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# Treasury auctions, uniform or discriminatory?: an agent-based approach♣

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**Abstract**

This study explores the use of the agent based computational economics (ACE) technique to address the question of how a Treasury should auction its securities. In particular, this study explores whether a Treasury should use a discriminatory-price rule or a uniform-price rule. Buyers are modeled as profit seekers that are capable of submitting strategic bids via reinforcement learning. The buyers' profits are determined by auction prices and ex-post competitive resale prices. Experimental designs focus on four treatment variables: (1) the buyers' learning representation; (2) market structures; (3) volatility of security prices in the secondary market; and (4) relative capacity (RCAP). Experimental findings show that security price volatility in the secondary market has little effect on market outcomes. However, market outcomes are sensitive to market structures, RCAP, and the buyers' learning representation. The two different auction rules result in different, persistent, systematically patterned market outcomes. Moreover, these findings help to explain why discrepancies have arisen among previous Treasury auction studies.

*JEL Classification:* C63, G28

*Keywords*: Agent-based computational economics; Treasury auctions; Discriminatory and uniform-price auction rules.

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## 1. Introduction

In 1992, the U.S. Treasury changed the way of auctioning its securities from discriminatory to uniform pricing, especially for two-year and five-year notes. Debates surrounding this initiative continue. Most of the theoretical studies surveyed by McAfee and McMillan (1987) conclude that the uniform-price rule generates higher Treasury revenues. This conclusion is also supported by some empirical researchers, for example, by Umlauf (1993), Cammack (1991), Feldman and Reinhart (1995), and Tenorio (1993).

Other researchers refute this idea. Mainly, they argue that such a conclusion is based on improper theoretical models. Back and Zender (1993) found that the single-unit auction model is not sufficient for describing the bidding game observed in a Treasury auction. Later, Binmore and Swierzbinski (1999) pointed out that theoretical support for the use of a uniform-price auction is largely based on the use of the Vickery auction model. Fabra (2002) argued that representing repeated auctions with a single-shot auction model could hinder the complete picture of how the game is played, especially in representing agents' strategic behavior. In general, these critics argue the need for special attention to the modeling technique. Any model that relaxes crucial real-world market features is in great risk of generating misleading conclusions.

Malvey, Archibald, and Flyn (1995) discuss the complexity of contrasting the effects of different auction rules. Using U.S. Treasury data from 1992 – 1995, they found it difficult to differentiate the effects of changing the auction rule. Many factors such as economic outlook, expectations of interest rates, and different market participants' objectives have substantial impacts on market outcomes. These factors hinder the effort to understand the actual effects of different auction rules. Unfortunately, such factors are usually unobservable and often idiosyncratic to different agents. The ability to control for these factors is critical to obtain unbiased conclusions.

Bartollini and Cottarelli (1997) report surprising results. They conducted a survey of forty-two countries that hold organized Treasury auctions. These countries include both industrial and developing countries. Two of these countries only use a uniform-price auction rule, two use both, and the rest only use a discriminatory-price auction rule. Moreover, the authors observed that six countries that earlier converted to a uniform-price rule switched back to a discriminatory one. This observation shows evidence of a significant gap between theory and practice in Treasury auction research. While a uniform-price rule receives favorable support from the research community, the discriminatory-price rule is the dominant form of auction rule applied all around the world.

In view of these problems, Binmore and Swierzbinski (2000) strongly suggest the use of parallel Treasury auction experiments with human subjects and with computational adaptive agents. Answering this call, we develop a computational Treasury auction framework in which buyers, equipped with learning ability, participate in an experimental environment that is similar to the human-subject laboratory environment used by Smith (1967) and Goswami, Noe, and Rebello (1996).

Surprising patterns are emerges persistently from most of the experiments. There is a ***cross-point*** that determines the profit level of the Treasury under the two auction rules. When the RCAP value is relatively low (excess supply capacity), the discriminatory-price auction rule results in higher profits for the Treasury. On the other hand, at a relatively high RCAP value (excess demand capacity), the Treasury earns higher profits under the uniform-price auction rule. More over, this cross-point is sensitive to the changes in treatment value. When a treatment value is changed, the cross-point is shifted with a direction that depends on how the treatment value is changed. The cross-point existence and its sensitiveness to market treatments might be used to explain why discrepancies have arisen among previous Treasury auction studies.

This paper is organized as follows. Section 2 provides details about the computational market to be used in this study. Section 3 presents the computational experimental design. Section 4 reports the findings of the computational experiments. Section 5 discusses these findings. Concluding remarks about this study are presented in Section 6.

## 2. The computational market model

### *2.1 Overview*

As stated earlier, this computational market setup is designed to be similar to the market set-up for the human-subject experiments conducted by Smith (1967). There are $n$ buyers, $n = 1, 2 \ldots N,$ that actively participate in submitting competitive bids in a repeated sealed-bid Treasury auction for a particular type security. Each buyer seeks to maximize its net profits from the purchase of securities in the auction and the resale of these securities in a secondary market at a competitively determined price with known mean value $V$. Each buyer $n$ has a fixed demand capacity, $Q_{n,}$ representing the maximum quantity of securities that this buyer is willing to purchase in view of secondary market conditions. In each auction round, the Treasury offers $\boldsymbol{S}$ units of the security for sale. The Treasury's reservation price for each security unit is $C$, meaning it will not sell the unit at a price below this level.

Motivated by Goswami et al. (1996), buyer in each auction round is required to submit a linear demand function. To assure that one unit is the lowest quantity traded in this auction, bidders are required to submit their bids in a step function form with one as the lowest quantity bid.

In this study, each buyer n's demand function is assumed to take the following form:

$$p_n(q_i) = b_n - a_n q_i \;,\; q_i = 1,2, .. Q_n. \text{ and } a_n \geq 0 \,,\; b_n > 0 \,, \tag{1}$$

where

$p_n(q_i)$ = buyer $n$'s bid price at the quantity level $q_i$;

$a_n$ = slope of buyer $n$'s demand function;

$b_n$ = ordinate of buyer $n$'s demand function .

Figs. 1a and 1b show how profit is calculated. In general, it depends on buyers' bids, the auction rule, and various fundamental market conditions. Following is an example of how to calculate profits under different auction rules. Without loss of generality, the quantity grids are assumed to be very small. Thus, individual and aggregate demand curves are illustrated as continuous curves. Suppose there are two buyers that participate in a Treasury auction that has $S$ securities available for sale. In the depicted auction round, buyer 1 and buyer 2 bid $d_1$ and $d_2$, respectively, and $V$ denotes the ex-post unit price of securities in the secondary market.

[*Fig. 1a and Fig 1b. are about here*]

Suppose both bids are accepted. Then, as shown in Fig. 1b, the Treasury forms the aggregate demand curve $D$. Equilibrium price $P^*$ and equilibrium quantity $S$ are located at the intersection between the aggregate demand curve $D$ and the supply curve vertical at the available quantity $S$.

Under the uniform-price auction rule, the unit price is the same for all units sold, $P^*$. Suppose buyer 1 and buyer 2 end up purchasing $q_1$ and $q_2$, where $q_1 + q_2 = S$. Recalling that the buyers' profits are dependent on the secondary market unit price $V$, under this auction rule the profits for buyer 1 and buyer 2, $(\pi^u_1, \pi^u_2)$, are calculated as follows:

$$\pi^u_1 = (V - P^*)\, q_1 \;; \tag{2}$$

$$\pi^u_2 = (V - P^*)\, q_2 \,. \tag{3}$$

On the other hand, under the discriminatory-price auction rule, for each unit purchased, a buyer pays his actual bid price. This rule is also known as pay-as-you-bid. Fig. 1a shows how much each buyer will have to pay under this rule. Again suppose buyer 1 and buyer 2 end up purchasing $q_1$ and $q_2$, where $q_1 + q_2 = S$. Instead of paying $P^*$ for each unit, buyer 1 and buyer 2 are obliged to pay the amount indicated by the areas $0b_1r_1q_1$ and $0b_2r_2q_2$, respectively. Let $\Phi_1 = 0b_1r_1q_1$ and $\Phi_2 = 0b_2r_2q_2$. Then the profits of buyer 1 and buyer 2 under the discriminatory-price auction rule are as follows:

$$\pi^d_1 = Vq_1 - \Phi_1 \; ; \tag{4}$$

$$\pi^d_2 = Vq_2 - \Phi_2 \, . \tag{5}$$

Treasury profits in the uniform-price auction $\Pi^u$ and in the discriminatory-price auction $\Pi^d$ are thus calculated as follows:

$$\Pi^u = (P^* - C)S \; ; \tag{6}$$

$$\Pi^d = \Phi_1 + \Phi_2 - CS \, . \tag{7}$$

*2.3 Roth-Erev reinforcement learning algorithm*

The reinforcement learning algorithm used by buyers is a key component in this computational market study. The basic idea of reinforcement learning is as follows. When an action produces favorable results, the tendency to implement this action should be strengthened. On the other hand, when an action produces unfavorable results, the tendency to implement it should be weakened (Sutton and Barto, 1998).[1]

The main learning algorithm considered for buyers in this study is the reinforcement learning algorithm developed by Roth and Erev (1998). A distinct philosophy that makes this reinforcement learning algorithm different from others is that it is aiming to mimic how real humans learn. Thus, many important principles in the psychological literature are incorporated into the algorithm.

Roth and Erev (1998) conducted human subject experiments for a specifically designed game. The game allows multiple players to interact in a competitive and strategic environment in which any player's action could directly affect other players' payoffs. They observed that not only are the successful choices in the past more likely to be employed in the future, but also similar actions will be selected more often as well. Moreover, recent experience is likely to have a more significant impact than past experience in shaping current and future behavior. They called these two principles the *experimentation effect* and the *recency (forgetting) effect*, respectively.

[1] In the psychological learning literature, this principle is known as the *law of effect*

Combining these observations and psychological principles in learning, the Roth-Erev algorithm has three main parameters: a *scaling parameter* $s(1)$; a *recency parameter* $r$; and an *experimentation parameter* $e$. To understand how the players utilize this algorithm, the algorithm will now be illustrated for a group of buyers participating in a Treasury auction.

In each auction round, each buyer has $K$ possible actions. Each action $k$, $k \subset K$, represents a unique linear demand function. For simplicity, assume that $K$ is the same for all buyers. In the first auction round, each buyer assigns an equal "propensity value" to each of its actions $k$, given by $q_{ik}(1) = s(1)X/K$, where $X$ is a rough measure of the average profit that a buyer can achieve in any given auction round. Concurrently, in the first auction round, each buyer $i$ assigns an equal choice probability to each action $k$, given by $p_{ik}(1) = 1/K$. Note that $\Sigma p_{ik}(1) = 1$ for each buyer $i$.

Suppose that, in the first auction round, buyer $i$ chooses and submits action $k'$ to the auction. Once the Treasury accepts bids from all buyers, it clears the market. At the end of the auction round, it communicates back the auction results to the buyers in the form of security prices and quantities. At this point the Treasury can calculate its profits, given any particular auction pricing rule. However, the buyers will know their profits only after the security price in the secondary market is established. Now assume that the security price in the secondary market is known, and let $\pi_{ik}(T)$ be the profits of buyer $i$ at the end of the $T^{th}$ round when it submits action $k'$. Buyer $i$'s propensity for action $k$ at the beginning of the $(T+1)^{th}$ round, $q_{ik}(T+1)$, is then obtained by the following equation:

$$q_{ik}(T+1) = (1-r)\, q_{ik}(T) + E(i,k,k',T,K,e) \qquad (8)$$

where $r$, $e$, and $E(.)$ respectively denote the recency parameter, the value of the experimentation parameter, and the update function that contains information about the experience in playing in the market from past trading activity. It is worth mentioning here that the recency parameter $r$ lessens the weight of past experiences, in accordance with the forgetting effect. The complete form of the update function $E(\bullet)$ is as follow:

$$E(i,k,k',T,e) = \begin{cases} \pi(i,k',T)(1-e), & k = k' \\ \pi(i,k',T)\dfrac{e}{K-1}, & \mathrm{k} \neq \mathrm{k'} \end{cases} \qquad (9)$$

The selection of $k'$ in round $T+1$ is strengthened or weakened, based on the profit $\pi(i,k',T)$ that buyer $i$ acquired in the subsequent selection, with a weight of $(1-e)$. Also, $E(\bullet)$

updates the propensity of other feasible actions by using a weight of *e/(K-1)*. This mechanism of updating both selected and unselected actions adheres to the experimentation principle of learning. These new set of propensity values will determine the new probability distribution of action choice in the successive auction round. The probability that buyer *i* chooses action *k* in auction round $T+1$, $p_{ik}(T+1)$, can now can be written as:

$$p_{ik}(T+1) = \frac{q_{ik}(T+1)}{\sum_{m=1}^{K} q_{im}(T+1)} \tag{10}$$

When the number of auction rounds is large, each buyer will have many chances to test different action choices. Thus, each buyer should eventually be able to deduce which action to choose in this market. The initially assigned uniformly distributed probability of choosing any action should tend to evolve to a different form of distribution that peaks at the action that gives the highest profit to the buyer.

Roth and Erev tried to calibrate the parameters *r* and *e* to reflect real human learning based on human-subject experiments. They noticed that, when *r* and *e* were set to *0.1* and *0.2,* respectively, the algorithm generated the closest results to real humans. They called these parameter settings the *best-fit* learning parameter values.

Koesrindartoto (2001) conducted a sensitivity study to test the effects of these learning parameter values on the overall market outcomes. By changing the value of the experimentation parameter *e,* it was found that the overall level of market efficiency was significantly changed[2].

A special case of learning behavior is observed when $e = (K-1)/K$. By substituting this value into equation (8), the updating function becomes $E(i, k', k, T, K, e)=\pi( i, k',T)/K$ for all *k* values. Recall that the initial propensity values were assigned to have the same value for all feasible actions *k*. Thus, by using $e=(K-1)/K$, the propensity values for any auction round as shown in equation (7) will be the same for all feasible actions *k*. Thus, in any auction round, the probability to choose any action as shown in equation (9) will never change. Buyers will never update their probabilities, hence they loose their capability of learning. This behavior is comparable to the zero-intelligent traders described by Gode and Sunder (1996). These traders choose actions randomly from a given set of feasible actions.

[2] When *e* was systematically increased from 0 to 1, the market efficiency decreased monotonically from about 94 percent to 20 percent. This same effect was observed under three different market structures.

In the present study, parameter values for the Roth-Erev reinforcement learning algorithm will be specified to represent three different learning behaviors: no learning; best-fit learning; and no-bias learning. These parameter value specifications are explained in the next section.

## 3. The computational experimental design

### *3.1 General setup*

In every auction round, the Treasury sells *100* securities. Each security has a *$10* reservation value. Each buyer possesses *20* possible bid actions (linear demand functions), generated from a combination of five different intercept values and four different slope values. At any given amount of quantity demanded, the buyers' linear demand function parameters (slope and intercept) are assigned such that price negative price bidding is never occurs.

For each experiment, two sets of tests are conducted on different auction rules. Experiments conducted under the discriminatory-price rule will have index **A**, and experiments conducted under the uniform-price rule will have index **B**.

### *3.2 Experiment treatments*

One of the advantages of the use of the ACE method is that it enables us to isolate a particular market condition by using it as an experimental treatment factor. The importance of any circumstance is determined by how significantly market outcomes are changed when the corresponding treatment factor is changed. This method is similar to the comparative statics approach that is commonly used in analytical modeling. We employ three market conditions as the treatments for the experiment, as follows:

(1) Relative capacity

*Relative capacity (RCAP)* is defined as a ratio of total demand capacity to total supply capacity. The higher the RCAP level, imply the more the excess demand capacity. Unlike in Smith (1967) and which employed RCAP as a given market parameter, this study will systematically vary this value in all market experiments. RCAP is set to have eight different values, which are *0.6, 0.72, 0.84, 0.96, 1.08, 1.20, 1.32* and *1.44.* As the supply quantity is held constant at *100* units, the total demands for the respective RCAP ratio values are *60, 72, 84, 96, 108, 120, 132*, and *144*. Individual buyer demand at a particular coverage ratio depends on the particular market structure.

(2) Learning representation

Koesrindartoto (2001) showed that, by modifying a specific learning parameter for the Roth-Erev (1998) learning algorithm, traders could have significantly different learning representations. The objective of this experiment is to study the importance of buyer learning capability in this Treasury auction market setup. To address this objective, we employ three different learning representations: *no learning; best fit learning*; and *no bias learning*[3]. Section 2.3 explains carefully the detailed implementation of these behaviors in terms of the Roth-Erev (1998) algorithm.

(3) Market structures

Different market structures will represent different combination between the number and the type of buyers. While market structure 1 represents a market that has *many small buyers*, market structure 3 characterizes a market that consists of *small number* of *big buyers*. Market structure 2 is designated to represent a market that is composed of a *mixed type of buyers*.

(4) Volatility of the security price in the secondary market

This treatment is important because each buyer's goal is to maximize profits by purchasing securities in the auction market and then reselling them in the secondary market. As what we observe in practice, this secondary market price is uncertain for buyers when they bid in the auction process. To capture this market situation, we introduce three levels of volatility for the security price in the secondary market: high volatility; medium volatility; and no volatility.

[*Table 1 is about here*]

### *3.3 Experiment 1 : learning representation effects*

The experimentation parameter *e* in the Roth–Erev learning algorithm will be used as the main treatment factor. Three different values, i.e. *19/20, 0.2*, and *0.0*, represent when buyers are behaving under *no learning*, *best-fit learning,* and *no-bias learning,* respectively. Market configurations such as the number of buyers and their maximum demand capacity follow **market structure 1** as shown in Table 1. It is also assumed that the security price in the secondary market is **constant (no volatility).**

[3] In their study, Gode and Sunder (1993) stressed that the importance of learning is determined by how significant market outcomes changed as participants' learning behavior altered.

*3.4 Experiment 2 : the effects of different market structures*

Market structure will be used as the main treatment factor in this experiment. Table 1 summarizes the experimental design for different market structures. It is assumed that buyers employ **no-bias learning** and that the security price in the secondary market is **constant (no volatility).**

*3.5 Experiment 3 : volatility of the security price in the secondary market*

Three volatility levels for the security price in the secondary market are used as treatments: no volatility; medium volatility; and high volatility. In the no-volatility situation, the security price in the secondary market is fixed at *$20*. For medium and high volatility conditions, the security prices are drawn randomly from a uniform distribution over the fixed intervals *[$18,$22]* and *[$15, $25],* respectively. This experiment is conducted under **market structure 1** and with buyers assumed to employ **no-bias learning**.

## 4. Experimental findings

Table 2 reports average profit for both the Treasury and the buyers in all experiments. Each cell in this table relates to a unique experiment as explained in section *3.2 – 3.4.* Each table cell reports the average of data for *100* auction runs. These *100* auction runs were conducted by using *100* different seeds for the pseudo-random number generator. Profits level reported in each run were acquired from the final auction round, which is the *200$^{th}$* auction round. Table 3 shows the standard deviation for the corresponding data cell shown in Table 2. The average and standard deviation of the profits were calculated across all *100* runs for each table cell.

Results from the experiments display a staggering degree of regularity, especially the convergence of the profits measure.  From 224 experiments, only 8 experiments show a standard deviation as high or higher than the average value. In Table 2, this situation is marked by an asterisk (*) symbol.

[*Table 2 is about here*]

The standard deviation is caused by two main sources: buyer learning behavior; and the volatility of the security price in the secondary market. However, these sources are affecting the market in different way. As shown in the experiment 1 Table 3, buyer learning behavior affects the standard deviation of both the Treasury and buyers' profit levels.  On the other hand, as shown in experiment 3 Table 3, volatility in the secondary markets only affects the standard deviation of the buyers' profit levels.

[*Table 3 is about here*]

To gain intuitive understanding of the results shown in Table 1, from now on reports and analysis will be based on the graphical presentation as shown in Figs. 2a – 4d. Also, Figs. 5a – 7f show stylized presentations of Table 2. Unlike in Figs. 2a – 4d that represent different experiments under a particular auction rule, Figs. 5a – 7d show the comparison of different auction rules under a particular experiment.

Figs. 2a - 2d show the effects of different learning representations under a particular auction rule. While these figures show persistent trends, profit levels are significantly different when the learning representation changes. Thus, it is apparent that the learning representation has important effects in this computational market.

Under the discriminatory-price auction rule, the profits obtained by buyers when they employ no-bias learning strictly dominate the profits they obtain when they employ best-fit learning.

[*Figs. 2a through 2d are about here*]

This confirms the conjecture that no-bias learning is superior to best-fit learning. Interestingly, when the uniform-price rule is used, the opposite results are observed, especially as RCAP increases from *1.0* to *1.44*. In this situation, the profits obtained by buyers when they employ no-bias learning weakly dominate the profits they obtain when they employ best-fit learning.

From experiments 1A and 1B in Table 3, the standard deviation of profits is lowest when buyers employ no-bias learning. In addition, at RCAP values higher than 1.0, when buyers employ *best-fit* learning, the standard deviation of buyer profits under the uniform-price rule is significantly higher than under the discriminatory-price rule

Results for experiment 2A are shown in Figs. 3a and 3b. In general, under the discriminatory-price rule, the effect of changing the market structure is inconclusive. However, under the uniform-price rule, the profit levels obtained under market structure 3 are significantly different from the two other market structures. As shown in Fig. 3c, the Treasury profit level is weakly dominated when market structure 3 is used. At the same time, as shown in Fig. 3d, the average buyer profit level weakly dominates the average buyer profit level under the two other market structures.

[*Figs. 3a through  3d are about here*]

Table 3 shows that the standard deviations in experiments 2A and 2B are the smallest. The reason for this is that the two main sources of variability are at a minimum because buyers are set to employ no-bias learning and the security price in the secondary market is held constant.

As heterogeneity of the buyers is introduced in market structure 2, profits trends for both the Treasury and the buyers become less persistent. Figs. 3a – 3d show some reverse trends are observed when market structure 2 is employed. More precisely, those occur as RCAP changes from 1.32 to 1.44.

Figs. 4a - 4d show the effects of volatility of the security price in the secondary market. In all of these graphs, trends are persistent and the differences in profit levels are not significant for all volatility conditions. Table 3 for experiment 3A shows that the discrepancy in Treasury profit levels across treatments is very small.

[*Figs. 4a through 4d are about here*]

However, significant standard deviations are observed when volatility is introduced into the secondary market security price. Changing the volatility from none to a medium level causes the standard deviation of the buyers' profit levels to increase from about 0 to 55. In addition, increasing the volatility level from medium to high causes the standard deviation of buyers' profit levels to increase on average by twice.

## 5. Discussion

### *5.1 The effects of learning representation*

The data strongly suggest that learning representation is highly relevant to the market outcomes. This conclusion is similar to earlier research conducted by Koesrindartoto (2001). Using a double auction market setup, he showed that the learning representation plays a significant role in market efficiency outcomes.  Based on Koesrindartoto (2001) results, it can be conjectured that that when the learning representation changes from no learning to best-fit, and from best-fit to no-bias, buyers are improving their learning behavior. Fig. 2b confirms this supposition. Under the discriminatory-price rule, when buyers employ no-bias learning, the buyers' profit level is strictly higher as compared to when buyers use best-fit learning.

As mentioned earlier, when buyers employ best-fit learning, the standard deviation of the buyers' profit levels is significantly higher under the uniform-price rule. Therefore, standard deviation information is needed to interpret experiment 1B. Statistical tests show that, under the uniform-price rule, we cannot reject the hypothesis that profits levels are the same whether buyers employ best-fit or no-bias learning.

One possible interpretation is that when buyers employ best-fit learning, the uniform-price rule provides a more challenging learning environment. Even after a significant amount of learning time (auction rounds), buyers do not choose a particular action persistently. Any attempts by a buyer to converge to a particular action will be taken advantage of by other buyers. Thus, convergence is difficult to achieve. As a result, higher volatility in outcomes is observed.

This observation is comparable to the earlier finding in a human-subject experiment conducted by Smith (1967). Smith consistently observed a higher variance in bids when a uniform-price rule was used. Smith also observed that this discrepancy in bid variance tended to widen as the proportion of rejected bids increased. Table 3 experiment 1B shows that, when buyers employ best-fit learning, the bid variance increases as RCAP increases.

However, the two other learning representations, i.e., no learning and no-bias learning, do not follow this pattern. Under the no-learning representation, the standard deviation of profit levels is persistently high under both uniform-price and discriminatory-price auction rules. In contrast, under no-bias learning, this profit standard deviation is persistently low under both auction rules. This shows that the aim of Roth-Erev (1998) to represent how humans learn by employing a *best-fit* learning representation is quite successful.

*5.2 The effects of market structures*

Figs. 3a and 3b show that under discriminatory auction, changing market structure i.e. changing bidders' market power is not considerably affecting Treasury and bidders' profit. However, Figs. 3c and 3d show different results when uniform-price rule is used. Profits obtained in structure 3 by Treasury and bidders are significantly different compare to two other market structures. From the treasury point of view, this suggests that it is easier to detect the change of market structures or market power when uniform-price rule is used.

Compares with the two other market structures, market structure 2 offers a unique situation as heterogeneous buyers are introduced. Buyers are differentiated by their capacity capacities. As shown in Table 1, at any RCAP value, higher capacity buyers submit maximum possible demand about twice as much as lower capacity buyers.

Figs. 3a-3d show trend inconsistencies under both market structure 2 and market structure 3. This implies that, when market structure is changed from structure 1, i.e. when bidders obtain greater market power, market outcomes become harder to predict. Introducing heterogeneous buyers in the market makes prediction even harder. Therefore, these findings suggest that market structure has an important effect on market outcomes.

*5.3 The effects of different volatility for the security price in the secondary market*

Figs. 4a - 4d**,** Table 2 and Table 3 show that, on average, the volatility of the security price in the secondary market does not significantly affect the profit levels for the buyers or for the Treasury. However, it has a considerable effect on the standard deviation of the buyers' profit levels. This standard deviation rises as the volatility is increased.

The lower standard deviation in Treasury profits is caused by convergence in the buyers' action choices. The buyers' action choices are insensitive to the volatility in the secondary market. Meanwhile, a significant standard deviation is observed in buyers' profit levels, mainly caused by volatility of the security price in the secondary market. This suggests that buyers absorb all the effects from changing the volatility.

The effects of changing the auction rule under a particular volatility level for the security price in the secondary market are shown in Figs. 7a – 7f**.** These figures show a strong similarity. Since the effects of the security price volatility are insignificant, other market conditions such as the use of no-bias learning and the use of market structure 1 must be shaping the market outcomes. Thus, it is not surprising to see that Figs. 7c - 7f are similar to Figs. 7a and 7b **.**

*5.4 Discriminatory-price vs. uniform-price auction rules*

This subsection focuses on the extent to which discriminatory-price versus uniform-price auction rules affect the profit levels earned by the Treasury and the buyers. To achieve this objective, the observed data is represented in a stylized way. Figs. 5a-7f show the stylized representation of Table 2, which compares market outcomes under the two different auction pricing rules.

An astonishing degree of regularity is observed in almost all the experiments. Except under the treatments of *no learning* and *market structure 3*, there exists a unique ***cross-point*** that determines the relative profit level of the Treasury under the two auction rules. When the RCAP value is relatively low, the discriminatory-price auction rule results in higher profits for the Treasury. On the other hand, at a relatively high RCAP value, the Treasury earns higher profits under the uniform-price auction rule. This implies that, at high RCAP ratio, the Treasury will benefit if a uniform-price rule is used. While this finding seems to agree with earlier research that claims the uniform-price rule will give the Treasury higher profit earnings, it is seen here that the claim is only true under certain market conditions.

[*Figs. 5a through 4f are about here*]

To be more precise, the position of the cross-point is influenced by the learning representation specified for the buyers. Figs. 5c and 5e show that, as the learning representation for buyers is changed from no learning to best-fit and from best fit to no-bias learning, i.e., changed to a better form of learning, the cross-point shifts to a lower RCAP value.[4] From the Treasury's point of view, to ensure higher profits when changing the auction rule from discriminatory-price to uniform-price, a higher RCAP ratio is needed when buyers are employing less superior learning.

Nonetheless, more careful evaluation has to be conducted when a best-fit learning representation is used. Not only the profit levels are slightly different, as shown in Figs. 5c and 5d, but also the profit standard deviation is significantly high, as presented in Table 2. Therefore, statistical testing is needed to draw any conclusions. As shown in Table 3, we cannot reject the hypothesis that the profits earned by the Treasury is the same when discriminatory or uniform-price rule is used. However, based on the observed trends, it is expected that hypothesis could be rejected when coverage ratio has significantly higher value.

Figs. 6a and 6c shows that, when the market structure changes from market structure 1 to market structure 2, the cross-point location for Treasury profits is unaffected. However, when market structure 3 employed, the uniqueness of the cross-point no longer exists. When the buyers obtain significant market power, it becomes difficult to predict which auction rule will give the Treasury a higher profit. In other words, the Treasury loses its ability to gain profits by changing the auction rules once the market becomes tight. Let us now refer to Figs. 6a, 6c, and 6e**.** Treasury profits gains from switching to a uniform-price rule decrease as the market structure changes from structure 1 to structure 3. Similarly, from the buyers' perspective, the decrease in profit caused by the auction rule change is less when the market is tight.

[*Figs. 6a through 6f are about here*]

[4] Under the no-learning representation, the cross-point might be observed at a very high value of RCAP.

As shown in Figs. 7a–7f, relative profit levels are virtually unchanged, and the cross-point is intact, under different volatility levels. The use of market structure 1 and no-bias learning dominates the outcomes of this experiment. Therefore, the conclusion from Figs. 7a-7f is similar to Figs. 5e and 5f: At a higher RCAP value, the Treasury will benefit when a uniform-price rule is used.

[*Figs. 7a through 7f are about here*]

From buyers' point of view, similar regularities are also persistently present. In all cases, the uniform-price auction results in a higher profit peak. This implies that if buyers are allowed to coordinate among themselves, a uniform-price rule will benefit them the most as their maximum profit is higher. Moreover, the slope on the buyers' profit is steeper in uniform-price rule, which suggests that the marginal profit increase from getting to the coordination point is higher compared to when the discriminatory-price rule is used. This suggests that, even though any coordination or collusive behaviors are prohibited in the market, the incentive for such conduct is more rewarding under the uniform-price rule.

Arupatan (2000) reaches a similar conclusion: namely, collusive behavior is embedded in the system. As regarding profits, buyers will be better off under either form of auction if they engage in some collusive behavior.

The theoretical study conducted by Fabra (2002) reaches a somewhat different conclusion. Within an infinitely repeated auction game, two symmetric firms compete to fulfill a stochastic demand. Under the uniform-price auction, the profitability of defections decreases at the same time toriahe future value of cooperation increases. On the other hand, under the discriminatory-price auction, collusive behavior among firms does not result in higher profits. The study concludes that a uniform-price auction (weakly) facilitates tacit collusion.

An earlier human-subject experiment conducted by Goswami et al. (1996) also obtained a similar finding. They use a similar market setup except that no secondary market is available. Experiments compared the market outcomes from both auction rules and under two communication conditions. The first communication condition is that bidders are prohibited from communication, and the second communication condition is when the bidders are allowed to communicate. They conclude that, when communication is allowed, the Treasury might obtain lower revenues when they employ a uniform-price auction rule.

This result can be explained by using our findings shown in Figs. 5d–7f, especially the ones that compare the buyers' profit under two different auction rules. Let us suppose the initial market clearing outcomes when communication is prohibited are located at the right side of the cross-point.

In this case the Treasury profits the most when a uniform-price rule is used, as they extract more profit from the buyers. However, when communication among buyers is allowed, the buyers will coordinate their bids to obtain maximum profit. As shown in Figs. 5d – 7f, in this case the buyers obtain higher profits when a uniform-price rule is used. Simultaneously, under coordinated bids, the Treasury earns lower profits compared to when the discriminatory-price rule is used.

These findings suggest that relative Treasury profits level under discriminatory-price and uniform-price auction rules are sensitive to the buyers' learning behavior, market structure, and the buyers' collusive behavior. This might be a possible explanation to the question raised in Bartollini and Cottarelli (1997) regarding the limited number of countries worldwide that using a uniform-price rule, despite the fact it is widely praised in theoretical research. Overly simplified and static modeling of the actual market situation in theoretical research could be one of the explanations. In the real world, from one Treasury auction to the next the market structure always differs. Moreover, participant expectations, objectives, and degree of collusive behavior are also always changing.

Discrepancy of results when comparing different auction rules are observed for many different countries. For example, in the study of Zambia by Tenorio (1997), and in the study of Mexico by Umlauf (1997), evidence is provided supporting the Treasury's use of a uniform-price rule: namely, small profit increases are predicted. However, the conclusions of researchers looking at different time series data for the U.S. are inconsistent. For example, Malvey et al. (1998), using 1991-1998 U.S. data, conclude that Treasury profits will increase moderately when a uniform-price rule is used, but Simon (1994) using 1971 - 1973 U.S. data concludes the opposite. Nyborg and Sunderasan (1997) using 1992 –1993 U.S. data for 2-year and 5-year notes conclude that the extent and direction of change in Treasury profits from such a switch in pricing rule depends on the type of securities being auctioned.

### *5.5 Aggregate demand and buyer strategies*

Understanding buyers' behavior is critical in explaining the observed results. This section conducts a detailed analysis of the experiments under the following three maintained assumptions: Market structure 1; no-bias learning; and no volatility in security prices in the secondary market. Under this treatment, outcome differences that would be generated by asymmetric types of buyers, learning inefficiency, or variance in security prices in the secondary market are eliminated. Therefore, any remaining outcome differences will solely reflect differences in the buyers' adaptive behaviors under the two different auction rules.

[*Fig. 8 is about here*]

Fig. 8 shows experimentally generated aggregate demand under different RCAP values when both uniform-price and discriminatory-price auction rules are used. Let us define infra-marginal and extra-marginal bids. *Infra-marginal* bids are the winner bids. Along the aggregate demand curve, these are the bids located on the left side of the clearing point. On the other hand, *extra-marginal bids* are non-winning bids; they are located to the right of clearing point. For each RCAP value, focusing on the infra-marginal part of the aggregate demand curve, the winning bid prices under the uniform-price rule are at least as high as the corresponding winning price bids under the discriminatory-price rule. However, two different interesting patterns emerge for different RCAP values. For the tested range of RCAP values varying from 0.6 to 0.96, i.e., when excess supply capacity occurs, increases in RCAP do not affect aggregate demand considerably. Under the uniform-price rule, the clearing point remains at the Treasury reservation value; and under the discriminatory-price rule, the level and the shape only slightly change. However, for the tested range of RCAP values varying from 1.08 to 1.44, i.e., when excess demand capacity occurs, increases in RCAP do affect aggregate demand considerably. Under both auction rules, the aggregate demand curve shifts upward and becomes flatter.

These experimental findings can be explained by considering the individual choices of the buyers and the resulting market outcomes as depicted in Tables 4 through 7.

[*Table 4 is about here*]

Recall that, in each auction round, each buyer must submit a bid in the form of a linear demand function. The range of possible linear demand function bids for each buyer in each auction round is listed in Table 4. These demand functions differ by intercept and slope, with lower numbered demand functions corresponding to higher intercept values. Tables 5 through 7 display actual experimental findings under different RCAP values and under the two different auction rules. Table 5 shows the actual buyer choices of their demand functions, Table 6 displays the actual average unit price paid by buyers, and Table 7 gives the actual quantities obtained by buyers.

[*Table 5 is about here*]

Consider, next, the findings reported in Table 5. These findings show that, for each tested RCAP value, buyers on average always choose a lower numbered (higher intercept) demand function under the uniform-price rule than under the discriminatory-price rule. However, dividing the results between RCAP values less than 1.0 (excess supply capacity) and RCAP values greater than 1.0 (excess demand capacity), an interesting additional regularity is seen. Under either auction rule, the standard deviations of the buyers' choices are higher for RCAP values less than 1.0 than for RCAP values greater than 1.0. This effect is particularly strong for the uniform-price auction rule.

[*Table 6 is about here*]

Now consider Table 6, which displays actual unit prices paid by buyers together with standard deviations. The extreme smallness of the standard deviations indicates a high degree of uniformity across price outcomes for buyers even under the discriminatory-price auction rule. However, an interesting difference arises for RCAP values less than 1.0 versus RCAP values greater than 1.0.

With RCAP values less than 1.0, the price obtained by each buyer (averaged over 100 runs) is at least as high under the discriminatory-price auction rule as under the uniform-price auction rule. Under the discriminatory-price auction rule, while about half the buyers manage to achieve a price equal to the Treasury's reservation price, the remaining buyers pay higher prices. Under the uniform-price auction rule, the market-clearing price coincides with the Treasury's reservation price. In contrast, with RCAP values greater than 1.08, the price obtained by each buyer (averaged over 100 runs) is at least as high under the uniform-price auction rule as under the discriminatory-price auction rule. RCAP=1.08 is a "switch point" in the sense that the latter result holds except for buyer 3.

In short, evaluated in terms of Treasury revenues, Table 6 shows that the "best" auction rule switches from discriminatory to uniform as RCAP varies from below 1.0 to greater than 1.0.

[*Table 7 is about here*]

Finally, consider the actual quantities obtained by buyers, as displayed in Table 7. Under RCAP values less than 1.0, each buyer purchases a quantity equal to its maximum capacity regardless of the type of auction rule. Under RCAP values greater than 1.0, buyers are constrained to quantities less than their maximum capacities. However, these constrained quantities are the same on average across the different auction rules.

## 6. Concluding remarks

This paper attempts to complement ongoing research by using an ACE approach to explore how a Treasury should auction its securities. The particular question addressed is whether a Treasury should use a discriminatory-price or uniform-price auction rule. The basic model used in this study was chosen to be comparable to the earlier research, especially the research conducted using human-subject experiments.

This study replicates results found in many earlier studies but also provides possible explanations for inconsistencies arising across different studies using different models and data. It turns out that the important key is to incorporate sufficient detail about the actual market situation. While the market rules for participating in Treasury auction are relatively simple, the detailed dynamics regarding how buyers participate in the auction are quite complex. Buyers are facing an always-changing market situation. Buyers' objectives, buyers' expectations regarding the resale price of securities on secondary markets, the competitiveness of the markets, the opportunity for collusive behaviors, and other factors all vary from one auction session to another. These dynamic situations are very difficult to model using the analytical method. On the other hand, simplifying the dynamics by ignoring them might end up with misleading conclusion.

This study shows that Treasury profit is sensitive to the buyers' learning behavior, market structure, and the protocols governing the determination of prices. This suggests that it is crucial to include the details of the market situation to analyze a Treasury auction. Klemperer (2000) reaches a similar conclusion in his study of auction design in the European telecommunications industry. He concludes that, even if one particular auction type fits perfectly in one country, results will not necessarily be the same for other countries. Local circumstances play a critical role in determining auction outcomes.

The use of the ACE method enables us to combine the strength of the theoretical approach in modeling profit-seeking strategically-interacting agents with the realistic setup of experimental design widely used in human-subject experiments. The ACE method not only permits solid microfoundations but also helps to capture the complex dynamics of real-world auctions. The ACE method provides a potentially useful tool for economists who wish to study market design. As Roth (2002) states, the challenge for economists interested in market design is not only to study general market features but also to take responsibility for detailed market implementation.

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**A. Market structure 1**

Number of buyers = 12
Number buyer's type = 1 (Type 1 only)

| RCAP | Total demand capacity | Individual demand capacity |
|---|---|---|
| 0.60 | 60 | 5 |
| 0.72 | 72 | 6 |
| 0.84 | 84 | 7 |
| 0.96 | 96 | 8 |
| 1.08 | 108 | 9 |
| 1.20 | 120 | 10 |
| 1.32 | 132 | 11 |
| 1.44 | 144 | 12 |

**B. Market structure 2 :**

Number of buyers = 8
Number buyers' type = 2 (Type 1 and type 2)
Number of type 1 = 4
Number of type 2 = 4

| RCAP | Total demand capacity | Individual demand capacity | |
|---|---|---|---|
| | | type 1 | type 2 |
| 0.60 | 60 | 10 | 5 |
| 0.72 | 72 | 12 | 6 |
| 0.84 | 84 | 14 | 7 |
| 0.96 | 96 | 16 | 8 |
| 1.08 | 108 | 18 | 9 |
| 1.20 | 120 | 20 | 10 |
| 1.32 | 132 | 22 | 11 |
| 1.44 | 144 | 24 | 12 |

**C. Market structure 3**

Number of buyers = 4
Number buyer's type = 1 (Type 1 only)

| RCAP | Total demand capacity | Individual demand capacity |
|---|---|---|
| 0.60 | 60 | 15 |
| 0.72 | 72 | 18 |
| 0.84 | 84 | 21 |
| 0.96 | 96 | 24 |
| 1.08 | 108 | 27 |
| 1.20 | 120 | 30 |
| 1.32 | 132 | 33 |
| 1.44 | 144 | 36 |

Table 1. Three market structures.

| | Experiment 1 A (Discriminatory) | | | | | | Experiment 1 B (Uniform-Price) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *No Learning* | | *Best Fit* | | *No Bias* | | *No Learning* | | *Best Fit* | | *No Bias* | |
| **RCAP** | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury |
| 0.60 | 344 | 256 | 468 | 132 | 528 | 72 | 599 | 2 | 598 | 2 | 600 | 0 |
| 0.72 | 422 | 298 | 572 | 148 | 638 | 82 | 836 | 4 | 715 | 5 | 720 | 0 |
| 0.84 | 493 | 347 | 664 | 176 | 742 | 98 | 960 | 0 | 839 | 1 | 840 | 0 |
| 0.96 | 555 | 405 | 752 | 208 | 863 | 97 | 952 | 47 | 955 | 5 | 960 | 0 |
| 1.08 | 555 | 445 | 676 | 324 | 736 | 264 | 828 | 172 | 875 | 125 | 668 | 332 |
| 1.20 | 483 | 517 | 555 | 445 | 655 | 345 | 785 | 215 | 654 | 346 | 450 | 550 |
| 1.32 | 458 | 542 | 406 | 594 | 517 | 483 | 785 | 215 | 421 | 579 | 250 | 750 |
| 1.44 | 436 | 564 | 255 | 745 | 266 | 599 | 741 | 259 | 253 | 747 | 250 | 750 |

| | Experiment 2 A (Discriminatory) | | | | | | Experiment 2 B (Uniform-Price) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Market 1* | | *Market 2* | | *Market 3* | | *Market 1* | | *Market 2* | | *Market 3* | |
| **RCAP** | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury |
| 0.60 | 528 | 72 | 518 | 82 | 519 | 81 | 600 | 0 | 600 | 0 | 599 | 2 |
| 0.72 | 638 | 82 | 666 | 54 | 644 | 76 | 720 | 0 | 720 | 0 | 720 | 0 |
| 0.84 | 742 | 98 | 698 | 142 | 808 | 32 | 840 | 0 | 840 | 0 | 840 | 0 |
| 0.96 | 863 | 97 | 874 | 222 | 845 | 115 | 960 | 0 | 960 | 0 | 960 | 0 |
| 1.08 | 736 | 264 | 778 | 222 | 768 | 233 | 668 | 332 | 750 | 250 | 1000 | 0 |
| 1.20 | 655 | 345 | 658 | 342 | 751 | 249 | 450 | 550 | 563 | 438 | 627 | 373 |
| 1.32 | 517 | 483 | 384 | 616 | 449 | 551 | 250 | 750 | 182 | 818 | 511 | 489 |
| 1.44 | 266 | 599 | 522 | 478 | 365 | 635 | 250 | 750 | 250 | 750 | 333 | 667 |

| | Experiment 3 A (Discriminatory) | | | | | | Experiment 3 B (Uniform-Price) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *No Volatility* | | *Medium Volatility* | | *High Volatility* | | *No Volatility* | | *Medium Volatility* | | *High Volatility* | |
| **RCAP** | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury |
| 0.60 | 528 | 72 | 531 | 72 | 521 | 77 | 600 | 0 | 603 | 0 | 598 | 0 |
| 0.72 | 638 | 82 | 633 | 90 | 643 | 81 | 720 | 0 | 724 | 0 | 724 | 0 |
| 0.84 | 742 | 98 | 767 | 77 | 749 | 87 | 840 | 0 | 845 | 0 | 836 | 0 |
| 0.96 | 863 | 97 | 866 | 100 | 867 | 108 | 960 | 0 | 965 | 0 | 975 | 0 |
| 1.08 | 736 | 264 | 729 | 277 | 708 | 290 | 668 | 332 | 685 | 321 | 748 | 250 |
| 1.20 | 655 | 345 | 601 | 404 | 598 | 395 | 450 | 550 | 601 | 404 | 274 | 719 |
| 1.32 | 517 | 483 | 546 | 459 | 501 | 484 | 250 | 750 | 180 | 825 | 146 | 839 |
| 1.44 | 266 | 599 | 441 | 565 | 368 | 639 | 250 | 750 | 180 | 825 | 144 | 863 |

Table 2. Actual average profit calculation obtained by treasury and buyers in auctions under 100 different random seed numbers

| | Experiment 1 A (Discriminatory) | | | | | | Experiment 1 B (Uniform-Price) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *No Learning* | | *Best Fit* | | *No Bias* | | *No Learning* | | *Best Fit* | | *No Bias* | |
| **RCAP** | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury |
| 0.60 | 40 | 40 | 32 | 32 | 0 | 0 | 9 | 9 | 11 | 11 | 0 | 0 |
| 0.72 | 57 | 57 | 45 | 45 | 2 | 2 | 10 | 10 | 23 | 23 | 0 | 0 |
| 0.84 | 58 | 58 | 43 | 43 | 1 | 1 | 19 | 19 | 5 | 5 | 0 | 0 |
| 0.96 | 74 | 74 | 51 | 51 | 4 | 4 | 0 | 0 | 24 | 24 | 0 | 0 |
| 1.08 | 81 | 81 | 57 | 57 | 0 | 0 | 82 | 82 | 83 | 83 | 8 | 8 |
| 1.20 | 92 | 92 | 57 | 57 | 0 | 0 | 73 | 73 | 106 | 106 | 0 | 0 |
| 1.32 | 89 | 89 | 44 | 44 | 0 | 0 | 84 | 84 | 149 | 149 | 0 | 0 |
| 1.44 | 96 | 96 | 32 | 32 | 0 | 0 | 96 | 96 | 121 | 121 | 0 | 0 |

| | Experiment 2 A (Discriminatory) | | | | | | Experiment 2 B (Unifrom-Price) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Market 1* | | *Market 2* | | *Market 3* | | *Market 1* | | *Market 2* | | *Market 3* | |
| **RCAP** | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury |
| 0.60 | 0 | 0 | 0 | 0 | 7 | 7 | 0 | 0 | 0 | 0 | 15 | 15 |
| 0.72 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.84 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.96 | 4 | 4 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1.08 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 8 | 0 | 0 | 0 | 0 |
| 1.20 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 16 | 16 |
| 1.32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1.44 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | Experiment 3 A (Discriminatory) | | | | | | Experiment 3 B (Uniform-Price) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *No Volatility* | | *Medium Volatility* | | *High Volatility* | | *No Volatility* | | *Medium Volatility* | | *High Volatility* | |
| **RCAP** | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury | Buyers | Treasury |
| 0.60 | 0 | 0 | 33 | 0 | 70 | 0 | 0 | 0 | 33 | 0 | 70 | 0 |
| 0.72 | 2 | 2 | 39 | 1 | 84 | 4 | 0 | 0 | 39 | 0 | 84 | 0 |
| 0.84 | 1 | 1 | 46 | 4 | 96 | 3 | 0 | 0 | 46 | 0 | 96 | 0 |
| 0.96 | 4 | 4 | 52 | 4 | 115 | 2 | 0 | 0 | 52 | 0 | 115 | 0 |
| 1.08 | 0 | 0 | 55 | 1 | 112 | 0 | 8 | 8 | 55 | 2 | 111 | 2 |
| 1.20 | 0 | 0 | 55 | 0 | 117 | 0 | 0 | 0 | 55 | 0 | 117 | 12 |
| 1.32 | 0 | 0 | 55 | 2 | 112 | 8 | 0 | 0 | 55 | 0 | 112 | 0 |
| 1.44 | 0 | 0 | 55 | 0 | 114 | 0 | 0 | 0 | 55 | 0 | 114 | 0 |

Table 3. Standard deviation of the actual average profit calculation obtained by Treasury and buyers under 100 different random seed numbers

| | | Slope | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | RCAP | | | | | | | |
| **Strategy** | **Intercept** | 0.6 | 0.72 | 0.84 | 0.96 | 1.08 | 1.2 | 1.32 | 1.44 |
| 1 | 20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 20 | -0.67 | -0.56 | -0.48 | -0.42 | -0.37 | -0.33 | -0.30 | -0.28 |
| 3 | 20 | -1.33 | -1.11 | -0.95 | -0.83 | -0.74 | -0.67 | -0.61 | -0.56 |
| 4 | 20 | -2.00 | -1.67 | -1.43 | -1.25 | -1.11 | -1.00 | -0.91 | -0.83 |
| 5 | 20 | -2.67 | -2.22 | -1.90 | -1.67 | -1.48 | -1.33 | -1.21 | -1.11 |
| 6 | 17.5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 7 | 17.5 | -0.50 | -0.42 | -0.36 | -0.31 | -0.28 | -0.25 | -0.23 | -0.21 |
| 8 | 17.5 | -1.00 | -0.83 | -0.71 | -0.63 | -0.56 | -0.50 | -0.45 | -0.42 |
| 9 | 17.5 | -1.50 | -1.25 | -1.07 | -0.94 | -0.83 | -0.75 | -0.68 | -0.63 |
| 10 | 17.5 | -2.00 | -1.67 | -1.43 | -1.25 | -1.11 | -1.00 | -0.91 | -0.83 |
| 11 | 15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 12 | 15 | -0.33 | -0.28 | -0.24 | -0.21 | -0.19 | -0.17 | -0.15 | -0.14 |
| 13 | 15 | -0.67 | -0.56 | -0.48 | -0.42 | -0.37 | -0.33 | -0.30 | -0.28 |
| 14 | 15 | -1.00 | -0.83 | -0.71 | -0.63 | -0.56 | -0.50 | -0.45 | -0.42 |
| 15 | 15 | -1.33 | -1.11 | -0.95 | -0.83 | -0.74 | -0.67 | -0.61 | -0.56 |
| 16 | 12.5 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 17 | 12.5 | -0.17 | -0.14 | -0.12 | -0.10 | -0.09 | -0.08 | -0.08 | -0.07 |
| 18 | 12.5 | -0.33 | -0.28 | -0.24 | -0.21 | -0.19 | -0.17 | -0.15 | -0.14 |
| 19 | 12.5 | -0.50 | -0.42 | -0.36 | -0.31 | -0.28 | -0.25 | -0.23 | -0.21 |
| 20 | 12.5 | -0.67 | -0.56 | -0.48 | -0.42 | -0.37 | -0.33 | -0.30 | -0.28 |
| 21 | 10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 4. Possible choices of linear demand function strategies under different RCAP values

| Buyer | RCAP 0.6 | | 0.72 | | 0.84 | | 0.96 | | 1.08 | | 1.2 | | 1.32 | | 1.44 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. |
| 1 | 21 | 21 | 18 | 21 | 17 | 19 | 19 | 20 | 1 | 17 | 7 | 16 | 6 | 12 | 3 | 11 |
| 2 | 10 | 15 | 8 | 19 | 18 | 21 | 20 | 20 | 2 | 18 | 2 | 11 | 2 | 8 | 6 | 8 |
| 3 | 19 | 21 | 13 | 21 | 12 | 18 | 2 | 21 | 3 | 12 | 3 | 12 | 6 | 12 | 2 | 11 |
| 4 | 13 | 14 | 14 | 17 | 16 | 17 | 21 | 21 | 4 | 18 | 8 | 13 | 6 | 13 | 6 | 11 |
| 5 | 9 | 17 | 10 | 19 | 20 | 21 | 16 | 18 | 11 | 13 | 2 | 13 | 6 | 12 | 1 | 11 |
| 6 | 17 | 20 | 11 | 20 | 3 | 13 | 10 | 20 | 13 | 16 | 6 | 13 | 2 | 12 | 3 | 3 |
| 7 | 3 | 10 | 1 | 14 | 21 | 21 | 21 | 21 | 13 | 19 | 3 | 16 | 5 | 13 | 1 | 11 |
| 8 | 14 | 21 | 1 | 10 | 6 | 14 | 7 | 21 | 11 | 13 | 2 | 12 | 6 | 12 | 2 | 11 |
| 9 | 2 | 17 | 3 | 18 | 10 | 21 | 20 | 20 | 11 | 18 | 3 | 16 | 6 | 7 | 1 | 3 |
| 10 | 16 | 21 | 20 | 21 | 4 | 21 | 18 | 10 | 11 | 14 | 8 | 16 | 2 | 12 | 2 | 4 |
| 11 | 3 | 19 | 19 | 21 | 6 | 17 | 18 | 19 | 12 | 16 | 2 | 16 | 6 | 13 | 6 | 11 |
| 12 | 13 | 21 | 10 | 21 | 17 | 21 | 19 | 20 | 2 | 18 | 8 | 16 | 6 | 11 | 2 | 7 |
| Average | 11.7 | 18.1 | 10.7 | 18.5 | 12.5 | 18.7 | 15.9 | 19.3 | 7.8 | 16.0 | 4.5 | 14.2 | 4.9 | 11.4 | 2.9 | 8.5 |
| Stdev | 6.4 | 3.6 | 6.6 | 3.4 | 6.5 | 2.9 | 6.2 | 3.0 | 4.9 | 2.4 | 2.6 | 2.0 | 1.8 | 1.9 | 2.0 | 3.4 |
| Max | 21.0 | 21.0 | 20.0 | 21.0 | 21.0 | 21.0 | 21.0 | 21.0 | 13.0 | 19.0 | 8.0 | 16.0 | 6.0 | 13.0 | 6.0 | 11.0 |
| Min | 2.0 | 10.0 | 1.0 | 10.0 | 3.0 | 13.0 | 2.0 | 10.0 | 1.0 | 12.0 | 2.0 | 11.0 | 2.0 | 7.0 | 1.0 | 3.0 |

Table 5 Actual buyers choices of demand functions (choices are presented in number as shown in Table 4)
over 100 different random seeds numbers.
Note: In 100 observations, each buyer's choice variance is very small.

| | RCAP | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Buyer** | 0.6 | | 0.72 | | 0.84 | | 0.96 | | 1.08 | | 1.2 | | 1.32 | | 1.4 | |
| | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. |
| 1 | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 | 11.5 | 10.0 | 11.0 | 13.3 | 12.1 | 15.5 | 12.5 | 17.5 | 14.1 | 17.5 | 15.0 |
| 2 | 10.0 | 13.0 | 10.0 | 11.6 | 10.0 | 10.0 | 10.0 | 11.0 | 13.3 | 11.7 | 15.5 | 15.0 | 17.5 | 14.8 | 17.5 | 15.3 |
| 3 | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 | 11.8 | 10.0 | 10.0 | 13.3 | 14.2 | 15.5 | 14.1 | 17.5 | 14.1 | 17.5 | 15.0 |
| 4 | 10.0 | 13.5 | 10.0 | 12.2 | 10.0 | 12.2 | 10.0 | 10.0 | 13.3 | 11.7 | 15.5 | 13.2 | 17.5 | 14.1 | 17.5 | 15.0 |
| 5 | 10.0 | 12.3 | 10.0 | 11.6 | 10.0 | 10.0 | 10.0 | 11.8 | 13.3 | 13.3 | 15.5 | 13.2 | 17.5 | 14.2 | 17.5 | 15.0 |
| 6 | 10.0 | 11.5 | 10.0 | 11.3 | 10.0 | 13.7 | 10.0 | 11.0 | 13.3 | 12.5 | 15.5 | 13.2 | 17.5 | 14.2 | 17.5 | 15.9 |
| 7 | 10.0 | 14.5 | 10.0 | 13.3 | 10.0 | 10.0 | 10.0 | 10.0 | 13.3 | 11.7 | 15.5 | 12.5 | 17.5 | 14.1 | 17.5 | 15.0 |
| 8 | 10.0 | 10.0 | 10.0 | 14.0 | 10.0 | 13.0 | 10.0 | 10.0 | 13.3 | 13.3 | 15.5 | 14.1 | 17.5 | 14.2 | 17.5 | 15.0 |
| 9 | 10.0 | 12.3 | 10.0 | 11.9 | 10.0 | 10.0 | 10.0 | 11.0 | 13.3 | 11.7 | 15.5 | 12.5 | 17.5 | 16.0 | 17.5 | 15.7 |
| 10 | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 | 13.0 | 13.3 | 12.7 | 15.5 | 12.5 | 17.5 | 14.2 | 17.5 | 15.8 |
| 11 | 10.0 | 11.7 | 10.0 | 10.0 | 10.0 | 12.2 | 10.0 | 11.4 | 13.3 | 12.5 | 15.5 | 12.5 | 17.5 | 14.1 | 17.5 | 15.0 |
| 12 | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 | 11.0 | 13.3 | 11.7 | 15.5 | 12.5 | 17.5 | 15.0 | 17.5 | 15.9 |
| Average | 10.0 | 11.6 | 10.0 | 11.3 | 10.0 | 11.2 | 10.0 | 10.9 | 13.3 | 12.4 | 15.5 | 13.1 | 17.5 | 14.4 | 17.5 | 15.3 |
| Stdev | 0.0 | 1.6 | 0.0 | 1.4 | 0.0 | 1.4 | 0.0 | 0.9 | 0.0 | 0.8 | 0.0 | 0.8 | 0.0 | 0.6 | 0.0 | 0.4 |

Table 6. Actual average per-unit price paid by buyers in auction under 100 different random seed numbers
Note: In 100 observations, variance of average unit price is very small.

| Buyer | RCAP 0.6 | | 0.72 | | 0.84 | | 0.96 | | 1.08 | | 1.2 | | 1.32 | | 1.44 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. | Unif. | Disc. |
| 1 | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 9.0 | 9.0 | 10.0 | 7.6 | 9.2 | 9.5 | 5.5 | 7.8 |
| 2 | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 9.0 | 8.1 | 10.0 | 10.0 | 10.6 | 10.0 | 3.7 | 7.7 |
| 3 | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 9.0 | 9.0 | 8.4 | 10.0 | 9.2 | 9.3 | 11.1 | 7.7 |
| 4 | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 7.0 | 8.1 | 5.0 | 9.8 | 8.4 | 4.3 | 2.4 | 7.3 |
| 5 | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 9.0 | 9.0 | 10.0 | 9.7 | 8.3 | 9.2 | 12.0 | 7.8 |
| 6 | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 6.0 | 9.0 | 10.0 | 9.8 | 10.8 | 9.1 | 5.4 | 11.4 |
| 7 | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 6.0 | 5.3 | 8.5 | 8.1 | 2.0 | 4.3 | 12.0 | 7.2 |
| 8 | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 9.0 | 9.0 | 10.0 | 10.0 | 8.4 | 9.1 | 11.2 | 4.8 |
| 9 | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 9.0 | 8.2 | 8.1 | 7.2 | 8.7 | 11.0 | 12.0 | 11.8 |
| 10 | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 9.0 | 8.2 | 5.0 | 6.3 | 10.7 | 9.1 | 11.1 | 7.4 |
| 11 | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 9.0 | 9.0 | 10.0 | 5.6 | 6.5 | 4.3 | 2.6 | 7.3 |
| 12 | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 9.0 | 8.1 | 5.0 | 5.9 | 7.2 | 11.0 | 11.1 | 12.0 |
| Average | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 8.3 | 8.3 | 8.3 | 8.3 | 8.3 | 8.3 | 8.3 | 8.3 |
| Stdev | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.2 | 1.0 | 2.1 | 1.7 | 2.4 | 2.5 | 4.0 | 2.2 |
| Max | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 9.0 | 9.0 | 10.0 | 10.0 | 10.8 | 11.0 | 12.0 | 12.0 |
| Min | 5.0 | 5.0 | 6.0 | 6.0 | 7.0 | 7.0 | 8.0 | 8.0 | 6.0 | 5.3 | 5.0 | 5.6 | 2.0 | 4.3 | 2.4 | 4.8 |

Table 7. Actual quantity obtained by buyers in auction under 100 different random seed numbers
Note: In 100 observations, variance of quantity obtained by each buyer is very small.

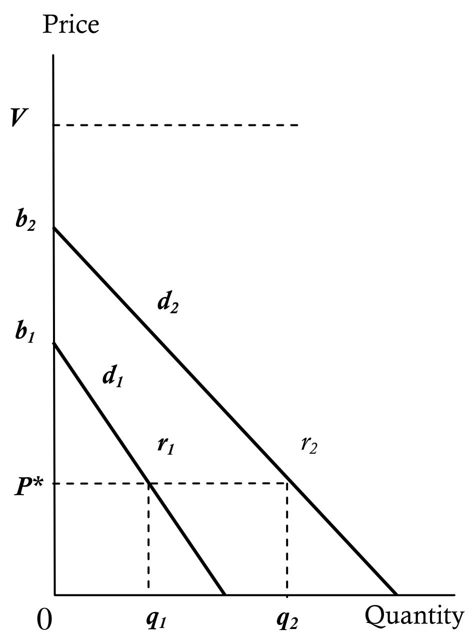


Fig. 1a  Individual demand curve.

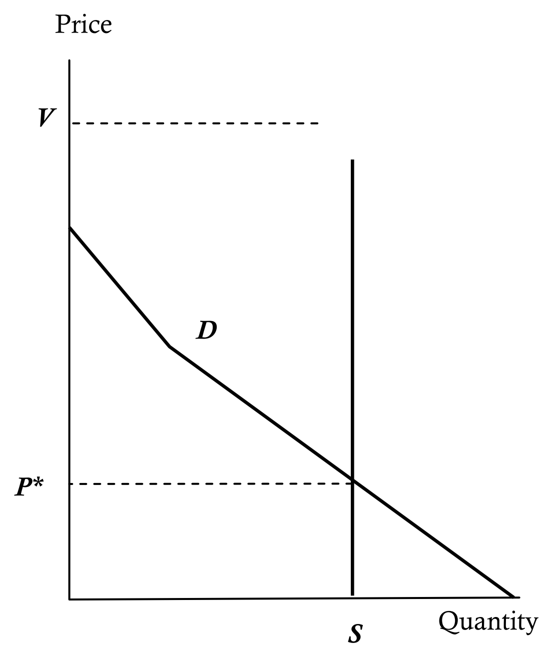


Fig. 1b  Aggregate  demand and supply curve .

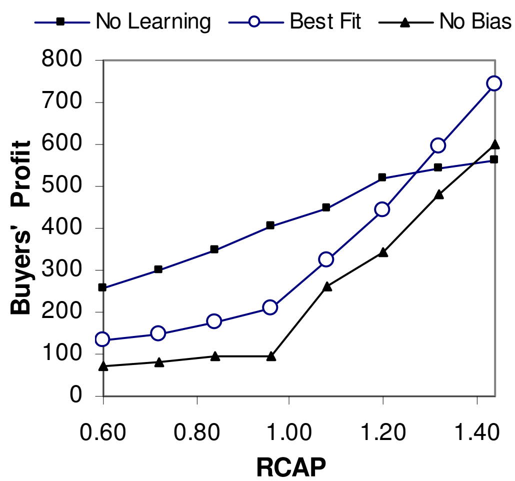


Fig. 2a Treasury profits in experiment 1A (discriminatory-price rule).

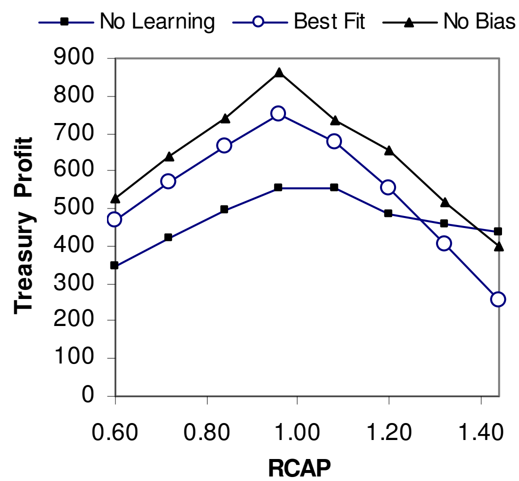


Fig. 2b Buyers' profits in experiment 1A (discriminatory-price rule).

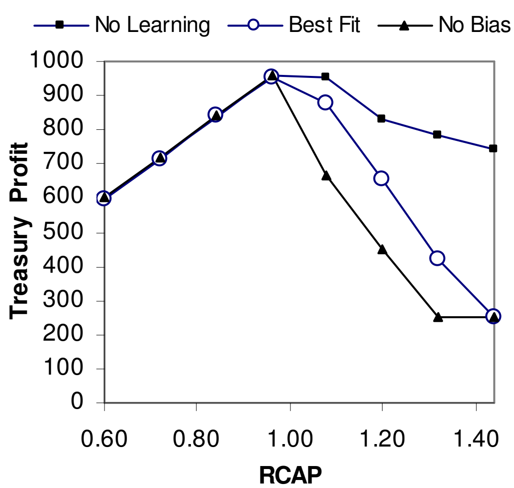


Fig. 2c Treasury profits in experiment 1B (uniform-price rule).

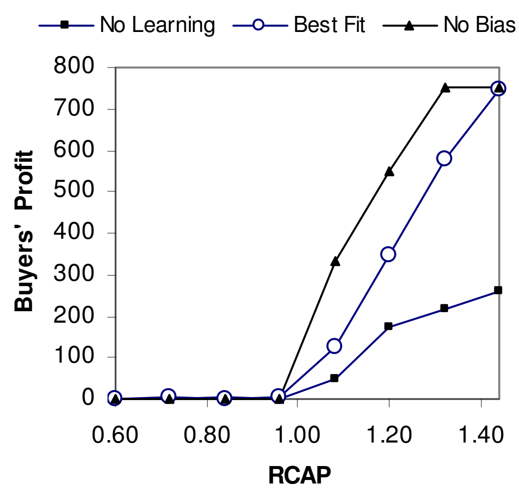


Fig. 2d Buyers' profits in experiment 1B (uniform-price rule).

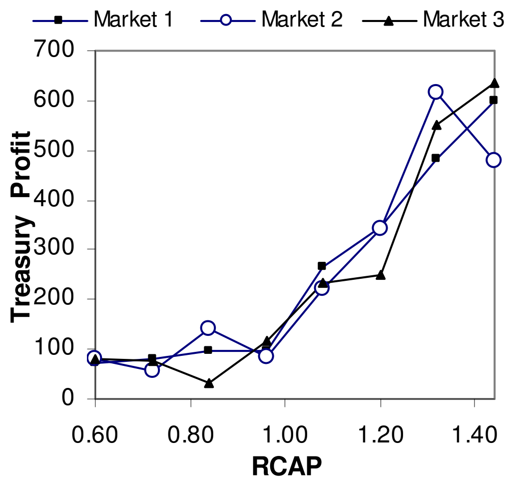


Fig. 3a Treasury profits in experiment 2A (discriminatory-price rule).

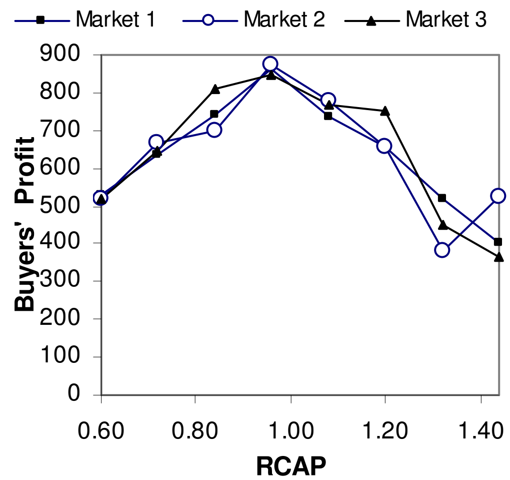


Fig. 3b Buyers' profits in experiment 2A (discriminatory-price rule).

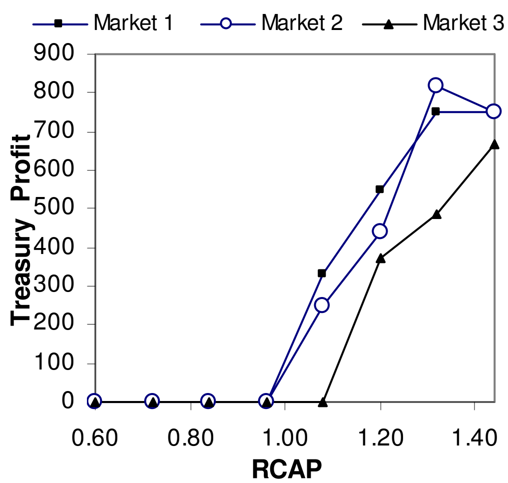


Fig. 3c Treasury profits in experiment 2B (uniform-price rule).

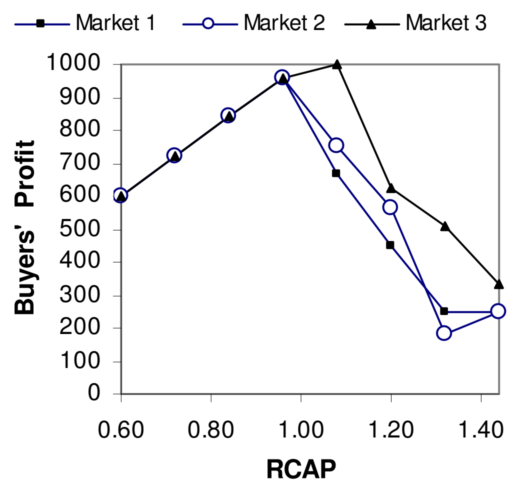


Fig. 3d Buyers' profits in experiment 2B (uniform-price rule).

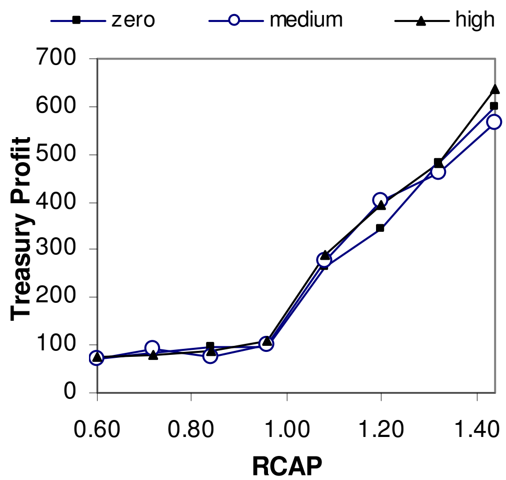


Fig. 4a Treasury profits in experiment 3A.
(discriminatory-price rule).

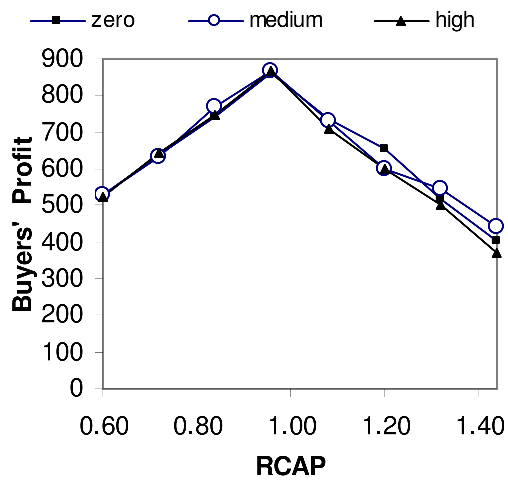


Fig. 4b Buyers' profits in experiment 3A.
(discriminatory-price rule).

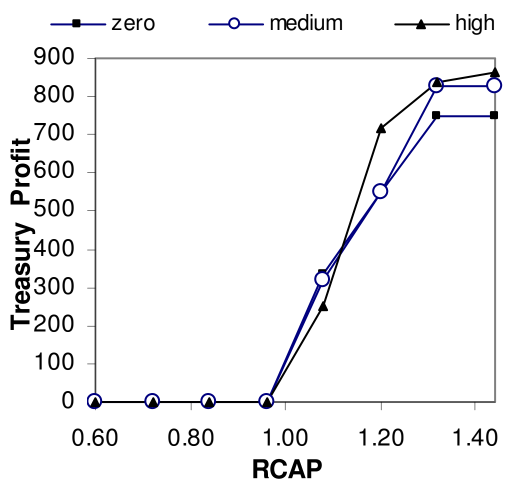


Fig. 4c Treasury profits in experiment 3B
(uniform-price rule).

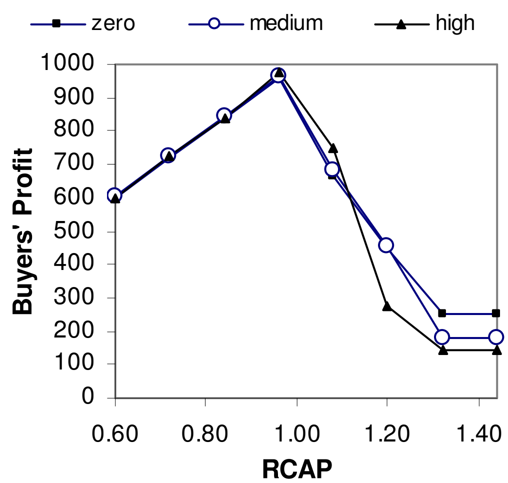


Fig. 4d Buyers' profits in experiment 3B
(uniform-price rule).

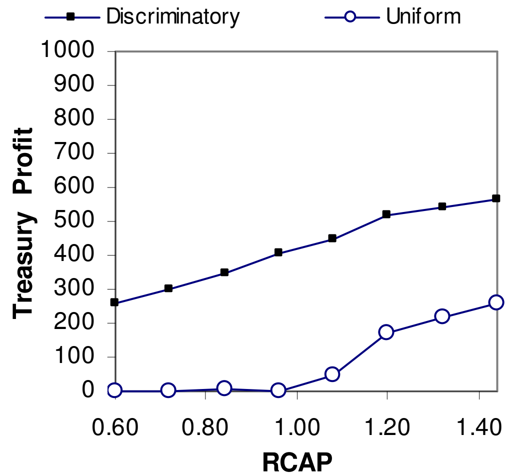


Fig. 5a Treasury profits with *no-learning* buyers.

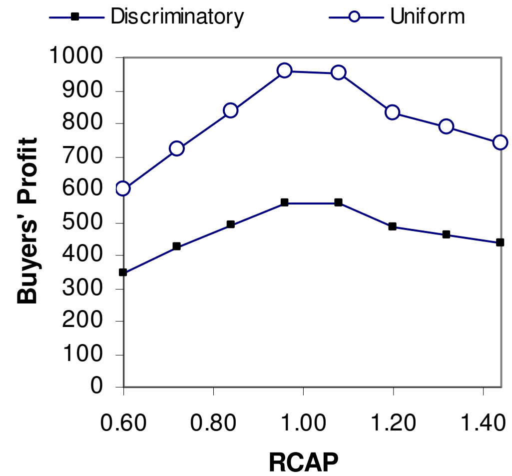


Fig. 5b Buyers' profits with *no-learning* buyers.

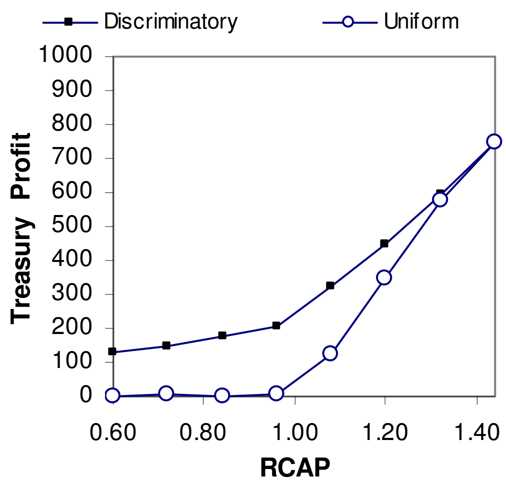


Fig. 5c Treasury profits with *best fit* learning buyers.

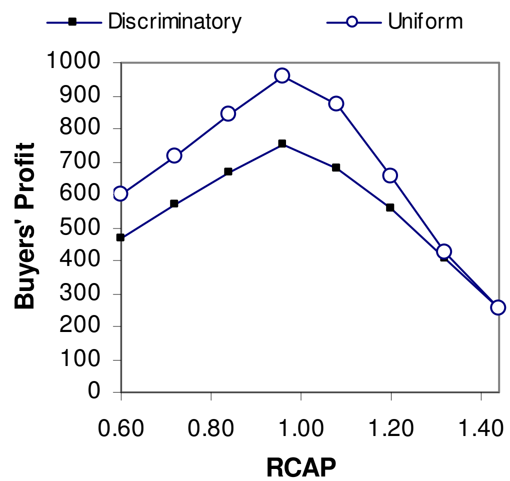


Fig. 5d Buyers' profits with *best fit* learning buyers.

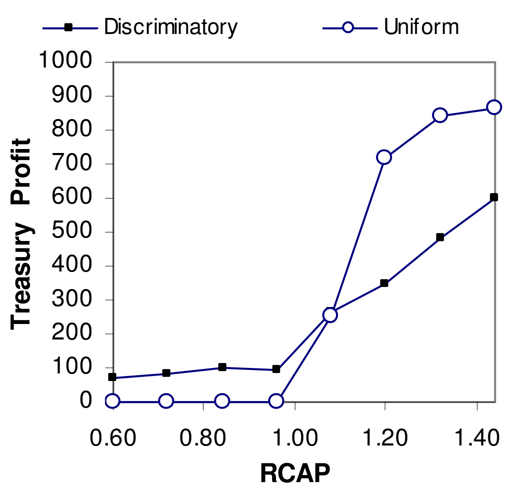


Fig. 5e Treasury profits with *no-bias* learning buyers.

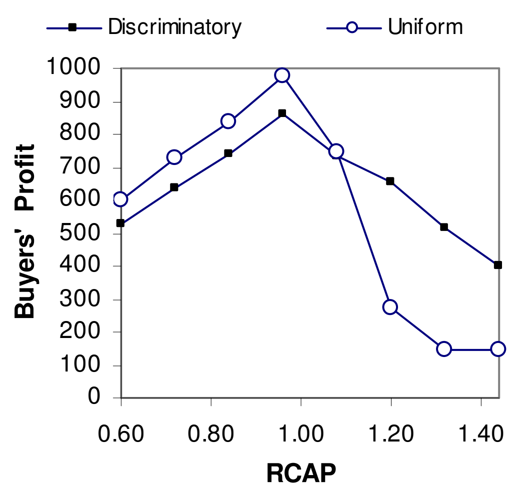


Fig. 5f Buyers' profits with *no-bias* learning buyers.

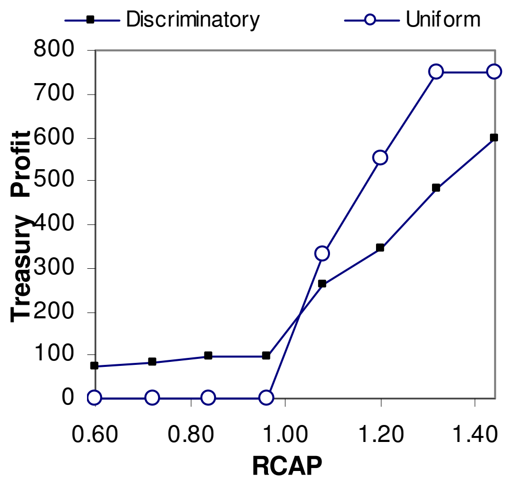


Fig. 6a Treasury profits with *market structure 1.*

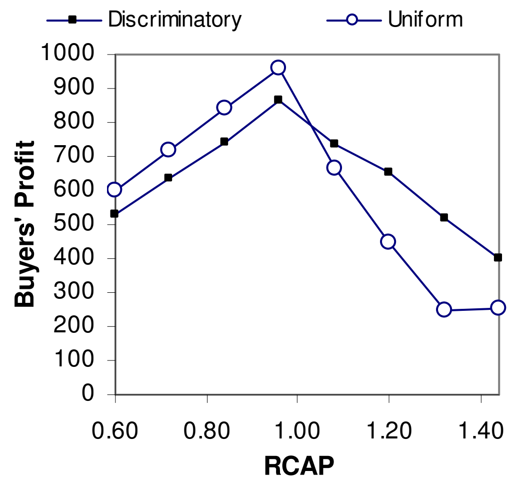


Fig. 6b Buyers' profits with *market structure 1.*

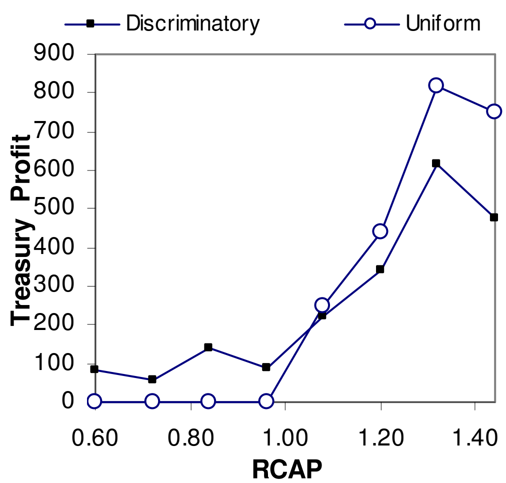


Fig. 6c Treasury profits with *market structure 2.*

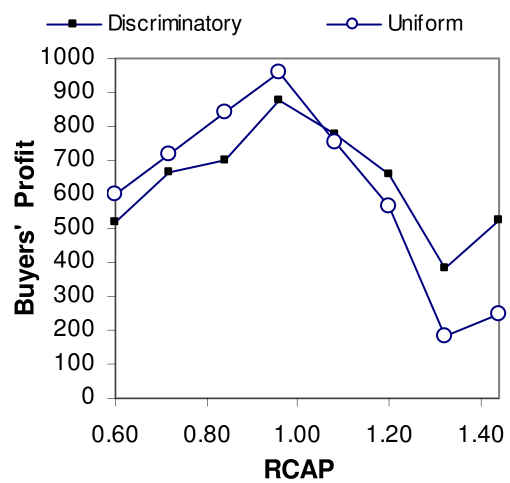


Fig. 6d Buyers' profits with *market structure 2.*

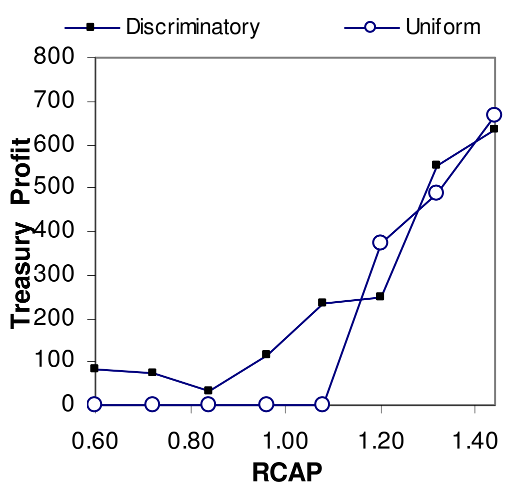


Fig. 6e Treasury profits with *market structure 3.*

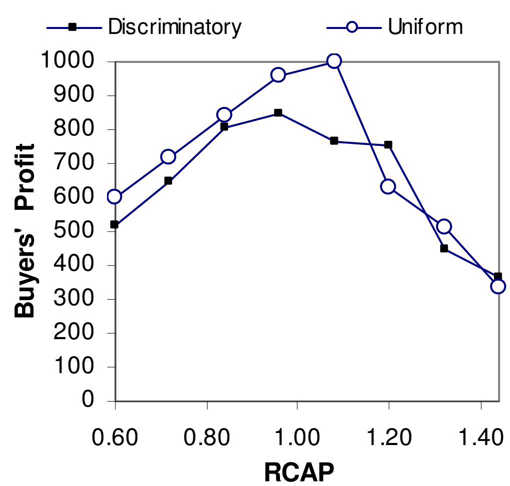


Fig. 6f Buyers' profits with *market structure 3.*

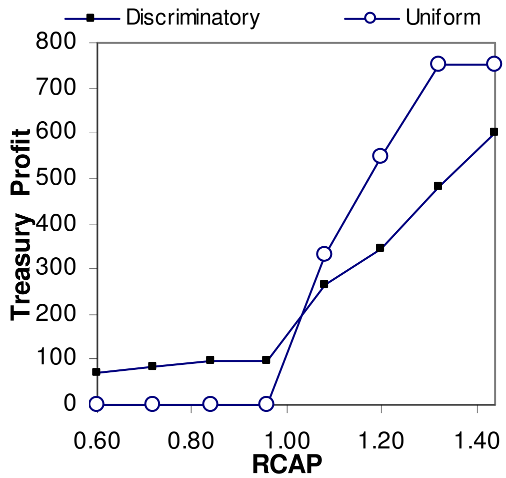


Fig. 7a Treasury profits with *no volatility.*

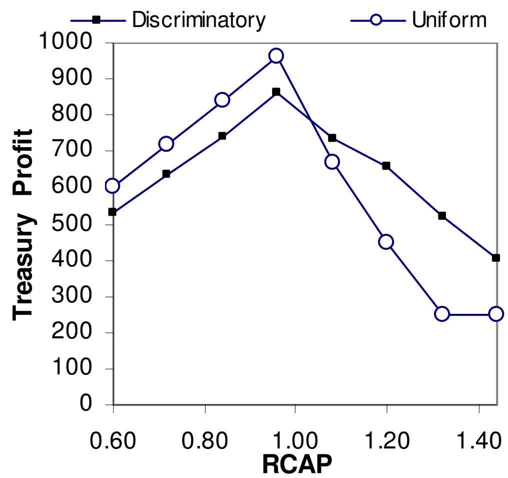


Fig. 7b Buyers' profits with *no volatility.*

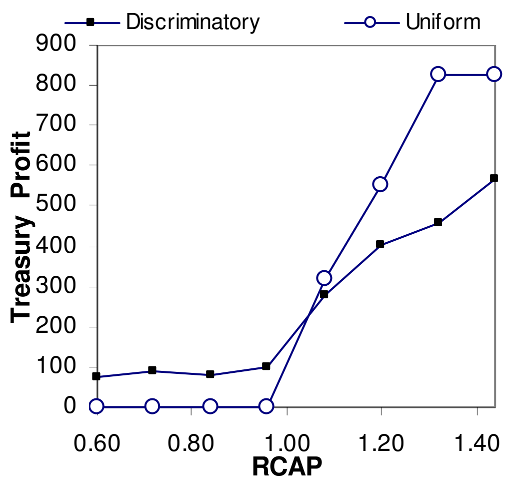


Fig. 7c Treasury profits with *medium volatility.*

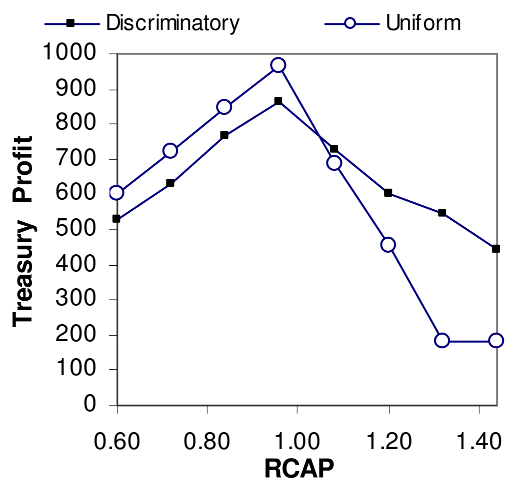


Fig. 7d Buyers' profits with *medium volatility.*

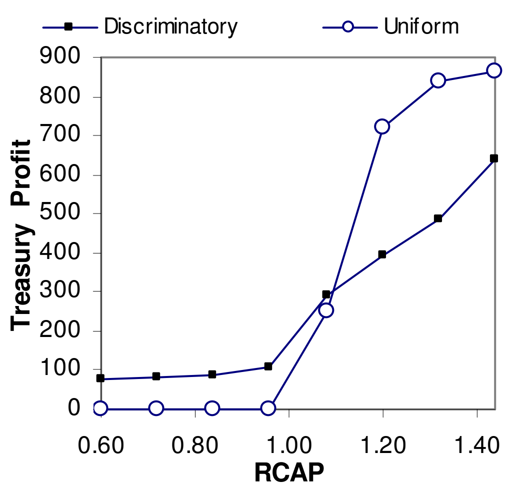


Fig. 7e Treasury profits with *high volatility.*

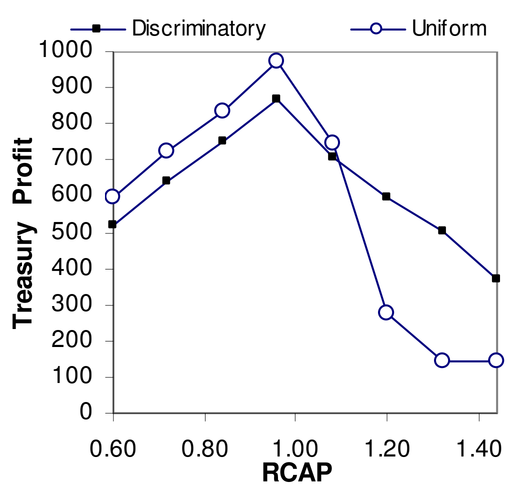


Fig. 7f Buyers' profits with *high volatility.*

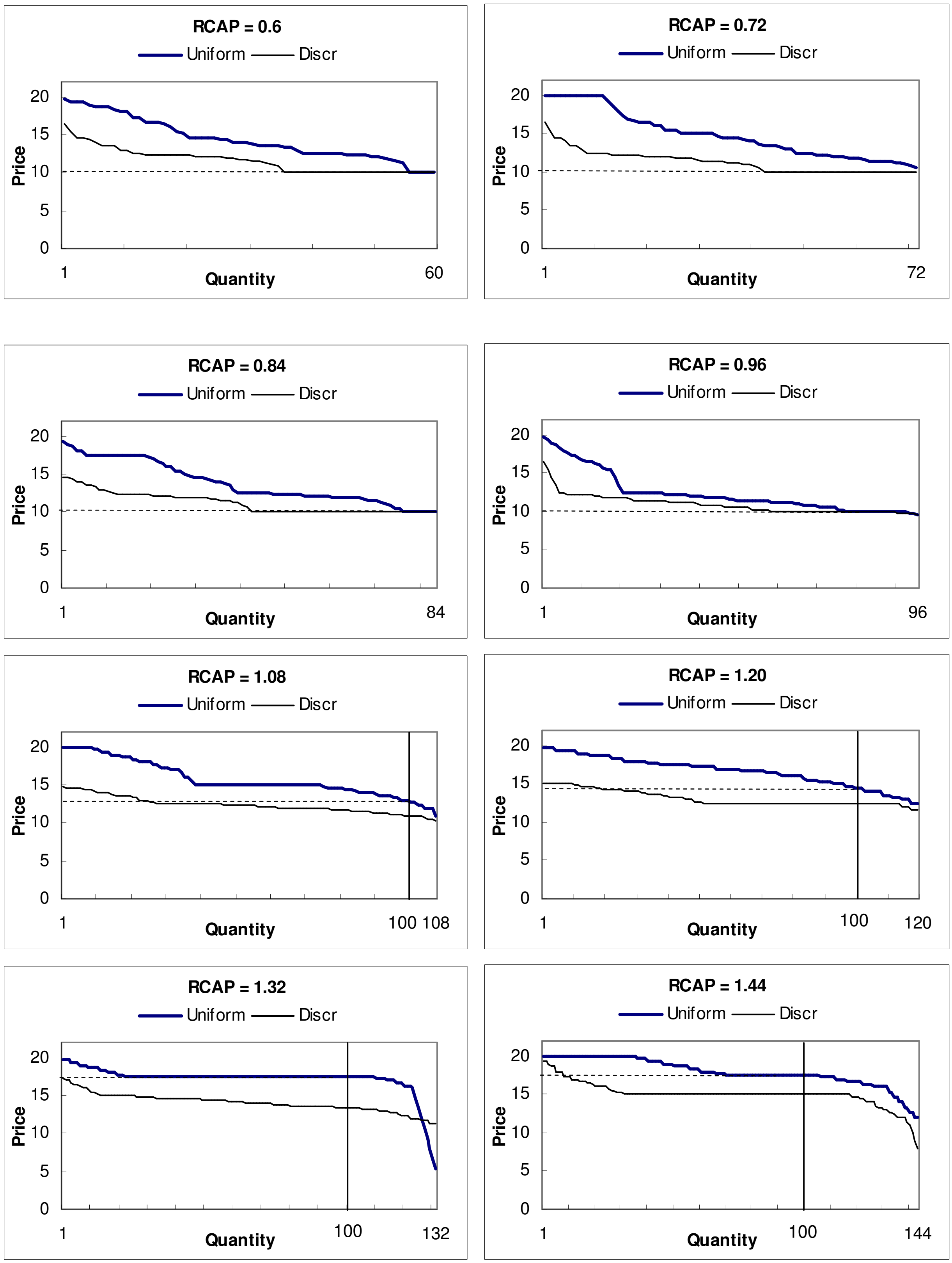


Fig. 8 Average value of actual aggregate demand under market structure 1; no-bias learning; no volatility at secondary market under 100 different random seed numbers.