

# Notes on Wilhite

(*Computational Economics*, 2001)

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<https://www2.econ.iastate.edu/tesfatsi/>

### Main Reference:

Allen Wilhite, “**Bilateral Trade and ‘Small World’ Networks,**” *Computational Economics* 18 (2001), 49-64

<https://www2.econ.iastate.edu/tesfatsi/SmallWorldNetworksBilateralTrade.Wilhite.pdf>

**NOTE:** For details about the C++ Program implementation, contact the author  
Allen Wilhite at [wilhitea@email.uah.edu](mailto:wilhitea@email.uah.edu)

# Small-World Networks & Economics

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A number of ACE researchers have begun to consider small-world networks in relation to economic processes.

For example, Wilhite (2001) uses an ACE model of a bilateral exchange economy to explore the consequences of restricting trade to four different types of networks, including a small-world trade network.

# Small-World Network

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As defined by Watts and Strogatz (1998), a *Small-World Network* is a network that can be represented as a simple connected graph  $G$  exhibiting two properties:

- \* *Global Reach*: The presence of “short cut” connections between vertices results in short characteristic path length  $L(G)$ .
- \* *Local Connectivity*: Each vertex of  $G$  is linked to a relatively well-connected set of neighboring vertices, resulting in a large value for the clustering coefficient  $C(G)$ .

# Wilhite's Basic Approach

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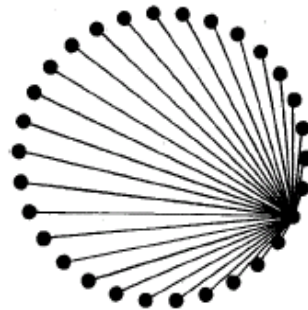
Examine the trade-off between market efficiency and transaction costs under four types of trade networks:

- \* *Completely connected trade network* (every trader can trade with every other trader);
- \* *Locally disconnected trade network* (disjoint trade groups);
- \* *Locally connected trade network* consisting of trade groups aligned around a ring with a 1-trader overlap at each meeting point;
- \* *Small-world trade network* constructed from the locally connected trade networks by permitting from 1 to 5 randomly specified short-cut trade links between members of non-adjacent trade groups.

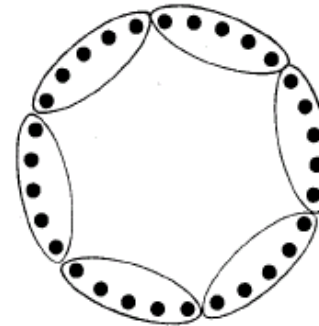
# Four Possible Trade Networks

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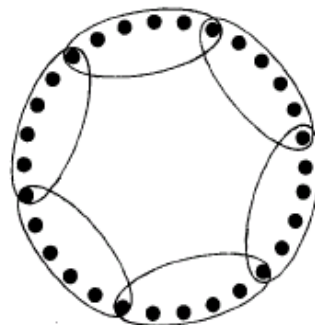


Panel a:  
Global Network  
Trade routes for one trader

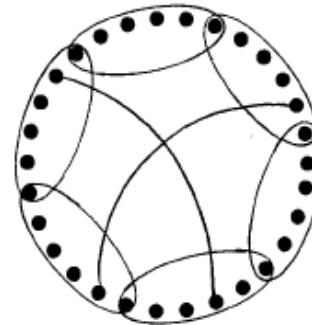


Panel b:  
Local Disconnected Network  
six groups, five agents per group

**Note:** Depicted links are for a typical trader in the global network



Panel c:  
Local Connected Network  
six groups, six agents per group



Panel d:  
Small-world Network  
two crossover agents

Figure 1. Sketches of four networks (population: 30 agents).

# Basic Wilhite Model

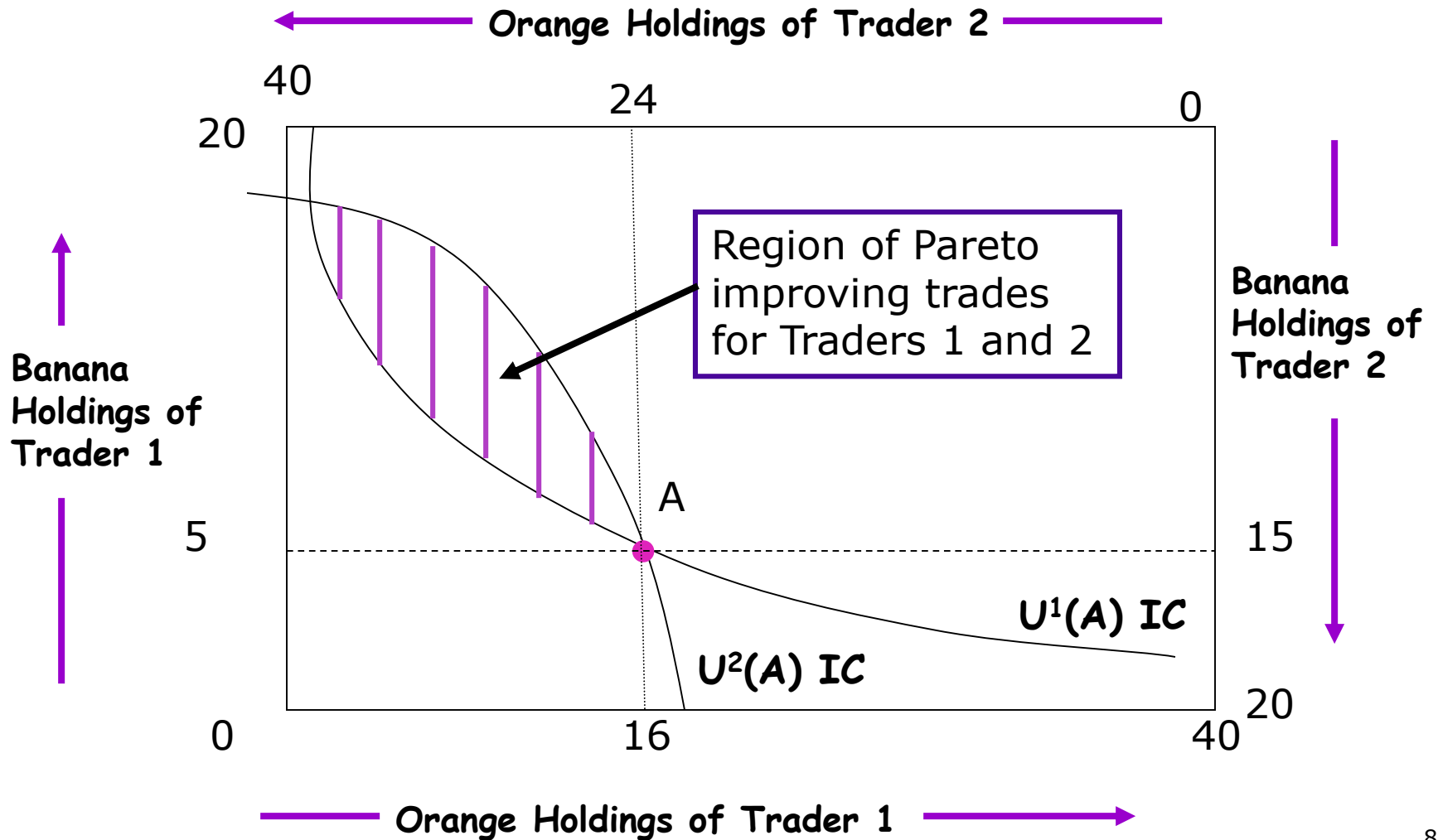
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- Exchange economy with 2 durable (non-perishable) goods: (1) a good  $g_1$  that must be traded in whole units; and (2) an “infinitely divisible” good  $g_2$ .
- 500 traders *initially* endowed with *random positive* amounts of  $g_1$  and  $g_2$  ( $g_2$  = numeraire good, i.e., the price of  $g_1$  is expressed as units of  $g_2$  per unit of  $g_1$ )
- Each trader  $i$  has the same form of utility function measuring preferences for the two goods:  
$$U^i = g_1^i g_2^i, i = 1, \dots, 500$$
- Traders are rational, non-strategic, and myopic agents who try to improve their utility in each period by voluntary feasible trades of  $g_1$  and  $g_2$ .

# Basic Model...Continued

- Opportunity for mutually beneficial trade exists for two traders if they have different reservation prices (“marginal rates of substitution” MRS) giving the rates at which they are *just willing* to exchange  $g_2$  for  $g_1$ .
- To see the intuition for this, consider an “Edgeworth Box” pure exchange economy with two goods - oranges  $g_1$  and bananas  $g_2$  in fixed supplies 40 and 20 – that are currently allocated between Traders 1 and 2 at point A.
  - $MRS^i(A) = [\Delta U^i(A)/\Delta g_1^i]/[\Delta U^i(A)/\Delta g_2^i] = \Delta g_2^i/\Delta g_1^i = g_2^i/g_1^i$
  - $U^i(A)$  IC = Indifference Curve of Trader  $i$  passing through A
  - $MRS^i(A) = -1$  times the slope of Trader  $i$ 's indifference curve passing through A

# Edgeworth Box Illustration





# Basic Model...Continued

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- ❑ Each trader is limited to trades within its own particular neighborhood as determined by the trade network.
- ❑ By random selection (without replacement), each trader in each neighborhood searches for trade partners within its neighborhood and selects a trade partner offering a “best” mutually beneficial price (if such a trade partner exists).
- ❑ Reservation prices are assumed to be truthfully revealed.

# Basic Model...Continued

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□ Whenever a suitable pair of trade partners  $i$  and  $j$  is determined, the two traders trade at the following “split the difference” price as long as the resulting trades are feasible and mutually beneficial:

$$p_{i,j} = \frac{g_2^i + g_2^j}{g_1^i + g_1^j} \quad i, j \in \{1, \dots, 500\}.$$

**Note:**  $p_{i,j} = \text{MRS}^i \cdot [G_1^i] + \text{MRS}^j \cdot [1 - G_1^i]$ ,  
where  $G_1^i = g_1^i / [g_1^i + g_1^j]$

# Experimental Design:

## Four Distinct Trade Networks

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### *Global network:*

(i)	number of groups:	1
(ii)	agents in each group:	500
(iii)	end-agents overlap?	no
(iv)	number of crossovers:	0

### *Local disconnected network*

(i)	number of groups:	50
(ii)	agents in each group:	10
(iii)	end-agents overlap?	no
(iv)	number of crossovers:	0

### *Local connected network:*

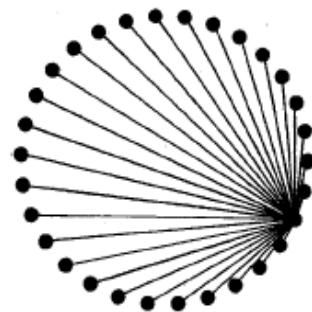
(i)	number of groups:	50
(ii)	agents in each group:	11
(iii)	end-agents overlap?	yes
(iv)	number of crossovers:	0

### *Small-world network*

(i)	number of groups:	50
(ii)	agents in each group:	11
(iii)	end-agents overlap?	yes
(iv)	number of crossovers:	5

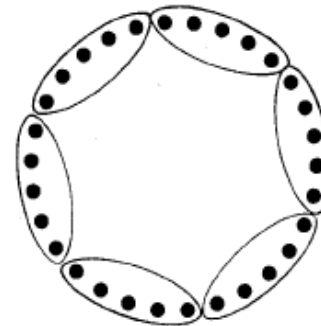
# 50 runs with the same 500-trader populations were conducted for each of the four tested trade networks

Illustrative depiction of the four tested trade networks for a 30-tradereconomy:

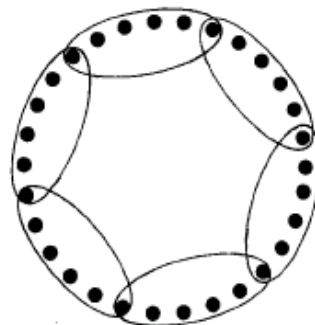


Panel a:  
Global Network  
Trade routes for one trader

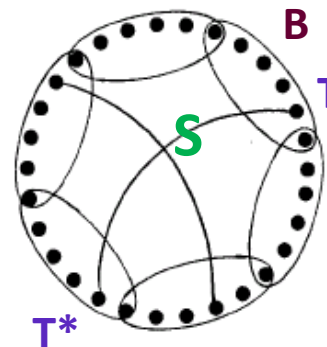
*Note:* Depicted links are for a typical trader in the global network



Panel b:  
Local Disconnected Network  
six groups, five agents per group



Panel c:  
Local Connected Network  
six groups, six agents per group



Panel d:  
Small-world Network  
two crossover agents

## *Note:*

The short-cut **S** adds one more trader **T\*** to group **B**, but only trader **T** in **B** can directly trade with **T\***.

Figure 1. Sketches of four networks (population: 30 agents).

# Key Questions Examined

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*H1: Price convergence:* Is there a significant difference in the dispersion of prices across each trade network?

*H2: Speed of convergence:* Do the different trade networks require a significantly different number of rounds of trading to reach their steady state?

*H3: Number of trades:* Is there a difference in the number of trades it takes for each network to reach its steady state?

*H4: Search:* Is there a difference in the amount of search and negotiation in each trade network?

# Key Experimental Findings

Note: *Round* = One pass through all traders as initiators of trades

*Equilibrium* = No more mutually beneficial trade opportunities

Table I. Average equilibrium characteristics.<sup>a</sup>

	Prices (standard deviation)	Rounds	Total trades	Total searches
Global network	1.0046 (0.00168)	8.08	1953.38	2,015,960 *
Local disconnected network	1.0396 (0.2771)	7.02	1727.7	31,590
Local connected network	1.0048 (0.0146)	497.14	93,975.72	2,734,270
Small-world network	1.0045 (0.00724)	242.54	45,944.56	1,236,954

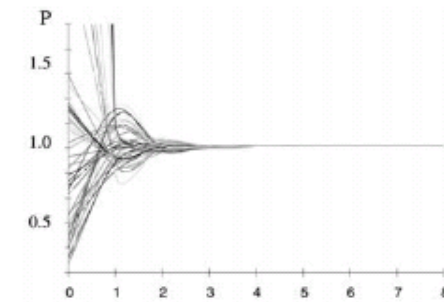
<sup>a</sup> Averages calculated from 50 simulations of each network configuration.

\*  $2,015,960 = [500 \times 499 \times 8.08] = 500 \text{ traders seeking best price from } 499 \text{ other traders for } 8.08 \text{ rounds}$

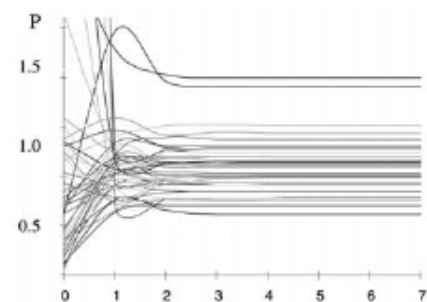
# Price Convergence in the Four Networks

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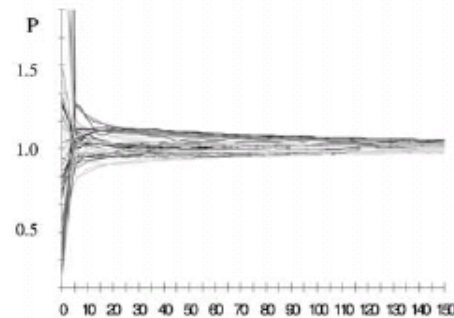
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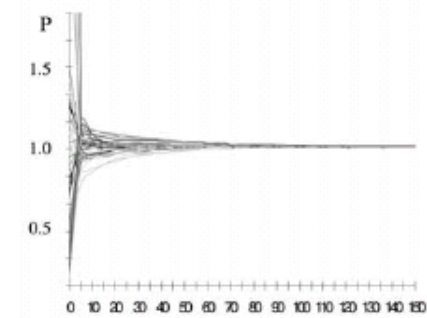
Global



Local Disconnected



Local Connected



Small-World with  
five crossover agents

Small-World with  
5 Crossover Traders

rounds

rounds

# Price Convergence for Different Numbers {0, 1, 3, 5} of Crossover Traders

BILATERAL TRADE AND 'SMALL-WORLD' NETWORKS

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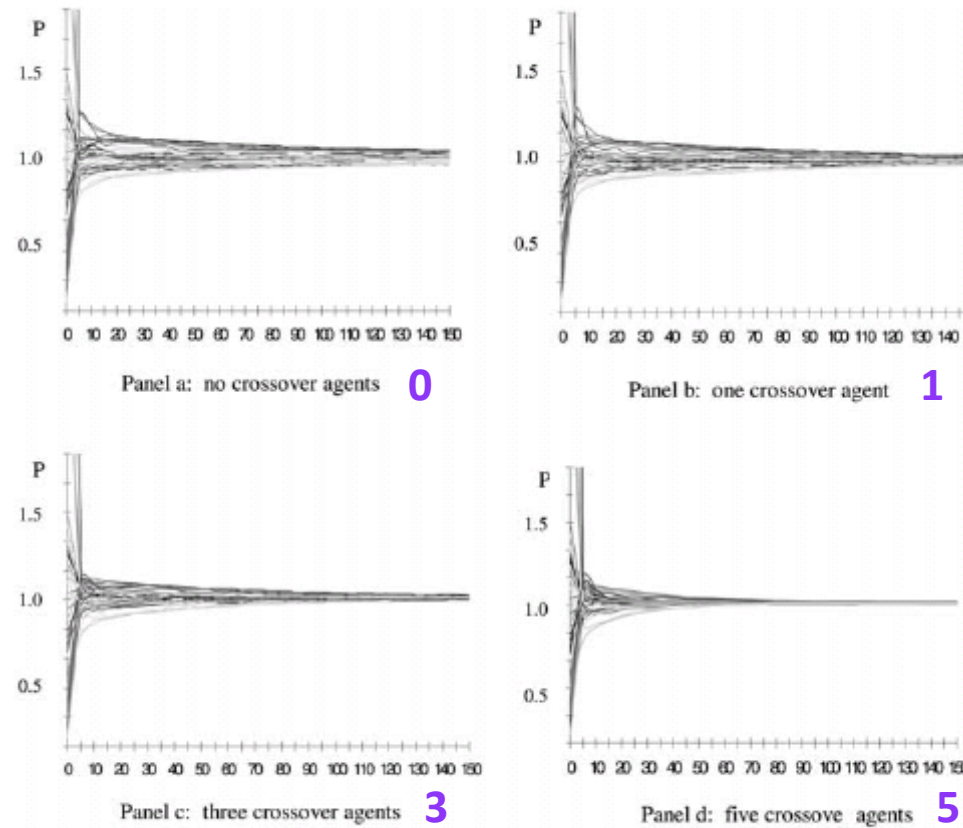


Figure 3. Dynamic price adjustment: adding crossover agents.



# Relative Wealth of Crossover Traders

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*Table III.* Wealth of crossover agents.<sup>a</sup>

	Agent A	Agent B	Agent C	Agent D	Agent E
0 crossovers	1929.61	887.73	1267.45	670.246	2059.95
1 crossover	<b>1997.17</b>	874.167	1269.23	675.557	2059.83
2 crossovers	1990.07	<b>1385.97</b>	1267.61	677.555	2058
3 crossovers	1984.94	1388.68	<b>1318.54</b>	677.591	2062.74
4 crossovers	1991.28	1389.3	1308.88	<b>985.451</b>	2066.52
5 crossovers	1981.82	1376.47	1307.04	981.252	<b>2105.27</b>

<sup>a</sup> The boldfaced number is the point at which that particular agent established a bridge to another trade group. Wealth from population #23, seed number 18847.

# Summary of Key Findings

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- ❑ The small-world trade network with 5 crossover traders provides most of the market-efficiency advantages of the completely connected trade network while retaining most of the transaction cost economies of the locally connected trade network.
- ❑ Wilhite's findings also suggest that there *might* exist private micro-level incentives for the *formation* of small-world trade networks.
- ❑ Specifically, the traders in the locally connected network who become crossover traders in the small-world trade network tend to amass greater wealth.

# Extensions?

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- \* For economic-social networks it is not satisfactory to focus solely on the implications of a fixed or exogenously-varied network structure.
- \* Feedback mechanisms at work in economic-social networks can result in endogenous changes in the network structure over time.
- \* **EXAMPLE:** Whom you have traded with in the past, and with what regularity, can affect how you behave in current trade transactions, which in turn can affect whom you choose and refuse to trade with in the future. For illustration, see the link below to the **Trade Network Game (TNG) Lab** that permits traders to choose and refuse their trade partners in each trading round. This choice and refusal of trade partners has substantial effects on trade network formation over time.

**TNG Lab Homepage:**

<https://www2.econ.iastate.edu/tesfatsi/tnghome.htm>