5 and 10 and very strong linear dependencies for values greater than 30.

Variance-decomposition proportions

To uncover the form of the collinear relationships Belsley (1991) employs variance-decomposition proportions. Variance-decomposition proportions use the variance of each estimated regression coefficient decomposed into a sum of terms that are associated with an eigenvalue. The variance-covariance matrix of the OLS estimator (i.e., $\mathbf{b}=(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$) is $\mathbf{s}^2(\mathbf{X}^T\mathbf{X})^{-1}$ where \mathbf{s}^2 is the variance of the error term of the linear regression. Using appropriate matrix manipulations Belsley demonstrates that the variance-covariance matrix of \mathbf{b} , $\mathbf{V}(\mathbf{b})$ can be re-expressed as:

$$\mathbf{V}(\mathbf{b}) = \mathbf{s}^2(\mathbf{X}^T\mathbf{X})^{-1} = \mathbf{s}^2\mathbf{V}\mathbf{D}^{-2}\mathbf{V}^T \quad (6)$$

where \mathbf{D} is a diagonal matrix with non-negative diagonal elements \mathbf{m}_k ($k=1,\dots,p$), the square root of an eigenvalue λ_k . For the k th component of \mathbf{b}

$$\text{Var}(b_k) = \mathbf{s}^2 \sum_j \frac{\mathbf{n}_{kj}^2}{\mathbf{m}_j^2} \quad (7)$$

for $k,j=1,\dots,p$, and where $\mathbf{V} \equiv \mathbf{v}_{kj}$. Equation (7) decomposes the variance of each regressor into the sum of components each associated with a single eigenvalue. As the eigenvalue appears in the denominator of eq.(7) elements associated with near-dependencies will be large relative to other elements. Thus, finding a high proportion of the variance of two or more regressors concentrated in elements associated with the same small eigenvector provides evidence that the near dependence causes OLS problems.

Following Belsley (1991) the k, j th variance-decomposition proportion are calculated as follows:

$$\mathbf{f}_{kj} \equiv \frac{\mathbf{n}_{kj}^2}{\mathbf{m}_j^2} \quad (8)$$

and

$$\mathbf{f}_k \equiv \sum_{j=1}^p \mathbf{f}_{kj} \quad (9)$$

for $k=1,\dots,p$.

Then the variance-decomposition proportions are:

$$\mathbf{p}_{jk} \equiv \frac{\mathbf{f}_{kj}}{\mathbf{f}_k} \quad (10)$$

for $k,j = 1,\dots,p$. The k, j th variance-decomposition proportion is that fraction of the variance of the k th regressor associated with the j th component of the decomposition in eq.(7).

3.2.2 Results

Before estimation of the condition indexes and the variance-decomposition proportions, we need to ensure that \mathbf{X} is correctly specified following the conditions specified by Belsley (1991). Firstly, because data used in this study are logged it is necessary to e scale this data, i.e.

data are scaled so that the geometric mean of the data equals e , the base of the natural logarithm.⁶ Secondly, it is necessary to column scale \mathbf{X} . Without column scaling, condition indexes do not provide stable information about the degree of collinearity present in data. The general approach to column scaling is to standardise the data such that each column has equal length. It is normal practice to scale each column of \mathbf{X} to have unit length. The reason why it is necessary to undertake these two data manipulations is that the numerical properties of \mathbf{X} are not invariant to scale effects (e.g., units of measurement) and the variance-decomposition proportions and the condition indexes will be severely affected.⁷

We calculate the condition indexes and the variance-decomposition proportions simultaneously. For a high condition index (i.e., greater than 30)⁸ two or more high π_{jk} (as a rule of thumb high is assumed to be greater than 0.5) indicate which regressors are collinear. Note, when two condition indexes are close (e.g., 28 and 31) then the π_{jk} can be compounded and as such we need to consider the sum of π_{jk} , associated with the two condition indexes.

To tease out the impact of collinearity on the data we present two sets of results - with and without the time trend. The resulting condition indexes and variance-decomposition proportions are presented in Tables 1 and 2.

Table 1: Condition Indexes and Variance-Decomposition Proportions Matrix (No Trend)

<i>USA Data</i>		<i>Variance-Decomposition Proportions Matrix</i>		
Condition Indexes		CONSTANT	LABOUR	CAPITAL
h1 = 1.00		0.002	0.001	0.003
h2 = 5.73		0.082	0.000	0.182
h3 = 26.31		0.916	0.999	0.815
<i>MASS Data</i>		<i>Variance-Decomposition Proportions Matrix</i>		
Condition Indexes		CONSTANT	LABOUR	CAPITAL
h1 = 1.00		0.005	0.002	0.005
h2 = 4.18		0.126	0.000	0.149
h3 = 17.64		0.869	0.998	0.846
<i>NSW Data</i>		<i>Variance-Decomposition Proportions Matrix</i>		
Condition Indexes		CONSTANT	LABOUR	CAPITAL
h1 = 1.00		0.009	0.000	0.000
h2 = 6.50		0.757	0.005	0.012
h3 = 41.52		0.234	0.994	0.988
<i>VIC Data</i>		<i>Variance-Decomposition Proportions Matrix</i>		
Condition Indexes		CONSTANT	LABOUR	CAPITAL
h1 = 1.00		0.001	0.000	0.001
h2 = 7.27		0.040	0.000	0.046
h3 = 58.73		0.959	1.000	0.953
<i>NZ Data</i>		<i>Variance-Decomposition Proportions Matrix</i>		
Condition Indexes		CONSTANT	LABOUR	CAPITAL
h1 = 1.00		0.002	0.001	0.002
h2 = 11.23		0.400	0.000	0.440
h3 = 22.78		0.598	0.999	0.558

Table 2: Condition Indexes /Variance-Decomposition Proportions Matrix (With Trend)

<i>USA Data</i>		<i>Variance-Decomposition Proportions Matrix</i>			
Condition Indexes	TREND	CONSTANT	LABOUR	CAPITAL	
h1 = 1.00	0.000	0.001	0.000	0.000	
h2 = 4.93	0.003	0.043	0.003	0.001	
h3 = 28.60	0.014	0.886	0.814	0.001	
h4 = 86.74	0.983	0.070	0.183	0.998	
<i>MASS Data</i>		<i>Variance-Decomposition Proportions Matrix</i>			
Condition Indexes	TREND	CONSTANT	LABOUR	CAPITAL	
h1 = 1.00	0.001	0.003	0.001	0.001	
h2 = 4.39	0.010	0.122	0.001	0.026	
h3 = 20.03	0.346	0.444	0.295	0.886	
h4 = 21.10	0.643	0.431	0.703	0.087	
<i>NSW Data</i>		<i>Variance-Decomposition Proportions Matrix</i>			
Condition Indexes	TREND	CONSTANT	LABOUR	CAPITAL	
h1 = 1.00	0.000	0.001	0.000	0.000	
h2 = 5.44	0.012	0.056	0.000	0.000	
h3 = 46.32	0.574	0.369	0.018	0.943	
h4 = 48.67	0.414	0.575	0.982	0.056	
<i>VIC Data</i>		<i>Variance-Decomposition Proportions Matrix</i>			
Condition Indexes	TREND	CONSTANT	LABOUR	CAPITAL	
h1 = 1.00	0.000	0.000	0.000	0.000	
h2 = 5.27	0.009	0.011	0.001	0.000	
h3 = 58.36	0.413	0.960	0.497	0.068	
h4 = 77.97	0.578	0.029	0.503	0.932	
<i>NZ Data</i>		<i>Variance-Decomposition Proportions Matrix</i>			
Condition Indexes	TREND	CONSTANT	LABOUR	CAPITAL	
h1 = 1.00	0.002	0.001	0.001	0.000	
h2 = 5.28	0.142	0.018	0.004	0.000	
h3 = 25.86	0.011	0.173	0.904	0.264	
h4 = 31.25	0.845	0.809	0.092	0.736	

Table 1 shows the collinearity diagnostics of the data without a time trend. The condition index (h_3) results indicate that for NSW and VIC that collinearity is a potentially serious problem, but less so for USA and NZ. However, for MASS the impact of collinearity on the data appears minimal. For both NSW and VIC labour and capital appear to be very strongly related a point borne out by Figure 1. For NZ and USA the linear dependence between capital and labour is less pronounced but still relatively significant.

Turning to Table 2 the inclusion of a time trend produces several important changes in the collinearity diagnostics. As we can see for all data except MASS the highest condition index (h_4) is greater than 30 indicating potentially severe collinearity. All the condition indexes are higher than reported in Table 1 and certainly for the USA, the inclusion of the time trend has

exacerbated the potential problems of collinearity. The importance of these findings especially in relation to the addition of the time trend will become apparent when the estimation results are examined.⁹

3.3 Time series properties

It has become common place in time-series econometrics to assess whether or not data are stationary and, if necessary, cointegrated. As Rao (1994) notes using OLS to estimate relationships with variables that are not stationary (i.e., have a unit root) can yield misleading results. This is known as the spurious regression problem. Indeed as Douglas estimated in levels it is necessary that data are stationary of order zero. Furthermore, when including a time trend in a regression, if data are non-stationary, the time trend may make interpretation of the regression estimates difficult. For example, if output, labour and capital are difference stationary and we estimate the relationship in levels it is possible that a time trend will be statistically significant even if there is no technical change. The significance can simply be the result of a common trend. It therefore becomes less clear how to interpret the time trend. However, if output, labour and capital are trend stationary, then estimation in levels with a time trend is the appropriate way to proceed. Indeed we should follow this procedure even when we are not interested in technical change.¹⁰

A standard approach in applied research to examine stationarity properties of data is to employ the Augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1981). The ADF test employs a regression of the following form:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 T + \sum_{j=1}^p d_j \Delta y_{t-j} + e_t \quad (11)$$

where $t=1 \dots n$, y_t is the natural logarithm of the data series of interest (i.e., output, labour and capital), y_{t-1} is a lagged value, T is a time trend and Δy_{t-1} are lagged first differences of order p where p is determined so that estimation is free of first order autocorrelation. It is the coefficient on y_{t-1} that provides the test of the unit root. Under the null hypothesis of difference stationary and a unit root $\alpha_1=1$ against a trend stationary root of $\alpha_1 < 1$. The results of the ADF tests are presented in Table 3.

The results in Table 3 indicate that there is a mix of stationary (I(0)) and non-stationary (I(1)) data, suggesting that none of the data sets examined by Douglas can yield meaningful parameter estimates when estimated in levels. For example, the NSW results indicate that the data need to be first differenced for statistically meaningful estimation to proceed.

However, Perron (1989) observed that the ADF test is biased in favour of identifying data as difference stationary if there are structural discontinuities. For all the data series examined here there is a strong likelihood that such structural discontinuities are present e.g., the First World War and the Great Depression. Perron proposed three extensions to the ADF regression, which counter the bias of not being able to reject the existence of a unit root (i.e., difference stationary) and allow structural breaks in the time series to be identified. The three extensions are based on the inclusion of dummy variables in eq.(11).

Table 3: ADF Test Results

Country	Variable	Test Statistic (\mathbf{a}_1)	Lags#	Order
USA	Output	-4.04*	0	I(0)
	Labour	-2.19	0	I(1)
	Capital	-3.34*	2	I(0)
MASS	Output	-2.16	0	I(1)
	Labour	-1.17	0	I(1)
	Capital	-2.01	1	I(2)
NSW	Output	-2.01	0	I(1)
	Labour	-1.96	0	I(1)
	Capital	-2.32	1	I(1)
VIC	Output	-1.84	0	I(1)
	Labour	-1.70	0	I(1)
	Capital	-1.89	0	I(2)
NZ	Output	-3.30*	0	I(0)
	Labour	-0.87	0	I(2)
	Capital	-3.55*	0	I(0)

Notes: * Reject Null Hypothesis at 10% level of significance
Number of lags to ensure errors are uncorrelated

$$y_t = \mathbf{a}_0 + \mathbf{a}_1 y_{t-1} + \mathbf{a}_2 T + \mathbf{a}_3 Du + \mathbf{a}_4 DTB + \sum_{j=1}^p \mathbf{d}_j \Delta y_{t-j} + \mathbf{e}_t \quad (12)$$

$$y_t = \mathbf{a}_0 + \mathbf{a}_1 y_{t-1} + \mathbf{a}_2 T + \mathbf{a}_5 DT^* + \sum_{j=1}^p \mathbf{d}_j \Delta y_{t-j} + \mathbf{e}_t \quad (13)$$

$$y_t = \mathbf{a}_0 + \mathbf{a}_1 y_{t-1} + \mathbf{a}_2 T + \mathbf{a}_3 Du + \mathbf{a}_4 DTB + \mathbf{a}_6 DT + \sum_{j=1}^p \mathbf{d}_j \Delta y_{t-j} + \mathbf{e}_t \quad (14)$$

where $DTB=1$ if $t= Tb+1$, 0 otherwise, $Du = 1$ if $t> Tb$, 0 otherwise, $DT^*=t-Tb$, and $DT=t$ if $t> Tb$ and 0 otherwise, and Tb is the time break identified in the time series. Equation (12) includes a sudden jump in the data immediately after Tb , eq.(13) captures a trend change, and eq.(14) includes both. By estimating all three equations we can reassess the null hypothesis of difference stationary versus the alternative of trend stationary.

We do not undertake the Perron (1989) procedure for all the data. Instead given the ADF results we examine those data that if found to be trend stationary as opposed to difference stationary would mean that we could estimate the production function in levels. The results are presented in Table 4.

The results in Table 4 indicate that the USA labour series is difference stationary, assuming that the structural discontinuity occurred in 1914. For NSW again assuming that the structural discontinuity occurred in 1914 we find that the capital and labour series are trend sta-

tionary but that the output series is still difference stationary. Finally, for the NZ labour series if we assume that T_b occurs at the onset of the Great Depression then there is statistical evidence that this series is trend stationary. The implication of these results is that estimating the data in levels is only likely to yield statistically meaningful results for NZ.

Table 4: Perron (1989) Unit Root Test Results

Country	Variable	Model	Test Statistic	Lag#
USA	Labour	A	-2.49	2
		B	-2.05	2
		C	-2.24	2
NSW	Output	A	-2.04	2
		B	-2.04	2
		C	-2.07	2
NSW	Capital	A	-3.64*	2
		B	-5.71***	2
		C	-4.56**	2
NSW	Labour	A	-3.46*	1
		B	-3.28	1
		C	-4.54**	1
NZ	Labour	A	-3.56*	2
		B	-2.32	2
		C	-9.35	2

Notes: * Reject Null Hypothesis at 10% level of significance
 ** Reject Null Hypothesis at 5% level of significance
 *** Reject Null Hypothesis at 1% level of significance
 # Number of lags to ensure errors are uncorrelated

If the NSW capital series and the USA labour series were stationary then these data would also yield meaningful results when estimated in levels. We do need to be cautious in interpreting the time-series results because it is well known that these tests have low power and that this is particularly pronounced in small samples (Enders, 1995). Therefore, it becomes statistically difficult to distinguish between difference stationary and trend stationary processes. Furthermore, Leybourne *et al.* (1998) demonstrate the existence of a 'converse Perron phenomenon', with Dickey-Fuller tests spuriously rejecting the unit root hypothesis in the presence of breaks under the null, which occur early in the sample period. With these caveats in mind we are very circumspect about drawing any firm conclusions from our time-series tests. Therefore, we proceed with estimating the data in levels but are mindful of the time-series properties we have identified.

4. OLS ESTIMATION AND RESULTS

We begin by presenting estimates for the data using OLS. Each data set is estimated without and with a time trend to proxy technological change.

4.1 No time trend

The results of the estimation based on eq.(4) excluding the time trend are presented in Table 5.

**Table 5: Least Squares Estimation
Cobb-Douglas Functional Form With No Technical Change**

Parameters	USA	MASS	NSW	VIC	NZ
α	-0.191 (0.43)	-0.287 (0.309)	-0.034 (0.085)	-0.341 (0.834)	0.505 (0.634)
β_1	0.812* (0.144)	0.846* (0.113)	0.683* (0.093)	0.839* (0.342)	0.34* (0.134)
β_2	0.231* (0.063)	0.218* (0.054)	0.329* (0.084)	0.229 (0.166)	0.602* (0.086)
R^2	0.9542	0.9574	0.9933	0.9366	0.9027
F	240.676*	405.614*	1859.606*	156.168*	93.803*
DW	1.4987	0.4064	1.6068	0.534	1.0451
F Test ($\beta_1 + \beta_2 = 1$)	0.23	0.913	0.37	0.13	0.414

Values in brackets are standard errors

* significantly different from zero at the 5% level of significance

** significantly different from zero at the 10% level of significance

In every case (except for the VIC β_2), β_1 (Labour) and β_2 (Capital) are statistically different from zero, and the constant term, α , is not statistically different from zero. The insignificance of β_2 for VIC is potentially a result of the collinearity previously identified (see Table 1). For all data the sum of the labour and capital estimates is close to one and we were unable to reject the null hypothesis of CRS. It was this combination of results that lead Douglas to make the claims about the results lending deductive support for the existence of *Laws of Production*.

An interesting feature of the estimates of the factor shares for capital and labour presented in Table 5 is the divergence between NSW, NZ and the other data. For the USA, MASS, and VIC the contribution of the labour input is more significant than capital which is a reflection of the rapid growth in the capital series not matched by the increases in output shown in Figure 1. Handsaker and Douglas (1937) suggest that the difference in factor shares for NSW happen because the capital index was lowered by the inclusion of depreciated capital at higher price levels (p.4, fn.4). They also note that the capital index for VIC overstates the actual growth in capital during the data period (p.23). Douglas (1934) noted that the capital series for NSW exhibited a rate of growth significantly lower than that found for the USA and MASS data sets. Douglas (1964) reports results for NSW that yield estimates of the factor shares that are significantly closer to those of MASS, USA and VIC by altering the capital series in terms of how capital is depreciated (see Table II, note b, p.16). The estimates are 0.78 for capital with

a standard error of 0.12, and 0.2 for labour with a standard error of 0.08. Unfortunately it is not possible to estimate the production function with this adjusted data set for NSW as it is not available in any published work of Douglas. Finally, the Durbin-Watson test statistics in Table 5 point to either a problem of first order autocorrelation or possibly spurious regressions (Phillips, 1986) for all data except NSW and USA. These test statistics to a certain extent bear out the findings of the time-series analysis.

4.2. With a time trend

The results with a time trend included in the estimation are presented in Table 6.

**Table 6: Least Squares Estimation
Cobb-Douglas Functional Form With Technical Change**

Parameters	USA	MASS	NSW	VIC	NZ
α	2.84* (1.362)	1.983* (0.29)	-0.15 (0.5)	-2.175 (1.536)	-2.0513** (1.13)
δ	0.048* (0.021)	0.0267* (0.0028)	-0.0013 (0.0055)	-0.0248 (0.0176)	-0.013* (0.005)
β_1	0.913* (0.134)	0.631* (0.064)	0.697* (0.113)	0.799* (0.335)	0.403* (0.119)
β_2	-0.54 (0.34)	-0.111* (0.045)	0.343* (0.106)	0.697** (0.369)	0.904* (0.139)
R^2	0.9622	0.9882	0.993	0.9397	0.9262
F	195.929*	1001.848*	1188.83*	110.14*	84.649*
DW	1.6014	0.971	1.614	0.6529	1.467
F Test ($\beta_1 + \beta_2 = 1$)	4.37*	50.52*	0.11	1.97	3.63**

Values in brackets are standard errors

* significantly different from zero at the 5% level of significance

** significantly different from zero at the 10% level of significance

The inclusion of a time trend to proxy technical change significantly alters the estimates. With respect to labour, all the estimates are significant but there are some changes in magnitude compared with those in Table 5, for example, the MASS labour estimate. However, economically more important changes occur with respect to the capital estimates. For the USA and MASS, the sign on the coefficients is negative, with the estimate being statistically significant for MASS. Clearly these results make no sense in the context of estimating a production function. For NSW, VIC and NZ the capital estimates are positive and statistically significant, although weaker for VIC.

Another change in the results is that the constant term is significantly different from zero for the USA, MASS and NZ. Also the time trend is positive and statistically different from zero for the USA and MASS implying technical progress. However, given the negative estimate for capital and the time-series properties reported earlier it is difficult to place much faith in these estimates. For NSW, VIC and NZ the time trend estimates are negative, but only in the case of NZ, is the time trend significantly different from zero.

In the case of NSW and VIC there is marginal supporting empirical evidence in literature for these results. For example, Pope (1987, Table 2.2, p.36) reports estimates of total factor productivity for Australia that are low for the years 1901-1914, much higher for 1920-1925 but negative between 1925 and 1930. In the data used by Douglas, the war years 1914-1918 are also included and the net effect in combination with the significant reduction in productivity experienced at the end of the 1920s may well explain why estimates of technical change indicate regress. In the case of NZ the estimate is statistically significant. Unlike the NSW and VIC the NZ data include the 1930s. For this reason the impact of the Great Depression is the mostly likely explanation for why technical regress is observed in the data.

In terms of the whether or not the various equations are subject to CRS, the F-tests imply that NSW and VIC are subject to CRS at the five percent level of significance. However, we reject the null hypothesis for the USA and MASS at the five percent level of significance and for NZ at the ten percent level of significance. The Durbin-Watson test statistics are similar to those presented in Table 5 although in the case for NZ the test statistic now lies in the inconclusive region. This result for NZ may be attributed to the inclusion of the time trend, a necessary addition to the regression as the time-series properties indicated that the data are trend stationary.

4.3. SUMMARY

OLS estimation suggests that NSW and NZ provide marginal support for claims by Douglas about the 'Laws of Production'. Note these findings have to be treated cautiously given the mixed results about the time-series properties of the data as well as the potential influence of collinearity. In the case of the USA and MASS the inclusion of the time trend as a proxy for technical change has significantly altered the capital estimates. Not only are some of the capital estimates economically implausible, but also the severity of collinearity in the USA data is exacerbated. To overcome the problems that collinearity introduces for estimation we now employ GME. However, it is likely that it is the construction of the capital series as opposed to the problems associated with collinearity that drive the results for MASS and the USA.

5. GME ESTIMATION AND RESULTS

5.1. GME Theory

The reason for employing GME is that it can cope with problems of collinearity that are problematic for conventional econometric estimation. Golan, Judge and Miller (1996) and Fraser (2000) have examined the robustness of parameter estimates using GME when data are collinear. Golan, Judge and Miller show that GME is a feasible alternative method of estimation when faced by collinear data. A higher degree of precision is achieved in terms of the estimation when using GME compared to alternative estimation methodologies. Fraser used GME

to estimate a set of demand equations, where the problem of collinearity has long been recognised.

In addition to dealing with collinearity, GME has been shown to be very useful in dealing with many other types of difficulty encountered in applied research e.g., to estimate under-identified equations and non-standard estimation problems. For example, Lansink, Silva and Stefanou (2001) used GME to undertake frontier estimation that allows for firm specific production function estimates to be derived without the need to make strong distributional assumptions. Golan, Perloff and Shen (2001) used GME to develop a single stage estimation procedure that allows the recovery of unknown parameter estimates for a nonlinear censored demand system for many goods without the need to make distributional assumptions.

GME works by expressing the unknown parameters (**b**) and disturbances (**e**) of the standard econometric problem in terms of discrete probability distributions. Estimation is based on the work of Shannon (1948) and Jaynes (1957a,b) on maximum entropy, where entropy measures uncertainty or missing information. Shannon defined a unique function to measure entropy that measures the uncertainty (state of knowledge) we have about the occurrence of a collection of events. If **X** is a random variable with possible outcomes **X**₁....**X**_K, then the entropy of the distribution of the probabilities **p**₁....**p**_K such that they sum to unity is given by the measure H(**p**) where

$$H(\mathbf{p}) = - \sum_{k=1}^K p_k \ln p_k \quad (15)$$

and $0 \cdot \ln(0) = 0$. H(**p**) reaches a maximum when $p_1 = p_2 = \dots = p_K = 1/K$ and is equal to zero when a particular $p_k = 1$. Hence expected information from an outcome is greatest when the probabilities are uniform. Jaynes (1957a,b) used the work of Shannon to develop maximum entropy methods as a means to recover the unknown **p**.

Although used by economists, the Shannon measure of entropy requires the unknown outcomes to have the properties of a probability distribution. With the conventional econometric problem, $\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{e}$, both **b** and **e** are real-valued vectors. It was Golan and Judge (1992) who proposed a means by which to overcome this difficulty.

To explain how GME works let us begin with the standard matrix regression representation

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{e} \quad (16)$$

where **y** is a (*T*×1) vector, **X** is the (*T*×*K*) design matrix, both of which are observed, **b** is a (*K*×1) vector of unknowns, and **e** is a (*T*×1) vector of disturbances. Judge and Golan (1992) showed that, by bounding **b** and **e**, it is possible to construct a finite and discrete supports for **b** and **e**. This reformulation allows us to re-express the classical regression model as follows:

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{e} = \mathbf{X}\mathbf{Z}\mathbf{p} + \mathbf{V}\mathbf{w} \quad (17)$$

where

$$\mathbf{b} = \mathbf{Z}\mathbf{p} \quad (18)$$

and

$$\mathbf{e} = \mathbf{V}\mathbf{w} \quad (19)$$

where **Z** is a (*K*×*KM*) matrix of known support values for **b**, **p** is a *KM* vector of unknown probabilities, and *M* is the number of support points. Similarly, **V** is a (*T*×*TJ*) matrix of known sup-

port values for \mathbf{e} and \mathbf{w} is a vector of probability weights ($T \times J \times 1$), where J is the number of support values chosen for each error w_t .

Using this re-parameterisation it is possible to reformulate the classical regression model as a GME problem as follows:

$$\text{Maximise } H(\mathbf{p}, \mathbf{w}) = - \sum_{m=1}^M \sum_{k=1}^K p_{km} \ln p_{km} - \sum_{j=1}^J \sum_{t=1}^T w_{tj} \ln w_{tj} \quad (20)$$

subject to

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{e} = \mathbf{XZ}\mathbf{p} + \mathbf{V}\mathbf{w} \quad (21)$$

$$\sum_{k=1}^K p_{km} = 1 \quad m=1, \dots, M \quad (22)$$

$$\sum_{t=1}^T w_{tj} = 1 \quad j=1, \dots, J \quad (23)$$

Equation (20) is the objective function, eq.(21) is the model constraint, the relationship normally estimated, using OLS for example, and eqs (22) and (23) are the additivity constraints. The GME specification has \mathbf{b} and \mathbf{e} expressed as linear combinations of the unknown probabilities. It is assumed that all \mathbf{p} and \mathbf{w} are equally likely to occur - the objective function is equivalent to the sum of Shannon entropies on the parameter and error distributions.

The specification of bounded supports is a necessary part of the estimation process. For the error term a symmetric representation centred on zero assuming a discrete uniform distribution is normally used. With \mathbf{Z} it is necessary that the support contains the true value of \mathbf{b} , and wide bounds may be used without extreme risk consequences if *a priori* information is minimal. This should ensure that the estimates of \mathbf{V} and \mathbf{Z} contain the true \mathbf{e} and \mathbf{b} . The number of elements in M is set at five and J at three. Finally, the GME results presented in this paper report asymptotic standard errors.¹¹

5.2 RESULTS

The GME estimation employs eqs (20) to (23), where eq.(21) is the log linear specification given in eq.(5). The support values and weights used are as follows. \mathbf{Z} was set symmetrically around zero taking values, [-100, -50, 0, 50, 100] with equal weighting, with each element in \mathbf{p} equal to 0.2. \mathbf{V} was also set symmetrically around zero taking values [-5,0,5] again with equal weighting in \mathbf{w} . The bounds were selected to be sufficiently wide so as to ensure that the true values of the various parameters are included. The bounds were selected as a result of undertaking sensitivity analysis that used the entropy score recovered as part of the estimation process to judge the fit of the model as well ensuring that the estimates derived are stable.

The GME results are presented in Table 7.

Table 7: GME Estimation
Cobb-Douglas Functional Form With Technical Change

Parameters	USA	MASS	NSW	VIC	NZ
α	0.859 (1.308)	1.753* (0.277)	-0.009 (0.461)	-0.548 (1.432)	-0.704 (0.698)
δ	0.018 (0.02)	0.025* (0.003)	-0.00019 (0.005)	-0.009 (0.016)	-0.009** (0.005)
β_1	0.882* (0.132)	0.669* (0.061)	0.667* (0.104)	0.63* (0.313)	0.356* (0.11)
β_2	-0.072 (0.323)	-0.095* (0.043)	0.338* (0.097)	0.493 (0.345)	0.815* (0.127)

Values in brackets are asymptotic standard errors

* significantly different from zero at the 5% level of significance

** significantly different from zero at the 10% level of significance

Support values

α, δ, β_1 and β_2 - $Z \in [-100, 100]$, $M=5$.

ε - $V \in [-5, 5]$, $J=3$

Comparing the GME results with the OLS estimates in Table 6 we note that the signs on α , δ , β_1 and β_2 are the same. Not surprisingly, there is still a negative sign on the capital estimates for the USA and MASS. This almost certainly results from data series construction and the introduction of the time trend into the estimating equation, as opposed to the impact of collinearity.

There is a change in the magnitude of the capital estimate for the USA from -0.54 to -0.072, although for both OLS and GME it is statistically insignificant. Also the capital estimate for VIC is again statistically insignificant. The capital and labour estimates for NZ are slightly smaller and as a result almost sum to one. This result suggests that the NZ data set provide marginally more support for Douglas when using GME by reducing the impact of collinearity. The time trend estimates of technical change derived by GME are all smaller than for OLS with the largest change for the USA. There are also difference in terms of estimates is α with only MASS being significantly different from zero.

6. CONCLUSIONS

The analysis has examined the original Douglas time-series data sets from which it was claimed that there existed 'Laws of Production'. An examination of collinearity diagnostics, time-series properties and the use of OLS and GME to estimate the production functions has found that this claim is unsubstantiated for several of the data sets examined. The reason for arriving at this conclusion stems from a simple consideration of the time-series data plots. A re-examination of the data reveals that the capital series for the USA, MASS and VIC are difficult to explain

when related to changes in output. For these three regions a rapid growth in capital is not accompanied by a similar increase in output.

The only data sets that yield statistically consistent results for both OLS and GME are NSW and NZ, when GME is used. It is this that leads us to suggest the possibility of an Antipodean defence for the deductive research carried out by Douglas. However, we are not suggesting that other data sets will not yield results that support the theory of diminishing marginal productivity and CRS, that is if one is willing to ignore the many other criticisms of the neoclassical aggregate production.

The possibility of there being an Antipodean defence is, however, open to challenge in that the results presented in this paper are only for certain variants of the data used by Douglas. The changes made to the various data sets result in significant changes to the factor shares estimated. It would be interesting to see if the adjusted data for NSW discussed in Section 4 would yield deductive support for Douglas. However, we are left to conjecture that perhaps it is only a matter of chance - Bertrand Russell's monkeys at work perhaps - that the NSW data set used here yielded results that support Douglas.

Maybe the most important lesson to learn from this type of research is that, with the advantage of hindsight, even the most highly esteemed academics can overstate the robustness of their results. There is nothing new or original in this observation, but it is a result that should lead researchers to maybe display a degree of humility, no matter how important or far reaching they think their research and results.

ENDNOTES

1. Department of Economics and Finance, La Trobe University, Victoria 3086, Australia, Email: I.Fraser@latrobe.edu.au. The constructive comments of an anonymous referee, John King, David Prentice, Quirino Paris, and seminar participants at La Trobe University, University of California, Davis, the University of Arizona, and Manchester Metropolitan University on earlier versions of this paper are acknowledged. The author also acknowledges the generosity of a Hillman Endowment Award from the Department of Agricultural and Resource Economics, University of Arizona.
2. Douglas undertook over 20 cross-sectional studies that have been severely criticised. See Williams (1945) and McCombie (1998) for details.
3. The application of time series tests to historical data sets is not uncommon e.g., Cook (2000).
4. McCombie (1998) provides an excellent analysis and summary of many of the criticisms made of the research of Douglas by his contemporaries.
5. Cobb and Douglas (1928) were not the first economists to use the functional form named after them. Weber (1998) notes that Wicksell employed the Cobb-Douglas functional form in production analysis, twenty years earlier. Weber also notes that Pareto used the Cobb-Douglas functional form to represent Utility in 1892.
6. Footnote 3 on page 275 of Belsley (1991) provides a computational note on how to e-scale data.
7. Shazam (Version 8) includes the necessary code to standardise X before calculation of diagnostics.

8. A condition index between 15-30 results from a dependency that has an associated correlation between regressors of 0.9.
9. Note, if technical progress is defined as a logarithmic variable similar to labour and capital, the condition number is 30.34, which is significantly less than the estimate presented in Table 2.
10. Difference stationary implies, for example, that fluctuations in output permanently shape future output whereas trend stationary means that fluctuations are only transitory.
11. See Fraser (2000) Appendix A for details.

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